Baryon chiral extrapolation in SU(3)

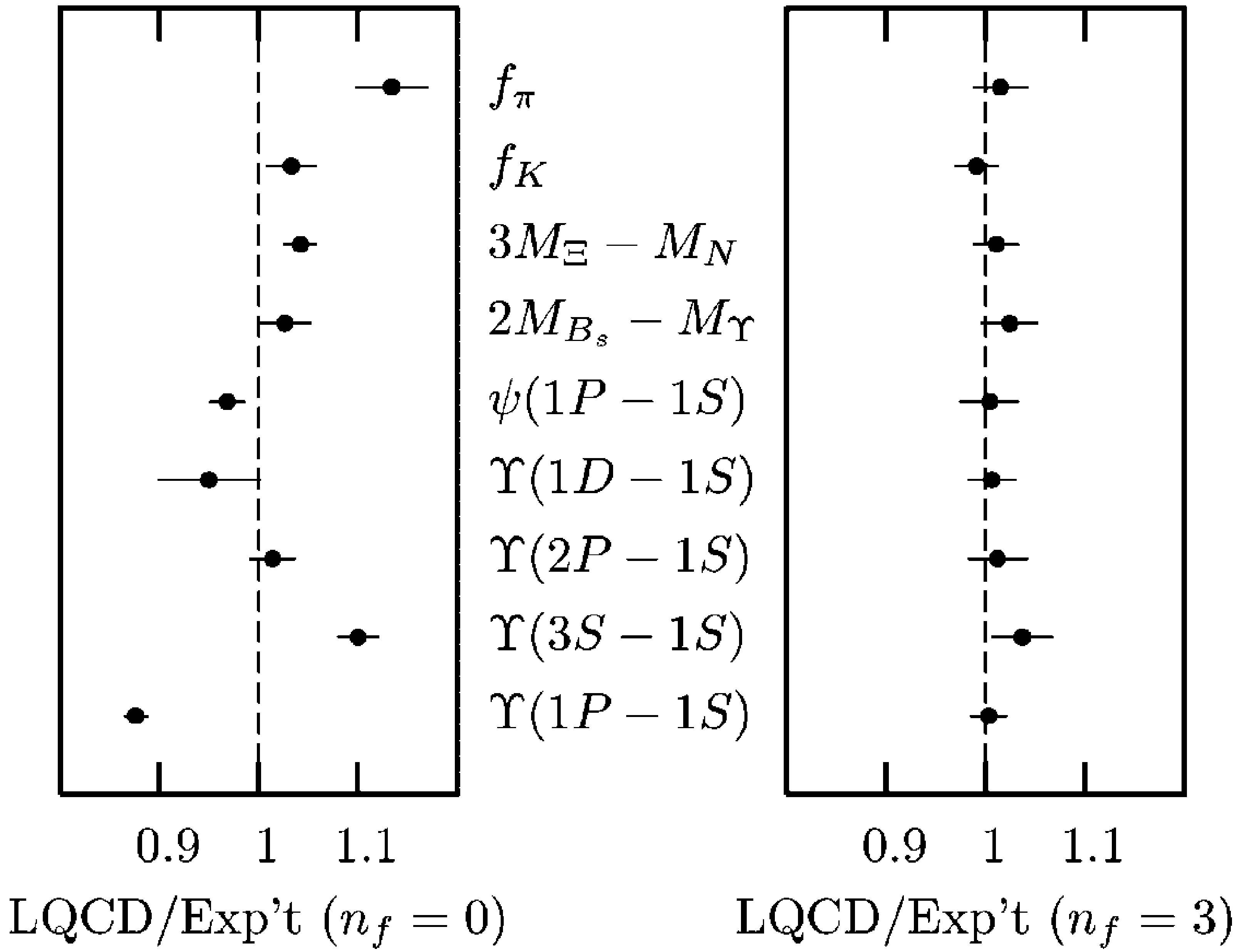
Ross D. Young

Chiral Dynamics 2009, University of Bern, Switzerland
6-10 July 2009

see Young & Thomas, arXiv:0901.3310

Outline

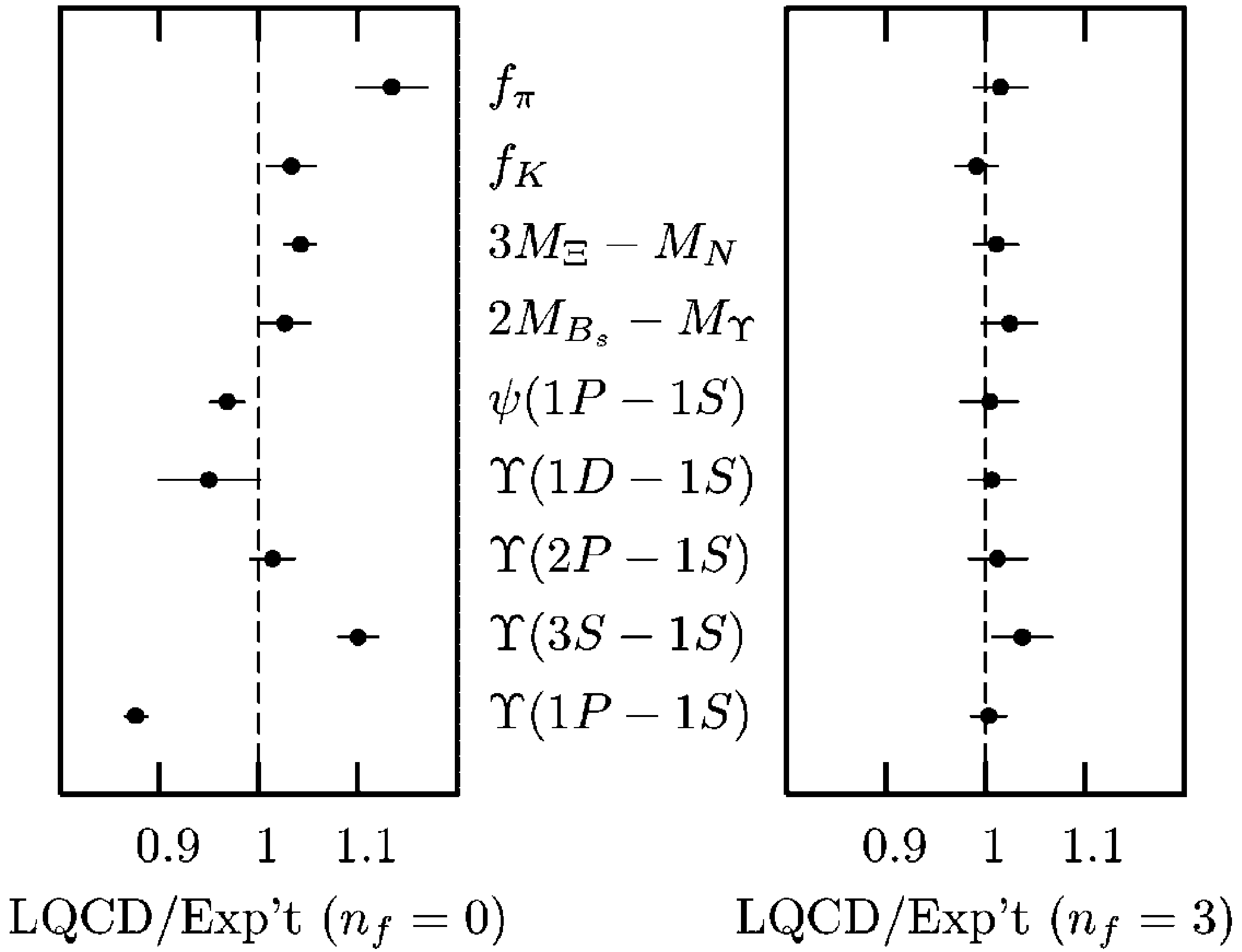
- Motivating the physics
- Chiral extrapolation problem
  - 3-flavour treatment of effective field theory (EFT)
  - poor convergence of SU(3) expansion
- Solution: Finite-Range Regularisation (FRR)
  - constrained by lattice
- New determination of the strangeness sigma term



