Threshold Pion Electroproduction at Large Momentum Transfers

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based on

V.M. Braun, D. Ivanov, A. Lenz and A. Peters, Phys.Rev.D75:014021,2007
V.M. Braun, D. Ivanov and A. Peters, Phys.Rev.D77:034016,2008

Chiral Dynamics 2009, Bern, 07.07.09

Electroproduction with Q^2 in a few GeV^2 range:

- **Tradition: excitation of nucleon resonances (transition form factors)**

$$\begin{aligned} e(l) + p(P) &\to e(l') + \Delta(1232)(P') \\ e(l) + p(P) &\to e(l') + N(1440)(P') \end{aligned}$$

- **Proposal: pion electroproduction close to threshold $W \to W_{\rm th}$**

$$\begin{aligned} e(l) + p(P) &\to e(l') + \pi^+(k) + n(P') \\ e(l) + p(P) &\to e(l') + \pi^0(k) + p(P') \end{aligned}$$

$$\begin{aligned} W^2 &= (P' + k)^2 \\ W_{\rm th} &= m_N + m_\pi \\ Q^2 &= -q^2 = -(\ell - \ell')^2 \end{aligned}$$

Hard ($pQCD$) and soft ($ChPT$) physics meet together !

Generalized Form Factors = S-wave Multipoles at Threshold

at the threshold

$$\langle \pi N | j_\mu^{\rm em} | p\rangle = -\frac{i}{f_\pi}\bar N(P_2)\gamma_5 \left\{ \left(\gamma_\mu q^2 - q_\mu \not{q}\right) \frac{1}{m_N^2} G_1^{\pi N}(Q^2) - \frac{i\sigma_{\mu\nu}q^\nu}{2m_N} G_2^{\pi N}(Q^2) \right\} N(P_2)$$

related to S-wave multipoles in the PWA, e.g. for $m_\pi = 0$

$$E_{0+}^{\pi N}(Q^2, W_{\rm th}) = \frac{\sqrt{4\pi\alpha_{\rm em}}}{8\pi} \frac{Q^2\sqrt{Q^2 + 4m_N^2}}{m_N^3 f_\pi} G_1^{\pi N}$$

$$L_{0+}^{\pi N}(Q^2, W_{\rm th}) = \frac{\sqrt{4\pi\alpha_{\rm em}}}{32\pi} \frac{Q^2\sqrt{Q^2 + 4m_N^2}}{m_N^3 f_\pi} G_2^{\pi N}$$

e.g. the differential cross section at threshold is given by

$$\frac{d\sigma_{\gamma^*}}{d\Omega_\pi}\Big|_{\rm th} = \frac{2|\vec{k}_f|W}{W^2 - m_N^2}\left[(E_{0+}^{\pi N})^2 + \epsilon \frac{Q^2}{(\omega_\gamma^{\rm th})^2}(L_{0+}^{\pi N})^2\right]$$

Chiral rotation

Spontaneous Breaking of Chiral Symmetry

- **In the chiral limit, $m_\pi/m_N \to 0$, the pion can be "rotated" away:**

$$|p\uparrow\rangle = \frac{\phi_s(x)}{\sqrt{6}}|2u_\uparrow d_\downarrow u_\uparrow - u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle + \frac{\phi_a(x)}{\sqrt{2}}|u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle$$

$$|p\uparrow\pi^0\rangle = \frac{\phi_s(x)}{2\sqrt{6}f_\pi}|6u_\uparrow d_\downarrow u_\uparrow + u_\uparrow u_\downarrow d_\uparrow + d_\uparrow u_\downarrow u_\uparrow\rangle - \frac{\phi_a(x)}{2\sqrt{2}f_\pi}|u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle$$

$$|n\uparrow\pi^+\rangle = \frac{\phi_s(x)}{\sqrt{12}f_\pi}|2u_\uparrow d_\downarrow u_\uparrow - 3u_\uparrow u_\downarrow d_\uparrow - 3d_\uparrow u_\downarrow u_\uparrow\rangle - \frac{\phi_a(x)}{2f_\pi}|u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle$$

Pobylitsa, Polyakov, Strikman; PRL87(2001)022001

- **allows one to "look" at the proton from a different "angle"**

The relevant degrees of freedom change with Q^2
$\Rightarrow$ **rich physics**
$\Rightarrow$ **rich theory**

$Q^2 < 0.1$ GeV^2: Chiral Perturbation Theory

- local effective low-energy theory
- systematic expansion in powers of m_π and $|q|$
- applicable for $|q| \sim m_\pi < 300$ MeV(?)

