Pole Dynamics in $K^-\pi^+$

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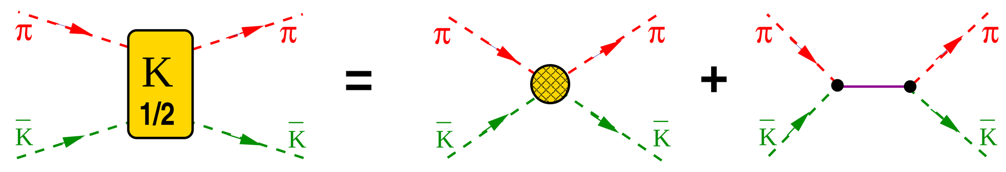
Abstract

We present a simple chiral model for the $J=0,\ I=1/2$, elastic $K\pi$ amplitude which allows a transparent determination of its poles while preserving the essential physics. In the case of the K-matrix approximation, the model yields a quadratic equation in s. The solutions to this equation can then be well approximated by polynomials of masses and coupling constants. This analytic structure clarifies the reason of why, depending on the values of one of the coupling constants, one may have one or two physical poles. The model yields a pole, associated with the κ, at $\sqrt{s}=(0.75-i\,0.24)$ GeV.

1 Introduction

The $K^-\pi^+$ elastic scaterring amplitude for $(J,I)=(1/2,0)$ is discribed by the tree level diagram



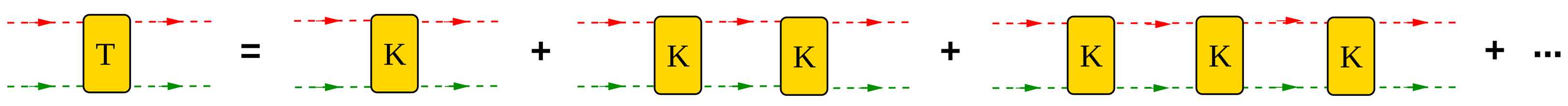
Contact[1] and resonant[2] terms are derived from $SU(3)\times SU(3)$ chiral effective lagrangians

$$\mathcal{L}^{(2)}=\frac{F^2}{4}\left\langle\nabla_\mu U^\dagger\,\nabla^\mu U+\chi^\dagger U+\chi U^\dagger\right\rangle+\ c_d\ \langle S\,u_\mu\,u^\mu\rangle+\ c_m\ \langle S\,\chi_+\rangle. \qquad (1)$$

- U is the pseudoscalar field,
- S represent scalar resonaces,
- c_d and c_m are scalar-pseudoscalar coupling constants.

2 Theory

The $(J,I)=(0,1/2)$ amplitude is unitarized considering all $K\pi$ buble loop interactions[3]



and the amplitude is written as

$$T_{1/2}(s)=\gamma^2(s)/D(s)\ ,\quad D(s)=[m_R^2-s+\gamma^2(s)\,\bar{R}_{1/2}(s)]-i\left[\gamma^2(s)\,\frac{\rho(s)}{16\pi}\right], \qquad (2)$$

- s is the usual Mandelstam variable and $\rho(s)=\sqrt{1-2\,(M_K^2+M_\pi^2)/s+(M_K^2-M_\pi^2)^2/s^2}$;
- m_R is the parameter present in the chiral lagrangian, called *nominal* resonance mass;
- $\bar{R}_{1/2}(s)$ is the function describing off-shell effects in the two-meson propagator, given by

