The cusp effect in $\eta' \to \eta\pi\pi$ decays

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1. Motivation

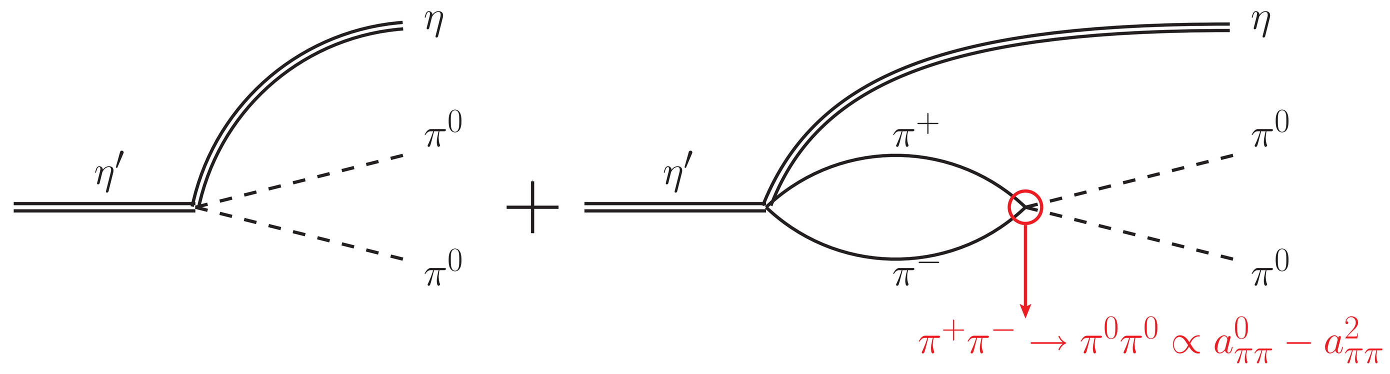
- Size of S-wave $\pi\pi$ scattering lengths is of central importance to the understanding of spontaneous symmetry breaking in QCD
- Prediction from 2-loop ChPT and Roy equation analysis [2]:

$$a^0_{\pi\pi} - a^2_{\pi\pi} = 0.265 \pm 0.004.$$

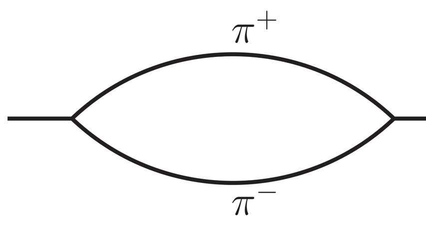
- Methods of experimental verification:
  - reactions on nucleons, e.g. $\pi N \to \pi\pi N$
  - $K^+ \to \pi^+\pi^- e^+ \nu_e$
  - pionium lifetime
  - cusp effect in $K^+ \to \pi^0\pi^0\pi^+$ and $\eta' \to \eta\pi^0\pi^0$
- Investigation of cusp effect in $K^+ \to \pi^0\pi^0\pi^+$ very precise method to extract S-wave $\pi\pi$ scattering lengths from experimental data [3-6]
- $\mathrm{BR}(K^+ \to \pi^+\pi^-\pi^+) > \mathrm{BR}(K^+ \to \pi^+\pi^0\pi^0)$ makes $K^+ \to \pi^0\pi^0\pi^+$ especially suited for a cusp analysis
- $\mathrm{BR}(\eta' \to \eta\pi^+\pi^-) = 2\,\mathrm{BR}(\eta' \to \eta\pi^0\pi^0) \Rightarrow \eta' \to \eta\pi\pi$ viable candidate for study of the cusp effect
- Upcoming experiments on η' decays: ELSA (Bonn), MAMI-C, WASA-at-COSY, KLOE-at-DAΦNE, BES-III

2. Origin of the cusp effect

- Cusp in invariant mass spectrum of the $\pi^0\pi^0$ pair generated by



- Calculation in appropriate framework shows

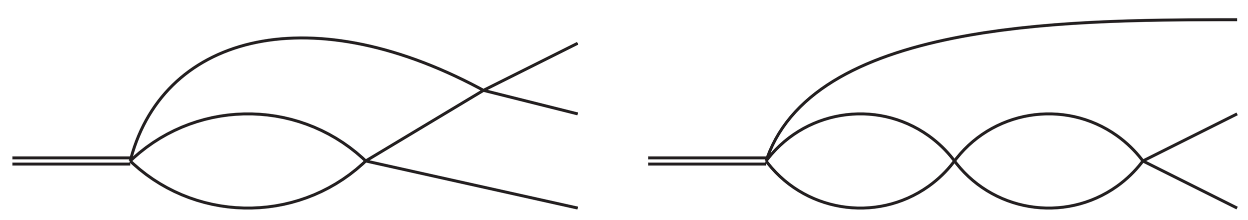


$$= \frac{i q_{+-}(s_3)}{8\pi\sqrt{s_3}} = \frac{i}{16\pi}\sqrt{1 - \frac{4M_\pi^2}{s_3}}$$

