S=+1 Pentaquarks in QCD Sum Rules

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Pentaquark Θ^+

Basic properties

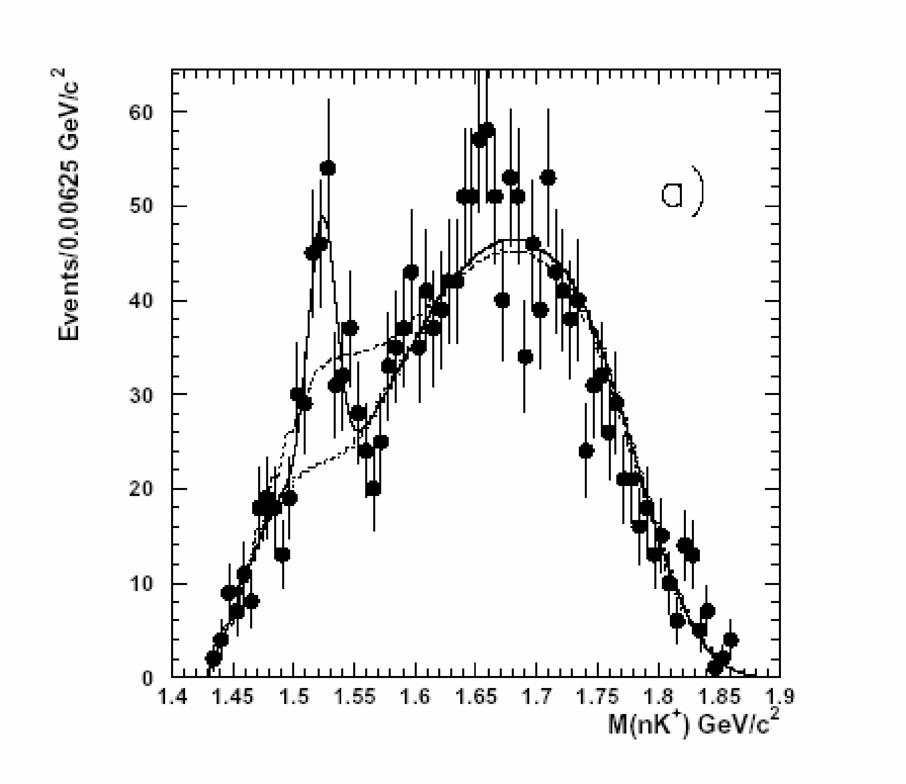
- B=1, S=1 → minimal quark content: 5 quarks $(uudd\bar{s})$
- No Isospin-partners Θ^0, Θ^{++} → I=0 (?)
- Narrow width: less than ~1 MeV
- Mass: ~1540 MeV

Why is it interesting?

- It is exotic.
- Why has it not been seen earlier?
- Why is it so narrow?

→ New dynamics in QCD ?

The SPring-8 experiment has reconfirmed a peak, so the question of the existence of Θ^+ is not settled yet.



T. Nakano *et al.*

Phys. Rev. C **79**, 025210 (2009).

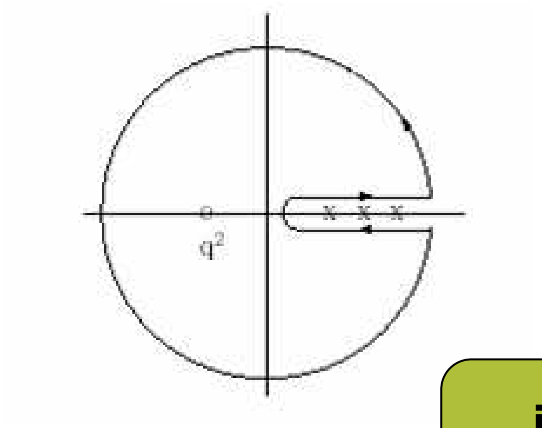
Also, there are still many theoretical questions that remain to be answered.

(Quantum numbers, narrow width, etc.)

QCD sum rules

In this method the properties of the two point correlation function is fully exploited:

$$\Pi(q) = i \int d^4x e^{iqx} \langle 0 | T\{\chi(x)\overline{\chi}(0)\} | 0 \rangle$$



$$\rightarrow \Pi(q^2) = \frac{1}{\pi} \int_{s_{min}}^{\infty} ds \frac{\mathrm{Im}\Pi(s)}{s - q^2 - i\epsilon}$$

is calculated "perturbatively"

spectral function of the operator χ

Borel transformation → Introduction of an unphysical parameter, the Borel mass

The concrete calculation (for I,J^P=0,3/2$^\pm$)

We use the following interpolating fields:

$$\eta^1_\mu(x) = \epsilon_{cfg}[\epsilon_{abc} u^T_a(x) C\gamma_5 d_b(x)][\epsilon_{def} u^T_d(x) C\gamma_\mu\gamma_5 d_e(x)] C\overline{s}^T_g(x),$$

$$\eta^2_\mu(x) = \epsilon_{cfg}[\epsilon_{abc} u^T_a(x) C d_b(x)][\epsilon_{def} u^T_d(x) C\gamma_\mu\gamma_5 d_e(x)]\gamma_5 C\overline{s}^T_g(x)$$

$$\longrightarrow \quad \eta_\mu(x) = \cos\theta\,\eta^1_\mu(x) + \sin\theta\,\eta^2_\mu(x)$$

