Resonance Chiral Theory and high energy constraints at NLO in $1/N_C$

Jaroslav Trnka[1,2], Juan Jose Sanz-Cillero[3]

[1] Department of Physics, Princeton University, New Jersey, USA

[2] Institute of Particle and Nuclear Physics, Charles University in Prague, Czech Republic

[3] Grup de Física Teórica and IFAE, Universitat Autónoma de Barcelona, 08193 Barcelona, Spain

Abstract

Resonance Chiral Theory is based on large N_C behavior of QCD within the effective Lagrangian formalism. It is an effective tool for description of the dynamics of mesons above the hadronic scale. RχT at LO (tree level) is well understood but there are just few studies of NLO effects. Further studies can answer the key question if RχT can be formulated as a renormalizable theory.

In our work, we study $S-P$ correlator at NLO in $1/N_C$. The key point is investigating of OPE constraints and LECs predictions.

Chiral Perturbation Theory

- Low energy effective field theory for QCD based on spontaneous symmetry breaking $SU(3)_R \times SU(3)_L$ to $SU(3)_V$
- Goldstone boson modes: essential building block $u(\phi) = \exp\left(i\frac{\phi}{\sqrt{2}F_0}\right)$ with

$$\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}$$

- Lagrangian organized in terms of derivatives

$$\text{Leading order } \mathcal{O}(p^2) \qquad \mathcal{L}_\chi^{(2)} = \frac{F^2}{4}\langle u^\mu u_\mu + \chi_+\rangle$$

- Valid for $p \ll M_R$, higher states effectively included

Resonance Chiral Theory

QCD in the limit of large N_C and RχT

- $1/N_C$ expansion (# resonance fields): LO - tree level diagrams
- Spectrum: Infinite tower of mesons; in limit $N_C \to \infty$ mesons are stable, non-interacting
- Massive $U(3)$ multiplets of the type $V(1^{--})$, $A(1^{++})$, $S(0^{++})$ and $P(0^{-+})$
- Single resonance approximation: only one resonance multiplet in each channel
- Interpolation between OPE (at high energies) and χPT (at low energies)

Resonance Lagrangian

- $1/N_C$ expansion (# resonances) vs. chiral expansion (# derivatives)
- Resonance Lagrangian $\mathcal{L}_{RChT} = \mathcal{L}_{GB}[\phi] + \mathcal{L}_R[R,\phi] + \ldots$
- LO $\mathcal{O}(p^4)$ Lagrangian (in sense of contribution to LECs)

$$\mathcal{L}_R = \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu}f_+^{\mu\nu}\rangle + \frac{iG_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu,u^\nu]\rangle + \frac{F_A}{2\sqrt{2}}\langle A_{\mu\nu}f_-^{\mu\nu}\rangle, \\ + c_d\langle S u^\mu u_\mu\rangle + c_m\langle S\chi_+\rangle + id_m\langle P\chi_-\rangle.$$

Matching with χPT

- Integrating out resonances at low energies

$$\int \mathcal{D}R \exp\left(i\int d^4x \mathcal{L}_R\right) = \exp\left(i\int d^4x \mathcal{L}_{\chi,R}\right)$$

  where the chiral Lagrangian is then $\mathcal{L}_\chi = \mathcal{L}_{GB} + \mathcal{L}_{\chi,R}$
- Saturation of $\mathcal{O}(p^4)$ and $\mathcal{O}(p^6)$ LECs, in our case

$$\mathcal{L} = \frac{L_8}{2}\langle\chi_-^2 + \chi_+^2\rangle + c_{38}\langle\chi_{+\mu}\chi_+^\mu\rangle$$

