Results from ETMC in the light-quark sector

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lattice QCD

Study QCD in a non-perturbative way

- Determine QCD parameters : α_S, Λ_{QCD}, quark masses, . . .
- Determine hadronic properties :
  - Spectrum of mesons and baryons
  - Hadronic structure : form factors, scattering lengths, . . .
- Constrain effective theories :
  - Chiral Perturbation Theory (χPT)
  - Heavy Quark Effective Theory (HQET)
- Constraints on Standard Model parameters : CKM
  - New Physics : precision in the non-perturbative determinations of hadronic matrix elements $\rightsquigarrow$ few percent
  - Control of systematic uncertainties in lattice QCD results

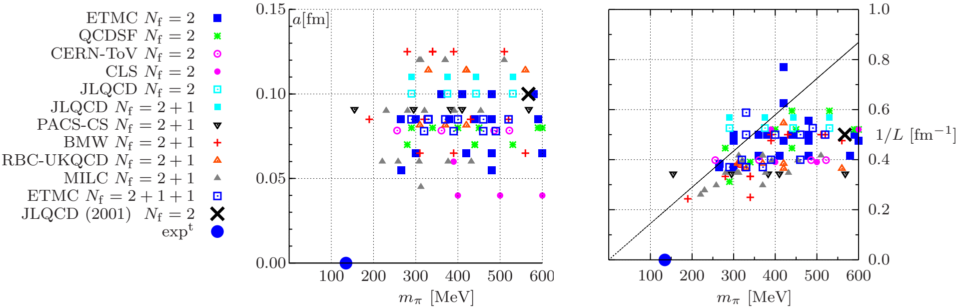
precision in lattice QCD results

- Control of systematic uncertainties
  - UV cutoff effects: lattice spacing a — $O(a)$ improvement, continuum limit
  - Finite Size Effects (FSE): lattice size L — $m_{\rm PS} L \gg 1$
  - Number of dynamical flavours ($u,d,s,c,\ldots$ quarks) — $N_f = 0;\ 2;\ 2+1;\ 2+1+1$
  - Range of quarks masses : simulation/physics — applicability of χPT, HQET
  - Operator renormalisation — non-perturbative
- Statistical errors
  - Improvement in Monte Carlo algorithms — Wilson type fermions
  - Supercomputers
- Outline
  - Light-quark physics from $N_f = 2$ and $N_f = 2+1+1$ dynamical simulations

lattice QCD: parameters landscape

- lattice spacing : a
- lattice size: L
- pion masses : m_π

(end 2008)



Wilson twisted mass lattice QCD (tmLQCD)

Lattice fermionic action [Frezzotti, Grassi, Sint, Weisz, 1999]

$$S_F^{\rm tmL} = a^4 \sum_x \bar{\chi}(x) \left[\gamma_\mu \tilde{\nabla}_\mu - r \frac{a}{2} \nabla^*_\mu \nabla_\mu + m_0 + i\gamma_5 \tau_3 \mu \right] \chi(x)$$

- automatic $\mathcal{O}(a)$ improvement of parity-even correlators in maximally twisted lattice QCD [Frezzotti, Rossi, 2003]
  - tuning of only one parameter: bare untwisted quark mass: $m_0 \longrightarrow M_{\rm cr}$
  - quark mass is then given by the twisted mass parameter : $M_q = \mu$
  - no tuning of operator-specific improvement coefficients
- low computational cost

But:

- explicit breaking of parity and isospin: the largest cut-off effects are in m_π^0
- however, the breaking is an $\mathcal{O}(a^2)$ effect in physical quantities