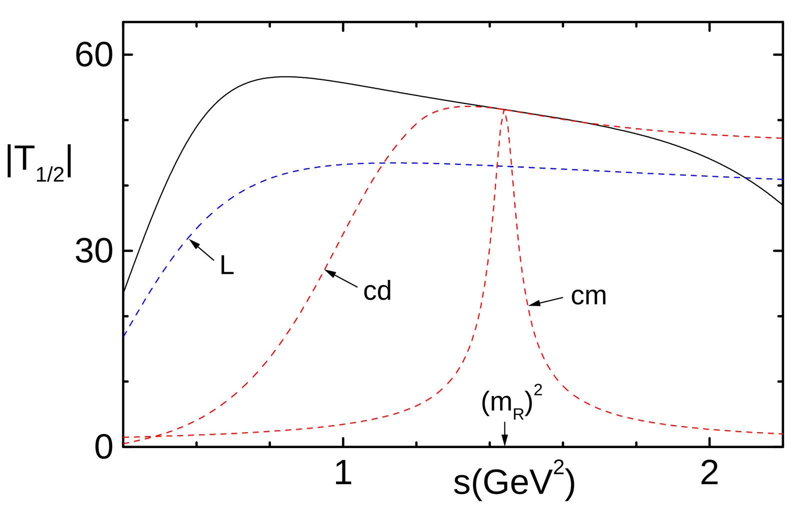
$$\begin{aligned}\bar{R}_{1/2}(s)&=-\Re\left[L(s)-L(m_R^2)\right]/16\pi^2\ ,\\ \Re L(s)&=\rho(s)\log\left[(1-\sigma)/(1+\sigma)\right]-2+\left[(M_K^2-M_\pi^2)/s\right]\log(M_K/M_\pi)\right]\ ,\\ \sigma&=\sqrt{|s-(M_K+M_\pi)^2|/|s-(M_K-M_\pi)^2|}\ ;\end{aligned} \qquad (3)$$

- $\bar{R}_{1/2}(m_R^2)=0$ by construction and therefore the phase shift is $\pi/2$ at $s=m_R^2$;
- $\gamma^2(s)$ is the function which incorporates chiral dynamics, given by

$$\begin{aligned}\gamma^2(s)\ =&\ \left\{(1/F^2)\ \left[\left(1-3\,\rho^2(s)/8\right)s-\left(M_\pi^2+M_K^2\right)\right]\ (m_R^2-s)\right\}_L\\ &+\left\{(3/F^4)\left[c_d\ \left(s-M_\pi^2-M_K^2\right)+c_m\ \left(4\,M_K^2+5\,M_\pi^2\right)/6\right]^2\right\}_R\ .\end{aligned} \qquad (4)$$

3 Amplitude

- full curve (black) is eq.(2),
- dashed curve L (blue) $\rightarrow c_d=c_m=0$,
- curve c_m (red) $\rightarrow$ Breit-Wigner shape,
- leanding order dominates at low-energies,
- all curves coincides at $s=m_R^2$.



c_d is **the** important parameter.

4 Poles

Poles are zeros in $D(s)$
- numerical solution $\longrightarrow$ exact but cumbersome
- analitical solution $\rightarrow$ approximation but transparent physics

Analitic equation:

• $m_\pi=0\ \rightarrow\ SU(2)$ limit ; • K-matrix approximation $\rightarrow\ \bar{R}_{1/2}(s)=0$