Bernard, Kaiser, Meissner; IJMP, E4 (1995)193
Drechsel, Tiator; J. Phys. G **18** (1992) 449

- **Subtlety: $q \to 0$ and $m_\pi \to 0$ limits do not commute**

Bernard, Kaiser, Meissner; PRL69 (1992)1877

Nambu, Lurié, Shrauner

$$E_{0+}^{(-)}(m_\pi = 0, q^2) = \frac{eg_A}{8\pi f_\pi}\left\{1 + \frac{q^2}{6}r_A^2 + \frac{q^2}{4m_N^2}\left(\kappa_v + \frac{1}{2}\right) + \frac{q^2}{128 f_\pi^2}\left(1 - \frac{12}{\pi^2}\right)\right\}$$

$$G_A(q^2) = g_A\left(1 + \frac{q^2}{6}r_A^2 + \ldots\right)$$

Experiment: $r_A = 0.65 \pm 0.03$ (elastic ep); $r_A = 0.59 + 0.04 \pm 0.05$ (pion el.prod)

$\mu = m_\pi/m_N \simeq 1/7$

Kroll, Ruderman

$$E_{0+}^{\pi^+ n}(q^2 = 0, W_{\rm th}) = \sqrt{2}\frac{eg_{\pi N}}{8\pi m_N}\left[1 - \frac{3}{2}\mu + O(\mu^2 \ln \mu^2)\right] = 26.6 \cdot 10^{-3}/m_\pi$$

exp: $27.9 \pm 0.5; 28.8 \pm 0.7$

$$E_{0+}^{\pi^0 p}(q^2 = 0, W_{\rm th}) = -\frac{eg_{\pi N}}{8\pi m_N}\mu\left\{1 - \mu\left[\frac{1}{2}(3 + \kappa_p) + \left(\frac{m_N}{4f_\pi}\right)^2\right]\right\}$$

$Q^2 \ll m_N^3/m_\pi$: Low-Energy Theorems (LET)

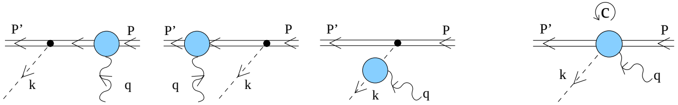
- **PCAC+Current Algebra (predate ChPT and QCD)**

$$\frac{Q^2}{m_N^2} G_1^{\pi^0 p} = \frac{g_A}{2} \frac{Q^2}{(Q^2 + 2m_N^2)} G_M^p \,, \qquad G_2^{\pi^0 p} = \frac{2 g_A m_N^2}{(Q^2 + 2m_N^2)} G_E^p \,,$$

$$\frac{Q^2}{m_N^2} G_1^{\pi^+ n} = \frac{g_A}{\sqrt{2}} \frac{Q^2}{(Q^2 + 2m_N^2)} G_M^n + \frac{1}{\sqrt{2}} G_A \,, \qquad G_2^{\pi^+ n} = \frac{2\sqrt{2} g_A m_N^2}{(Q^2 + 2m_N^2)} G_E^n \,,$$

Derivation does not imply $Q^2 \sim m_\pi^2$!

- **Pion emission from external legs + Chiral Rotation**



- **Threshold photoproduction of π^0 is suppressed compared to π^+**
- **The π^0/π^+–ratio is rapidly increasing with Q^2**

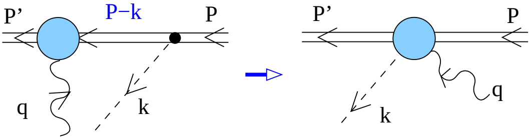
Vainshtein, Zakharov, NPB36(1972)589
Scherer, Koch, NPA534(1991)461

◇ **The $\mathcal{O}(m_\pi)$ corrections can be added**
◇ **but, no systematic way to treat $\mathcal{O}(m_\pi^2)$ terms (ChPT)**