$N_f = 2$ ensembles

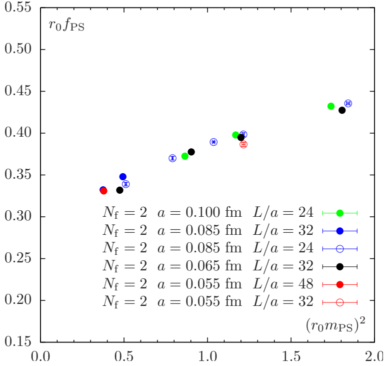
Ensemble	$\beta = \frac{6}{g_0^2}$	a (fm)	V/a^4	$m_{PS}L$	$a\mu_l$	m_{PS} (MeV)
D_1	4.20	0.055	$48^3 \cdot 96$	3.6	0.0020	270
D_2			$32^3 \cdot 64$	4.2	0.0065	480
C_1	4.05	0.065	$32^3 \cdot 64$	3.3	0.0030	310
C_2				4.6	0.0060	430
C_3				5.3	0.0080	500
C_4				6.5	0.0120	610
C_5			$24^3 \cdot 48$	3.5	0.0060	430
C_6			$20^3 \cdot 48$	3.0	0.0060	430
B_1	3.90	0.085	$24^3 \cdot 48$	3.3	0.0040	315
B_2				4.0	0.0064	390
B_3				4.7	0.0085	450
B_4				5.0	0.0100	490
B_5				6.2	0.0150	600
B_6			$32^3 \cdot 64$	4.3	0.0040	310
B_7				3.7	0.0030	270
A_2	3.80	0.100	$24^3 \cdot 48$	5.0	0.0080	410
A_3				5.8	0.0110	480
A_4				7.1	0.0165	580

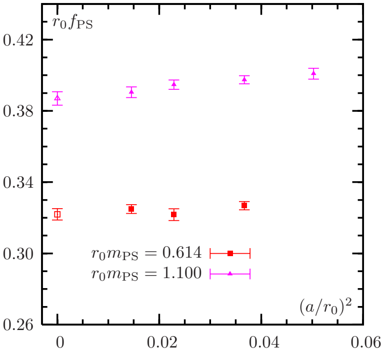
scaling to the continuum limit of $f_{\rm PS}$

$a = 0.055,\ 0.065,\ 0.085,\ 0.100$ fm

pion decay constant

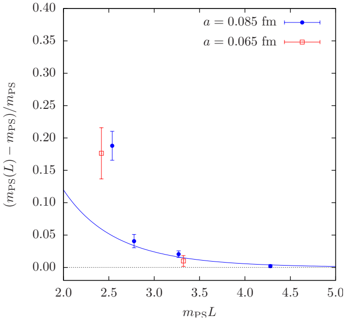
$$f_{\rm PS}\, p_\mu = \langle 0|\bar{u}\gamma_\mu\gamma_5 d|\pi(p)\rangle \quad \rightsquigarrow \quad f_{\rm PS} = \frac{2\mu}{m_{\rm PS}^2}\,|\langle 0|P^1(0)|\pi^\pm\rangle|$$

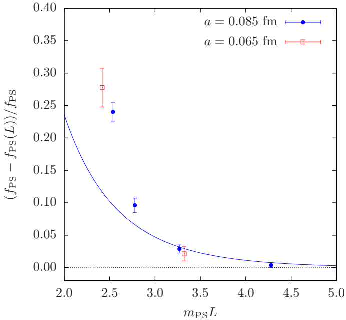




Finite size effects

- non negligible FSE since relative stat. error : $\sim 1\%$ on $m_{\rm PS}$ and $f_{\rm PS}$
- relative deviation : $R_O = (O_\infty - O_L)/O_\infty$





$m_{\rm PS} \approx 300$ MeV

- for $m_{\rm PS}L > 3$, data lies in the exponential FSE regime

finite size effects

Comparison of lattice data at several volumes to :

- NLO χPT : GL [Gasser, Leutwyler, 1987, 1988]
- resummed Lüscher formula : CDH [Colangelo, Dürr, Haefeli, 2005]
- relative deviation : $R_O = (O_\infty - O_L)/O_\infty$

	a (fm)	$m_{PS}L_1 \to m_{PS}L_2$	meas.(%) ($L_1 \to L_2$)	GL (%) ($L_1 \to \infty$)	CDH (%) ($L_1 \to \infty$)
m_{PS}	0.085	$3.3 \to 4.3$	-1.8	-0.6	-1.2
f_{PS}	0.085	$3.3 \to 4.3$	$+2.6$	$+2.6$	$+2.6$
m_{PS}	0.065	$3.0 \to 4.6$	-6.1	-1.9	-6.3
f_{PS}	0.065	$3.0 \to 4.6$	$+10.7$	$+7.0$	$+9.0$

