Analyticity constrained pion-nucleon analysis

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- Generalities of πN interaction
- The Karlsruhe analysis
- Recent activity with the expansion techniques
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Generalities of πN interaction

The pion-nucleon amplitude can be presented as

$$T_{\pi N} = \bar{u}'[A(\nu,t) + \frac{1}{2}\gamma^{\mu}(q+q')_{\mu}B(\nu,t)]u$$

where $$\nu = \frac{s-u}{4m} = \omega + \frac{t}{4m}.$$

$$C(\nu,t) = A(\nu,t) + \frac{\nu}{(1-t/4m^2)}B(\nu,t)$$

Optical theorem: $\mathrm{Im}\, C(\omega, t=0) = k_{\,\mathrm{lab}}\, \sigma$

Isospin: $C^{\pm} = \frac{1}{2}(C_{\pi^- p} \pm C_{\pi^+ p})$

Basic principles:

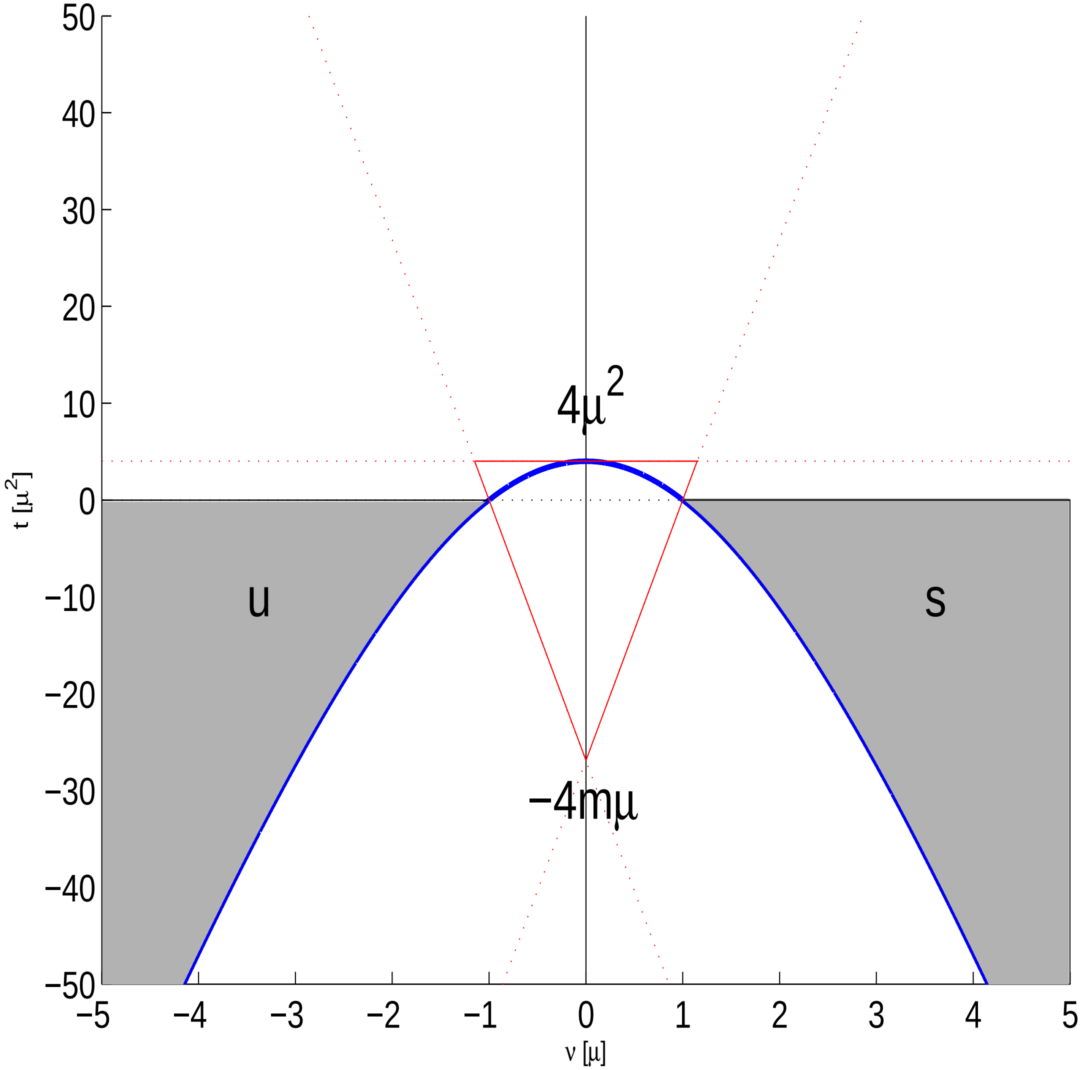
Analyticity, unitarity and crossing.

Fixed-t dispersion relations can be proven from first principles for $4\,\mu^2 > t > -18\,\mu^2 \simeq -0.35\,\mathrm{GeV}^2$.

Isospin is an approximate symmetry: violated by the electromagnetic interaction and $m_u \neq m_d$.