Using these currents, the 2-point function is calculated:

$$\begin{aligned}\Pi^{ij}_{\mu\nu}(q) &= i\int d^4x e^{iqx}\langle 0|T[\eta^i_\mu(x)\overline{\eta}^j_\nu(0)]|0\rangle \\ &= g_{\mu\nu}\Big[\hat{q}\Pi^{ij}_0(q^2) + \Pi^{ij}_1(q^2)\Big] + \ldots \qquad (\hat{q} \equiv q^\mu\gamma_\mu)\end{aligned}$$

chiral even part

chiral odd part

Importance of the Borel window

1. The OPE Convergence

$$\left|\frac{Dimension\ \ N\ \ terms}{OPE\ \ summed\ \ up\ \ to\ \ Dimension\ \ N}\right| \leq 0.1$$

2. The Pole Contribution

$$\frac{\int_0^{s_{th}} ds e^{-\frac{s}{M^2}} Im\Pi^{OPE}(s)}{\int_0^{\infty} ds e^{-\frac{s}{M^2}} Im\Pi^{OPE}(s)} \geq 0.5$$

It is very important that these two conditions are satisfied simultaneously to obtain reliable results from QCDSR calculations!

How to obtain a high Pole Contribution (1)

We use an approach similar to the old idea of the Weinberg spectral function sum rule:

$$\langle V_\mu(x)\overline{V}_\nu(0)\rangle - \langle A_\mu(x)\overline{A}_\nu(0)\rangle \;\simeq\; 0$$

$$(x \to 0)$$

→ **leading orders in the OPE expansion are suppressed!**

T. Kojo, A. Hayashigaki, D.Jido, Phys. Rev. C **74**, 045206 (2006)

In our case we calculate the difference of two (independent) correlators with different mixing angles to obtain a good suppression of the leading OPE orders:

$$\Pi_D(q^2,\phi) \equiv \Pi_i(q^2,\theta_1) - \Pi_i(q^2,\theta_2)$$

$$(\phi \equiv \theta_1 + \theta_2)$$

How to obtain a high Pole Contribution (2)

The sum and the difference of the used interpolating fields belong to specific chiral multiplets:

$$\begin{aligned}\xi_{1,\mu} &\equiv \eta_{1,\mu} + \eta_{2,\mu} \\ &= 2(u_R^T C d_R)[(u_L^T C \gamma_\mu d_R) - (u_R^T C \gamma_\mu d_L)] C \overline{s}_R^T \\ &\quad - 2(u_L^T C d_L)[(u_L^T C \gamma_\mu d_R) - (u_R^T C \gamma_\mu d_L)] C \overline{s}_L^T,\end{aligned} \qquad (\mathbf{3}, \overline{\mathbf{15}}) \oplus (\overline{\mathbf{15}}, \mathbf{3})$$

$$\begin{aligned}\xi_{2,\mu} &\equiv \eta_{1,\mu} - \eta_{2,\mu} \\ &= 2(u_R^T C d_R)[(u_L^T C \gamma_\mu d_R) - (u_R^T C \gamma_\mu d_L)] C \overline{s}_L^T \\ &\quad - 2(u_L^T C d_L)[(u_L^T C \gamma_\mu d_R) - (u_R^T C \gamma_\mu d_L)] C \overline{s}_R^T.\end{aligned} \qquad (\mathbf{8}, \mathbf{8})$$