HPQCD / UKQCD / MILC / Fermilab, PRL92,022001(2004)

2003

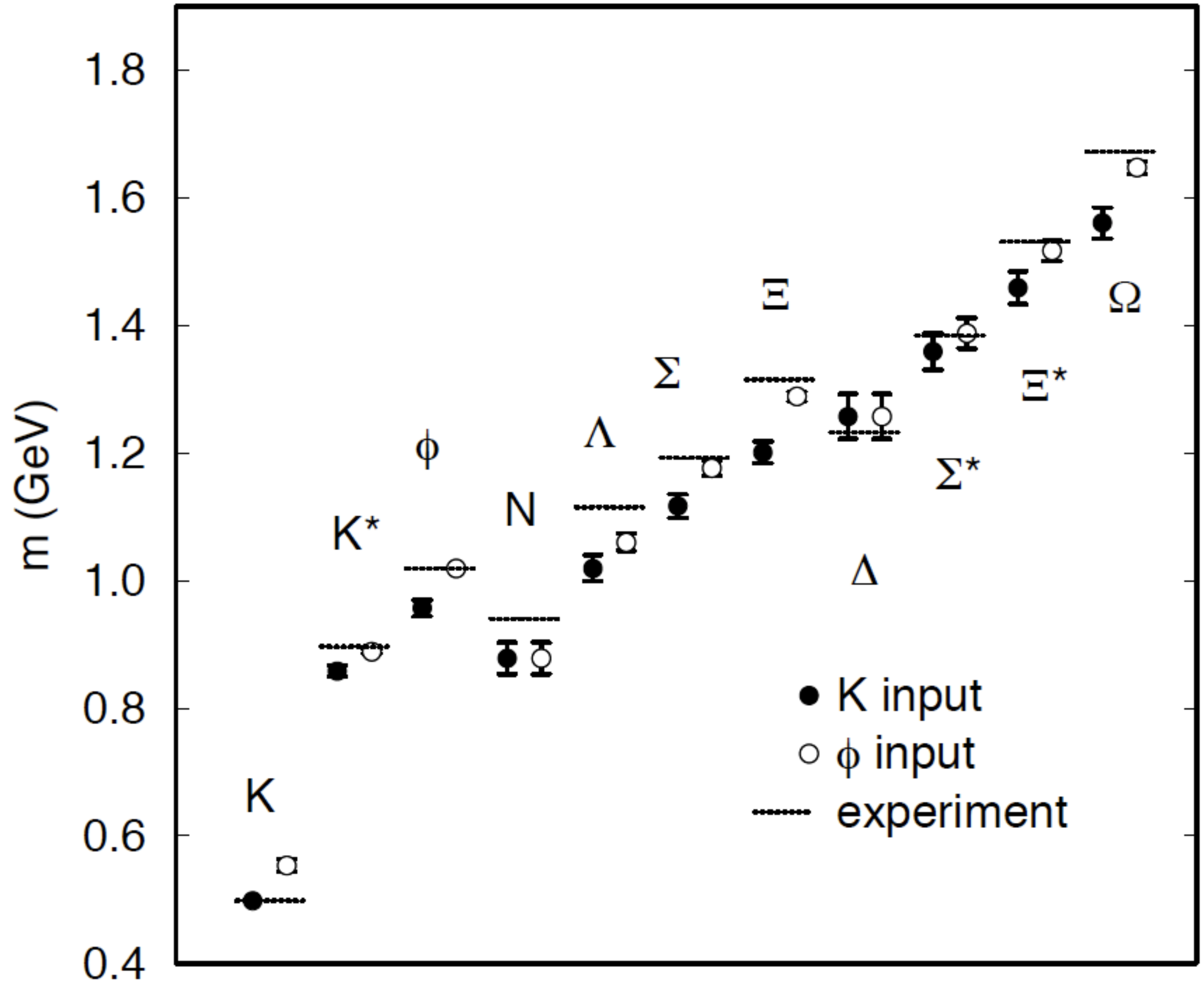
“High-Precision Lattice QCD Confronts Experiment”