Constraints from fixed-t analyticity and isospin invariance are strong enough to resolve the ambiguities of phase-shift analysis.

Mandelstam diagram:



The Karlsruhe analysis

G. Höhler, Landolt-Börnstein, Vol. 9 b2, ed. H. Schopper (Springer, Berlin, 1983).

Analysis in 3 stages

- fixed-t analysis
- fixed centre-of-mass angle analysis
- phase-shift analysis

which are performed iteratively until the amplitudes agree to about 3 %; solutions KH78 and KH80.

For the fixed-t and the fixed-θ_{CM} analyses the Karlsruhe group uses the expansion techniques.

Pietarinen's expansion for the amplitudes at fixed-t:

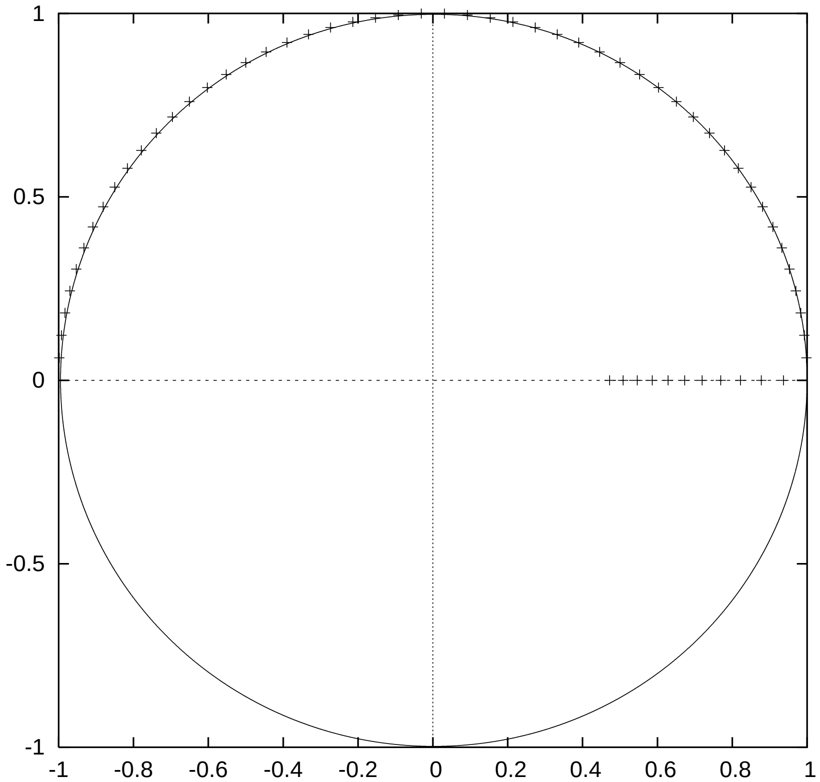
$$C^+(\nu,t) = C_N^+(\nu,t) + H(Z,t)\sum_{n=0}^{N} c_n^+ Z^n,$$

where H is adjusted to the asymptotic behaviour of the amplitude and

$$Z(\nu^2,t) = \frac{\alpha - \sqrt{\nu_{th}^2 - \nu^2}}{\alpha + \sqrt{\nu_{th}^2 - \nu^2}},$$

where $\alpha = 0.72$ GeV and $\nu_{th} = \mu + \frac{t}{4m}$.

This maps the physical region on the upper semicircle of the unit circle



The coefficients c_n^+ in the expansion can be determined by minimizing

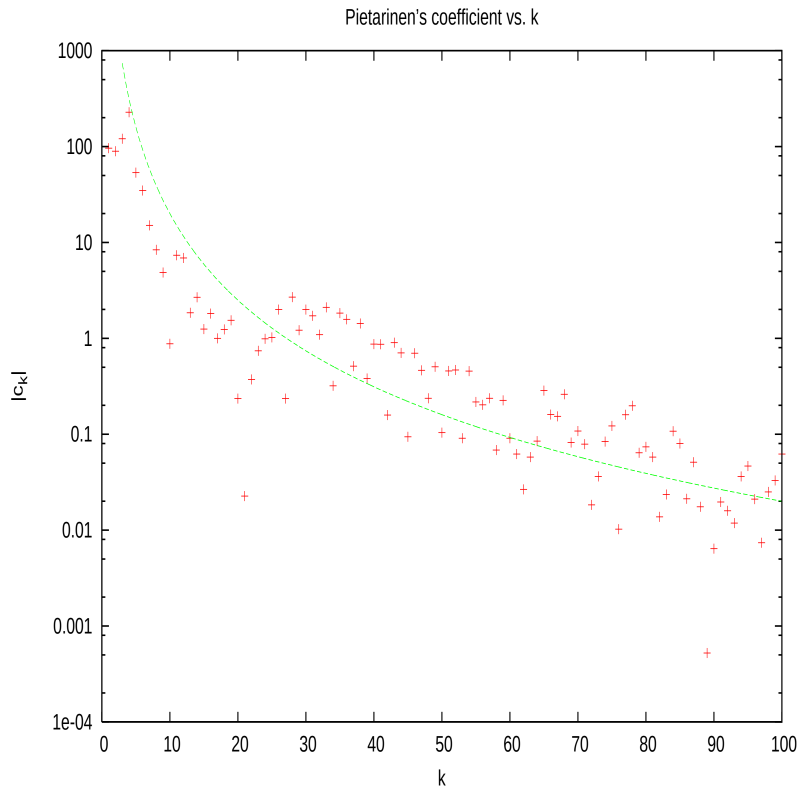
$$\chi^2 = \chi^2_{DATA} + \chi^2_{PW} + \chi^2_T (4\,\text{terms}),$$

