$\pi\pi$–*scattering length measurement from K -> 3π decays*

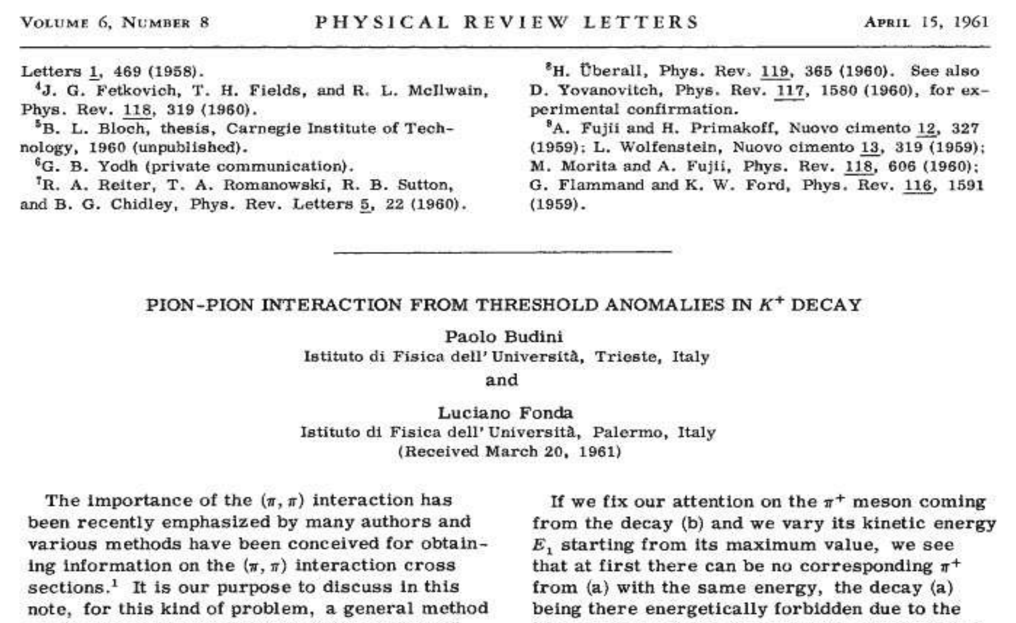
Sergio Giudici

University of Pisa and INFN

member of NA48 collaboration

Chiral Dynamics 2009
University of Bern, July 6-10 2009

Budini–Fonda prophecy (1961)



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Letters 1, 469 (1958).

[4]J. G. Fetkovich, T. H. Fields, and R. L. McIlwain, Phys. Rev. 118, 319 (1960).

[5]B. L. Bloch, thesis, Carnegie Institute of Technology, 1960 (unpublished).

[6]G. B. Yodh (private communication).

[7]R. A. Reiter, T. A. Romanowski, R. B. Sutton, and B. G. Chidley, Phys. Rev. Letters 5, 22 (1960).

[8]H. Überall, Phys. Rev. 119, 365 (1960). See also D. Yovanovitch, Phys. Rev. 117, 1580 (1960), for experimental confirmation.

[9]A. Fujii and H. Primakoff, Nuovo cimento 12, 327 (1959); L. Wolfenstein, Nuovo cimento 13, 319 (1959); M. Morita and A. Fujii, Phys. Rev. 118, 606 (1960); G. Flammand and K. W. Ford, Phys. Rev. 116, 1591 (1959).

PION-PION INTERACTION FROM THRESHOLD ANOMALIES IN K^+ DECAY

Paolo Budini
Istituto di Fisica dell' Università, Trieste, Italy
and
Luciano Fonda
Istituto di Fisica dell' Università, Palermo, Italy
(Received March 20, 1961)

The importance of the (π, π) interaction has been recently emphasized by many authors and various methods have been conceived for obtaining information on the (π, π) interaction cross sections.[1] It is our purpose to discuss in this note, for this kind of problem, a general method which allows in principle the determination of

If we fix our attention on the π^+ meson coming from the decay (b) and we vary its kinetic energy E_1 starting from its maximum value, we see that at first there can be no corresponding π^+ from (a) with the same energy, the decay (a) being there energetically forbidden due to the heavier mass of the charged with respect to the

“ A general method which allows in principle the determination of certain cross sections which are out of reach of direct experimental measurement [...]”

“ A threshold effect of the kind of a cusp will be observed in the spectrum [...] “

“ This argument can be reversed to give a qualitative estimate of $\sigma\ (\pi^+\pi^- \longrightarrow \pi^0\pi^0)$ once the threshold effect is found experimentally [...] ”

“The applicability of the method is naturally bound to the statistics available and to the energy resolution attainable [...] “

Thanks to who re-discovered this article

~ 40 years later.. NA48 experiment

1997 -2001: study of direct CP violation in K --> 2π
measure of Re(ε'/ε)
2003-2004 : Study of CP violation in K –> 3π and
Ke4 to measure ππ phase shift
(see Brigitte Bloch-Devaux talk in this conference)
2007 : study of lepton universality Ke2/Kμ2 Branching ratio

Final state photons ($\pi^0 \to \gamma\gamma$) are detected by an excellent Liquid Krypton Calorimeter with very good performances

$$\frac{\sigma(E)}{E} = \frac{0.09}{E_{[GeV]}} + \frac{0.032}{\sqrt{(E_{[GeV]})}} + 0.0042$$

$1 < E < 100$ GeV

IMPORTANT: in order to limit systematics error a frequently monitoring of Energy scale, Calibration and Resolution is required

How do we do that ?

Tools for Calorimeter Calibration

We use mesons of known mass, small lifetime, produced in a known position to calibrate our calorimeter

π^0--> $\gamma\gamma$, η--> $\gamma\gamma$ cross checked with η--> $3\pi^0$

as a by product of the method we measured

m_η = 547.843 ± 0.051MeV (imy PHD thesis work 10 years ago)

Only two calibration points: 0.135 GeV and 0.548 GeV

.. it would have been nice to have something in the middle ...