Low-Energy Theorems – *continued*

- expected to fail for $Q^2 \sim \frac{m_N^3}{m_\pi}$

since π cannot have small momentum w.r.t. the initial and final state protons simultaneously



at threshold

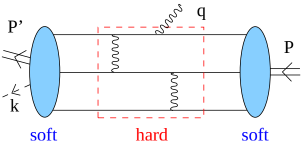
$$m_N^2 - (P-k)^2 \quad = \quad \frac{m_\pi}{m_N}\left[Q^2 + 2m_N^2\right]$$

⇒ **phenomenological Lagrangians to take into account nucleon resonances**
⇒ **or go over to quark-gluon description**

$Q^2 \gg m_N^3/m_\pi$: Perturbative QCD

QCD factorization for $Q^2 \gg \Lambda^3_{\rm QCD}/m_\pi$

Pobylitsa, Polyakov, Strikman, PRL87(2001)022001:



$$|p\uparrow\rangle = \frac{\phi_s(x)}{\sqrt{6}}|2u_\uparrow d_\downarrow u_\uparrow - u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle + \frac{\phi_a(x)}{\sqrt{2}}|u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle$$

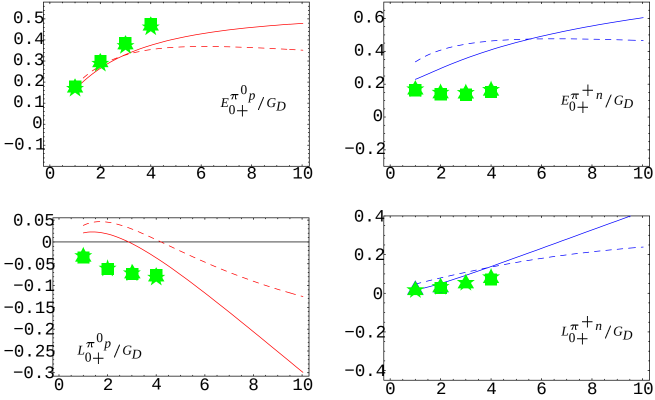
$$|p\uparrow \pi^0\rangle = \frac{\phi_s(x)}{2\sqrt{6}f_\pi}|6u_\uparrow d_\downarrow u_\uparrow + u_\uparrow u_\downarrow d_\uparrow + d_\uparrow u_\downarrow u_\uparrow\rangle - \frac{\phi_a(x)}{2\sqrt{2}f_\pi}|u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle$$

$$|n\uparrow \pi^+\rangle = \frac{\phi_s(x)}{\sqrt{12}f_\pi}|2u_\uparrow d_\downarrow u_\uparrow - 3u_\uparrow u_\downarrow d_\uparrow - 3d_\uparrow u_\downarrow u_\uparrow\rangle - \frac{\phi_a(x)}{2f_\pi}|u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle$$

- **Only for $G_1^{\pi N}$ (E_{0+})**
- **Probably unrealistic at reachable momentum transfers**

$Q^2 \sim m_N^3/m_\pi$: Light–Cone Sum Rules: $Q^2 \sim 1 - 10$ GeV^2

◇ **normalized to the dipole formula** $G_D = 1/(1 + Q^2/0.71)^2$



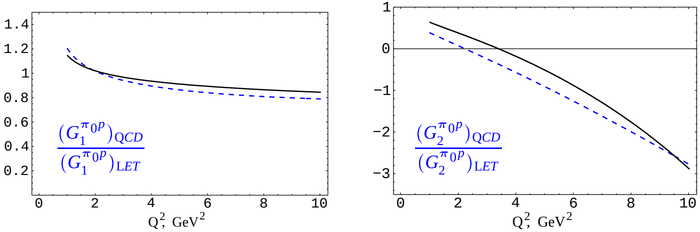
green dots: **MAID;** ■: $W = 1074$ **MeV,** ▲: $W = 1084$ **MeV,** ★: $W = 1094$ MeV

solid curves: **LCSRs using experimental elastic EM formfactors as input**

dashed curves: **pure LCSRs, no experimental input**

Light Cone Sum Rules –*continued*

Deviation from LET:

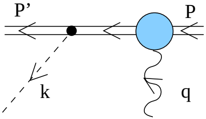


- **Reproduce LET for $Q^2 \sim 1$ GeV^2**
- **Reproduce pQCD for $Q^2 \to \infty$ (part of the NNLO α_s^2 contribution)**
- **No double counting of “soft” and “hard” contributions**
- **Tested: Electromagnetic and axial form factors, heavy meson decays, pion form factors**

Moving away from threshold

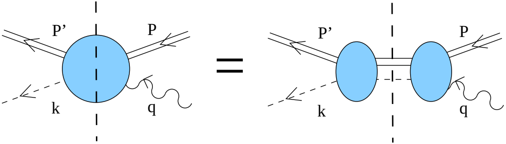
◇ **Higher partial waves**

- P-wave dominated by pion emission from the final state

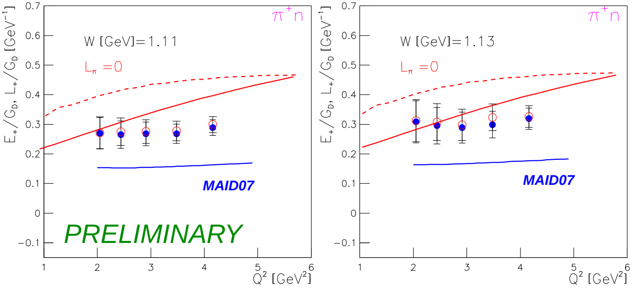


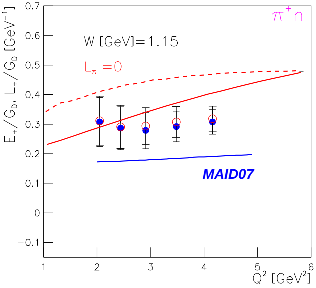
◇ **Energy dependence $E_{0+}(W)$, $L_{0+}(W)$, etc.**

- due to final state interactions



CLAS (preliminary) $ep \to e\pi^+ n$





Blue circle = assumption of $G_E^n = 0$

Red cicle = $G_E^n(Q^2) = -a\mu_n\tau G_D(Q^2)/(1+b\tau)$

with assumption $\mathrm{Im}(G_2^{n\pi+}) = 0$

Stat. Err. ONLY in data

Red Solid line =
LCSR using experimental
EM form factors as input

Red Dash line = pure LCSR

Kijun Park; DNP 2008
Oakland, CA (Oct. 23-26)

CLAS collaboration meeting,
June 11-13,2009

Summary

◇ **Use pion as a handle to "rotate" the nucleon wave function**

- **a novel object: generalized form factor; an overlap between usual and rotated WF**
- **check Low Energy Theorems (Nambu, ...) and transition to QCD**
- **new scale in QCD: $Q^2 \sim m_N^3/m_\pi$**
- **measure nucleon axial form factor (requires π^+)**
- **theory progress feasible, large community (ChPT, pQCD, PWA)**
- **an (almost) untouched terrain...**

! **no data at $Q^2 \sim 0.1 - 1$ GeV^2 MAMI?**

Perfectly suited for the JLab 12 GeV upgrade physics program

Supplementary Material

$Q^2 \sim 1 - 10$ GeV^2: Light Cone Sum Rules

Balitsky, V.B., Kolesnichenko '86–'88

- **consider**

$$T_\nu^{\pi N}(P', q) \quad = \quad i \int d^4x\, e^{-iqx} \langle N(P')\pi^a(k)|T\{j_\nu^{\rm em}(x)\bar\eta_p(0)\}|0\rangle$$

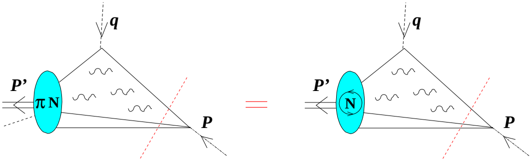
$$\eta_p(x) \quad = \quad \epsilon^{ijk}\left[u^i(x)C\gamma_\mu u^j(x)\right]\,\gamma_5\gamma^\mu d^k(x)\,, \qquad \langle 0|\eta_p|N(P)\rangle = \lambda_p m_N N(P)$$