where χ^2_{DATA} and χ^2_{PW} refer to the contributions from data and the existing partial wave solution respectively. The convergence and smoothing is taken care by a convergence test function

$$\chi^2_T = \lambda \sum_{n=0}^{N} (c_n^+)^2 (n+1)^3,$$

which is added to the χ^2 expression with similar terms for the C^- and $B^\pm$ amplitudes.

The coefficients c_n are expected to go as n^{-3} for large n and for the C^+-amplitude we obtain:



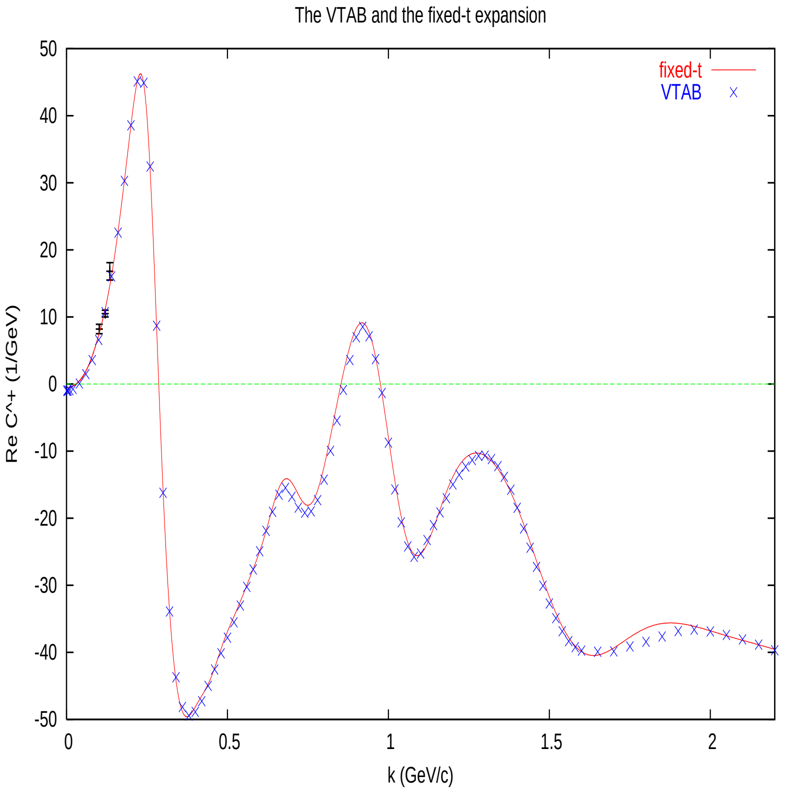
For the expansion techniques for the fixed centre-of-mass angle analysis, see e.g. G. Höhler et al., Handbook of Pion-Nucleon Scattering (1979).

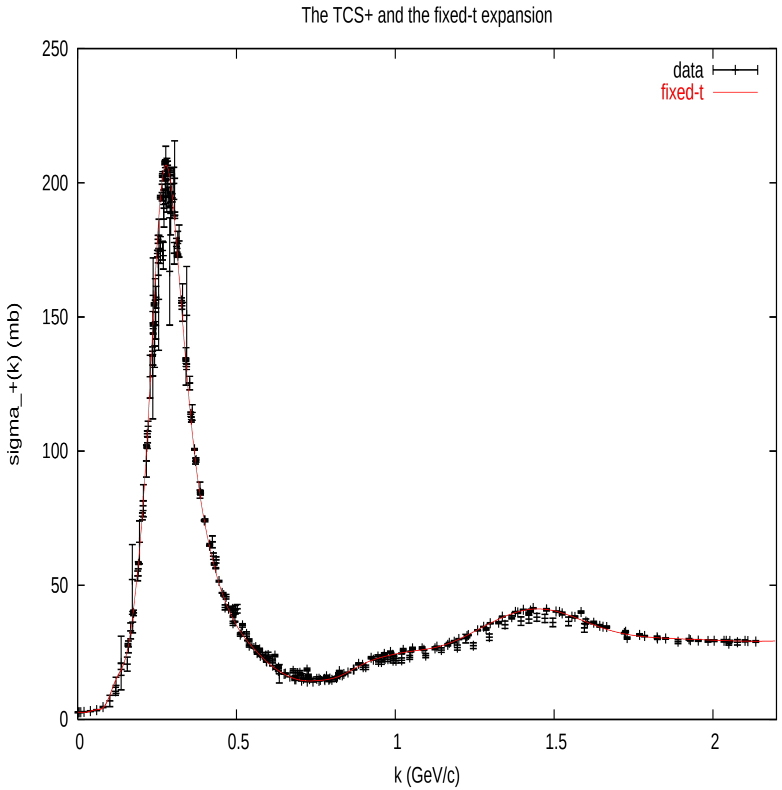
The KH78 analysis covers the range $k_{lab} = 0.$ - 10. GeV/c and the KH80 analysis the range $k_{lab} = 0.$ - 0.5 GeV/c.