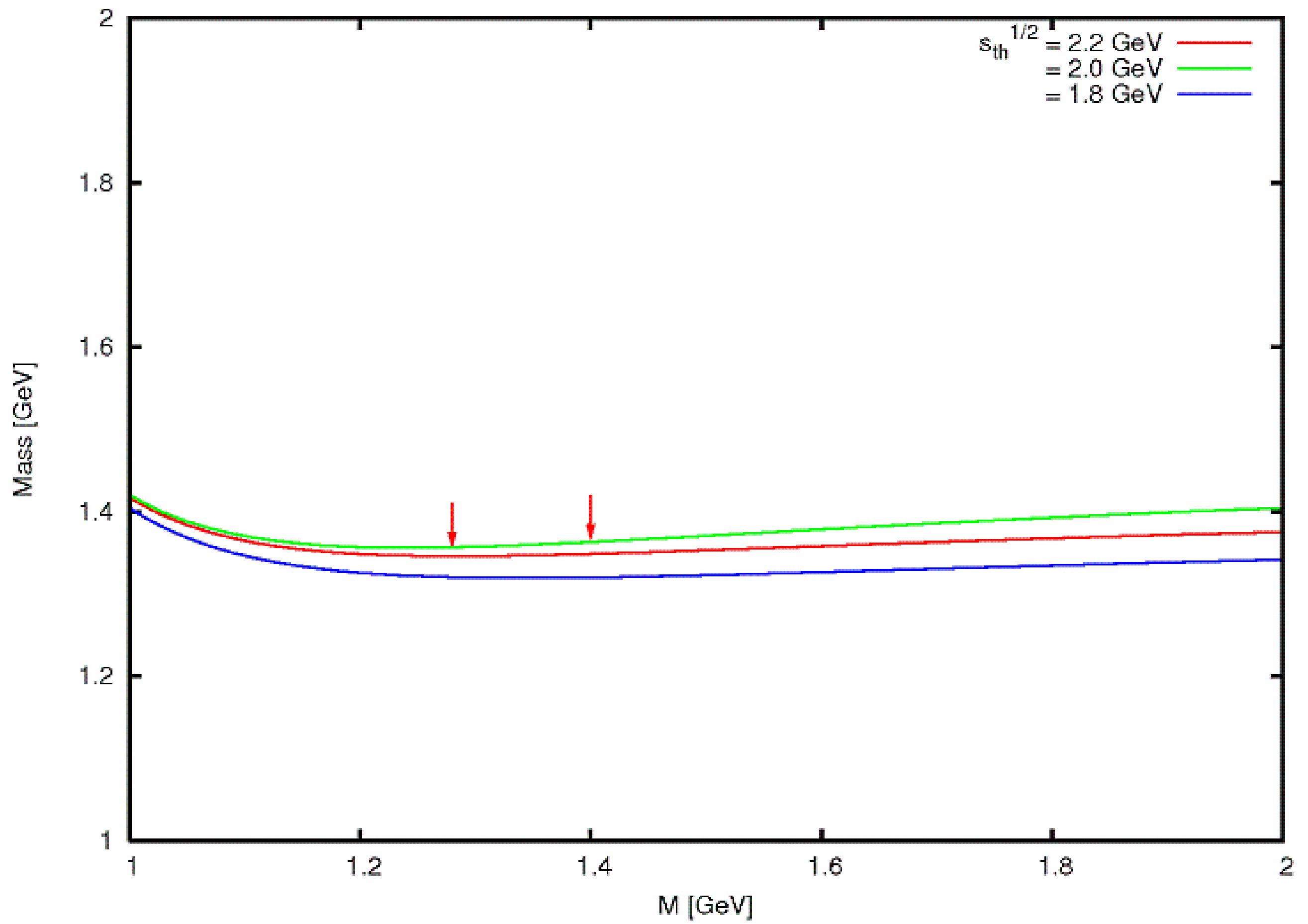
$$\begin{aligned}\Pi_D(q^2,\phi) = &\frac{1}{2}\Big\{\cos\phi[\langle \xi_1 \overline{\xi_1}\rangle - \langle \xi_2 \overline{\xi_2}\rangle] \\ &- \sin\phi[\langle \xi_1 \overline{\xi_2}\rangle + \langle \xi_2 \overline{\xi_1}\rangle]\Big\}\end{aligned}$$

1) A sufficiently wide Borel window exists.

2) The calculated pentaquark mass should only weakly depend on the Borel mass M and the threshold parameter s_{th}.

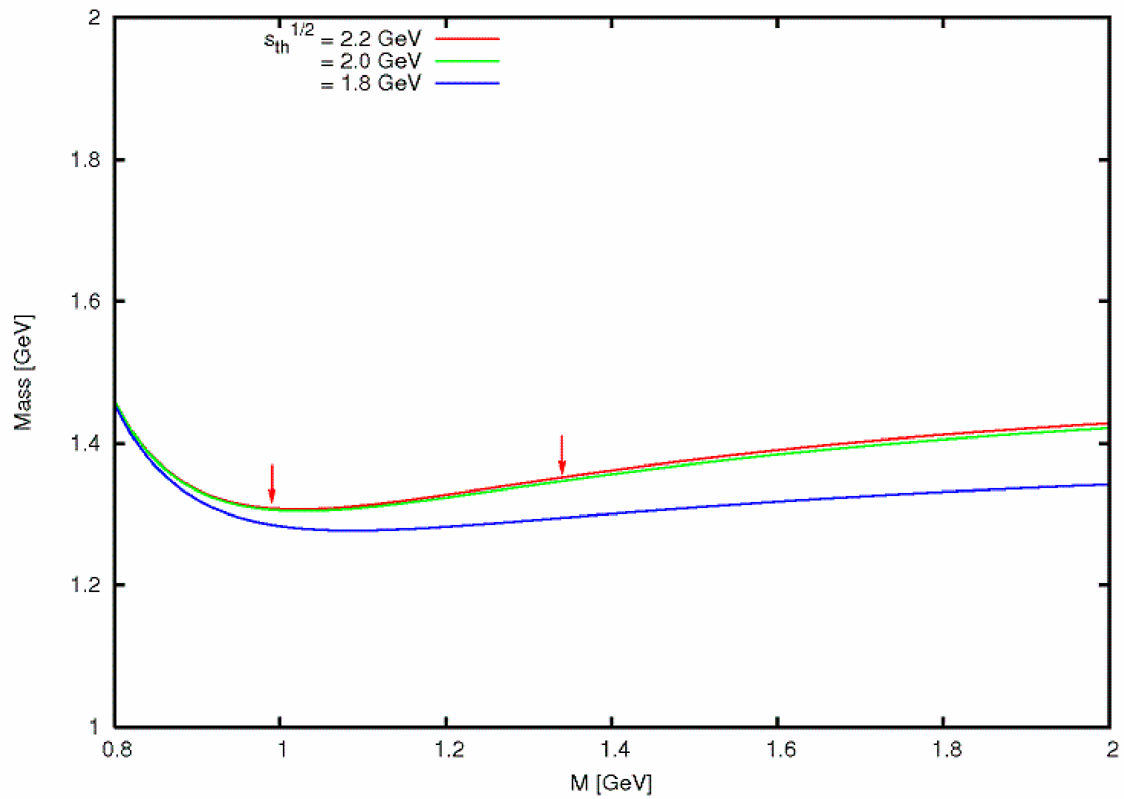
$\rightarrow$ $\boxed{\phi = 0.063}$

Results (Chiral even part)



$$\longrightarrow \; m_{\Theta^+} = 1.4 \pm 0.2 \;\; \text{GeV}$$

Results (Positive Parity)