HPQCD / UKQCD / MILC / Fermilab, PRL92,022001(2004)

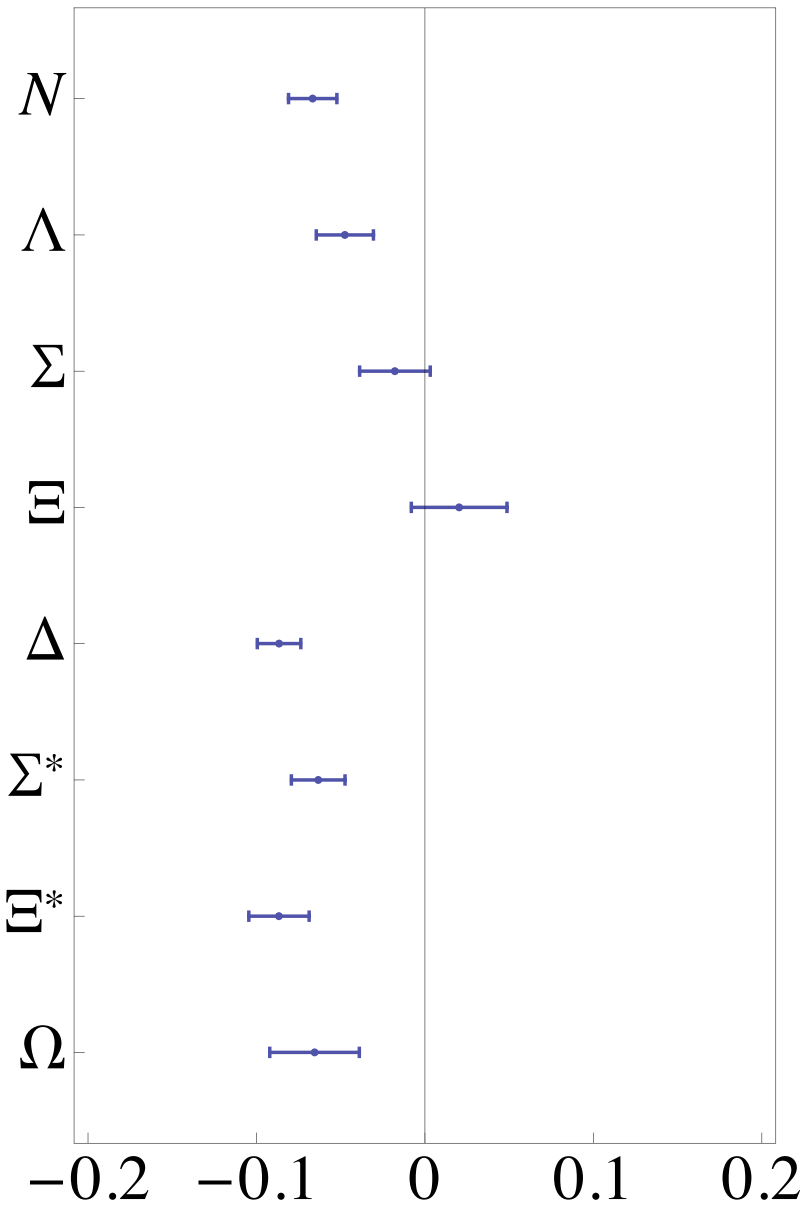
1999

"Quenched Light Hadron Spectrum"



CP-PACS, PRL84,238(2000)