In 2003 while the experiment was collecting $K^\pm$ --> $\pi^\pm \pi^0\pi^0$
I. Mannelli suggested to use pionium as a third calibration point

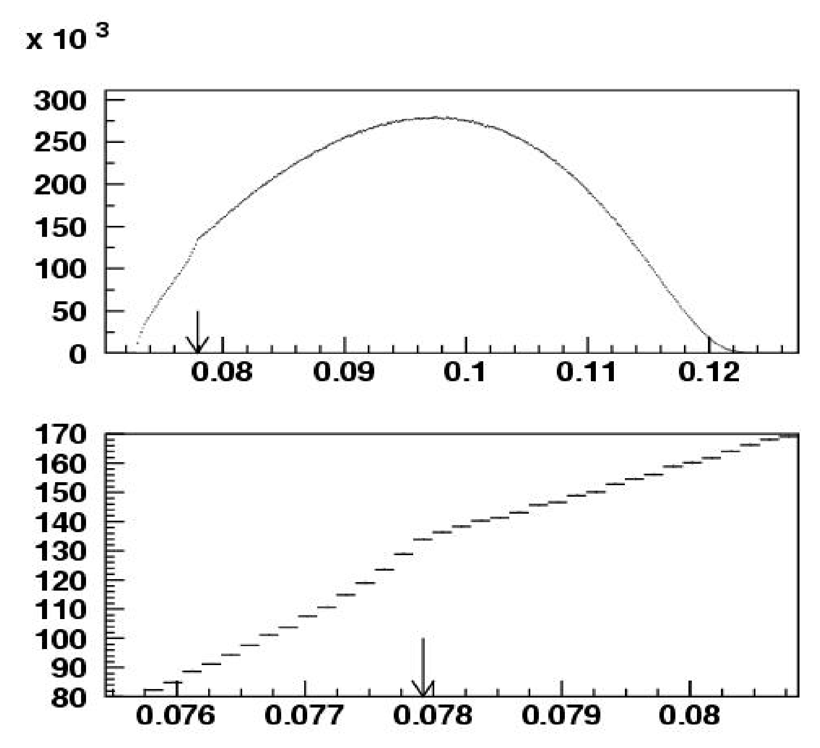
An excess O(1000) events due to pionium formation was expected on top of the natural $m_{\pi\pi}$ distribution. Estimation based on

$$\frac{\Gamma(K^+\rightarrow\pi^+ A_{2\pi})}{\Gamma(K^+\rightarrow\pi^+\pi^+\pi^-)} \simeq 10^{-5}$$

(Z.K.Silagadze , JETP Lett.60:689-693, 1994)

pionium mass = 0.278 Gev --> opportunity for energy calibration check
experimental pionium width --> opportunity for resolution check

experimental $m_{\pi\pi}$ distribution



$(m_{\pi\pi})^2$ distribution from a sample of ~60 M $K^{\pm} \dashrightarrow \pi^{\pm}\pi^0\pi^0$ events

Puzzling ! a small pionium peak was expected and a cusp is found !!!

What is this ?

First answer came from Nicola Cabibbo

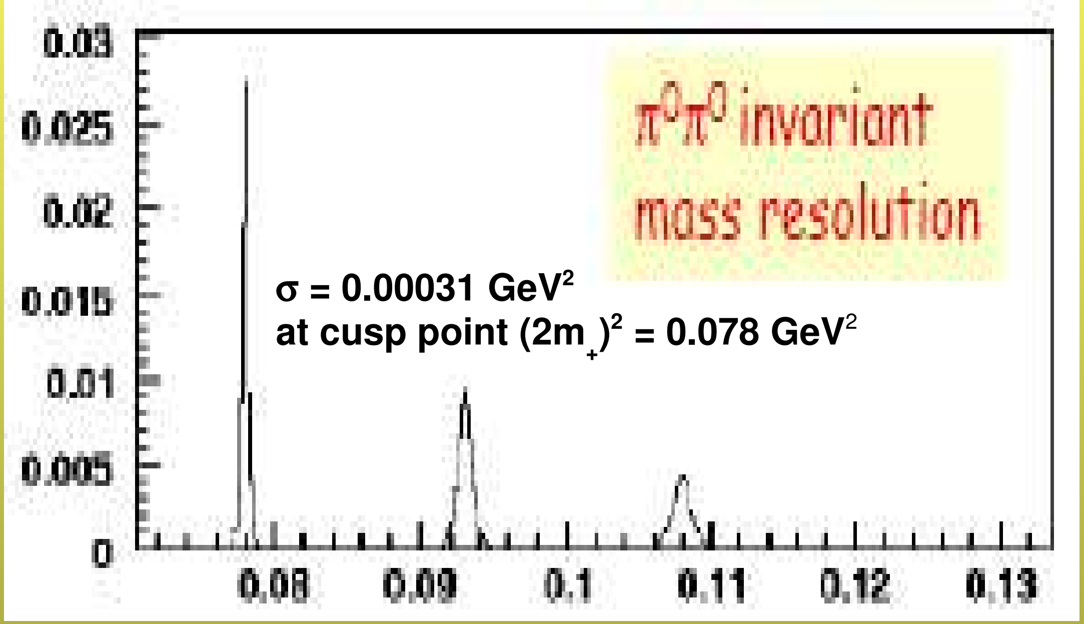
The cusp is due to $\pi\pi$ charge exchange final state rescattering and from the sudden change of the derivative one can measure the pion scattering length

N. Cabibbo , PRL 93 , 121801, (2004)

However the cusp is located at $(2m_{\pi})^2 = 0.078$ GeV ^{2} suggesting that we were facing a real physics process ... not an instrumental effect!

None of us knew Budini Fonda's prophecy, Nicola as well - I believe !!!!

experimental $m_{\pi\pi}$ resolution



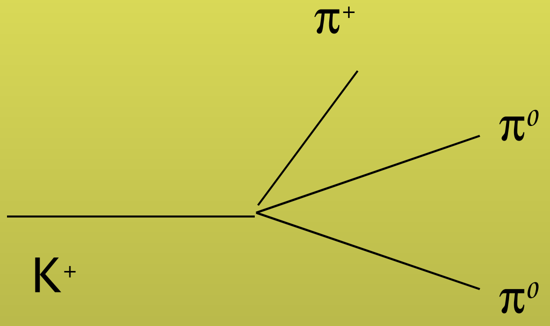
σ = 0.00031 GeV^2
at cusp point $(2m_+)^2$ = 0.078 GeV^2

LKR calorimeter performances , small Q-value and pi0s mass constraint allow to reach a very good resolution