In the negative parity channel no valid Borel window with a flat Borel mass curve is obtained.

$$\rightarrow \; IJ^P = 0\tfrac{3}{2}^+$$

The other quantum numbers (1,3/2$^{\pm}$ 0,1/2$^{\pm}$ 1,1/2$^{\pm}$)

The following interpolating fields are used:

$$\eta'^{1}_{\mu}(x) = \epsilon_{cfg}[\epsilon_{abc}u_a^T(x)C\gamma_5 d_b(x)][\epsilon_{def}u_d^T(x)C\gamma_\mu d_e(x)]C\overline{s}_g^T(x),$$
$$\eta'^{2}_{\mu}(x) = \epsilon_{cfg}[\epsilon_{abc}u_a^T(x)Cd_b(x)][\epsilon_{def}u_d^T(x)C\gamma_\mu d_e(x)]\gamma_5 C\overline{s}_g^T(x).$$

(IJ$^{\pi}$= 1,3/2$^{\pm}$)

$$\eta^{1}(x) = \epsilon_{cfg}[\epsilon_{abc}u_a^T(x)C\gamma_5 d_b(x)][\epsilon_{def}u_d^T(x)C\gamma_\mu\gamma_5 d_e(x)]\gamma^\mu\gamma_5 C\overline{s}_g^T(x),$$
$$\eta^{2}(x) = \epsilon_{cfg}[\epsilon_{abc}u_a^T(x)Cd_b(x)][\epsilon_{def}u_d^T(x)C\gamma_\mu\gamma_5 d_e(x)]\gamma^\mu C\overline{s}_g^T(x).$$

(IJ$^{\pi}$= 0,1/2$^{\pm}$)

$$\eta'^{1}(x) = \epsilon_{cfg}[\epsilon_{abc}u_a^T(x)C\gamma_5 d_b(x)][\epsilon_{def}u_d^T(x)C\gamma_\mu d_e(x)]\gamma^\mu C\overline{s}_g^T(x),$$
$$\eta'^{2}(x) = \epsilon_{cfg}[\epsilon_{abc}u_a^T(x)Cd_b(x)][\epsilon_{def}u_d^T(x)C\gamma_\mu d_e(x)]\gamma^\mu\gamma_5 C\overline{s}_g^T(x).$$

(IJ$^{\pi}$= 1,1/2$^{\pm}$)

The rest of the calculations follows the same lines as in the isosinglet case.

Summary of all obtained Results

		Parity	
		$+$	-
$J = \frac{3}{2}$	$I = 0$	1.4 ± 0.2 GeV (KN P-wave)	no state found below 2.0 GeV (KN D-wave)
	$I = 1$	1.6 ± 0.3 GeV (KN P-wave)	no state found below 2.0 GeV (KN D-wave)
$J = \frac{1}{2}$	$I = 0$	no state found below 2.0 GeV (KN P-wave)	1.5 ± 0.3 GeV (?) (KN S-wave)
	$I = 1$	no state found below 2.0 GeV (KN P-wave)	1.6 ± 0.4 GeV (KN S-wave)

Why can`t we observe this state ?

The width is expected to be quite large.
Most probably these are not the experimentally observed states

Conclusion and Outlook

- Our results suggest that the $I,J^{\pi}=0,3/2^{+}$ seems to be the most probable candidate for the experimentally observed $\Theta^{+}(1540)$.
- To further improve the reliability of our results a quantitative evaluation of the KN scattering states is necessary.
- As we have obtained a spin 3/2 state with positive parity, the problem of the narrow width will need further consideration.

  Calculation of the width using the QCD sum rule approach would be interesting.

Backup Slides

The theoretical (QCD) side

The operator product expansion (OPE) is used:

$$i\int d^4x e^{iqx}\langle 0|T\{\chi(x)\overline{\chi}(0)\}|0\rangle = C_I(q^2)I+\sum_n C_n(q^2)\langle 0|O_n|0\rangle$$

$$\begin{aligned}\langle 0|O_n|0\rangle = &\langle 0|\overline{q}q|0\rangle,\\ &\langle 0|G^a_{\mu\nu}G^{a\mu\nu}|0\rangle,\\ &\langle 0|\overline{q}\sigma_{\mu\nu}\frac{\lambda^a}{2}G^{a\mu\nu}q|0\rangle,\\ &\langle 0|\overline{q}q\overline{q}q|0\rangle,\ldots\end{aligned}$$

The phenomenological (hadronic) side

Sharp resonance + continuum is assumed:

$$\frac{1}{\pi}Im\Pi(s) = \lambda^2\delta(s-m^2)+\theta(s-s_{th})\frac{1}{\pi}Im\Pi^{OPE}(s)$$