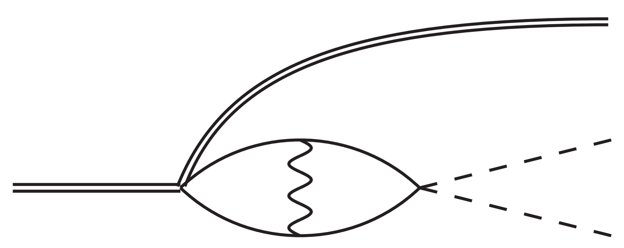
- Loop function real below $\pi^+\pi^-$ threshold $\Rightarrow$ interference with (real) tree contribution $\Rightarrow$ square-root behavior below threshold $\Rightarrow$ cusp
- Size of the cusp effect depends on value of $a^0_{\pi\pi} - a^2_{\pi\pi}$

3. The decay amplitude

- Calculations performed in a modified non-relativistic effective field theory $\Rightarrow$ manifestly covariant results, correct analytic structure in low-energy region
- $\mathcal{L} = \mathcal{L}_{\eta'} + \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi\eta}$
  - $\mathcal{L}_{\eta'} \Rightarrow$ Dalitz-plot distribution of $\eta' \to \eta\pi\pi$ at tree level
  - $\mathcal{L}_{\pi\pi}$ and $\mathcal{L}_{\pi\eta} \Rightarrow$ effective range expansion of the scattering amplitudes
- Consistent power counting: correlated expansion in $a_{\pi\pi}$, $a_{\pi\eta}$ and ϵ:
  - three-momenta $\sim \mathcal{O}(\epsilon)$
  - kinetic energies $T_i = p_i^0 - M_i \sim \mathcal{O}(\epsilon^2)$
  - masses $\sim \mathcal{O}(1)$
  - two-particle rescattering $\sim \mathcal{O}(a\epsilon)$
- Analytic representation of the decay amplitude has been obtained up to and including terms of $\mathcal{O}(a^2_{\pi\pi}\epsilon^6, a_{\pi\pi}a_{\pi\eta}\epsilon^2, a^2_{\pi\eta}\epsilon^2)$, see [1]
- Following two-loop topologies have been taken into account:



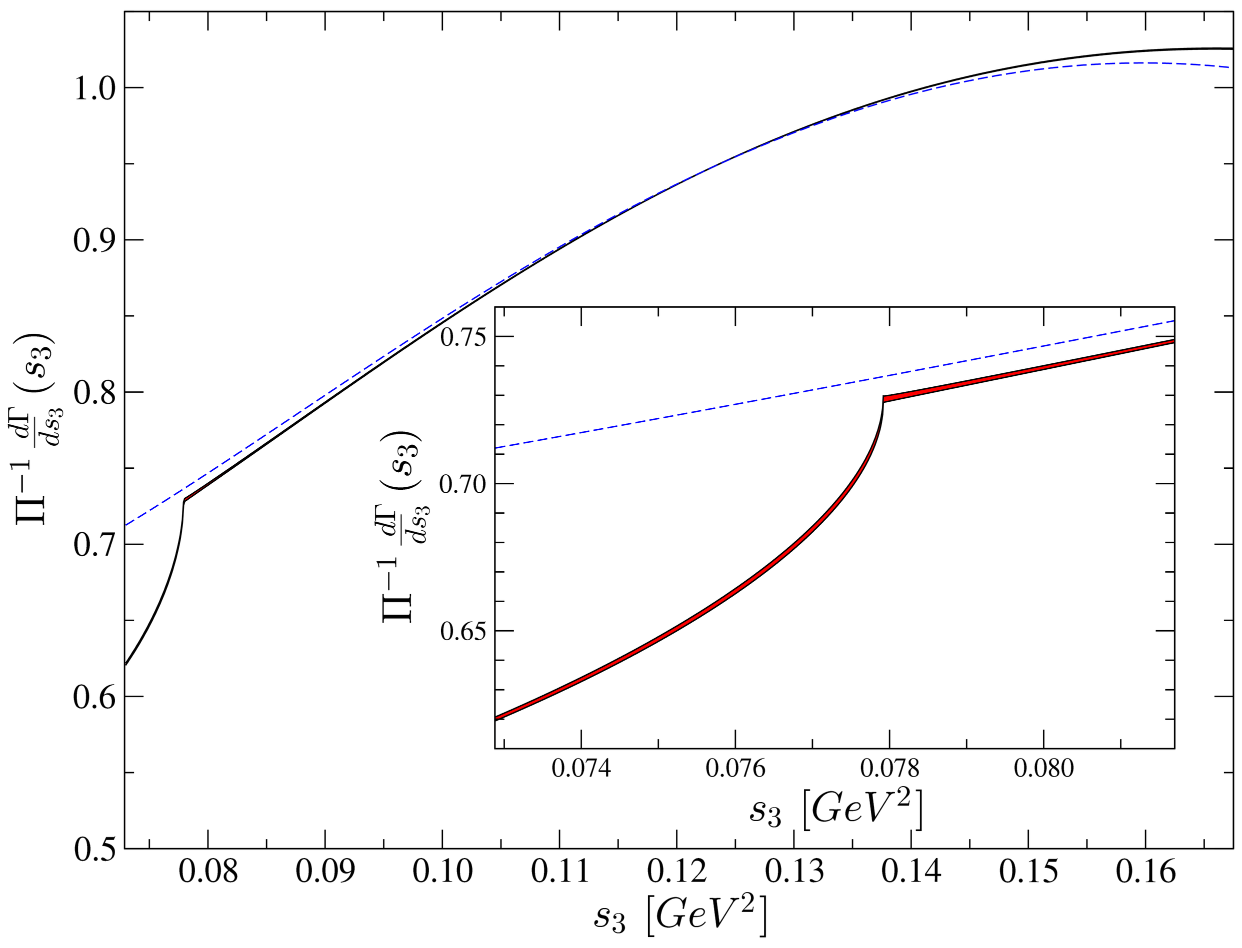
- Radiative corrections performed up to $\mathcal{O}(a_{\pi\pi}\log(\epsilon))$ in $\eta' \to \eta\pi^0\pi^0$ (virtual photon exchange) and up to $\mathcal{O}(e^2a^0)$ in $\eta' \to \eta\pi^+\pi^-$ (Bremsstrahlung)



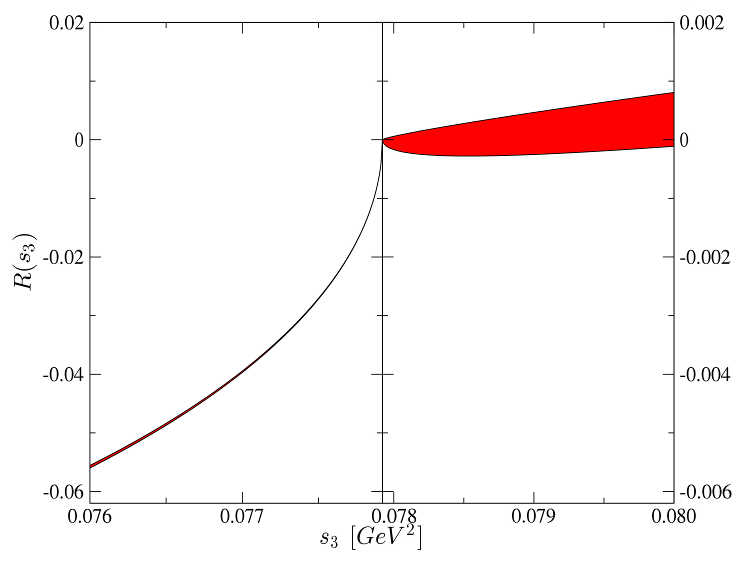
- Contributions from six-particle vertices and inelastic channels negligible