Partial wave dispersion relations (PWDR) have been checked separately (Hutt and Koch) as well as partial wave relations (Koch). Agreement has been found satisfactory.

For the details of the KA84 solution, see R. Koch, Z. Phys. **C29** (1985) 597.

For the forward direction, $t = 0$, with N=40 we have:





Recent activity with the expansion techniques

In Helsinki we have been working on a πN PWA for quite some time (together with Pekko Metsä).

The idea has been to build in the fixed-t constraints with the Pietarinen's expansion, i.e. in this respect to repeat the Karlsruhe analysis with an improved data base and more computing power. The latter has made it possible for us to pay more attention to the error analysis.

Our focus is more on the low-energy scattering to make contact to the ChPT domain.

The analysis of the forward data has been completed and published: V. Abaev et al., EPJ **A32** (2007) 321; P. Metsä, EPJ **A33** (2007) 349.

Goldberger-Miyazawa-Oehme sum rule:

$$C^-(\mu) = \frac{8\pi f^2}{\mu(1-(\frac{\mu}{2m})^2)} + 4\pi\mu J^- = 4\pi(1+\frac{\mu}{m})a^-_{0+}$$

where

$$\begin{aligned} J^- &= \frac{1}{4\pi^2}\int_0^\infty \frac{\sigma_{\pi^- p}(k) - \sigma_{\pi^+ p}(k)}{\omega}\,dk, \\ &= -1.060 \pm 0.030 \text{ mb.} \end{aligned}$$

This agrees exactly with the Höhler-Kaiser value of 1980, they did not, however, perform an error analysis.

If pionic hydrogen information is used, giving

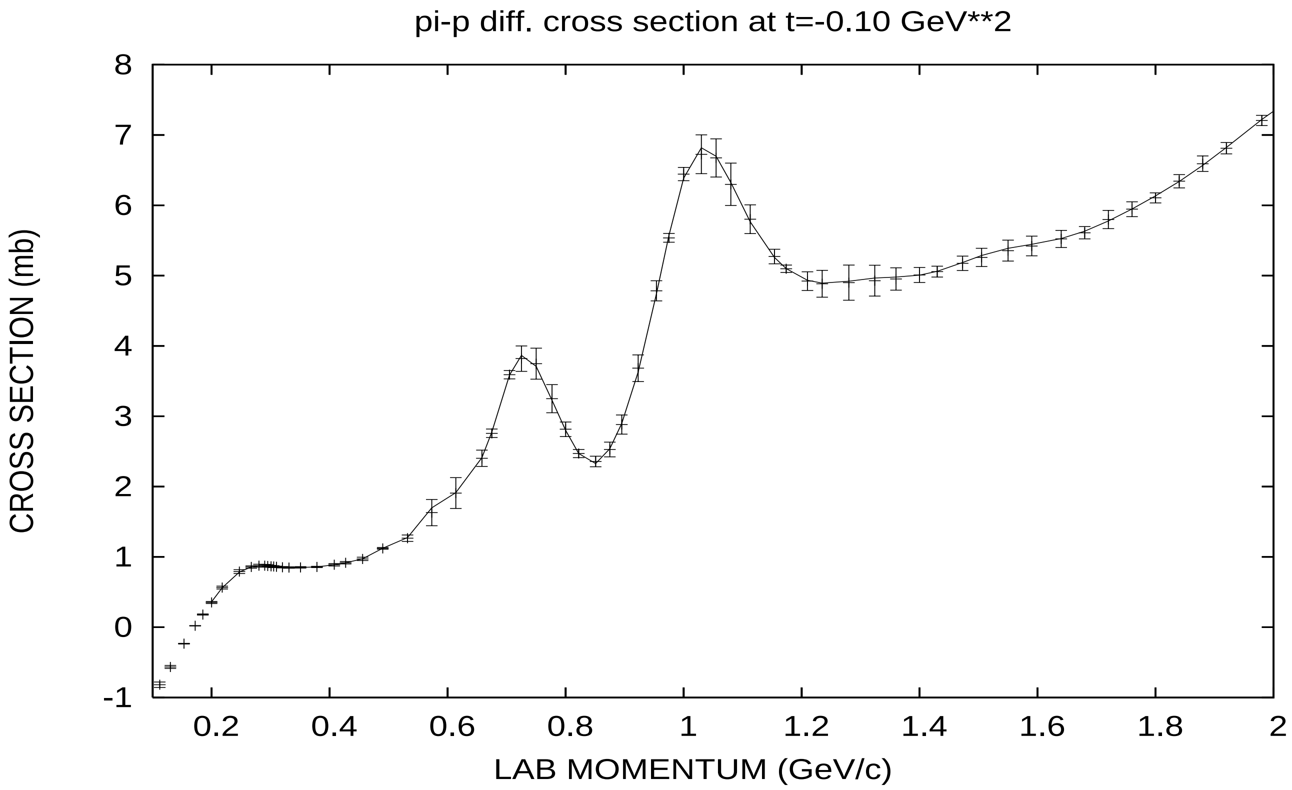
$a_{\pi^- p} = 0.0933 \pm 0.0029 \; 1/\mu,$