$\Downarrow$

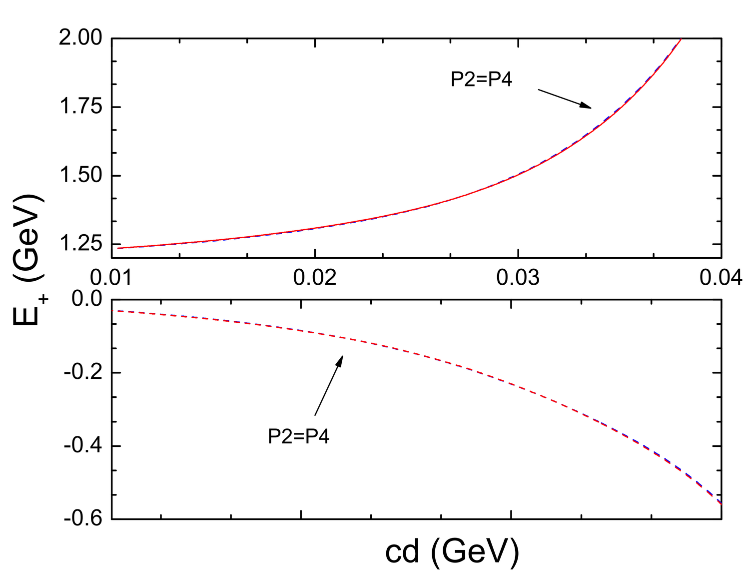
$D(s)$ becames a quartic function

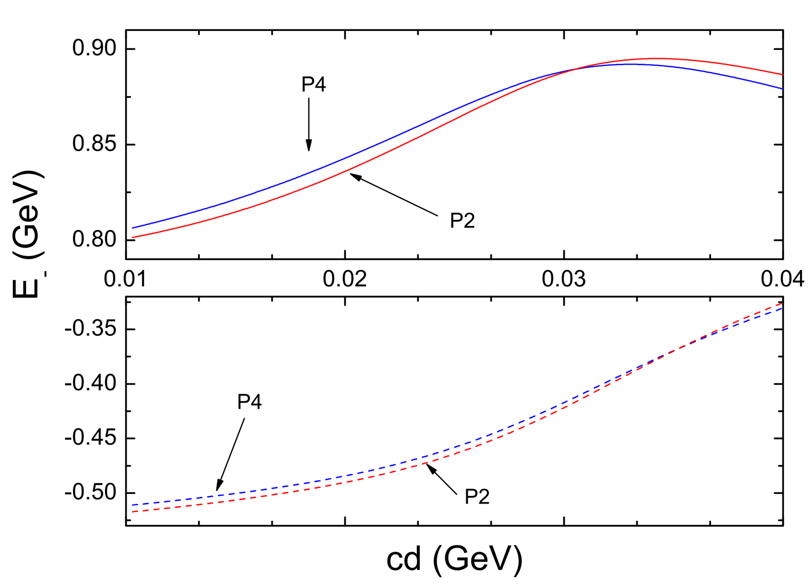
$$\begin{aligned}&\left(\frac{5}{8}-\frac{3c_d^2}{8}\right)s^4+\left[-(5m_r^2+7m_K^2)/8\ +\frac{c_d}{F^2}(9c_d-4c_m)m_K^2\ +i16\pi F^2\right]s^3\\ &+\left[(7m_r^2-m_K^2)\frac{m_K^2}{8}-(c_d-2c_m/3)(9c_d-2c_m)\,\frac{M_K^4}{F^2}\,m_r^2\ -i16\pi F^2\right]s^2\\ &+\left[(m_r^2\ +\ 3m_K^2)/8\ +3(c_d-2c_m/3)^2\,\frac{m_K^2}{F^2}\right]\,M_K^4 s-3m_r^2M_K^6/8=0\end{aligned} \qquad (5)$$

$\hookrightarrow$ only two physical poles.

Close to pole position $m_K^2/|s|\ll 1\ \Rightarrow$ quadratic function

$$A\,s^2+B\,s+C=0\begin{cases}A\ =[5/8\ -3c_d^2/F^2]\ ;\\ B\ =[-(5m_R^2\ +\ 7M_K^2)\ /8+c_d(9c_d-4c_m)\,\frac{M_K^2}{F^2}+i\,16\pi F^2]\ ;\\ C\ =[\,7M_K^2/8\ -\ i\,16\pi F^2]\ m_R^2\end{cases}$$





The coefficient $A=5/8\ -3c_d^2/F^2$ is very important

- $A=0\rightarrow\ c_d/F=\sqrt{5/24}=0.047\ \longrightarrow$ single solution

$$s_-(0)=\frac{[7M_K^2/5-i\ 128\pi\ F^2/5]}{1+\left[\frac{7M_K^2}{5}\ -8c_d(9c_d-4c_m)\frac{M_K^2}{F^2}\ -i\ 128\pi\frac{F^2}{5}\right]/m_R^2}\ . \qquad (6)$$

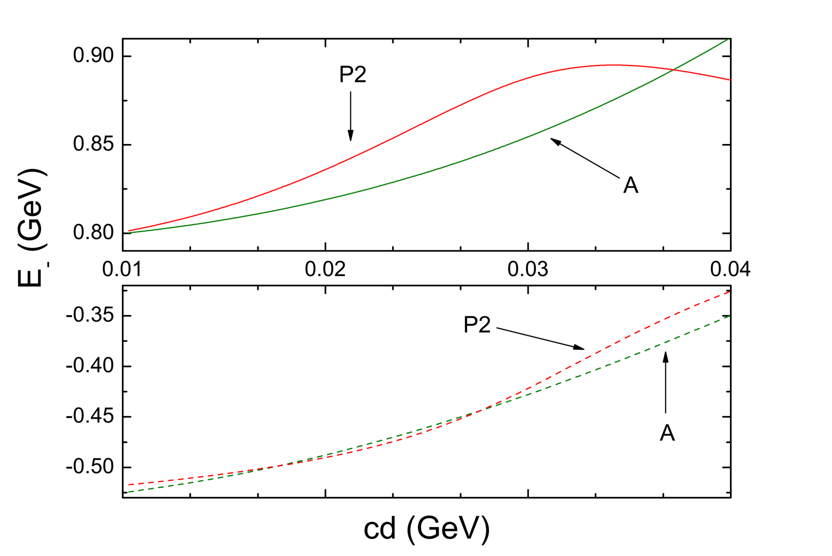
- $A=5/8\rightarrow\ c_d=0$: resonance $R\ \longrightarrow$ decoupled bound state in the real axis,