- CDH describes data in general better than GL but needs more parameters

chiral perturbation theory : $f_\pi, \; m_\pi$

- Use of χPT to describe the dependence on :
  - the quark mass μ
  - finite spatial size L

- Simultaneous fit to $N_f = 2$ χPT

$$m_{\rm PS}^2(L) = \chi_\mu \left[1 + \xi \ln(\chi_\mu/\Lambda_3^2) + T_m^{\rm NNLO} + a^2 D_m\right] \cdot \left(K_m^{\rm CDH}(L)\right)^2$$
$$f_{\rm PS}(L) = f_0 \left[1 - 2\xi \ln(\chi_\mu/\Lambda_4^2) + T_f^{\rm NNLO} + a^2 D_f\right] \cdot K_f^{\rm CDH}(L)$$

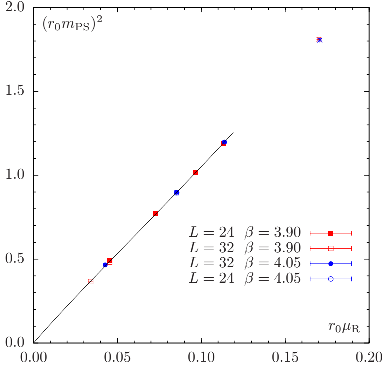
where $\chi_\mu = 2\widehat{B}_0 \mu_R \,, \qquad \mu_R = 1/Z_P\, \mu \,, \qquad \xi = \chi_\mu/(4\pi f_0)^2 \,, \qquad f_0 = \sqrt{2} F_0$

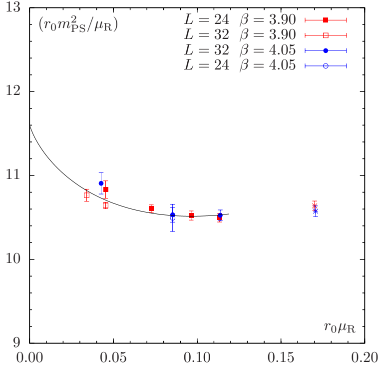
- data : $af_{\rm PS}$, $am_{\rm PS}$, $Z_{\rm P}$ and r_0/a
- parameters : $r_0 f_0,\ r_0 B_0,\ r_0\Lambda_3,\ r_0\Lambda_4,\ D_m,\ D_f,\ \{r_0/a(\mu=0)\}_\beta,\ \{D_{r_0}\}_\beta$
- derived quantities : $m_{u,d}$, $\langle \bar{q} q\rangle$, low-energy constants : $\bar{l}_{3,4} \equiv \log(\Lambda_{3,4}^2/m_{\pi^\pm}^2)$
- Finite size corrections : (CDH : Colangelo *et al.*, 2005)
- Mass dependence : NLO and NNLO (extra parameters : $r_0\Lambda_{1,2}$, k_M, k_F)
- Include $O(a^2)$ terms in the fits