together with a value for the s-wave $\pi^+ p$ scattering length,

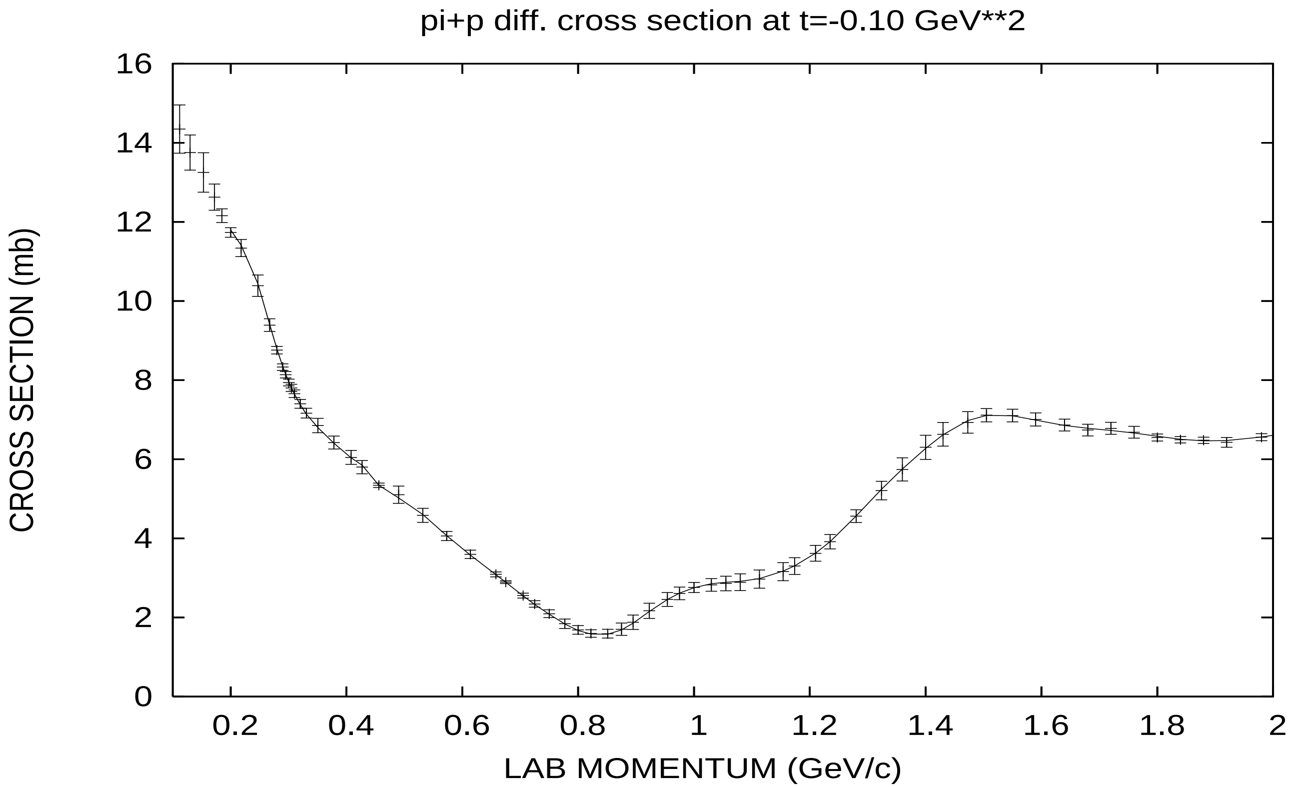
$a_{\pi^+ p} = -0.0764 \pm 0.0014 \; 1/\mu,$