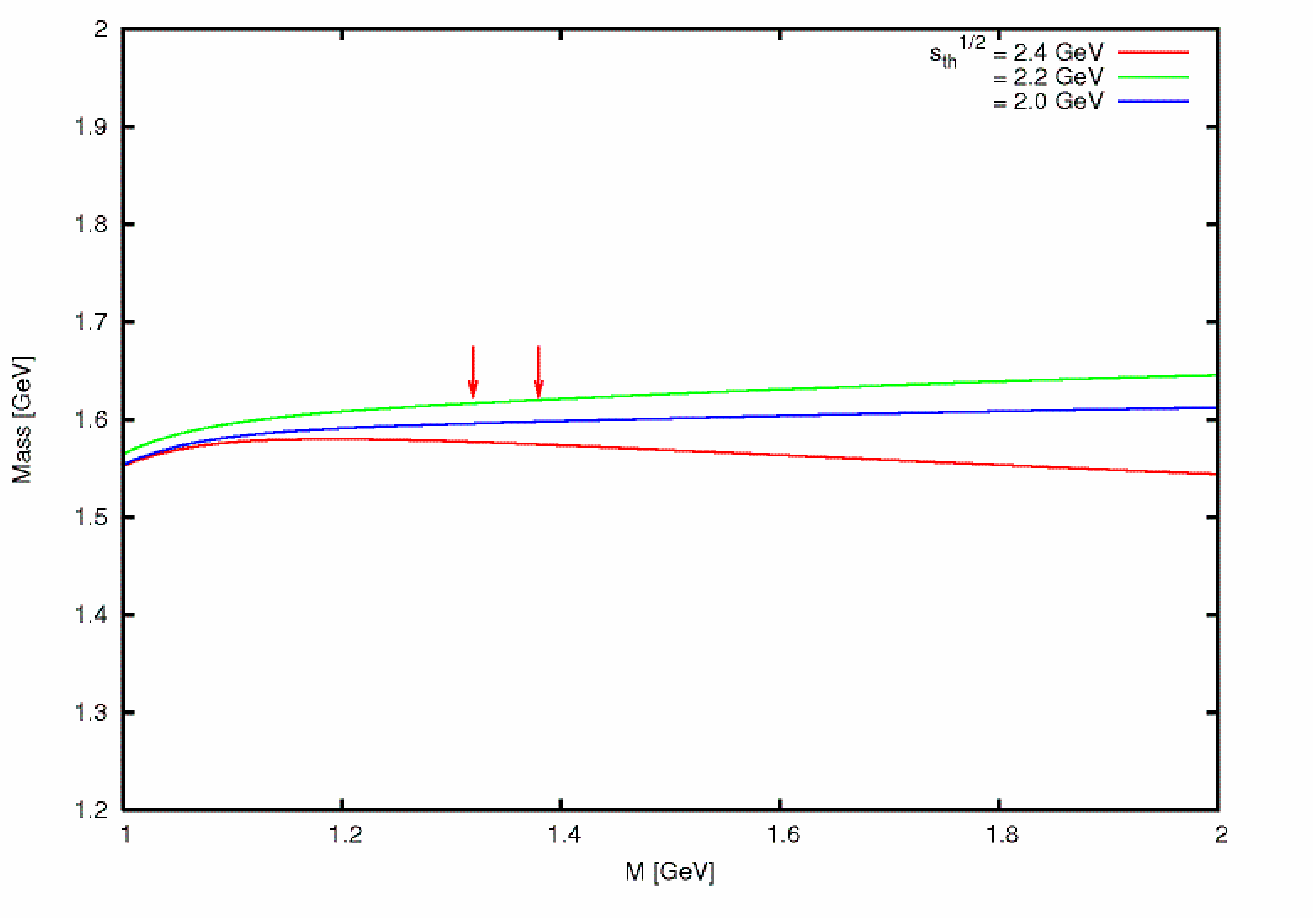
$$
\begin{aligned}
& -i\int d^4x e^{iqx}\langle 0|T[\eta_\mu^{I=0}(x)\bar{\eta}_\nu^{I=0}(0)]|0\rangle_{g_{\mu\nu}q'} \\
= & -\frac{1}{2^{19}3^2 5^2\pi^8}q^{10}\ln(-q^2)\cdot(\cos^2\theta^{I=0}+\sin^2\theta^{I=0}) \\
& -\frac{m_s\langle\bar{s}s\rangle}{2^{13}3^{3}\pi^{6}}q^6\ln(-q^2)\cdot(\cos^2\theta^{I=0}+\sin^2\theta^{I=0}) \\
& +\frac{\langle\frac{\alpha_s}{\pi}G^2\rangle}{2^{18}3^4 5\pi^6}q^6\ln(-q^2)\cdot(7\cos^2\theta^{I=0}+12\cos\theta^{I=0}\sin\theta^{I=0}+7\sin^2\theta^{I=0}) \\
& -\frac{\langle\bar{q}q\rangle^2}{2^9 3^2 5\pi^4}q^4\ln(-q^2)\cdot(\cos^2\theta^{I=0}-9\sin^2\theta^{I=0}) \\
& +\frac{m_s\langle\bar{s}g\sigma\cdot Gs\rangle}{2^{16}3^2 5\pi^6}q^4\ln(-q^2)\cdot(37\cos^2\theta^{I=0}+12\cos\theta^{I=0}\sin\theta^{I=0}+37\sin^2\theta^{I=0}) \\
& +\frac{5m_s\langle\bar{s}s\rangle\langle\frac{\alpha_s}{\pi}G^2\rangle}{2^{15}3^3\pi^4}q^2\ln(-q^2)\cdot(\cos^2\theta^{I=0}+\sin^2\theta^{I=0}) \\
& -\frac{\langle\bar{q}q\rangle\langle\bar{q}g\sigma\cdot Gq\rangle}{2^{12}3^3\pi^4}q^2\ln(-q^2)\cdot(26\cos^2\theta^{I=0}+7\cos\theta^{I=0}\sin\theta^{I=0}+198\sin^2\theta^{I=0}) \\
& +\frac{m_s\langle\frac{\alpha_s}{\pi}G^2\rangle\langle\bar{s}g\sigma\cdot Gs\rangle}{2^{17}3^4\pi^4}\ln(-q^2)\cdot(211\cos^2\theta^{I=0}+156\cos\theta^{I=0}\sin\theta^{I=0}+211\sin^2\theta^{I=0}) \\
& -\frac{m_s\langle\bar{q}q\rangle^2\langle\bar{s}s\rangle}{2^5 3^3\pi^2}\ln(-q^2)\cdot(\cos^2\theta^{I=0}-5\sin^2\theta^{I=0}) \\
& +\frac{\langle\bar{q}g\sigma\cdot Gq\rangle^2}{2^{15}3^4\pi^4}\ln(-q^2)\cdot(489\cos^2\theta^{I=0}+44\cos\theta^{I=0}\sin\theta^{I=0}+1959\sin^2\theta^{I=0}) \\