continuum χPT at NLO : $m^2_{\rm PS}$ vs. $\mu_{\rm R}$

$\beta = 4.05$: $a = 0.065$ fm
$\beta = 3.90$: $a = 0.085$ fm

NLO without $O(a^2)$ terms excluding heavier masses

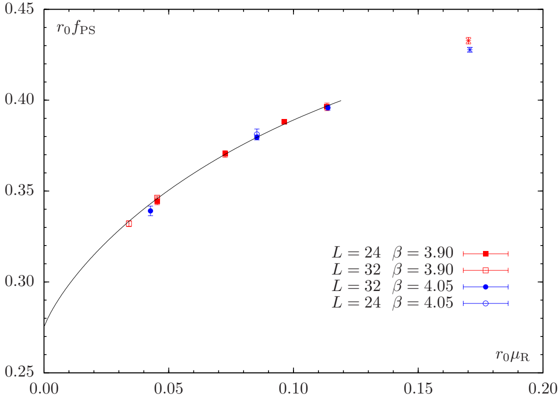




continuum χPT at NLO : f_{PS} vs. μ_R

$\beta = 4.05$: $a = 0.065$ fm
$\beta = 3.90$: $a = 0.085$ fm

NLO without $O(a^2)$ terms excluding heavier masses



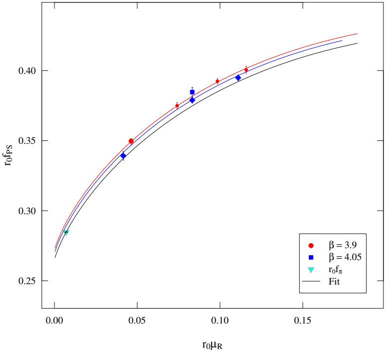
χPT fits : NLO

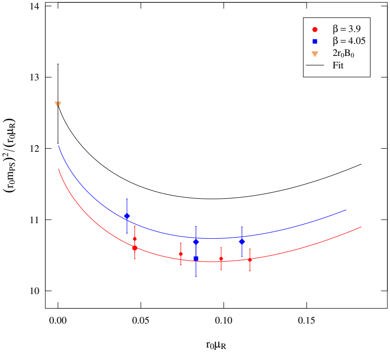
$$r_0 f_{\rm PS} = r_0 f_0 \left[1 - 2\xi \log(\chi_\mu/\Lambda_4^2) + (a/r_0)^2 D_f\right] K_f^{\rm CDH}(L)$$

$$(r_0 m_{\rm PS})^2 = \chi_\mu r_0^2 \left[1 + \xi \log(\chi_\mu/\Lambda_3^2) + (a/r_0)^2 D_m\right] \left(K_m^{\rm CDH}(L)\right)^2$$

NLO with O(a^2) terms excluding heavier masses

$\chi^2/dof = 19.6/17$





$\beta = 4.05$: $a = 0.065$ fm

$\beta = 3.90$: $a = 0.085$ fm

χPT fits : discretization effects

$$r_0 f_{\rm PS} = r_0 f_0 \left[1 - 2\xi \log(\chi_\mu/\Lambda_4^2) + (a/r_0)^2 D_f\right] K_f^{\rm CDH}(L)$$

- fit of $f_{\rm PS}$ and $m_{\rm PS}$ combining $a_1 = 0.055$, $a_2 = 0.065$, $a_3 = 0.085$ fm [PRELIMINARY]
- mass dependence : NLO higher masses ($m_{\rm PS} \sim 600$ MeV) not included
- volume dependence : CDH

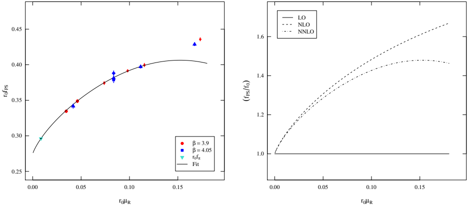
	$D_{m,f} = 0$	fit $D_{m,f}$	fit $D_{m,f}$
a_i	$a_{2,3}$	$a_{2,3}$	$a_{1,2,3}$
$\bar{l}_3$	3.38(7)	3.51(7)	3.47(6)
$\bar{l}_4$	4.62(3)	4.63(3)	4.59(3)
$\widehat{B}_0$ [GeV]	2.55(4)	2.89(14)	2.79(12)
f_0 [MeV]	121.62(7)	121.58(7)	121.65(6)
r_0 [fm]	0.449(3)	0.429(9)	0.439(6)
χ^2/dof	30.8/21	23.2/19	26.7/23

- values of $D_{m,f}$: $D_m = -1.08(95)$; $D_f = 0.70(56)$

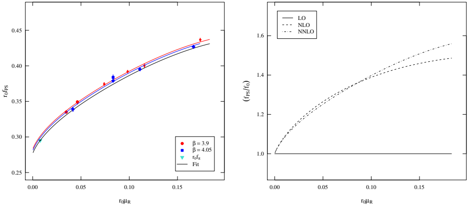
χPT fits : NNLO

$\beta = 4.05$: $a = 0.065$ fm
$\beta = 3.90$: $a = 0.085$ fm

NNLO excluding heavier masses $\chi^2/dof = 23.7/19$



NNLO including heavier masses $\chi^2/dof = 30.9/23$



NNLO : Input some knowledge on $\bar{l}_{1,2}$, k_M and k_F in the fit:
$\bar{l}_1 = -0.4 \pm 0.6$ $\bar{l}_2 = 4.3 \pm 0.1$ $k_M = k_F = 0 \pm 10$

Results : LEC, m_q, $\langle \bar{q}q \rangle$, ...