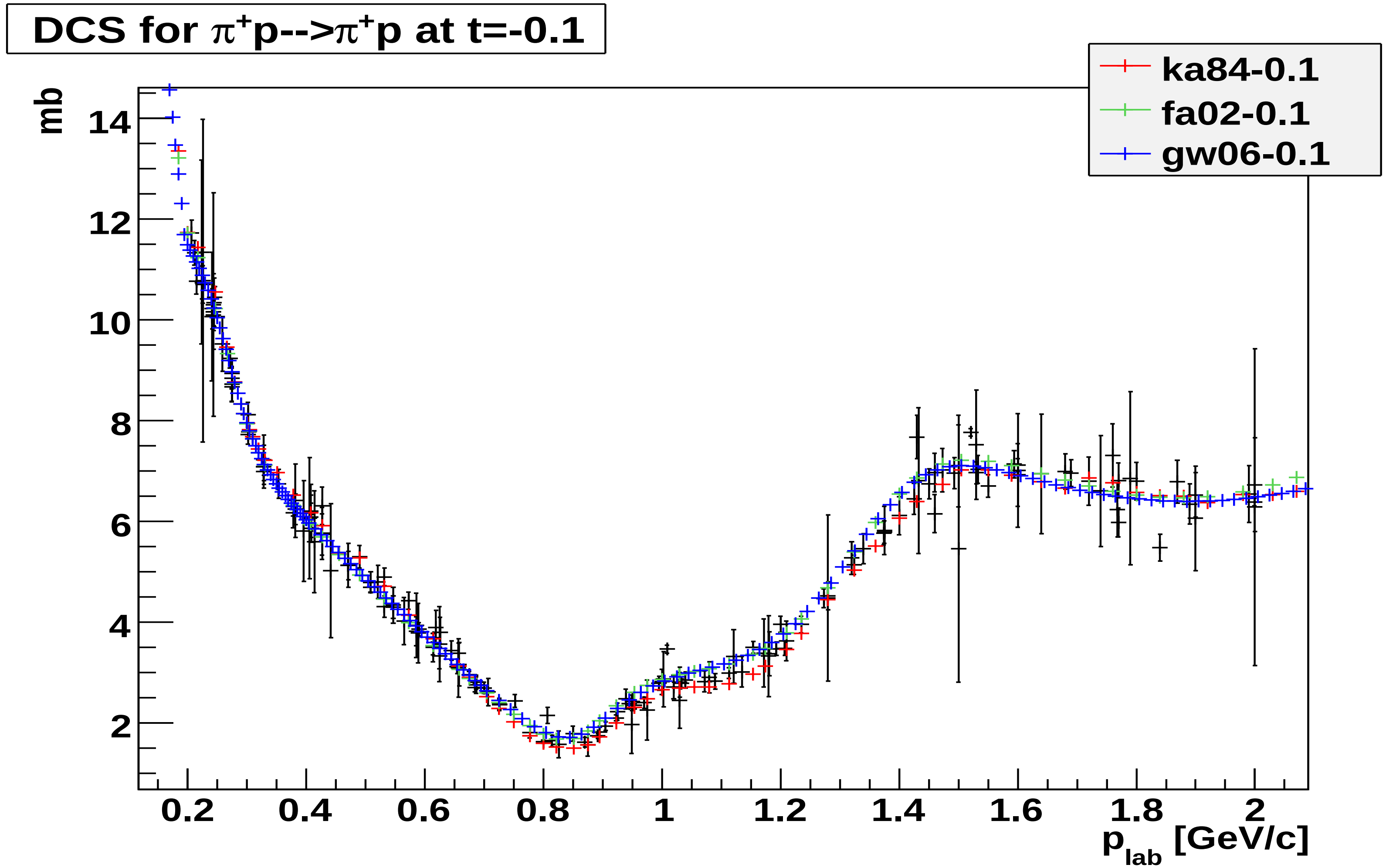
we get $f^2 = 0.075 \pm 0.002$ for the pion-nucleon coupling constant.