& +\frac{\langle\bar{q}q\rangle^2\langle\frac{\alpha_s}{\pi}G^2\rangle}{2^{11}3^4\pi^2}\ln(-q^2)\cdot(151\cos^2\theta^{I=0}+4\cos\theta^{I=0}\sin\theta^{I=0}+133\sin^2\theta^{I=0}) \\
& +\frac{m_s\langle\bar{q}q\rangle^2\langle\bar{s}g\sigma\cdot Gs\rangle}{2^8 3^3\pi^2 q^2}\cdot(17\cos^2\theta^{I=0}+\cos\theta^{I=0}\sin\theta^{I=0}-13\sin^2\theta^{I=0}) \\
& -\frac{5m_s\langle\bar{q}q\rangle\langle\bar{s}s\rangle\langle\bar{q}g\sigma\cdot Gq\rangle}{2^7 3^2\pi^2 q^2}\cdot\sin^2\theta^{I=0} \\
& -\frac{\langle\bar{q}q\rangle\langle\bar{q}g\sigma\cdot Gq\rangle\langle\frac{\alpha_s}{\pi}G^2\rangle}{2^{14}3^4\pi^2 q^2}\cdot(1365\cos^2\theta^{I=0}+65\cos\theta^{I=0}\sin\theta^{I=0}+849\sin^2\theta^{I=0}) \\
& +\frac{\langle\bar{q}q\rangle^4}{2\cdot 3^3 q^2}\cdot(\cos^2\theta^{I=0}-\sin^2\theta^{I=0}) \\
& +\frac{m_s\langle\bar{q}q\rangle\langle\bar{q}g\sigma\cdot Gq\rangle\langle\bar{s}g\sigma\cdot Gs\rangle}{2^{12}3^4\pi^2 q^4}\cdot(271\cos^2\theta^{I=0}+68\cos\theta^{I=0}\sin\theta^{I=0}-209\sin^2\theta^{I=0}) \\
& -\frac{m_s\langle\bar{s}s\rangle\langle\bar{q}g\sigma\cdot Gq\rangle^2}{2^{10}3^3\pi^2 q^4}\cdot(6\cos^2\theta^{I=0}+17\sin^2\theta^{I=0}) \\
& -\frac{m_s\langle\bar{q}q\rangle^2\langle\bar{s}s\rangle\langle\frac{\alpha_s}{\pi}G^2\rangle}{2^9 3^4 q^4}\cdot(21\cos^2\theta^{I=0}+35\sin^2\theta^{I=0}) \\
& +\frac{97\langle\bar{q}q\rangle^3\langle\bar{q}g\sigma\cdot Gq\rangle}{2^6 3^4 q^4}\cdot(\cos^2\theta^{I=0}-\sin^2\theta^{I=0})
\end{aligned}
$$