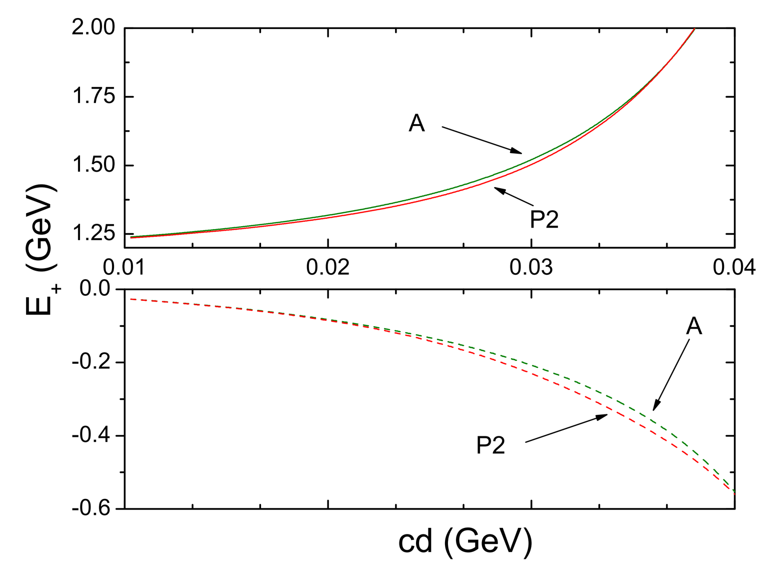
$$s_+(5/8)=m_R^2\quad\text{and}\quad s_-(5/8)=\left[7M_K^2/5-i\ 128\pi\ F^2/5\right]\ ; \qquad (7)$$

Analitic solution: (approximate),

$$\begin{aligned}s_+\ =&\ \frac{1}{A}\left\{\frac{5}{8}m_R^2-\frac{c_d}{F}\left(\frac{24c_d}{5F}-\frac{4c_m}{F}\right)M_K^2-\frac{3c_d^2}{m_R^2F^2}\left(1-\frac{24c_d^2}{5F^2}\right)\left(\frac{128\pi F^2}{5}\right)^2\right.\\ &\left.-i\,\frac{c_d}{F}\left[3\frac{c_d}{F}-\left(\frac{3c_d}{5F}-\frac{4c_m}{F}\right)\frac{M_K^2}{m_R^2}-\frac{3c_d}{F}\left(1-\frac{24c_d^2}{5F^2}\right)\left(\frac{128\pi F^2}{5m_R^2}\right)^2\right]\frac{128\pi F^2}{5}\right\},\end{aligned} \qquad (8)$$

$$s_-\ =\ \frac{7}{5}M_K^2\ +\frac{24m_R^2c_d^2}{5F^2}\left(\frac{128\pi F^2}{5m_R^2}\right)^2-\ i\ \left[1\ -\frac{24c_d^2}{5F^2}\left(\frac{128\pi F^2}{5m_R^2}\right)^2\right]\frac{128\pi F^2}{5}\ . \qquad (9)$$





5 Conclusion

We identify
- $\sqrt{s_+}\Longleftrightarrow K_0^*(1430)$
- $\sqrt{s_-}\Longleftrightarrow\kappa$

- if resonance R is absent $\begin{cases}\kappa & \longrightarrow \text{origin in contact interaction.}\\ K_0^*(1430) & \longrightarrow \text{absent.}\end{cases}$
- if $c_d=c_m=0$ $\begin{cases}\kappa & \longrightarrow \text{origin in contact interaction.}\\ K_0^*(1430) & \longrightarrow \text{bound state in the real axis at } s=m_R^2.\end{cases}$
- if $c_d\neq 0$ $\begin{cases}\kappa & \longrightarrow \text{origin in contact interaction.}\\ K_0^*(1430) & \longrightarrow \text{mass and width } \textit{increase} \text{ monotonically whith } A.\end{cases}$
- if $c_d/F=\sqrt{5/24}$ $\begin{cases}\kappa & \longrightarrow \text{origin in contact interaction.}\\ K_0^*(1430) & \longrightarrow \text{blows up; absent beyond this point.}\end{cases}$