Estimate systematic effects [PRELIMINARY]

- discretization
- NLO/NNLO
- FSE

$\bar{l}_3$	3.49(19)
$\bar{l}_4$	4.57(15)
$\widehat{B}_0$ [GeV]	2.77(19)
f_0 [MeV]	121.8(5)
$(-\langle \bar{q}q \rangle)^{1/3}$ [MeV]	274(6)
$m_{u,d}$ [MeV]	3.37(23)
r_0 [fm]	0.433(14)

B_0, $\langle \bar{q}q \rangle$ and $m_{u,d}$ are given in $\overline{\mathrm{MS}}$ at 2 GeV

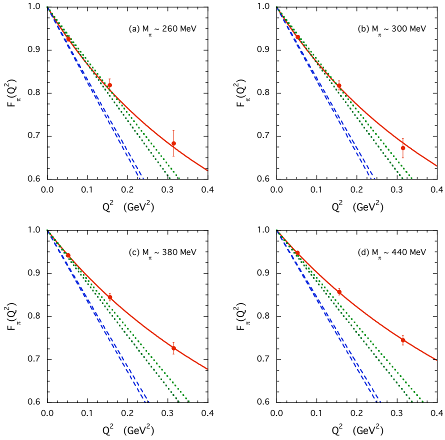
To constrain further the determination of the LEC : more data points or include in the fit other observables ...

electromagnetic form factor of the pion

[Frezzotti, Lubicz, Simula, 2008]

$$\langle \pi^+(p')|\widehat{V}_\mu|\pi^+(p)\rangle = F_\pi(q^2)\,(p+p')_\mu \; ; \qquad \text{where } q^2 = (p-p')^2$$

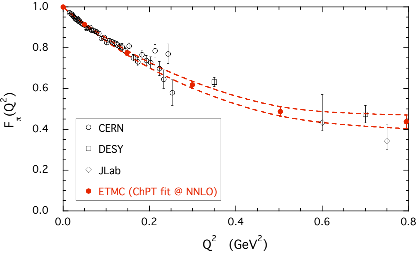
- $Q^2 = -q^2 \in [0.05,\ 0.80]$ GeV^2
- $m_{\rm PS} \in [270,\ 600]$ MeV
- $m_{\rm PS} \times L > 4$
- $a = 0.085$ fm
- NLO χPT [Gasser, Leutwyler, 1984] and vector meson dominance



When Q^2 increases, NLO order does not describe anymore the lattice data
$\rightsquigarrow$ NNLO [Bijnens, Colangelo, Talavera, 1998]

pion form factor : NNLO χPT $a = 0.085$ fm

- use $\langle r^2 \rangle_S^{\rm exp.}$ as input to stablize the fit
- $Q^2 = -q^2 \in [0.05,\ 0.80]$ GeV^2