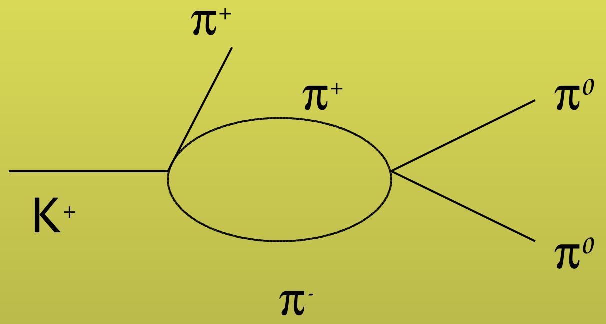
N. Cabibbo - G. Isidori approach

N. Cabibbo , PRL 93 , 121801, (2004) 1-loop calculation

N.Cabibbo and G. Isidori, JHEP 03, 021, (2005) 2-loops calculation

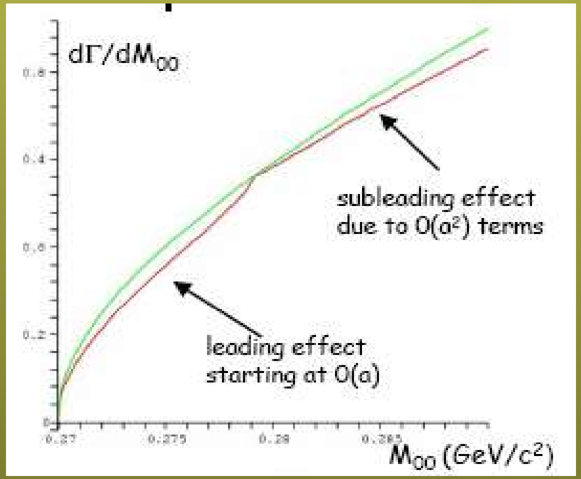


tree level direct emission



charge exchange loop

$M_0 = A_0 (1 + g_0\, u/2 + h' u^2/2 + k' v^2/2)$



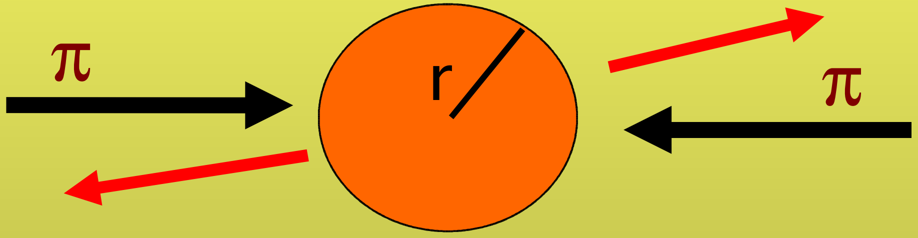
$$M_1 = i\,(2/3)\, m_+ (a_0 - a_2)(1 + \varepsilon/3) M_+ \sqrt{1 - \left(\frac{2m_+}{M_{00}}\right)^2}$$

$$\epsilon = \frac{(m_+^2 - m_0^2)}{m_+^2} = 0.065$$

M.Knecht and R.Urech, NPH. B519:329-360,1998

M_0 and M_1 interfere destructively below $2m_+$ threshold while add quadratically above

ππ scattering length for pedestrians



$$k = \frac{\sqrt{2mE}}{\hbar}$$

At low energy $kr << 1$ S-wave dominates total cross section
Isospin $I = 0,2$ constrained by Bose statistics

Scattering matrix $S|\pi\pi\rangle = \exp(2i\delta)|\pi\pi\rangle$
may be parametrized with 2 phases

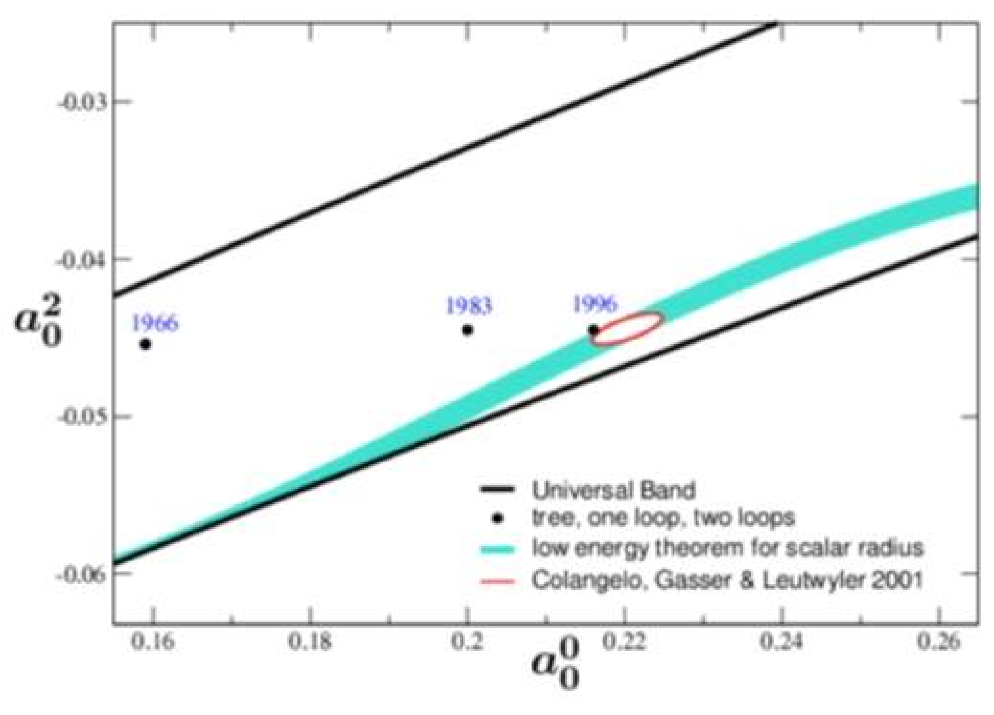
$$\delta_{0,2} = a_{0,2}k + O(k^{**}2)$$

a_0 , a_2 are called scattering lengths

At low energy the S-wave scattering lengths are essential

parameters of *Chiral Pertubation Theory* (CHPT)

a_0 and a_2 theoretical predictions



$$a_0 m_\pi = \frac{7 M_\pi^2}{32 \pi F_\pi^2} = 0.16$$

$$a_2 m_\pi = \frac{-M_\pi^2}{16 \pi F_\pi^2} = -0.045$$

S. Weinberg, PRL 17 (1966) 216

a0 = 0.220 +- 0.005
a2 = -0.0444 +- 0.0010
a0-a2 = 0.265 +- 0.004

Colangelo, Gasser, Leutwyler, PRL 86, 5008, (2001)

Analyticity and chiral symmetry predicts the constraint

$$a_2 = (-0.0444 \pm 0.0008) + 0.236 (a_0 - 0.22) - 0.61 (a_0 - 0.22)^2 - 9.9 (a_0 - 0.22)^3$$

Bern – Bonn approach

G. Colangelo, J. Gasser, B. Kubis, A. Rusetsky, PHL B638, 187, (2006)

Model based on a non-relativistic field theory framework
using two expansion parameter

a = generic $\pi\pi$ scattering length at threshold
ε = a formal parameter such that
pion momentum is of order $O(\varepsilon)$
pion kinetic energy is of order $O(\varepsilon^2)$

The present formulation includes terms up to $O(\varepsilon^4, a\varepsilon^3, a^2\varepsilon^2)$
corresponding to 1-loop and 2-loops calculation.
Valid over the full physical region