Ratio plot - Quenched QCD



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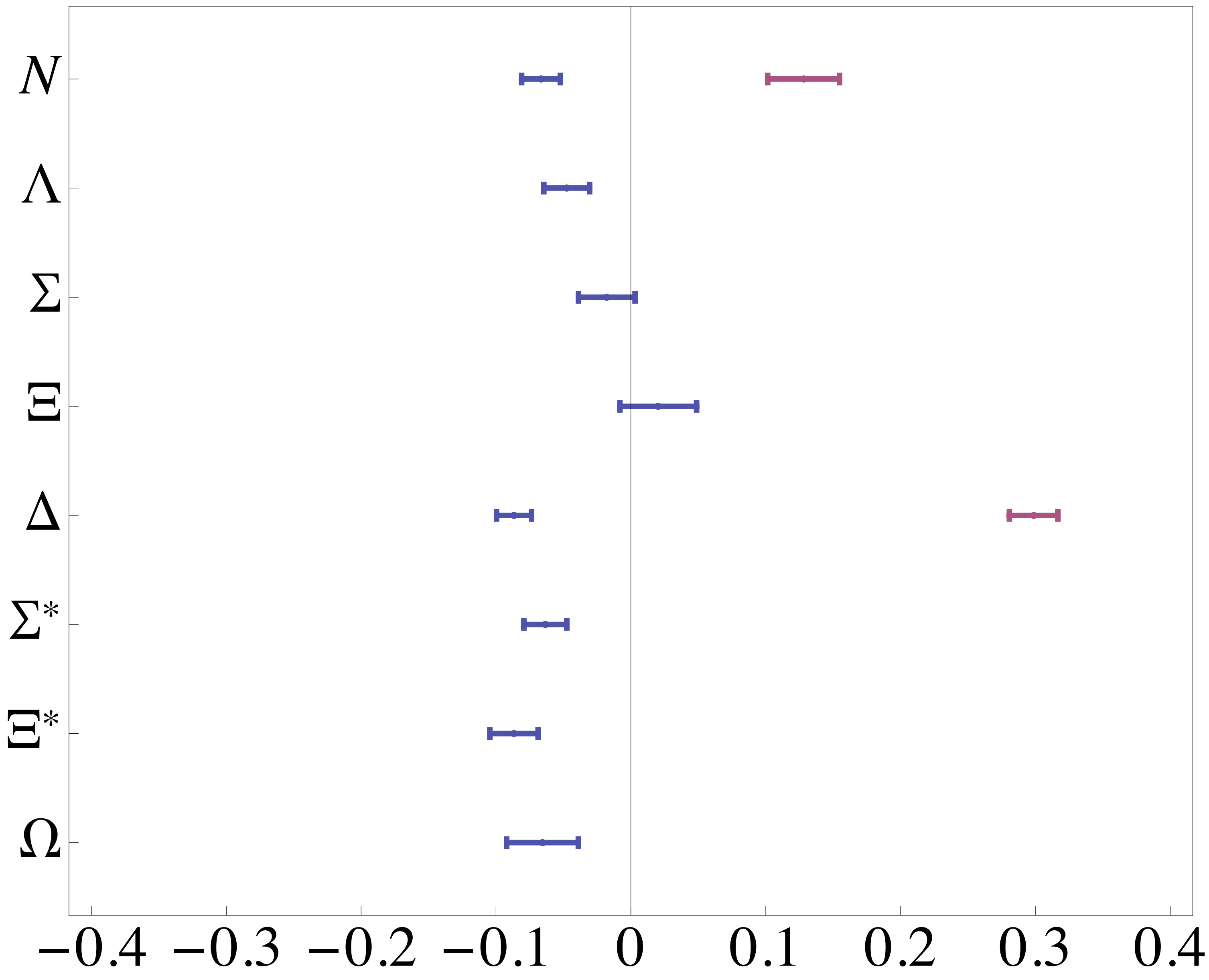
New (2004) FLIC fermion results



