SU(3) breaking in hadronic τ decays

Hadronic τ decay rate

Consider the physical quantity R_τ: (Braaten, Narison, Pich 1992)

$$R_\tau \equiv \frac{\Gamma(\tau^- \to \text{hadrons}\,\nu_\tau(\gamma))}{\Gamma(\tau^- \to e^-\bar{\nu}_e\nu_\tau(\gamma))} = 3.6350(94)\,.$$

R_τ is related to the QCD correlators $\Pi^{T,L}(z)$: $(z \equiv s/M_\tau^2)$

$$R_\tau = 12\pi \int\limits_0^1 dz\,(1-z)^2 \Big[(1+2z)\,\text{Im}\Pi^T(z) + \text{Im}\Pi^L(z)\Big]\,,$$

with the appropriate combinations

$$\Pi^J(z) = |V_{ud}|^2 \Big[\Pi_{ud}^{V,J} + \Pi_{ud}^{A,J}\Big] + |V_{us}|^2 \Big[\Pi_{us}^{V,J} + \Pi_{us}^{A,J}\Big]\,.$$

Hadronic τ decay moments

Additional information can be inferred from the moments

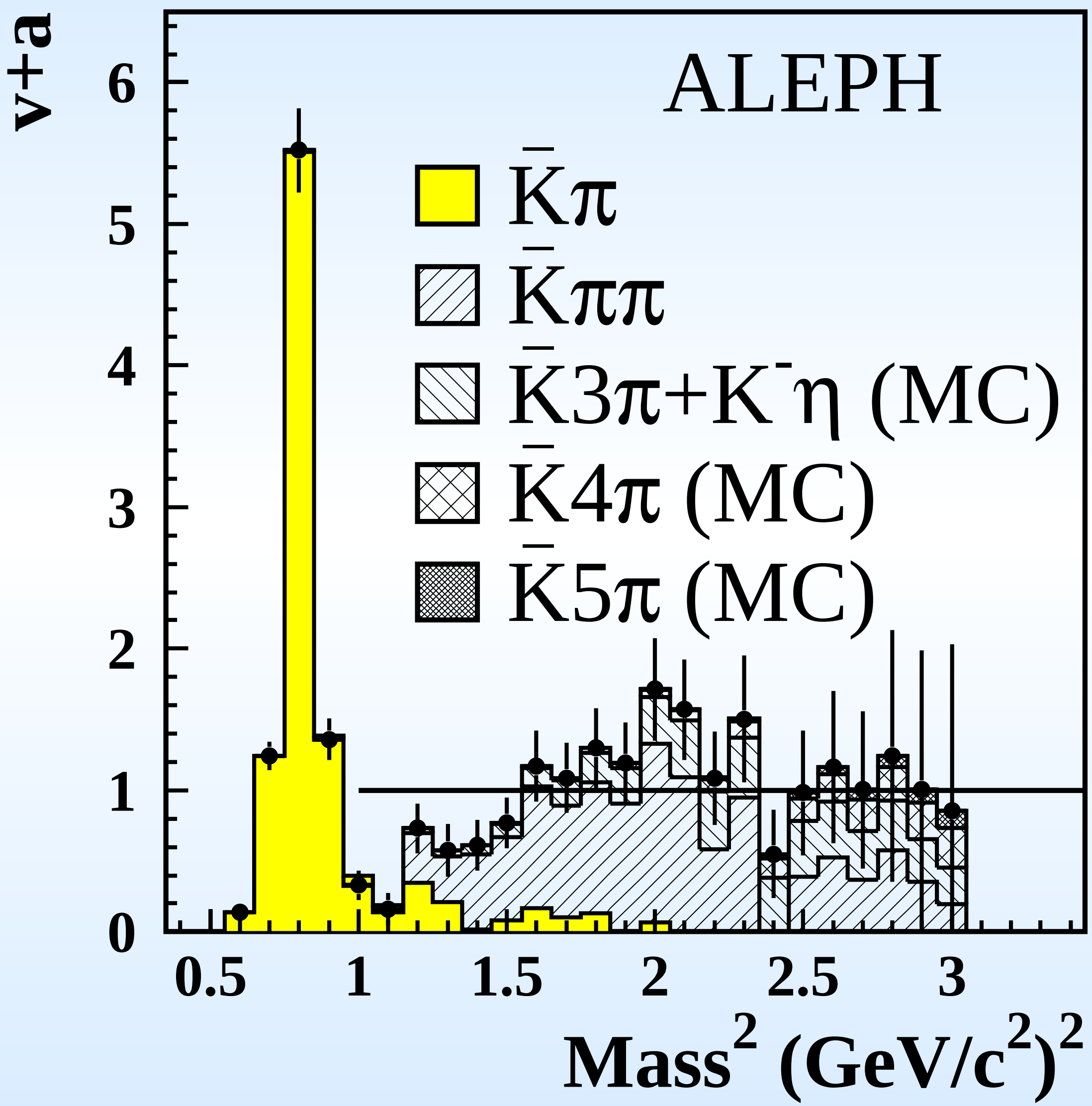
$$R_\tau^{kl} \equiv \int_0^1 dz\,(1-z)^k z^l \,\frac{dR_\tau}{dz} = R_{\tau,V}^{kl} + R_{\tau,A}^{kl} + R_{\tau,S}^{kl}\,.$$