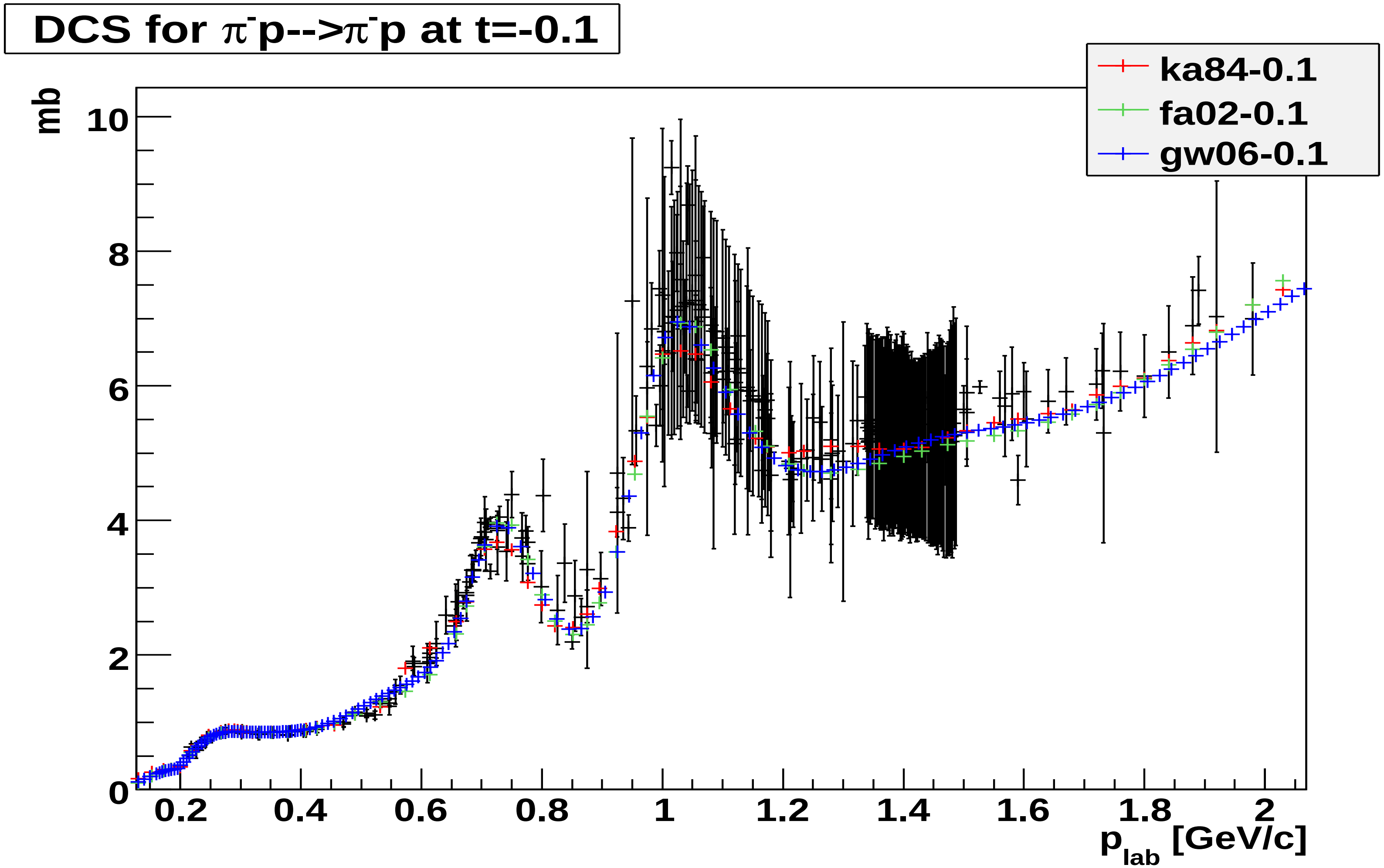
At $t = -0.1$ GeV^2 we have



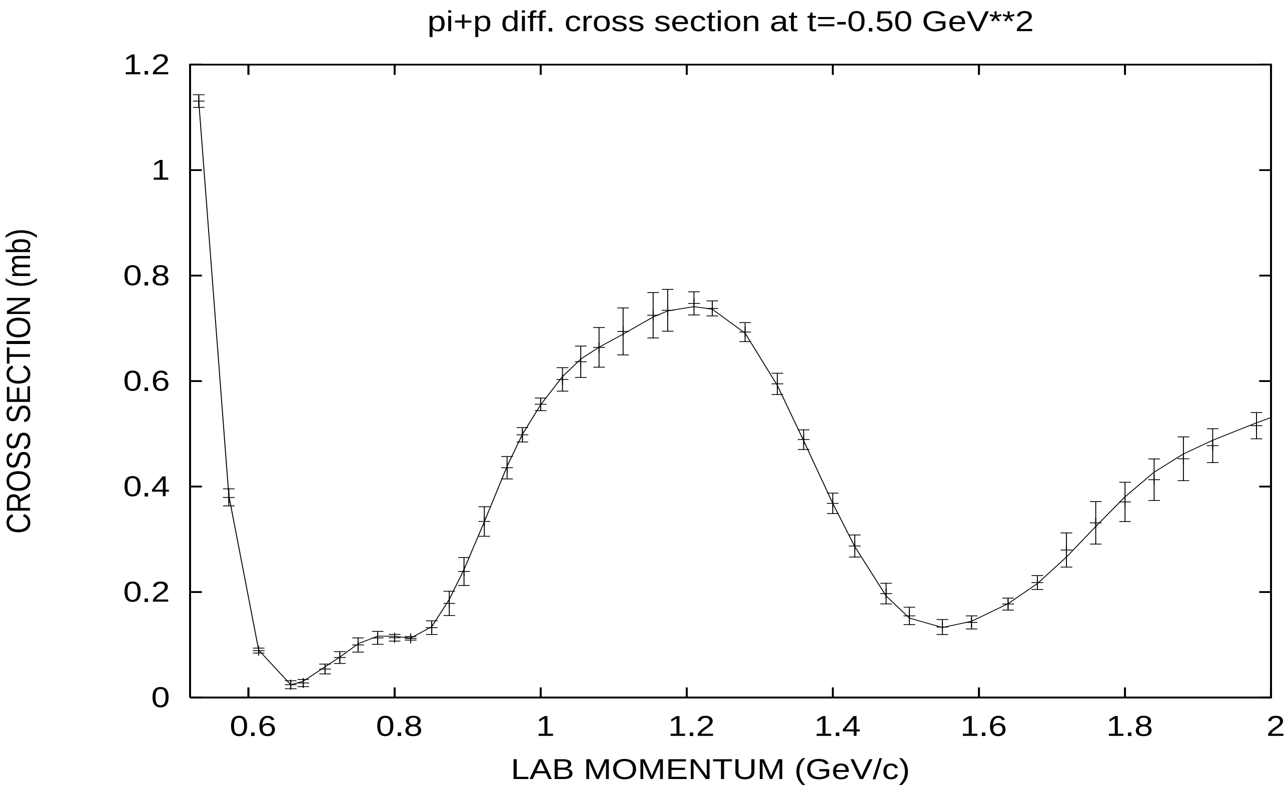
and similarly for the $\pi^+ p$ scattering