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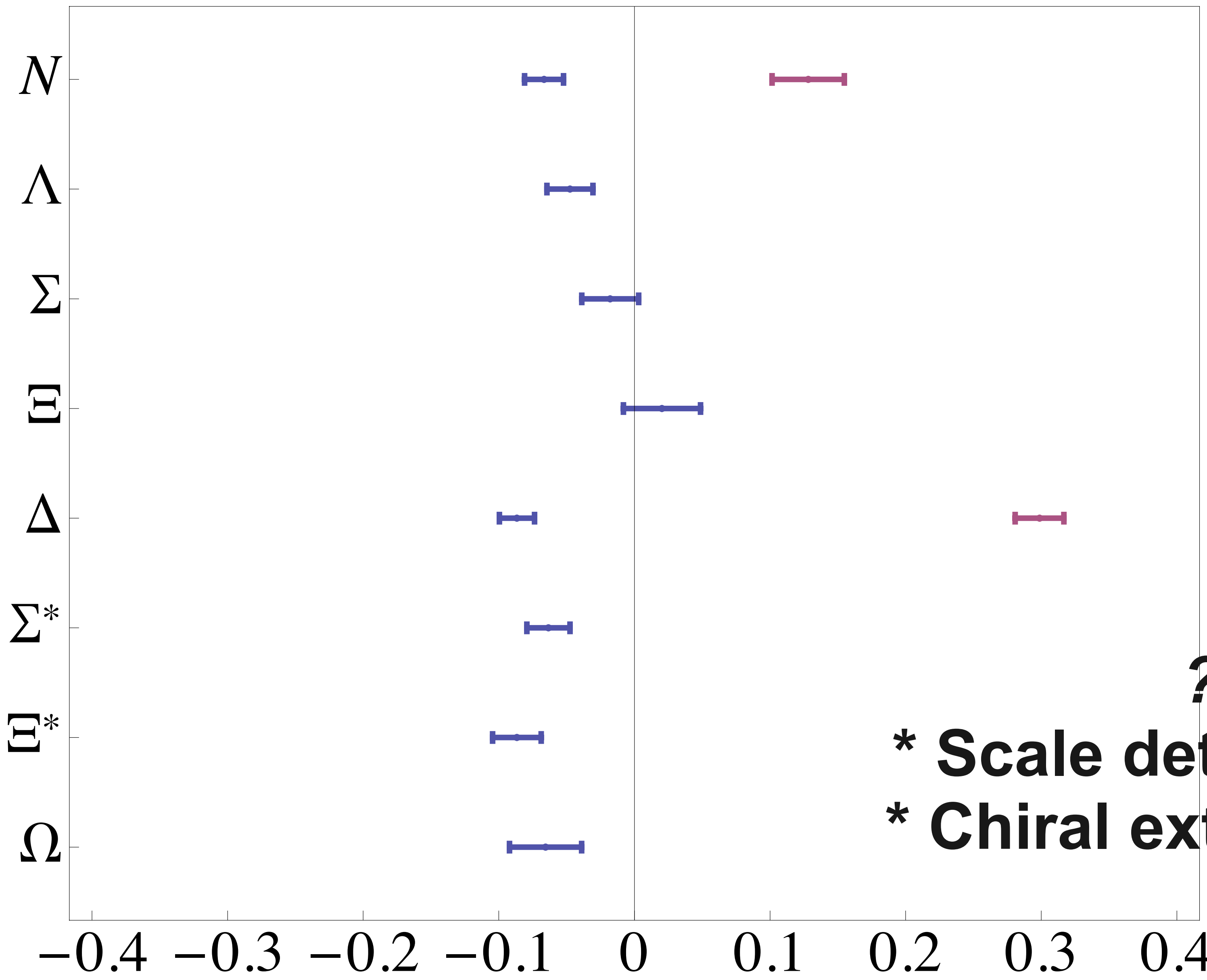


??