- take $P = P' + q - k$, $P^2 \sim -1$ **GeV**^{2} and make a matching between

(a) **The Operator Product Expansion in terms of pion-nucleon DAs**

$$\langle N(P')\pi^a(k)|T\{j_\nu^{\rm em}(x)\bar\eta_p(0)\}|0\rangle \quad = \quad \sum_{\rm twist} H_\nu(x^2, px) \otimes \langle 0|q(x_1)q(x_2)q(x_3)|N(P')\pi^a(k)\rangle^\dagger$$

(b) **The dispersion representation in terms of hadronic states**



◇ **Borel transformation to improve convergence**

Light Cone Sum Rules – *continued*

Good things:

- ♡ Reproduce LET for $Q^2 \sim 1$ GeV^2
- ♡ Reproduce pQCD for $Q^2 \to \infty$ (part of the NNLO α_s^2 contribution)
- ♡ No double counting of “soft” and “hard” contributions
- ♡ Tested: Electromagnetic and axial form factors, heavy meson decays, pion form factors

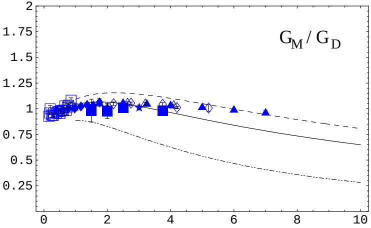
Not–so–good things:

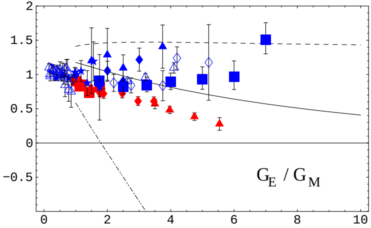
- ♢ Use nucleon distribution amplitudes as input — not so well known
- ♢ Calculation rather demanding, especially in NLO

Bad things:

- ♠ Approximation for the continuum contribution not improvable — irreducible error of order 20% for all Q^2

Light–Cone Sum Rules: Nucleon Electromagnetic form factors





choice of nucleon DAs:

solid:	**BLW model**
long dashes:	**asymptotic**
short dashes:	**CZ model**

Braun, Lenz, Wittmann; PRD73(2006)094019

A. Lenz; arXiv:0708.0633v1

Light–Cone Sum Rules: Pion–Nucleon Intermediate States

- **New: Semidisconnected pion-nucleon contributions in the intermediate state**

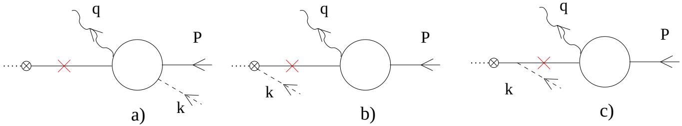


Figure: Schematic structure of the pole terms in the correlation function

b) and c) correspond to πN coupling to the Ioffe current

$$\langle 0|\, \eta_p(0)\, |N(P'-k)\pi(k)\rangle = \frac{i\lambda_1^p m_N}{2f_\pi}\left[1 - \frac{g_A}{P'^2 - m_N^2}(\not{P}' - \not{k} + m_N)\,\not{k}\right]\gamma_5 N(P'-k)\,.$$

In the threshold kinematics, with $\delta = m_\pi/m_N$

$$\begin{aligned} T_\nu^{\pi^0 p}(P,q) = \frac{i\lambda_1^p m_N}{f_\pi}\Bigg\{ & \frac{((1+\delta)\not{P} - \not{q} + m_N)\gamma_5}{m_N^2 - P'^2}\left[(\gamma_\nu q^2 - q_\nu \not{q})\frac{G_1^{\pi^0 p}}{m_N^2} - \frac{i\sigma_{\nu\mu}q^\mu}{2m_N}G_2^{\pi^0 p}\right] \\ & + \frac{1}{2}\frac{(1+\delta)\gamma_5(\not{P} - \not{q} + m_N)}{[m_N^2(1+\delta)^2 + \delta Q^2] - P'^2}\left[\gamma_\nu F_1^p - \frac{i\sigma_{\nu\mu}q^\mu}{2m_N}F_2^p\right] \\ & - \frac{(1+\delta)g_A(\not{P} - \not{q} + m_N)\gamma_5}{[m_N^2(1+\delta)^2 + \delta Q^2] - P'^2}\left[(\gamma_\nu q^2 - q_\nu \not{q})G_M^p - \frac{i\sigma_{\nu\mu}q^\mu}{2m_N}4m_N^2 G_E^p\right]\Bigg\} N(P) \end{aligned}$$