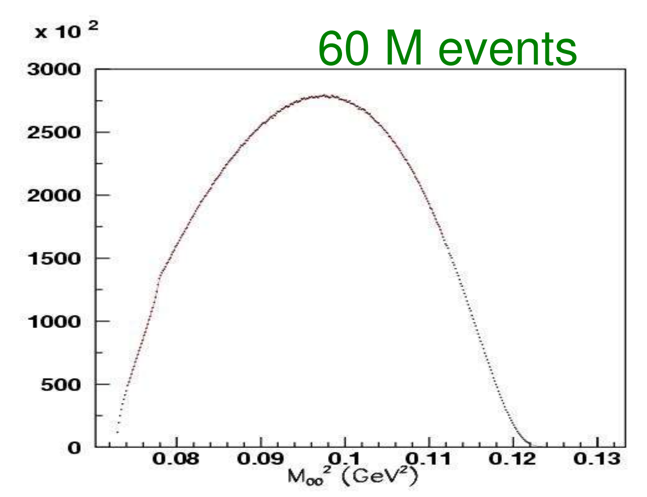
Bern-Bonn calculation is used to fit simultaneously
m_{00} distribution from $K^\pm \dashrightarrow \pi^\pm \pi^0\pi^0$ and
m_{++} distribution from $K^\pm \dashrightarrow \pi^\pm \pi^+\pi^-$

The Bern-Bonn group calculated radiative correction outside the cusp point

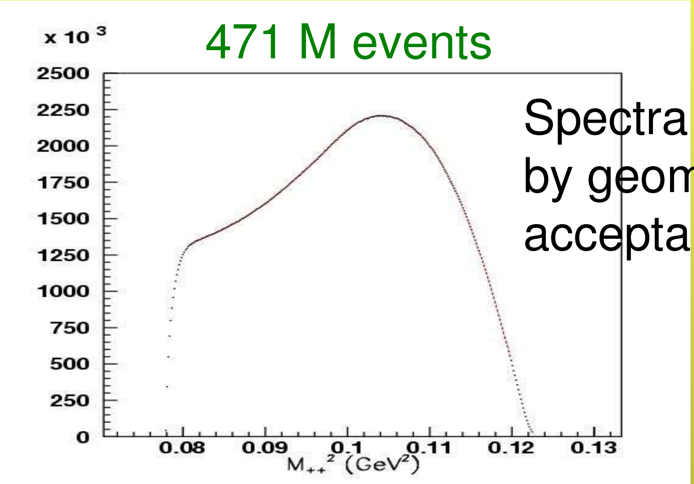
M. Bissegger, A. Fuhrer, J.Gasser, B. Kubis, A. Rusetsky NPH B806, 178 (2009)

So far B.B. approach provides the most complete description of rescattering effect

Fitting experimental data



m_{00} distribution



Spectra distorted by geometrical acceptance effects

m_{++} distribution

fitting region $0.074 < m_{00}^2 < 0.104 \quad GeV^2$ $\quad 0.081 < m_{++}^2 < 0.120 \quad GeV^2$

Small acceptance bins excluded to limit sensitivity on MC

Free parameters : g_0 , h_0 , g, h, k weak amplitude

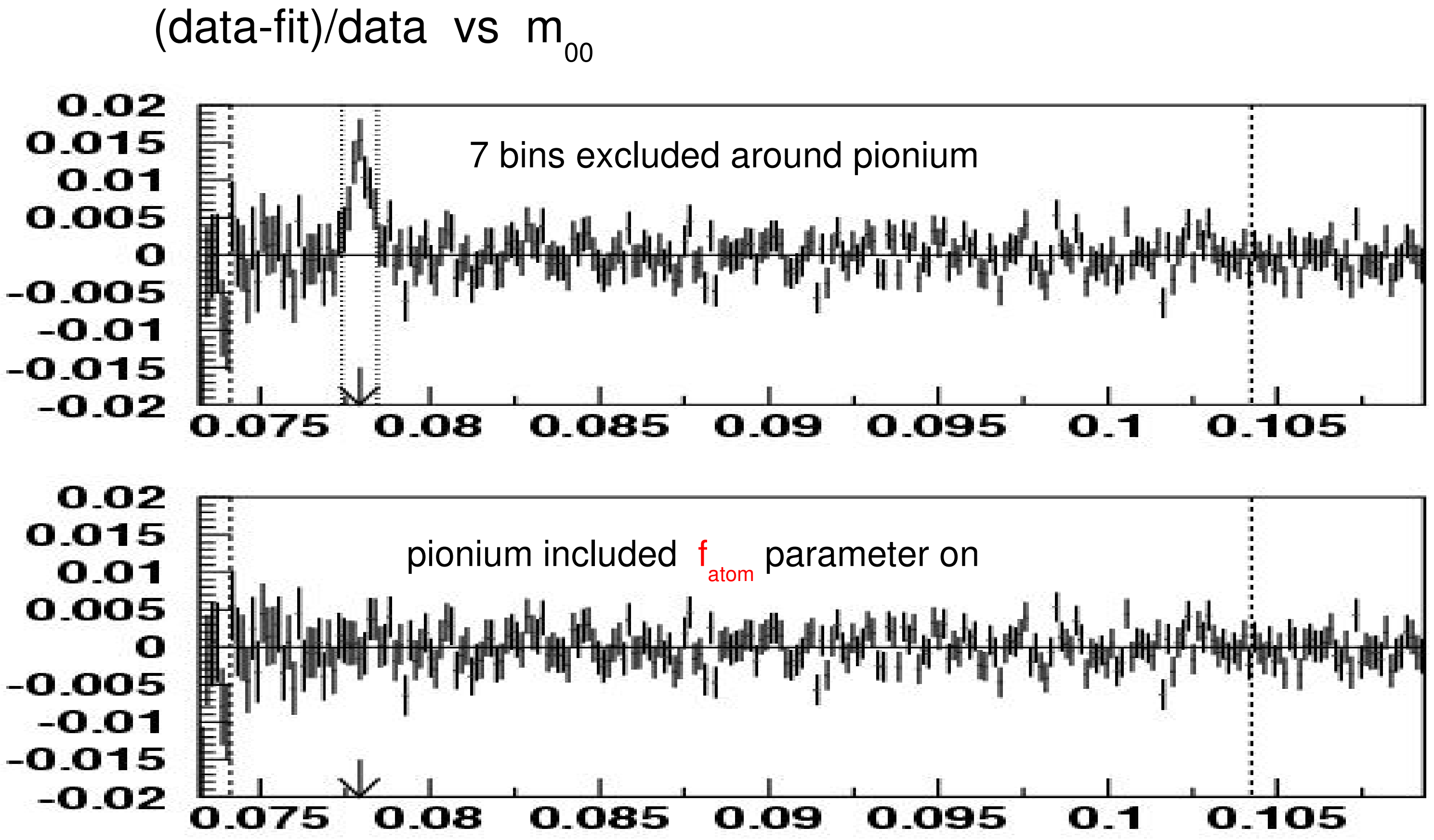
(a_0-a_2) , a_2 pion rescattering

f_{atom} amount of pionium in bin 51 (Threshold)