Theoretically, R_τ^{kl} can be expressed as:

$$R_\tau^{kl} = N_c\,S_{\rm EW}\Big\{(|V_{ud}|^2+|V_{us}|^2)\Big[1+\delta^{kl(0)}\Big] + \sum_{D\geq 2}\Big[|V_{ud}|^2\delta_{ud}^{kl(D)}+|V_{us}|^2\delta_{us}^{kl(D)}\Big]\Big\}\,.$$

$\delta_{ud}^{kl(D)}$ and $\delta_{us}^{kl(D)}$ are corrections in the Operator Product Expansion, the most important ones being $\sim m_s^2$ and $m_s\langle\bar{q}q\rangle$.

Strange spectral function



The sensitivity to strange quark effects can be enhanced by considering the flavour SU(3)-breaking difference:

(Pich, Prades; ALEPH 1998)

$$\delta R_\tau^{kl} \equiv \frac{R_{\tau,V+A}^{kl}}{|V_{ud}|^2} - \frac{R_{\tau,S}^{kl}}{|V_{us}|^2} = 3\, S_{\rm EW} \sum_{D\geq 2} \Big(\delta_{ud}^{kl(D)} - \delta_{us}^{kl(D)} \Big).$$

Flavour independent uncertainties drop out in the difference.

In previous analyses a sizeable part of the theoretical error was due to large α_s corrections in the longitudinal contribution.

This uncertainty could be greatly reduced by replacing badly behaved scalar/pseudoscalar correlators with phenomenology.

(Gámiz, MJ, Pich, Prades, Schwab 2003/04)

To start with, an experimental result for the SU(3)-breaking difference $\delta R_\tau^{\rm exp}$ will be provided.

The essential input here is: $R_{\tau,S} = 0.1609(38)$

(BaBar 2008; Maltman et al. 2009)

(For comparison in 2006: $R_{\tau,S} = 0.1686(47)$ (Davier et al. 2006))

This also yields: $R_{\tau,V+A} = R_\tau - R_{\tau,S} = 3.474(10)$

Except for inclusive τ decays, most recent V_{us} determinations are compatible with unitarity. (talk by Achim Denig)

Thus employ: $V_{ud} = 0.97425(23)$ (Towner, Hardy 2009)

$$\Rightarrow \quad \delta R_\tau^{\rm exp} = 0.494 \pm 0.086$$

SU(3) breaking $\delta R_\tau^{\rm th}$

The experimental value may be compared to the theoretical prediction: $\delta R_\tau^{\rm th} = 0.218(26)$ (Gámiz et al. 2005)

displaying the same 3σ discrepancy, also seen in the τ-based V_{us} determination.