S-P correlator

- Definition

$$\Pi_{S-P}^{ab}(p) = i\int d^4x e^{ip\cdot x}\langle 0|T[S^a(p)S^b(0) - P^a(p)P^b(0)]|0\rangle$$

  where $S^a = \bar{q}\frac{\lambda^a}{\sqrt{2}}q$, $P^a = i\bar{q}\frac{\lambda^a}{\sqrt{2}}\gamma_5 q$.
- Low energy expansion determined by χPT (independent on μ_χ)

$$\Pi(p^2)_{\chi PT} = B_0^2\left\{\frac{2F^2}{p^2} + 32L_8^r(\mu_\chi) + \frac{\Gamma_8}{\pi^2}\left(1 - \ln\frac{-p^2}{\mu_\chi^2}\right) \right. \\ \left. + \frac{p^2}{F^2}\left[32C_{38}^r(\mu_\chi) - \frac{\Gamma_{38}^{(L)}}{\pi^2}\left(1 - \ln\frac{-p^2}{\mu_\chi^2}\right) + \mathcal{O}(N_C^0)\right] + \mathcal{O}(p^4)\right\}$$

  where $\Gamma_8 = 5/48\,[3/16]$ and $\Gamma_{38}^L = -5L_5/6\,[-3L_5/2]$ in SU(3) [U(3)] χPT.
- At high energies $\Pi(p^2)$ must vanish like $1/p^4$
- In resonance region, one obtains at leading order

$$\frac{1}{B_0^2}\Pi(p^2)_{LO} = \frac{2F^2}{p^2} - 16\left(\frac{c_m^2}{p^2 - M_S^2} + \frac{d_m^2}{p^2 - M_P^2}\right)$$

Renormalization procedure

Subleading Lagrangian

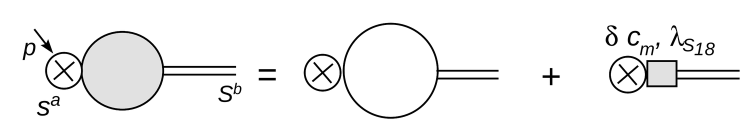
For renormalization of vertices we need to introduce

$$\mathcal{L}_{NLO} = \frac{X_R}{2}\langle R\nabla^4 R\rangle + \lambda_{18}^S\langle S\nabla^2\chi_+\rangle + i\lambda_{13}^P\langle P\nabla^2\chi_-\rangle$$

- Rewrite $k = k(\mu) + \delta k(\mu)$ where $\delta k(\mu)$ absorb infinities
- Finite parts of $k(\mu)$ are absorbed by meson field redefinitions , e.g. $c_m^{eff} = c_m - c_m X_S M_S^2 - M_S^2\lambda_{18}^S$

Renormalization of vertices

- Example 1: vertex sS



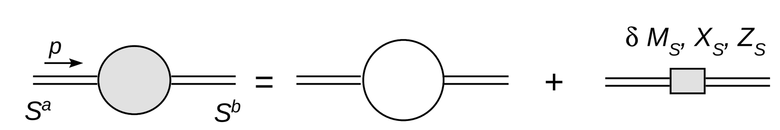
The vertex function is

$$\Phi_{sS}^{loop}(p^2) = -4B_0\left\{c_m(\mu) - \lambda_{18}^S(\mu)p^2 - \frac{1}{4B_0}\Phi_{sS}(p^2)\right\}$$

where

$$-\frac{1}{4B_0}\Phi_{sS}(p^2) = \frac{3c_d p^2}{64\pi^2F^2}\left(1 - \ln\frac{-p^2}{\mu^2}\right)$$

- Example 2: Scalar resonance propagator



The propagator has the form

$$\Delta_S = \frac{i}{p^2 - M_S^2} + \frac{i}{(p^2 - M_S^2)^2}\left\{-X_S(\mu)p^4 + \Sigma_S(p^2)\right\} + \ldots$$

where the self-energy

$$\Sigma_S(p^2) = -\frac{3c_d^2p^4}{16\pi^2F^4}\left[1 - \ln\frac{-p^2}{\mu^2}\right]$$

NLO correction

Up to considered order in $1/N_C$ we obtain

$$\frac{1}{B_0^2}\Pi(p^2) = \frac{16c_m^{\text{eff}\,2}}{M_S^{\text{eff}\,2} - p^2} - \frac{16c_m^2}{(M_S^2 - p^2)^2}\Sigma_S(p^2) - \frac{8c_m}{M_S^2 - p^2}\frac{1}{B_0}\Phi_{sS}(p^2) \\ - \frac{16d_m^{\text{eff}\,2}}{M_P^{\text{eff}\,2} - p^2} + \frac{16d_m^2}{(M_P^2 - p^2)^2}\Sigma_P(p^2) + \frac{8d_m}{M_P^2 - p^2}\frac{1}{B_0}\Phi_{pP}(p^2) \\ + \frac{2F^2}{p^2} + \frac{2F^2}{p^4}\Sigma_\phi(p^2) + \frac{2F\sqrt{2}}{p^2 B_0}\Phi_{p\phi}(p^2)$$