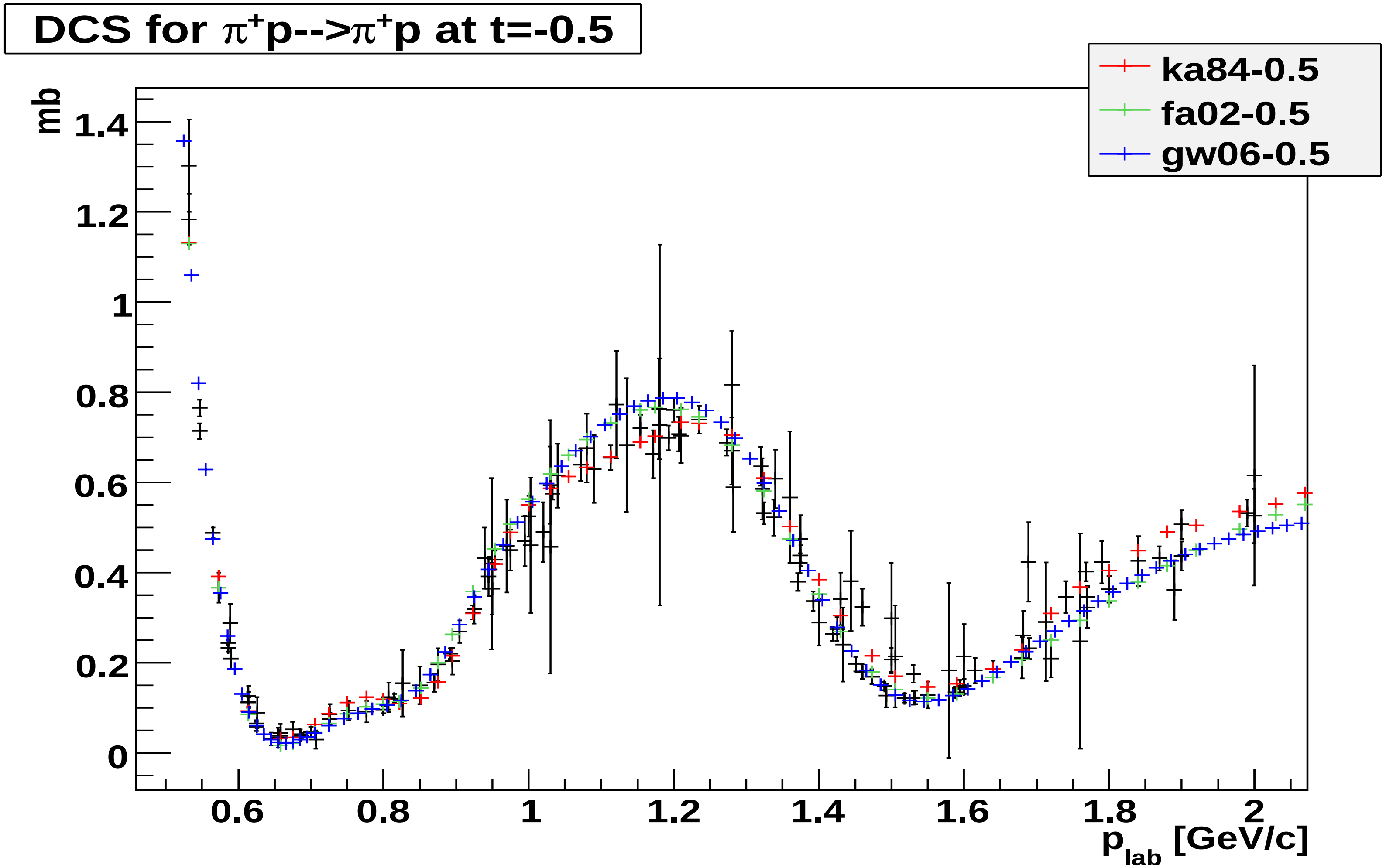



DCS for π+p-->π+p at t=-0.1
ka84-0.1
fa02-0.1
gw06-0.1
mb
14
12
10
8
6
4
2
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
p_lab [GeV/c]


DCS for π^-p-->π^-p at t=-0.1
ka84-0.1
fa02-0.1
gw06-0.1
mb
0
2
4
6
8
10
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
p_{lab} [GeV/c]

At $t = -0.5$ GeV^2 we have for the $\pi^+ p$ scattering




DCS for $\pi^+p \rightarrow \pi^+p$ at t=-0.5
ka84-0.5
fa02-0.5
gw06-0.5
mb
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0.6
0.8
1
1.2
1.4
1.6
1.8
2
p_{lab} [GeV/c]

Conclusions

- In general the KH analysis produces a consistent set of PWA's.
- The data set has changed since 1980.
- The GWU-VPI analysis includes fixed-t analyticity as well.
- At fixed-t KA84 and FA02 agree surprisingly well.
- How to perform the amalgamation of the experimental data will be essential.