→ Perturbative part, contributing mainly to the continuum

$$\longrightarrow$$

$$
\begin{aligned}
& \Pi(\tfrac{\phi}{2}+\tfrac{\pi}{4})-\Pi(\tfrac{\phi}{2}-\tfrac{\pi}{4}) \\
& = +\frac{\langle\frac{\alpha_s}{\pi}G^2\rangle}{2^{16}3^3 5\pi^6}q^6\ln(-q^2)\cdot\cos\phi \\
& \quad +\frac{\langle\bar{q}q\rangle^2}{2^8 3^2\pi^4}q^4\ln(-q^2)\cdot\sin\phi \\
& \quad +\frac{m_s\langle\bar{s}g\sigma\cdot Gs\rangle}{2^{14}3\cdot 5\pi^6}q^4\ln(-q^2)\cdot\cos\phi \\
& \quad -\frac{\langle\bar{q}q\rangle\langle\bar{q}g\sigma\cdot Gq\rangle}{2^{12}3^3\pi^4}q^2\ln(-q^2)\cdot(7\cos\phi+172\sin\phi) \\
& \quad +\frac{13m_s\langle\frac{\alpha_s}{\pi}G^2\rangle\langle\bar{s}g\sigma\cdot Gs\rangle}{2^{15}3^3\pi^4}\ln(-q^2)\cdot\cos\phi \\
& \quad +\frac{m_s\langle\bar{q}q\rangle^2\langle\bar{s}s\rangle}{2^4 3^2\pi^2}\ln(-q^2)\cdot\sin\phi \\
& \quad +\frac{\langle\bar{q}g\sigma\cdot Gq\rangle^2}{2^{14}3^4\pi^4}\ln(-q^2)\cdot(22\cos\phi+735\sin\phi) \\
& \quad +\frac{\langle\bar{q}q\rangle^2\langle\frac{\alpha_s}{\pi}G^2\rangle}{2^{10}3^4\pi^2}\ln(-q^2)\cdot(2\cos\phi-9\sin\phi) \\
& \quad +\frac{m_s\langle\bar{q}q\rangle^2\langle\bar{s}g\sigma\cdot Gs\rangle}{2^8 3^3\pi^2 q^2}\cdot(\cos\phi-30\sin\phi) \\
& \quad -\frac{5m_s\langle\bar{q}q\rangle\langle\bar{s}s\rangle\langle\bar{q}g\sigma\cdot Gq\rangle}{2^7 3^2\pi^2 q^2}\cdot\sin\phi \\
& \quad -\frac{\langle\bar{q}q\rangle\langle\bar{q}g\sigma\cdot Gq\rangle\langle\frac{\alpha_s}{\pi}G^2\rangle}{2^{14}3^4\pi^2 q^2}\cdot(65\cos\phi-516\sin\phi) \\
& \quad -\frac{\langle\bar{q}q\rangle^4}{3^3 q^2}\cdot\sin\phi \\
& \quad +\frac{m_s\langle\bar{q}q\rangle\langle\bar{q}g\sigma\cdot Gq\rangle\langle\bar{s}g\sigma\cdot Gs\rangle}{2^{10}3^4\pi^2 q^4}\cdot(17\cos\phi-120\sin\phi) \\
& \quad -\frac{11m_s\langle\bar{s}s\rangle\langle\bar{q}g\sigma\cdot Gq\rangle^2}{2^{10}3^3\pi^2 q^4}\cdot\sin\phi \\
& \quad -\frac{7m_s\langle\bar{q}q\rangle^2\langle\bar{s}s\rangle\langle\frac{\alpha_s}{\pi}G^2\rangle}{2^8 3^4 q^4}\cdot\sin\phi \\
& \quad -\frac{97\langle\bar{q}q\rangle^3\langle\bar{q}g\sigma\cdot Gq\rangle}{2^5 3^4 q^4}\cdot\sin\phi
\end{aligned}
$$