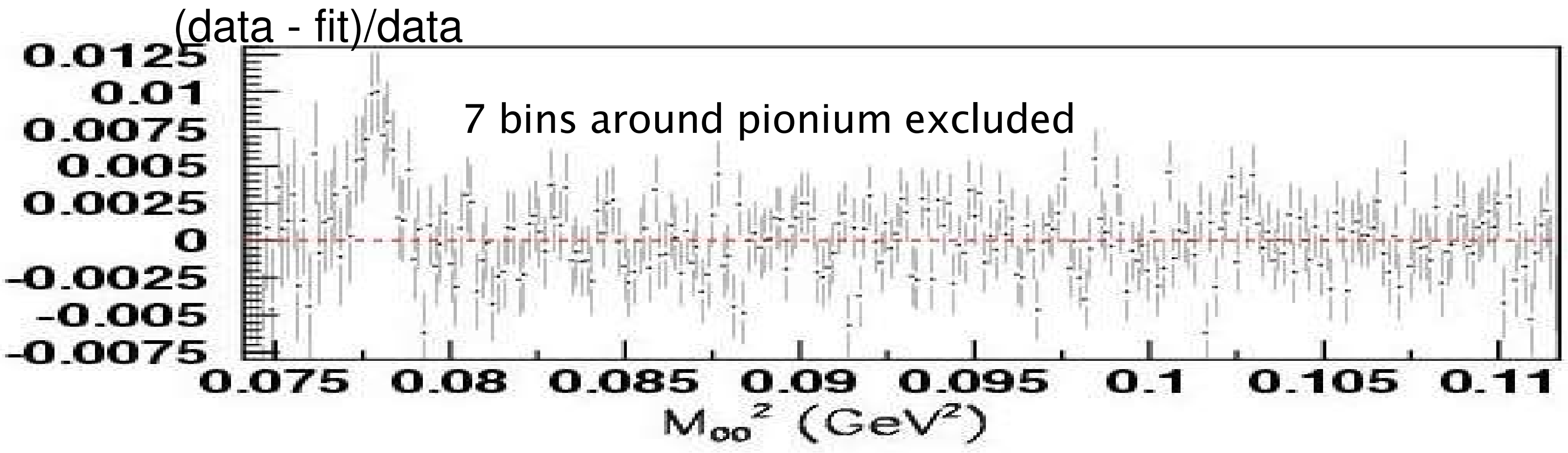
External Parameters : $\Gamma_{+-}/\Gamma_{00} = 3.175 \pm 0.050$ is taken from PDG

Relative detection efficiency are not know with sufficient precision

Cabibbo–Isidori fitting

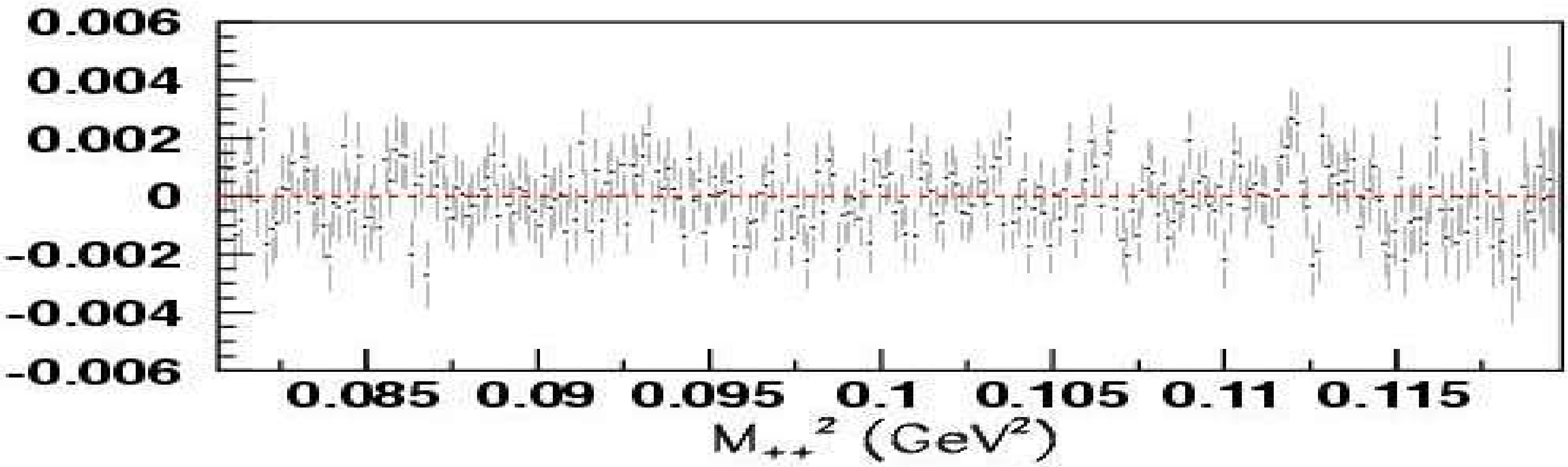

(data-fit)/data vs m_{00}

Bern Bonn residuals



(data - fit)/data

7 bins around pionium excluded



Fitting results

fit	χ^2/ndf	a_0-a_2	a_2	f_{atom}
CI	206.3/195	0.2727(46)	-0.0392(80)	0.0533(91)
CI (a)	201.6/189	0.2689(50)	-0.0344(86)	0.0533
CI (c)	210.6/196	0.2749(21)	-0.0413	0.0441(76)
CI (a,c)	207.6/190	0.2741(21)	-0.0415	0.0441
BB	462.9/452	0.2815(43)	-0.0693(136)	0.0530(95)
BB (a)	458.5/446	0.2775(48)	-0.0593(142)	0.0542
BB (c)	467.3/453	0.2737(26)	-0.0417	0.0647(76)
BB (a,c)	459.8/447	0.2722(27)	-0.0421	0.0647

RAD. CORR. OFF

(a) pionium fixed
(c) CHPT constraint ON
(a,c) both ...