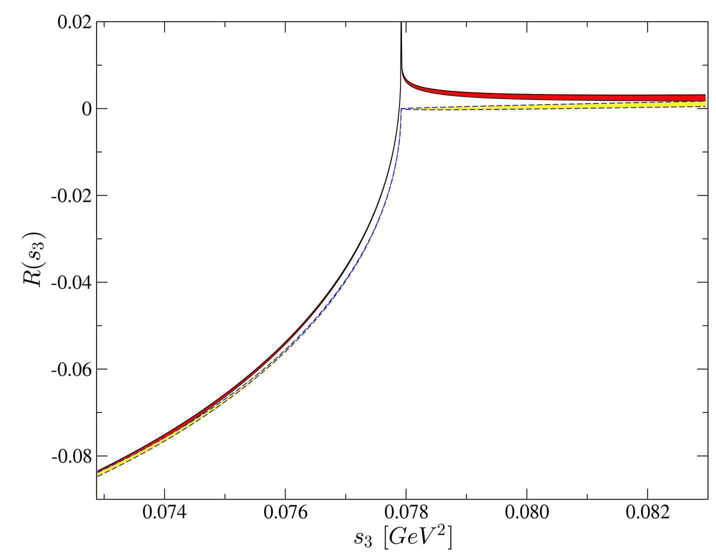
4. Prediction of the cusp

- Numerical input:
  - $\pi\pi$ threshold parameters as predicted in [2]
  - $\pi\eta$ threshold parameters from low-energy expansion of ChPT amplitude [7]
  - $\eta' \to \eta\pi\pi$ Dalitz plot parameters from VES collaboration [8]
- **Remark:** $\pi\eta$ threshold parameters very badly constrained by ChPT $\Rightarrow$ variation by as much as 150%
- $\pi\eta$ error bands can be significantly reduced by readjusting tree-level couplings so that the *full* amplitude squared yields the VES parameters
- Phase-space normalized decay rate $\frac{d\Gamma}{ds_3}$ of full and tree-level decay amplitude:



- **Integrated event deficit** $\approx 8\% \Rightarrow$ comparable to 13% of $K^+ \to \pi^0\pi^0\pi^+$ $\Rightarrow$ **pronounced cusp in $\eta' \to \eta\pi^0\pi^0$ decay spectrum**
- Close-up on cusp region without (left) and with (right) radiative corrections:



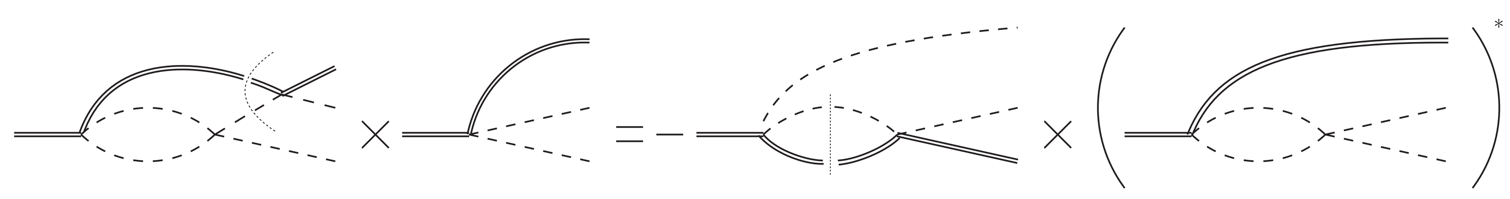


$$R(s_3) = \Pi^{-1}(s_3)\left[\frac{d\Gamma_{\rm full}}{ds_3} - \frac{d\Gamma_{\rm tree}}{ds_3}\right] - \Pi^{-1}(4M_\pi^2)\left[\frac{d\Gamma_{\rm full}}{ds_3} - \frac{d\Gamma_{\rm tree}}{ds_3}\right]_{s_3=4M_\pi^2}.$$

- Two-loop cusp highly suppressed with respect to one-loop cusp $\Rightarrow$ Threshold theorem:
  - $\mathcal{O}(a^2)$ effects $\sim 0.5\%$ relative to $\mathcal{O}(a)$
  - $\mathcal{O}(a^3)$ effects reduce one loop cusp by about 0.5%.
- **Cusp in $\eta' \to \eta\pi\pi$ entirely dominated by $\mathcal{O}(a)$ rescattering effects**

5. On the extraction of $\pi\eta$ parameters

- Only possibility: extraction from a cusp analysis in the $\pi\eta$ invariant mass spectrum s_1 of the decay rate
- One-loop cusp does not lie in physical region of s_1
- $\mathcal{O}(a^2)$-effects exactly cancel at threshold:



- **$\pi\eta$ threshold parameters cannot be determined in $\eta' \to \eta\pi\pi$ decays**

6. References

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