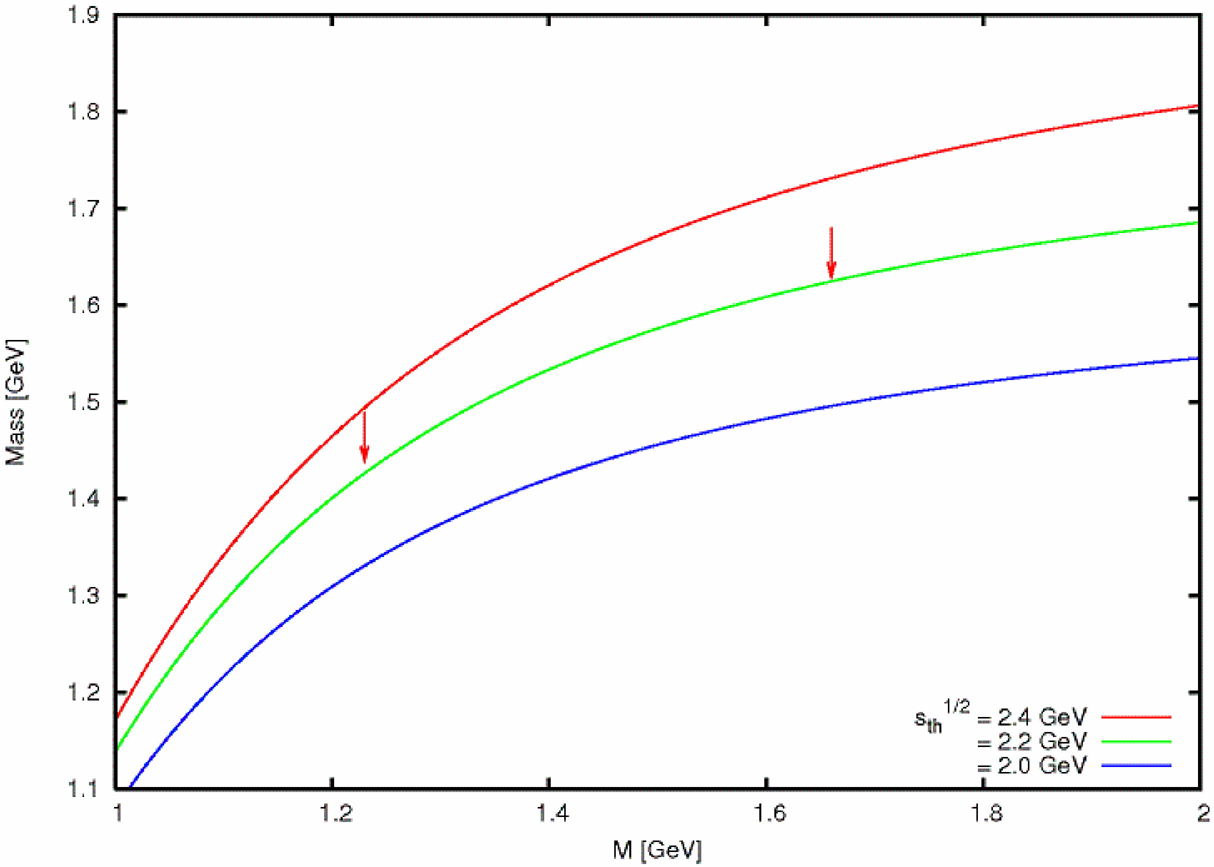
Results $(1,3/2^{\pm})$



$$\rightarrow \quad m = 1.6 \pm 0.3 \text{ GeV}$$

Parity projection $\rightarrow \quad IJ^P = 1\frac{3}{2}^+$

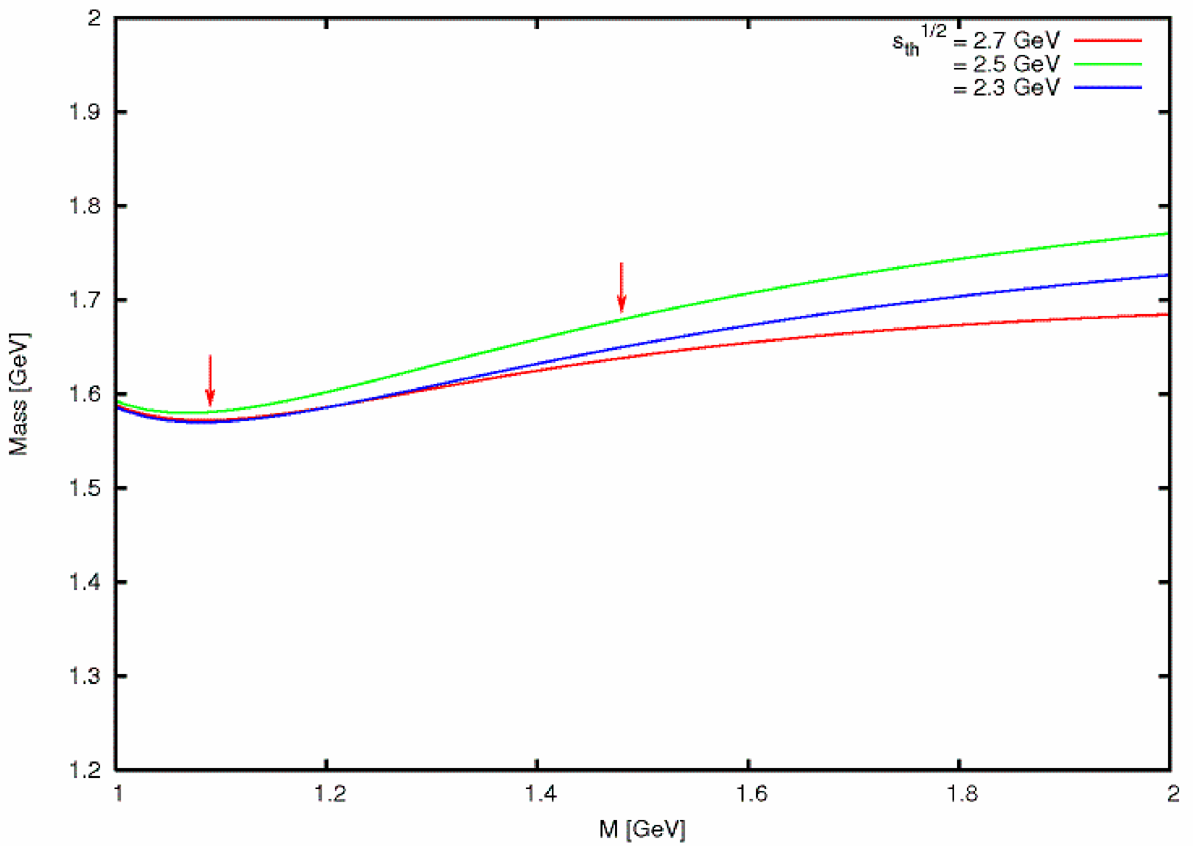
Results $(0,1/2^{\pm})$



$$\rightarrow \quad m = 1.5 \pm 0.3 \text{ GeV}$$

Parity projection $\rightarrow \quad IJ^P = 0\frac{1}{2}^-$

Results $(1,1/2^{\pm})$



$$\rightarrow \quad m = 1.6 \pm 0.4 \text{ GeV}$$

Parity projection $\rightarrow \quad IJ^P = 1\frac{1}{2}^-$