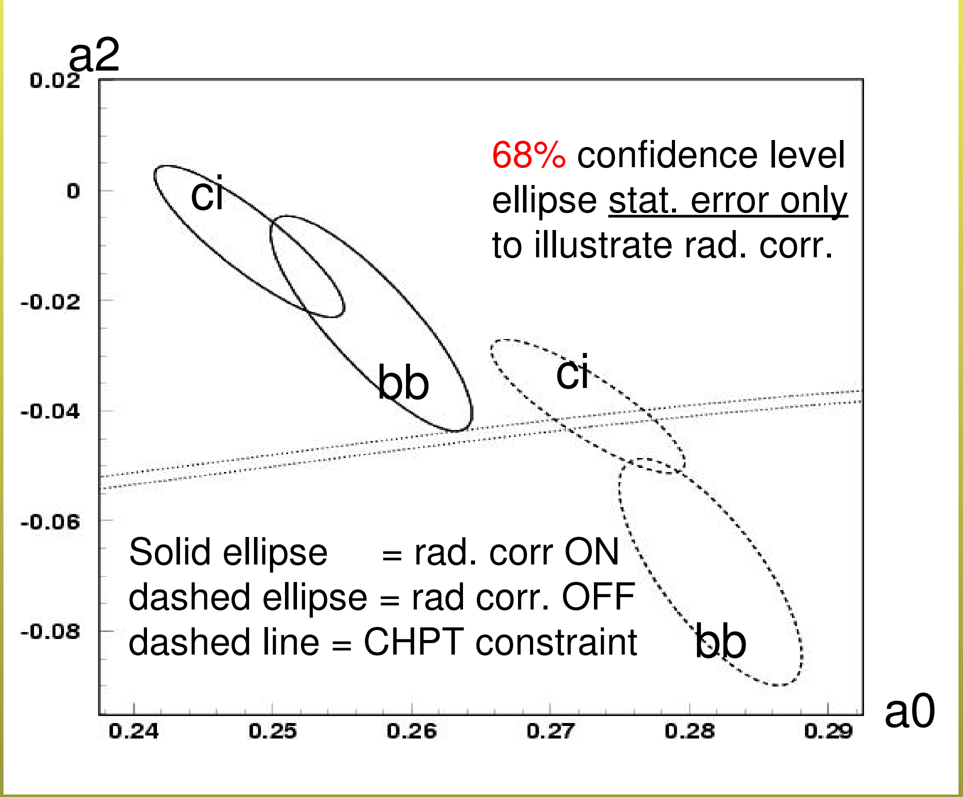
(..) stat. error
no (..) = fixed par.

CI	205.6/195	0.2483(45)	-0.0092(91)	0.0625(92)
CI (a)	202.9/189	0.2461(49)	-0.0061(98)	0.0625
CI (c)	222.1/196	0.2646(21)	-0.0443	0.0420(77)
CI (a,c)	219.7/190	0.2645(22)	-0.0444	0.0420
BB	**477.4/452**	**0.2571(48)**	**-0.0241(129)**	**0.0631(97)**
BB (a)	474.4/446	0.2544(51)	-0.0194(132)	0.0631
BB (c)	479.8/453	0.2633(24)	-0.0447	0.0538(77)
BB (ac)	478.1/447	0.2627(25)	-0.0449	0.0538

RAD. CORR. ON

FINAL RESULT

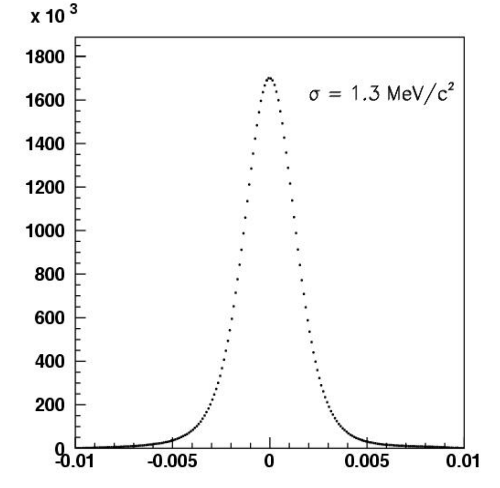
Fitting results



68% confidence level ellipse <u>stat. error only</u> to illustrate rad. corr.

Solid ellipse = rad. corr ON
dashed ellipse = rad corr. OFF
dashed line = CHPT constraint

Reconstructed K mass resolution

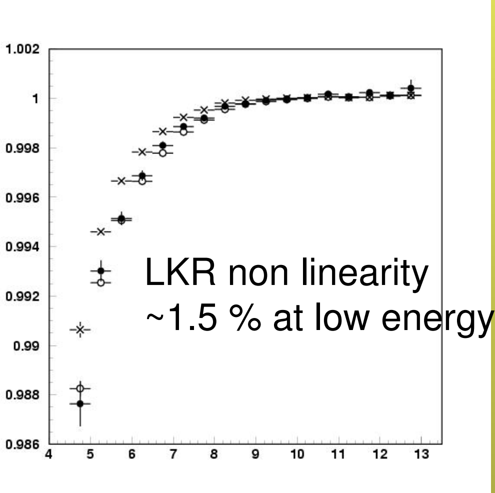


cut off for radiative correction
E_{cut} = 10 MeV
no significant change seen in the range 5 < E_{cut} < 20 MeV

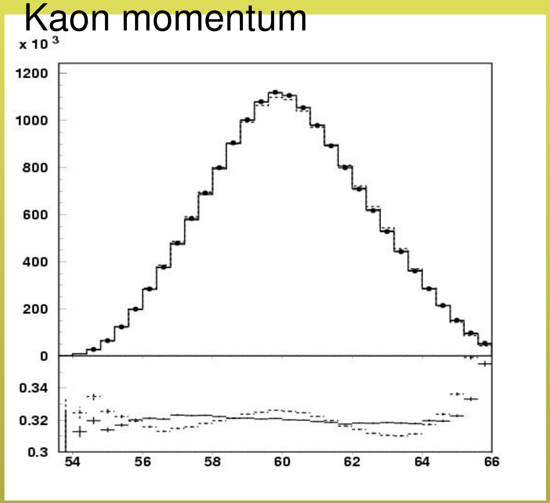
Somehow arbitrarily the BB radiative corrections have been applied to CI approach. This makes sense for Coulomb and bremsstrahlung corrections which are “universal”. It is not correct for direct photon emission (structure dependent), however its effect is small compared to previous ones. (discussion with Gino Isidori).

systematics and final result

source	a0-a2	a2
Acceptance (Z)	4	20
Acceptance (V)	1	2
Trigger efficiency	10	39
LKR resolution	9	29
LKR non linearity	12	67
Pk spectrum	13	32
MC	5	1
hadronic shower	10	18
TOTAL systematics	25	94
Statistical	48	129



LKR non linearity
~1.5 % at low energy


Kaon momentum

a0 -a2 = 0.2571 ± 0.0048 (stat.) ± 0.0025 (syst.) ± 0.014 (ext.)

a2 = -0.024 ± 0.013 (stat.) ± 0.009 (syst.) ± 0.002(ext.)

statistical correlation is -0.839

The external uncertainty is due to $\Gamma_{+-}/\Gamma_{00} = 3.175 \pm 0.050$ taken from PDG

Final result with CHPT constraint

$$(\mathbf{a}_0 - \mathbf{a}_2) = \mathbf{0.263 \pm 0.0024_{stat} \pm 0.0014_{syst} \pm 0.0019_{ext}}$$