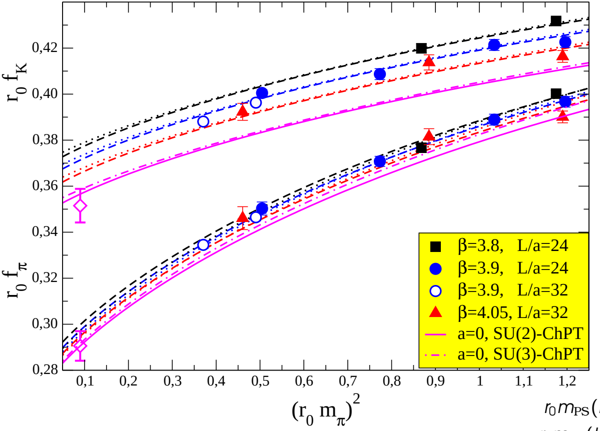


LEC	NNLO	non-lattice
$\widehat{B}_0$ (GeV)	$2.45 \pm 0.03 \pm 0.10$	—
f_0 (MeV)	$122.5 \pm 0.5 \pm 1.0$	—
$\bar{\ell}_1$	$-0.4 \pm 0.7 \pm 0.6$	-0.4 ± 0.6
$\bar{\ell}_2$	$4.3 \pm 0.6 \pm 0.4$	4.3 ± 0.1
$\bar{\ell}_3$	$3.2 \pm 0.4 \pm 0.2$	2.9 ± 2.4
$\bar{\ell}_4$	$4.4 \pm 0.1 \pm 0.1$	4.4 ± 0.2
$\bar{\ell}_6$	$14.9 \pm 0.6 \pm 0.7$	$16.0 \pm 0.5 \pm 0.7$
$r^r_M \cdot 10^4$	$-0.45 \pm 0.16 \pm 0.10$	—
$r^r_F \cdot 10^4$	$0.08 \pm 0.08 \pm 0.05$	—
$r^r_1 \cdot 10^4$	$-0.94 \pm 0.07 \pm 0.10$	-2.0
$r^r_2 \cdot 10^4$	$0.46 \pm 0.02 \pm 0.31$	2.1

- agreement with χPT fit of $m_{\rm PS}$ and $f_{\rm PS}$ using $a = \{0.055, 0.065, 0.085\}$ fm data.
- pion charge radius: $\langle r^2 \rangle \quad = 0.456 \pm 0.030 \pm 0.024$ fm^2
- experimental result: $\langle r^2 \rangle^{exp.} = 0.452 \pm 0.011$ fm^2

strange-quark sector : $f_{\rm PS}(\mu_l, \mu_l, \mu_s)$ vs. $m^2_{\rm PS}$

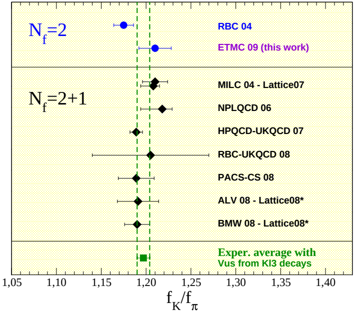
- $N_f = 2$ $\rightsquigarrow$ the strange quark is quenched : use of Partially Quenched PQχPT
- lattice spacing : $a \sim$ 0.065, 0.085, 0.100 fm

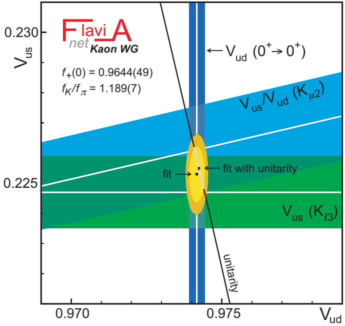


$r_0 m_{\rm PS}(l, s, s) = 1.63$

$r_0 m_{\rm PS}(l, s, s)^{\rm exp.} \approx 1.55$

strange-quark sector : f_K/f_π $K_{\ell 2}$ decay





ETMC :

$$f_K = 158.1 \pm 0.8 \pm 2.0 \pm 1.1 \text{ MeV}$$
$$f_K/f_\pi = 1.210(6)(15)(9)$$
$$|V_{us}|/|V_{ud}| = 0.2281(5)(35)$$
$$|V_{us}| = 0.2222(5)(34)$$
$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.00146(160)$$

Flavianet (Kaon WG global fit, 2008) :

$$|V_{us}|/|V_{ud}| = 0.2313(9)$$
$$|V_{us}| = 0.2253(9)$$
$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.00023(70)$$

$$N_f = 2 + 1 + 1$$

$u,\ d,\ s,\ c$

sea quarks

$N_f = 2 + 1 + 1$

- test QCD in realistic conditions
- repeat physical conditions of $N_f = 2$ simulations
- setup
  - $N_f = 2 + 1 + 1$ twisted mass
  - automatic $O(a)$ improvement [Frezzotti, Rossi, 2003]
  - non-degenerate quark masses :

$$m_{c,s} = 1/Z_P \, \mu_\sigma \pm 1/Z_s \, \mu_\delta$$

  - Iwasaki gauge action

$N_f = 2+1+1$ ensembles

- Range of masses:
  $m_\pi \in [270; 600]$ MeV
  $m_K \sim m_K^{\rm exp.}$
  $m_c \gtrsim 10 m_s$
- *e.g.* $\beta = 1.90$:
  $a \approx 0.085$ fm
  $m_{\rm PS} \times L \gtrsim 3.5$
  $L \approx 2.0$ and 2.7 fm
- Ensembles at two finer lattice spacings are being generated