Light–Cone Sum Rules: Pion–Nucleon Intermediate States — *cont.*

- **The semidisconnected πN contributions can be included in the continuum if**

$$m_\pi Q^2 > m_N(s_0 - m_N^2) \quad \Rightarrow \quad Q^2 > 7\ \text{GeV}^2 \qquad [\sim \Lambda^3_{\rm QCD}/m_\pi]$$

- **Otherwise they have to be taken into account explicitly**

$$\frac{Q^2}{m_N^2} G_1^{\pi^0 p} = \frac{e^{m_N^2/M^2}}{2\lambda_1^p} \mathbb{B}_{P'2}[\mathcal{A}^{\pi^0 p}](M^2,Q^2) - \frac{1}{2} e^{-\delta(2m_N^2+Q^2)/M^2}\left[F_1^p(Q^2) - \frac{g_A Q^2}{Q^2+2m_N^2} G_M^p(Q^2)\right]$$

$$G_2^{\pi^0 p} = -\frac{e^{m_N^2/M^2}}{\lambda_1^p} \mathbb{B}_{P'2}[\mathcal{B}^{\pi^0 p}](M^2,Q^2) + e^{-\delta(2m_N^2+Q^2)/M^2}\left[\frac{1}{2}F_2^p(Q^2) + \frac{2g_A m_N^2}{Q^2+2m_N^2} G_E^p(Q^2)\right]$$

$$\frac{Q^2}{m_N^2} G_1^{\pi^+ n} = \frac{e^{m_N^2/M^2}}{2\lambda_1^p} \mathbb{B}_{P'2}[\mathcal{A}^{\pi^+ n}](M^2,Q^2) - \frac{1}{\sqrt{2}} e^{-\delta(2m_N^2+Q^2)/M^2}\left[F_1^n(Q^2) - \frac{g_A Q^2}{Q^2+2m_N^2} G_M^n(Q^2)\right]$$

$$G_2^{\pi^+ n} = -\frac{e^{m_N^2/M^2}}{\lambda_1^p} \mathbb{B}_{P'2}[\mathcal{B}^{\pi^+ n}](M^2,Q^2) + e^{-\delta(2m_N^2+Q^2)/M^2}\left[\frac{1}{\sqrt{2}}F_2^n(Q^2) + \frac{2\sqrt{2}g_A m_N^2}{Q^2+2m_N^2} G_E^n(Q^2)\right]$$

where $\mathcal{A}(P'^2,Q^2)$ and $\mathcal{A}(P'^2,Q^2)$ are the invariant functions defined as

$$z^\nu \Lambda^+ T_\nu^{\pi N}(P,q) = \frac{i}{f_\pi}(pz+kz)\gamma_5 \left\{m_N \mathcal{A}(P'^2,Q^2) + \not{q}_\perp \mathcal{B}(P'^2,Q^2)\right\} N^+(P)$$

πN scattering phases

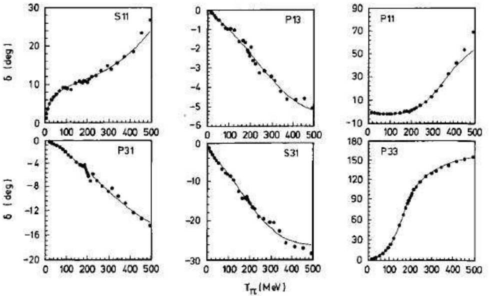


Figure from Nozawa *et al.*, PRC41(1990)213

Including P-waves:

for $W - W_{\rm th} \ll m_\pi$ **accept**

$$\langle N(P')\pi(k)|j_\mu^{em}(0)|p(P)\rangle =$$
$$= \quad -\frac{i}{f_\pi}\bar N(P')\gamma_5\left\{\left(\gamma_\mu q^2 - q_\mu \not{q}\right)\frac{1}{m_N^2}G_1^{\pi N}(Q^2) - \frac{i\sigma_{\mu\nu}q^\nu}{2m_N}G_2^{\pi N}(Q^2)\right\}N(P)$$
$$+\frac{ic_\pi g_A}{2f_\pi\left[(P'+k)^2)-m_N^2\right]}\bar N(P')\not{k}\gamma_5(\not{P}'+m_N)\left\{F_1^p(Q^2)\left(\gamma_\mu - \frac{q_\mu \not{q}}{q^2}\right)+\frac{i\sigma_{\mu\nu}q^\nu}{2m_N}F_2^p(Q^2)\right\}N(P)$$