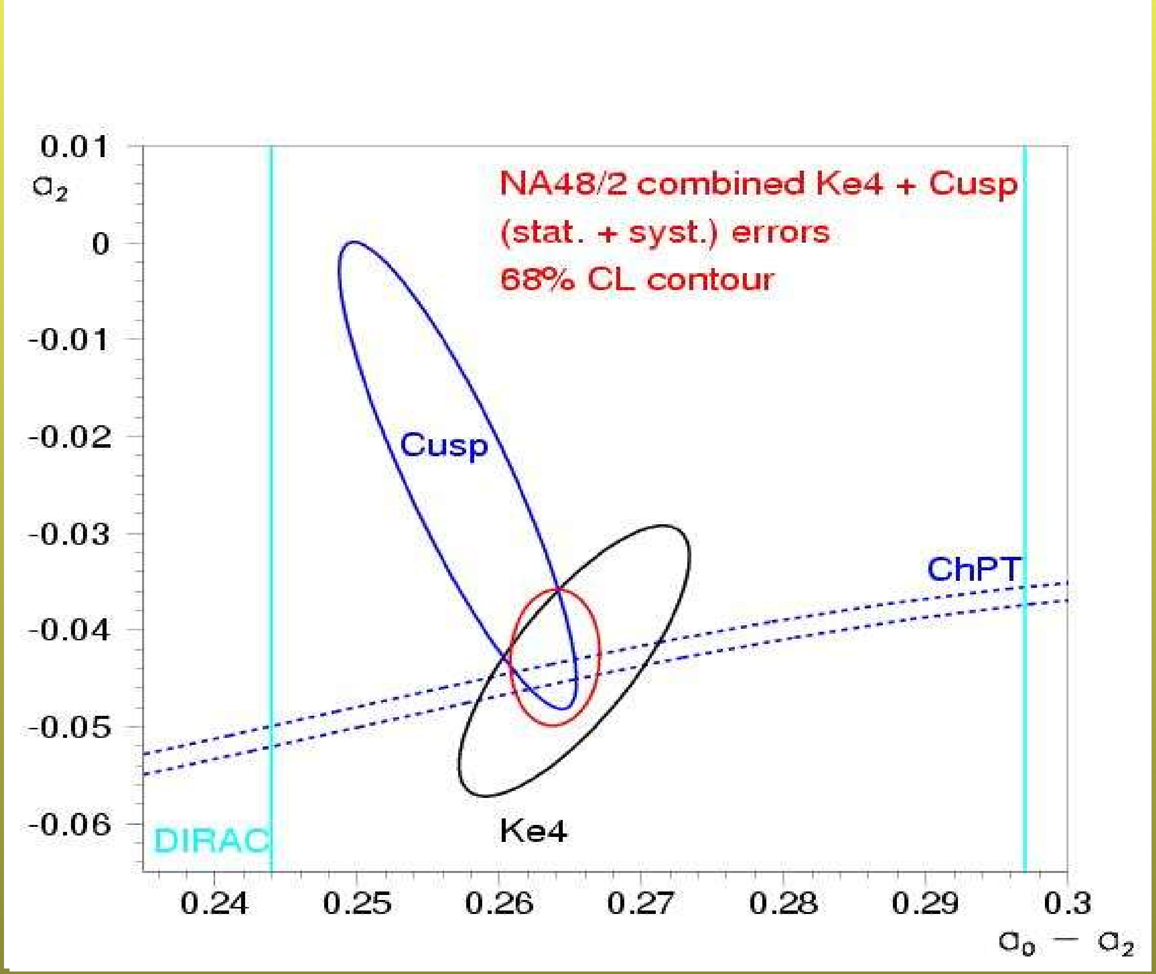
experimental precision comparable with the theoretical prediction

a0-a2= 0.265 ±0.004

One should also add a theoretical error accounting for the accuracy of the method used to extract scattering lengths from the cusp ... arising from truncating expansion at a given order O(a2) for C.I. approach and similar for BB one.

The estimation is in progress...

Results comparison

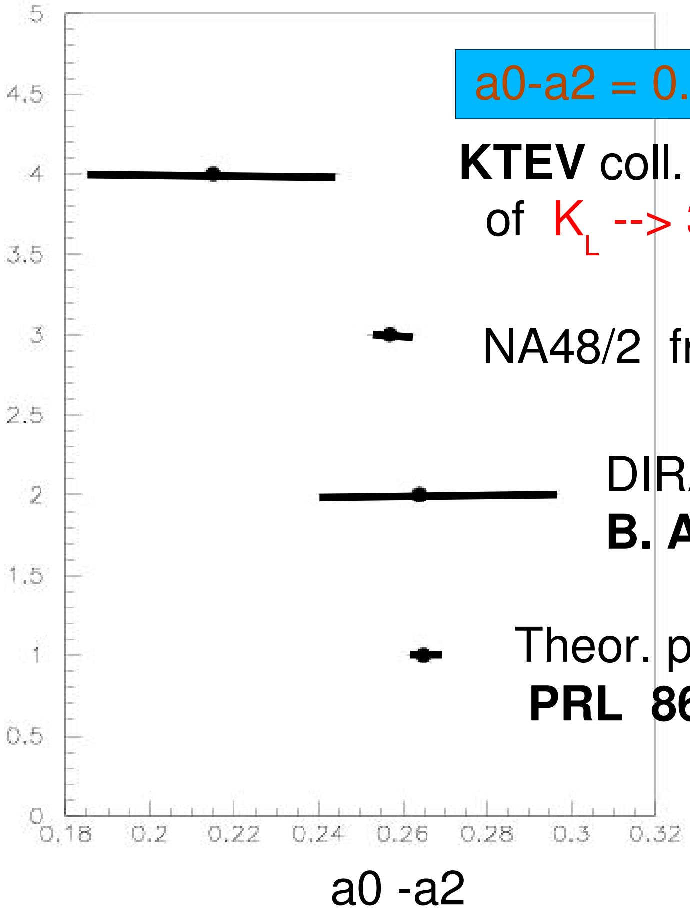


Na48/2 combined result
Cusp and Ke4

Cusp ellipse accounting for stat., syst. and ext. errors

Ke4 ellipse includes stat. and syst. error

a0-a2 comparison



a0-a2 = 0.215 ±0.014(stat)±0.025(syst.)±0.006(ext)

KTEV coll. from Cusp analysis based on C.I. approach
of $K_L \to 3\pi^0$ **E. Abouzaid et al., PR D78, 032009 (2008)**

NA48/2 from Cusp in K^+ , present analysis

DIRAC coll., from pionium lifetime measurement
B. Adeva et al. PRL B619 ,50, (2005)