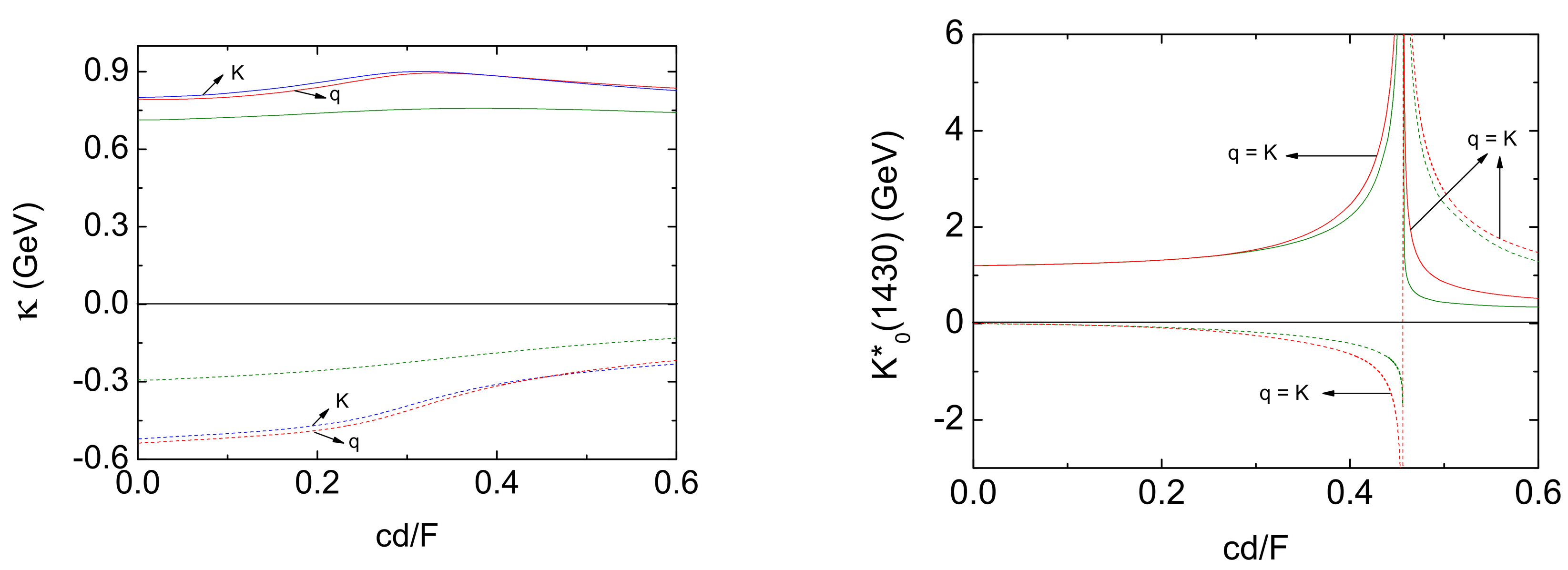


Figure 1: *Real (full) and imaginary (dashed) components of κ and $K_0^*(1430)$ functions. Green $\equiv$ eq(2); K and q are K-matrix and quadratic approximation.*

- Coefficient A appears explicitly in approximate function and explains behavior of numerical solutions $\Rightarrow$ **Essential of poles is maintained in approximation**;
- inclusion of the pion mass is not numerically important;
- off-shell effects in the two-meson propagator are numerically important.

Prediction:

$K_0^*(1430)$ pole at $[(1.414\pm 0.006)-i\,(0.145\pm 0.010)]$ GeV $\Longrightarrow$
- $m_R=1.1865\pm 0.079$ GeV
- $c_d=0.02786\pm 0.00078$ GeV

• κ pole at $(0.7505\pm 0.0010)-i\,(0.2363\pm 0.0023)$ GeV.

References

[1] G. Colangelo, J. Gasser and H. Leutwyler, Nucl. Phys. B **603** (2001) 125.
[2] A. Pich, E. Rafael, G Ecker, J. Gasser, Nucl. Phys. **B 321** (1989) 77.
[3] J.A. Oller and E. Oset, Nucl. Phys. A **620** (1997) 438.