- **S–wave: generalized form factors from LCSR**
- **P–wave: pion emission from the final state nucleon; exact in chiral limit**
- **Eventually can take into account the final state interactions**

$$G_1^{\pi N}(Q^2) \to G_1^{\pi N}(Q^2, W) \equiv G_1^{\pi N}(Q^2)[1 + i\, t_{\pi N}]$$

Structure Functions at $x_B \to 1$

$$F_1(W,Q^2) = \frac{\beta(W)}{(4\pi f_\pi)^2}\sum_{\pi^0,\pi^+}\left\{\frac{Q^2+4m_N^2}{2m_N^4}|Q^2 G_1^{\pi N}|^2 + \frac{c_\pi^2 g_A^2 W^2\beta^2(W)}{8(W^2-m_N^2)^2}Q^2 m_N^2 G_M^2\right\}$$

$$F_2(W,Q^2) = \frac{\beta(W)}{(4\pi f_\pi)^2}\sum_{\pi^0,\pi^+}\left\{\frac{Q^2}{m_N^4}\left(|Q^2 G_1^{\pi N}|^2 + \frac{m_N^2}{4}Q^2|G_2^{\pi N}|^2\right) + \frac{c_\pi^2 g_A^2 W^2\beta^2(W)Q^2 m_N^2}{4(W^2-m_N^2)^2}\left(\frac{Q^2G_M^2+4m_N^2G_E^2}{Q^2+4m_N^2}\right)\right\}$$

$$g_1(W,Q^2) = \frac{\beta(W)}{(4\pi f_\pi)^2}\sum_{\pi^0,\pi^+}\left\{\frac{Q^2}{2m_N^4}\left[|Q^2 G_1^{\pi N}|^2 - m_N^2 \mathrm{Re}(Q^2 G_1^{\pi N} G_2^{*,\pi N})\right] + \frac{c_\pi^2 g_A^2 W^2\beta^2(W)}{8(W^2-m_N^2)^2}Q^2 m_N^2 G_M F_1^p\right\}$$

$$g_2(W,Q^2) = -\frac{\beta(W)}{(4\pi f_\pi)^2}\sum_{\pi^0,\pi^+}\left\{\frac{Q^2}{2m_N^4}\left[|Q^2 G_1^{\pi N}|^2 + \frac{1}{4}Q^2 \mathrm{Re}(Q^2 G_1^{\pi N} G_2^{*,\pi N})\right] + \frac{c_\pi^2 g_A^2 W^2\beta^2(W)}{32(W^2-m_N^2)^2}Q^4 G_M F_2^p\right\}$$

$$\beta(W) = \frac{|\vec{k}_f|}{W}\,, \qquad\qquad x_B = \frac{Q^2}{Q^2+W^2-m_N^2}$$

SLAC E136

P. E. Bosted *et al.*; PRD49(1994)3091

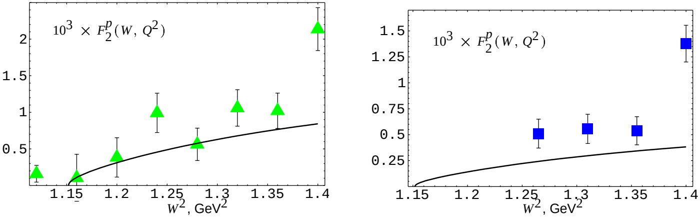


Figure: The structure function $F_2^p(W, Q^2)$ as a function of W^2 scaled by a factor 10^3 compared to the SLAC E136 data at the average value $Q^2 = 7.14$ GeV^2 (left panel) and $Q^2 = 9.43$ GeV^2 (right panel).