OPE at NLO

The high energy expansion can be written as

$$\frac{1}{B_0^2}\Pi(p^2) = \alpha_0^p p^2 + \alpha_0^l\ln\frac{-p^2}{\mu^2} + \alpha_0^p + \frac{\alpha_2^l}{p^2}\ln\frac{-p^2}{\mu^2} + \frac{\alpha_2^p}{p^2} + \frac{\alpha_4^l}{p^4}\ln\frac{-p^2}{\mu^2} + \frac{\alpha_4^p}{p^4} + \mathcal{O}\left(\frac{\ln -p^2}{p^6}\right)$$

where e.g. $\alpha_4^l = \frac{3}{8\pi^2F^2}\left[-\frac{24c_d^2c_m^2M_S^4}{F^2} - 4c_dc_mM_S^4 + 6c_d^2M_S^4 + 3G_V^2M_V^4\right]$

Different renormalization schemes

- Rewriting

$$\kappa = \hat{\kappa} + \Delta\kappa \qquad \text{where} \qquad \kappa = c_m, d_m, M_S^2, M_P^2$$

  where $\Delta\kappa$ is of NLO and $\hat{\kappa}$ satisfy large-N_C WSR.

$$8\hat{c}_m^2 - 8\hat{d}_m^2 - F^2 = 0, \qquad 8\hat{c}_m^2\hat{M}_S^2 - 8\hat{d}_m^2\hat{M}_P^2 = 0$$

- Mass scheme: fixing physical masses

$$\Delta M_R^2 = M_R^2 - M_R^{pole\,2} = -\mathrm{Re}\,\Sigma_R(M_R^2)$$

Low energy expansion

Expanding the result for small energies we obtain for LECs

$$L_8(\mu_\chi) = \frac{c_m^2}{2M_S^2} - \frac{d_m^2}{2M_P^2} + \tilde{L}_8 + \xi_{L_8} + \frac{\Gamma_8}{\pi^2}\ln\frac{\mu^2}{\mu_\chi^2},$$

$$C_{38}(\mu_\chi) = \frac{F^2c_m^2}{2M_S^4} - \frac{F^2d_m^2}{2M_P^4} + \xi_{C_{38}} - \frac{\Gamma_{38}^L}{\pi^2}\ln\frac{\mu^2}{\mu_\chi^2}$$

where

$$\xi_{L_8}(\mu) = -\frac{3c_dc_m}{32\pi^2F^2}\left(\ln\frac{M_S^2}{\mu^2} + \frac{1}{2}\right) + \frac{3c_d^2}{128\pi^2F^2}\left(\ln\frac{M_S^2}{\mu^2} + \frac{5}{6}\right) + \frac{3G_V^2}{256\pi^2F^2}\left(\ln\frac{M_V^2}{\mu^2} + \frac{5}{6}\right) + \frac{3c_d^2}{32\pi^2F^2}\ln\frac{M_S^2}{\mu^2} - \frac{3d_m^2}{32\pi^2F^2}\ln\frac{M_P^2}{\mu^2}$$

$$\xi_{C_{38}}(\mu) = \frac{3d_m^2}{64\pi^2M_P^2} - \frac{3c_d^2}{512\pi^2M_S^2} - \frac{3G_V^2}{1024\pi^2M_V^2} - \frac{3c_m^2}{64\pi^2M_S^2} + \frac{3c_dc_m}{96\pi^2M_S^2}$$

Numerical results: large NLO corrections $L_8 \sim 2\cdot 10^{-3}$, $C_{38} \sim 30\cdot 10^{-6}$

Open question: Are NLO corrections really large?

Not really, we omit important terms

- Lagrangians with 2 and 3 resonances
- SRA could not be valid and we have to consider whole tower of resonances

Yes, they are really large.

- Need for a procedure to incorporate large subleading corrections into RChT (e.g. broad widths).
- Possible other illustrations of large NLO correction effects: additional degrees of freedom in the resonance region