$\beta = \frac{6}{g_0^2}$	V/a^4	$a\mu_l$	$a\mu_\sigma$	$a\mu_\delta$
1.90	$32^3 \cdot 64$	0.0030	0.150	0.190
		0.0040		
		0.0050		
	$24^3 \cdot 48$	0.0040		
		0.0060		
		0.0080		
		0.0100		
	$20^3 \cdot 48$	0.0040		
	$24^3 \cdot 48$	0.0100	0.150	0.197
1.95	$32^3 \cdot 64$	0.0025	0.135	0.170
		0.0035		
		0.0055		
		0.0075		
	$24^3 \cdot 48$	0.0085		
stout 1.90	$24^3 \cdot 48$	0.0040	0.170	0.185
		0.0060		
		0.0080		

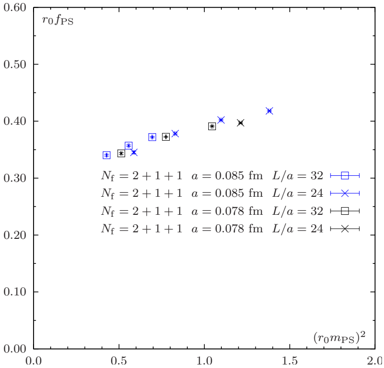
scaling to the continuum limit of f_{PS} and m_N

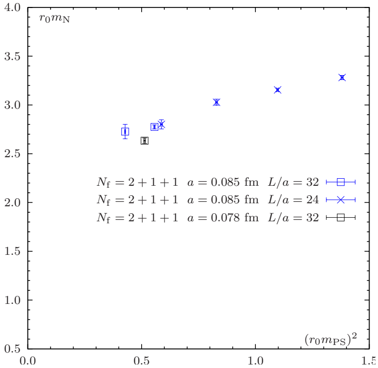
[PRELIMINARY]
$a = 0.078,\ 0.085$ fm

pion decay constant and nucleon mass

$$f_{PS} = \frac{2\mu}{m_{PS}^2} |\langle 0|P^1(0)|\pi^{\pm}\rangle|$$

nucleon mass





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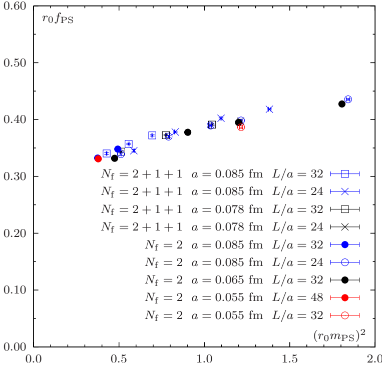

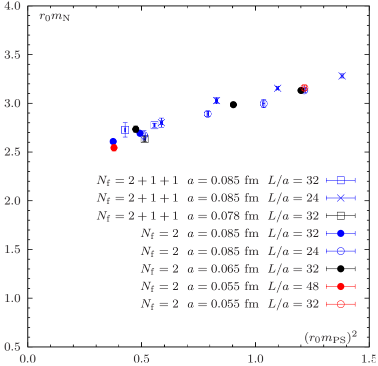

Conclusions

Summary :

- confront lattice QCD data to χPT : mass and volume dependence
- extraction of LEC, m_q and $\langle \bar{q}q \rangle$ with good statistical precision
- control of systematic errors

Other results from ETMC :

- meson and baryon spectrum
- $f_{\rm D}$, $f_{\rm D_s}$, $B_{\rm K}$, ...
- pion scattering lengths, ρ decay, K, D meson weak decays, PDF, ...
- $N_f = 2+1+1$: SU(2) and SU(3) χPT