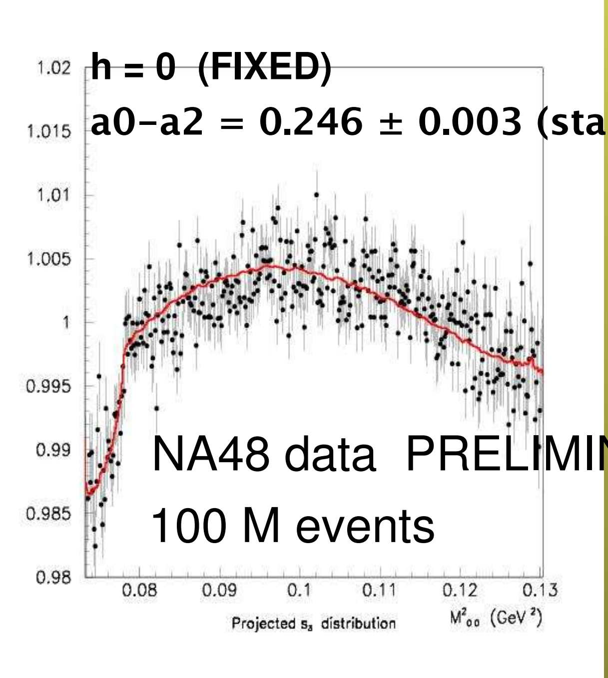
Theor. prediction: **G. Colangelo, J. Gasser, H. Leutwyler**
PRL 86, 5008, (2001)

CUSP in K_L-->$3\pi^0$

KL as well shows a cusp due to weak and strong amplitude interference

$M_0 = A_0 (1 + h u^2)$ tree level amplitude (PDG parametrization)

$M_1 \propto (a_0 - a_2) M_{+-0}$ 1 loop charge exchange

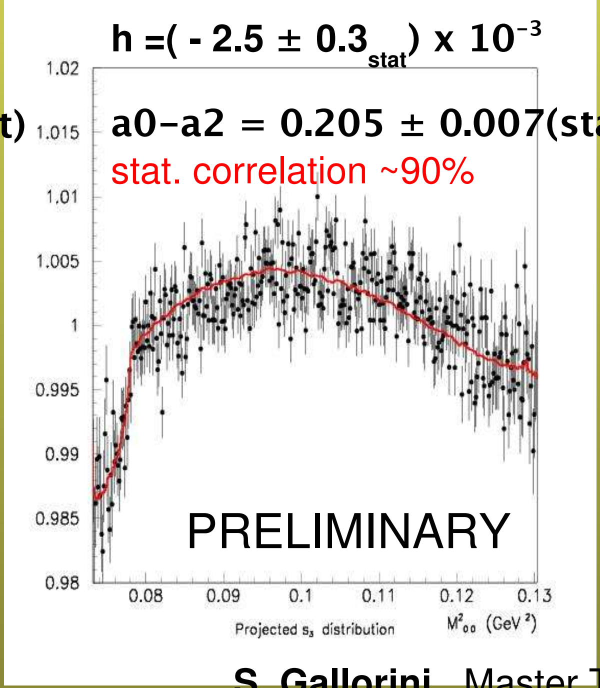


h = 0 (FIXED)

$a0-a2 = 0.246 \pm 0.003$ (stat.) ± 0.004 (ext)

NA48 data PRELIMINARY

100 M events



$h = (-2.5 \pm 0.3_{stat}) \times 10^{-3}$

$a0-a2 = 0.205 \pm 0.007$(stat.)± 0.004(ext)

stat. correlation ~90%

PRELIMINARY

S. Gallorini, Master Thesis ,Pisa University ,
(2008)

Fitting C.I. 2 loops calculation , a2 = -0.0044 fixed
N.B. systematics still to be determined.

CUSP in $K_L \to 3\pi^0$

KTEV measurement : $a0-a2 = 0.215 \pm 0.014(stat) \pm 0.025(syst.) \pm 0.006(ext)$

$h = [-2.09 \pm 0.62(stat.) \pm 0.72(syst.) \pm 0.28(ext)]10^{-3}$

NA48 PRELIMINARY: $a0-a2 = 0.205 \pm 0.007(stat) \pm$??? (syst.) ± 0.004 (ext)

$h = [-2.5 \pm 0.3(stat.)$ ± ??? (syst.) ± ??? (ext)] x 10^{-3}

Both measure a stat. correlation ~90% (CI approach)
NA48 obtain similar result with BB approach (Rad. Corr. not yet implemented)

Quadratic slope h disturbs the determination of scattering length and
a0-a2 takes smaller value with respect to other measurements and prediction

h may absorbs effectively a fraction of final state rescattering and once
we account for $\pi\pi$ scattering h should be set a zero
....but so far there is no theoretical argument to fix h=0 or at least to bound
$|h| < 10^{-3}$ in that case a0-a2 would be measured in a reasonable range

Conclusions

Budini – Fonda prophecy comes to reality ~45 years later

Physics is something one can re-discover...

The existing theoretical picture for pion – pion interaction have been quantitatively and experimentally validated through the Threshold Cusp effect in $K^+ \rightarrow 3\pi$ decays

Chiral Theory has been successfully tested at 2% precision level

We may thank

CERN – SPS for beam performances and for the high statistics collected ;

Technical staff of NA48 participating laboratories for data quality and experimental performances of the NA48 apparatus;

Theoreticians who guided us in the fitting procedure:
G. Isidori, G. Colangelo, J.Gasser, ,B.Kubis, A. Rusetsky

Unfortunately neutral K long meson cannot join the party ... !!!

CUSP VISIBILITY

two possible $\pi^+\pi^-$ pairs

$$R(K^+) \approx \frac{2M_{++-}\, M_{+00}}{(M_{+00})^2} = \frac{2M_{++-}}{M_{+00}}$$

$$\left\{\begin{array}{l} \mathbf{M}_{++-} : K^+ \to \pi^+\,\pi^+\,\pi^- \text{ matrix element} \\ \mathbf{M}_{+00} : K^+ \to \pi^+\,\pi^\circ\,\pi^\circ \text{ matrix element} \end{array}\right\}$$

$$R(K_L) \approx \frac{M_{+-0}\, M_{000}}{(M_{000})^2} = \frac{M_{+-0}}{M_{000}}$$

$$\left\{\begin{array}{l} \mathbf{M}_{+-0} : K_L \to \pi^+\,\pi^-\,\pi^\circ \text{ matrix element} \\ \mathbf{M}_{000} : K_L \to \pi^\circ\,\pi^\circ\,\pi^\circ \text{ matrix element} \end{array}\right\}$$

Calculate matrix elements at cusp point ($M_{pp} = 2m_+$)
from <u>measured</u> partial width ratios and slope parameters:

$$\mathbf{R}(K^+) \approx 6.1\ ;\ \mathbf{R}(K_L) \approx 0.47 \Longrightarrow \frac{\mathbf{R}(K_+)}{\mathbf{R}(K_L)} \approx 13$$

Cusp "visibility" is ~ 13 times higher in $K^+ \to \pi^+\pi^\circ\pi^\circ$ decays than in $K_L \to \pi^\circ\pi^\circ\pi^\circ$ decays