Differential Cross Section

For unpolarized protons, the virtual photon cross section is

$$d\sigma_{\gamma*} = \frac{\alpha_{\rm em}}{8\pi}\frac{k_f}{W}\frac{d\Omega_\pi}{W^2 - m_N^2}|\mathcal{M}_{\gamma*}|^2$$

with

$$|\mathcal{M}_{\gamma*}|^2 = M_T + \epsilon\, M_L + \sqrt{2\epsilon(1+\epsilon)}\, M_{LT}\cos(\phi_\pi) + \epsilon M_{TT}\cos(2\phi_\pi) + \lambda\sqrt{2\epsilon(1-\epsilon)}\, M'_{LT}\sin(\phi_\pi)$$

$$\begin{aligned}
f_\pi^2 M_T &= \frac{4\vec{k}_i^2 Q^2}{m_N^2}|G_1^{\pi N}|^2 + \frac{c_\pi^2 g_A^2 \vec{k}_f^2}{(W^2 - m_N^2)^2} Q^2 m_N^2 G_M^2 + \cos\theta \frac{c_\pi g_A |k_i||k_f|}{W^2 - m_N^2} 4Q^2 G_M \mathrm{Re}\, G_1^{\pi N} \\
f_\pi^2 M_L &= \vec{k}_i^2 |G_2^{\pi N}|^2 + \frac{4c_\pi^2 g_A^2 \vec{k}_f^2}{(W^2 - m_N^2)^2} m_N^4 G_E^2 - \cos\theta \frac{c_\pi g_A |k_i||k_f|}{W^2 - m_N^2} 4m_N^2 G_E \mathrm{Re}\, G_2^{\pi N} \\
f_\pi^2 M_{LT} &= -\sin\theta \frac{c_\pi g_A |k_i||k_f|}{W^2 - m_N^2} Q m_N \left[G_M \mathrm{Re}\, G_2^{\pi N} + 4G_E \mathrm{Re}\, G_1^{\pi N}\right] \\
f_\pi^2 M_{TT} &= 0\,, \\
f_\pi^2 M'_{LT} &= -\sin\theta \frac{c_\pi g_A |k_i||k_f|}{W^2 - m_N^2} Q m_N \left[G_M \mathrm{Im}\, G_2^{\pi N} - 4G_E \mathrm{Im}\, G_1^{\pi N}\right]
\end{aligned}$$

- ◇ $M_{TT} = 0$: **no D–wave; tests quality of the approximation**
- ◇ M'_{LT}: **single–spin asymmetry; arises because of FSI, calculable**

Miscellaneous Results

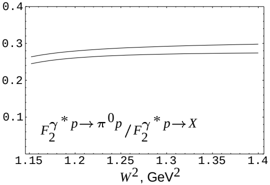


Figure: **The fraction of $\pi^0 p$ in $F_2^p(W, Q^2)$ for $Q^2 = 3$ GeV^2 (upper curve) and $Q^2 = 9$ GeV^2 (lower curve)**

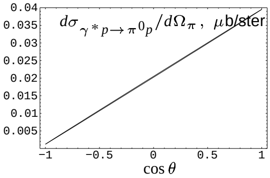


Figure: **Differential cross section $d\sigma_{\gamma^* p \to \pi^0 p}/d\Omega_\pi$ for $\phi_\pi = 135$ grad, $Q^2 = 4.2$ GeV^2 and $W = 1.11$ GeV**

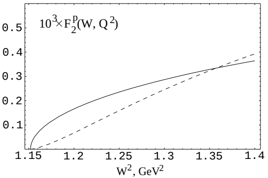


Figure: **S-wave (solid) vs. P-wave (dashed) for $F_2^p(W, Q^2)$ at $Q^2 = 7.14$ GeV^2**

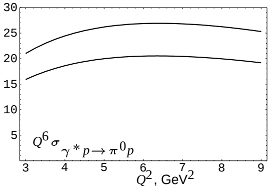


Figure: **Integrated cross section $Q^6 \sigma_{\gamma^* p \to \pi^0 p}$ for $W = 1.11$ GeV (lower curve) and $W = 1.15$ GeV (upper curve)**