A reconsideration of $\delta R_\tau^{\rm th}$ appears in order!

$$\delta R_\tau^{\rm th} = \delta R_\tau^{m^2} + \delta R_\tau^{D\geq 4} + \delta R_\tau^{\rm S+P}$$

The last term $\delta R_\tau^{\rm S+P}$ is rather well known from a phenomenological parametrisation: $\delta R_\tau^{\rm S+P} = 0.1544(37)$

$D\geq 4$ contributions in the OPE turn out small:
$\delta R_\tau^{D\geq 4} = 0.0034(28)$ (MJ 2006)

PseudoScalar correlators

Defining a sum rule for the longitudinal contribution alone:

$$R_{ij}^{kl,S/P} \equiv -24\pi^2 \int_0^1 dz\,(1-z)^{2+k}\,z^{l+1}\,\rho_{ij}^{S/P}(z)\,.$$

The dominant contribution stems from the pseudoscalar (us) spectral function, $\rho = \mathrm{Im}\Pi/\pi$, which can be parameterised as:

$$s^2\rho_{us}^P = 2f_K^2 M_K^4 \delta(s-M_K^2) + 2f_{K(1460)}^2 M_{K(1460)}^4 BW(s)\,.$$

	$R_{us}^{00,P}$	$R_{us}^{00,S}$	$R_{ud}^{00,P}$ $[10^{-3}]$
Theo:	-0.144 ± 0.024	-0.028 ± 0.021	-7.79 ± 0.14
Phen:	-0.135 ± 0.003	-0.028 ± 0.004	-7.77 ± 0.08

SU(3) breaking $\delta R_\tau^{\rm th}$

We can infer a phenomenological expectation for $\delta R_\tau^{m^2}$:

$$\delta R_\tau^{m^2}(\text{phen}) = 0.336 \pm 0.086$$

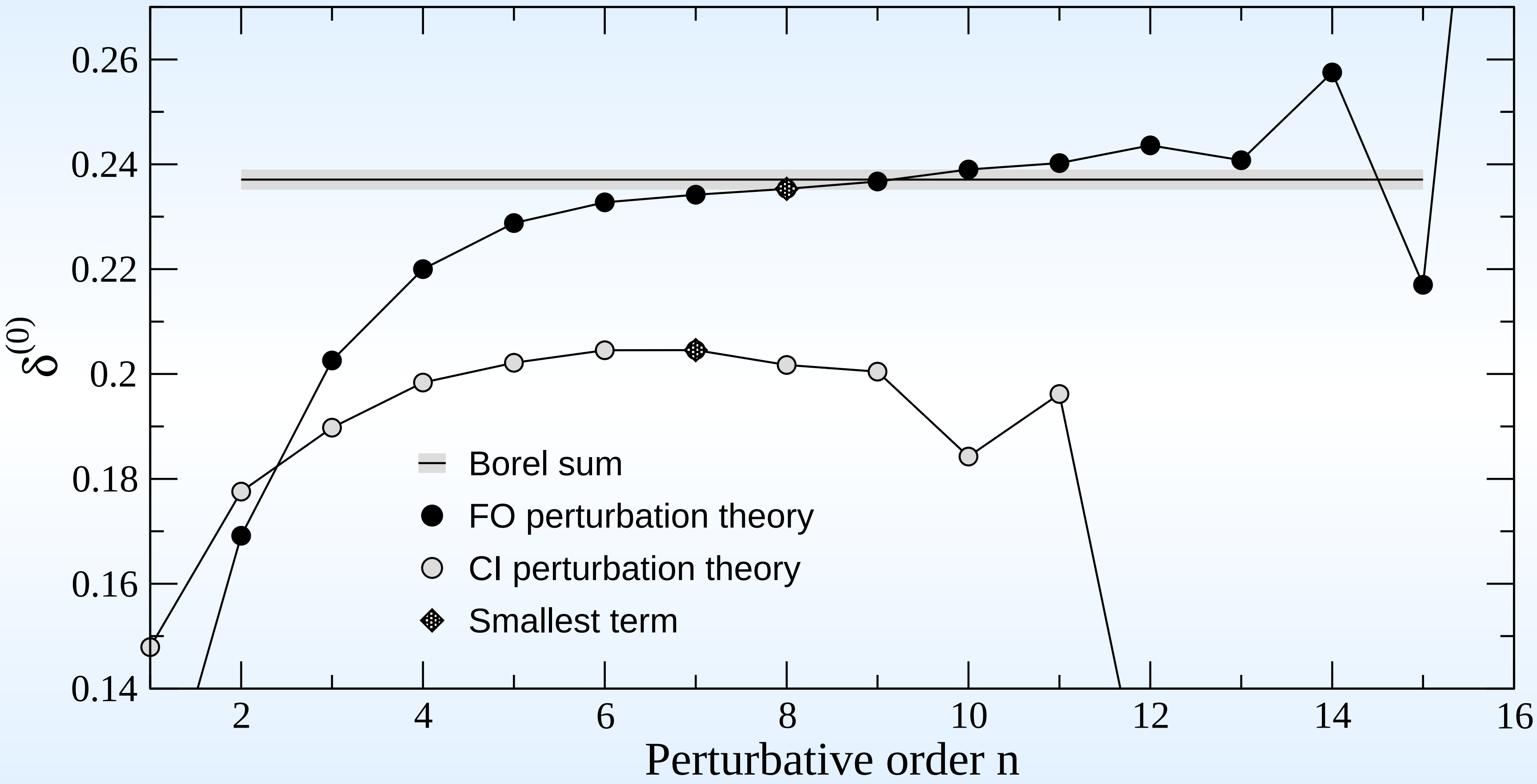
The corresponding theoretical expression reads:

$$\delta R_\tau^{m^2} = 18\, S_{\rm EW}\, \Delta^{V+A}(a_\tau)\, \frac{m_s^2(M_\tau)}{M_\tau^2} \left(1 - \frac{m_d^2}{m_s^2}\right)$$

with $(\alpha_s(M_Z) = 0.119)$:

$$\begin{array}{lcccc} & a_\tau^0 & a_\tau^1 & a_\tau^2 & a_\tau^3 \\ \Delta_{\rm FO}^{V+A} = & 1.0000 & +0.4119 & +0.2654 & +0.1615 = 1.8388 \\ \Delta_{\rm CI}^{V+A} = & 0.7688 & +0.2195 & +0.0075 & -0.0040 = 1.0232 \end{array}$$

Purely perturbative correction



(Beneke, MJ 2008)

SU(3) breaking $\delta R_\tau^{\rm th}$

As a rather conservative estimate, employ:

$$\Delta^{V+A}(a_\tau) = 1.84 \pm 0.82$$

Now, we can either determine the strange mass, or, with a given input m_s, compare the theory result to $\delta R_\tau^{m^2}(\rm phen)$.

The first option results in: $m_s(2\,\text{GeV}) = 170 \pm 60\,\text{MeV}$

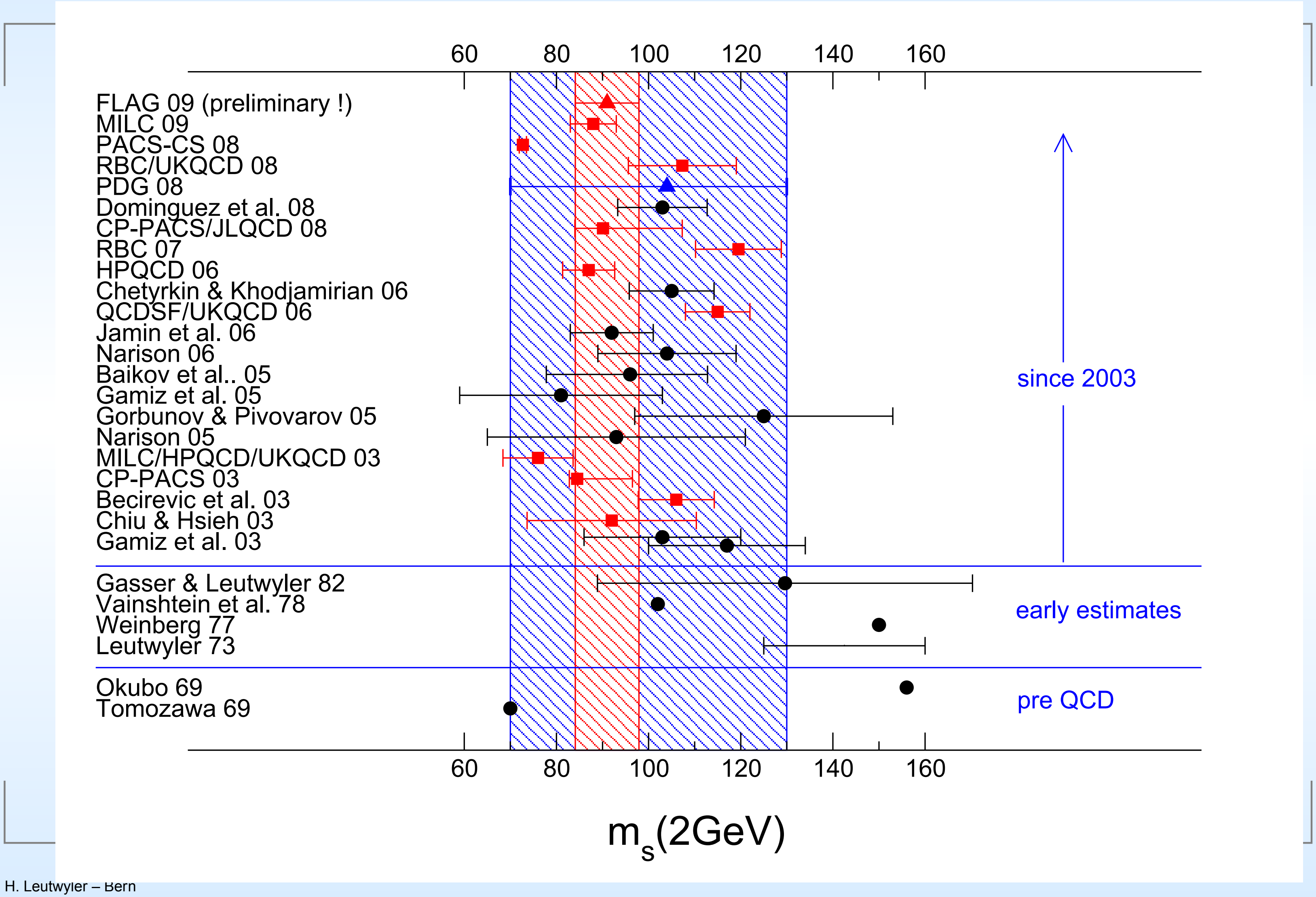
On the other hand employing $m_s(2\,\text{GeV}) = 95 \pm 10\,\text{MeV}$:

$$\delta R_\tau^{m^2}(\rm theo) = 0.103 \pm 0.052$$

The theoretical side alone does not seem to be able to resolve the current discrepancy to $\delta R_\tau^{m^2}(\rm phen) = 0.336 \pm 0.086$.

Strange quark mass

Mass of the strange quark



Conclusions

- Issue concerning Contour-Improved vesus Fixed-Order perturbation theory in the m^2-corrections to R_τ requires further investigation.
- Still, the theory side alone does not seem to be able to explain the discrepancy to the experimental result for δR_τ.
- Possible resolution from the experimental side would be larger $R_{\tau,S}$. Cross-check from $R_{\tau,V+A}$ feasible?

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Thank You for Your attention !