*** Scale determination**

*** Chiral extrapolation!**

Latest results in lattice QCD

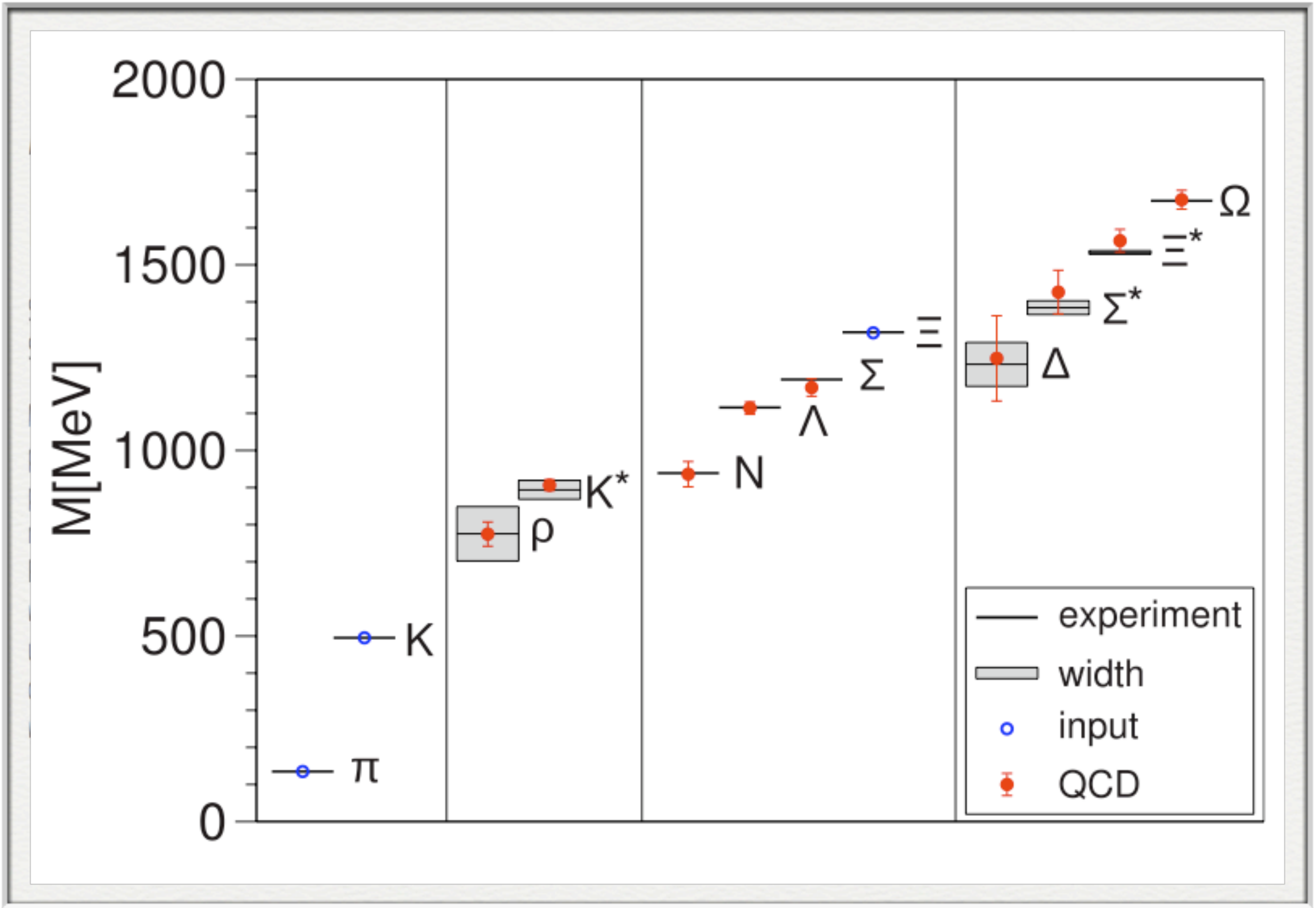
Ab Initio Determination of Light Hadron Masses

S. Dürr,[1] Z. Fodor,[1,2,3] J. Frison,[4] C. Hoelbling,[2,3,4] R. Hoffmann,[2] S. D. Katz,[2,3] S. Krieg,[2] T. Kurth,[2] L. Lellouch,[4] T. Lippert,[2,5] K. K. Szabo,[2] G. Vulvert[4]

More than 99% of the mass of the visible universe is made up of protons and neutrons. Both particles are much heavier than their quark and gluon constituents, and the Standard Model of particle physics should explain this difference. We present a full ab initio calculation of the masses of protons, neutrons, and other light hadrons, using lattice quantum chromodynamics. Pion masses down to 190 mega–electron volts are used to extrapolate to the physical point, with lattice sizes of approximately four times the inverse pion mass. Three lattice spacings are used for a continuum extrapolation. Our results completely agree with experimental observations and represent a quantitative confirmation of this aspect of the Standard Model with fully controlled uncertainties.

21 NOVEMBER 2008 VOL 322 **SCIENCE** www.sciencemag.org

Latest results in lattice QCD



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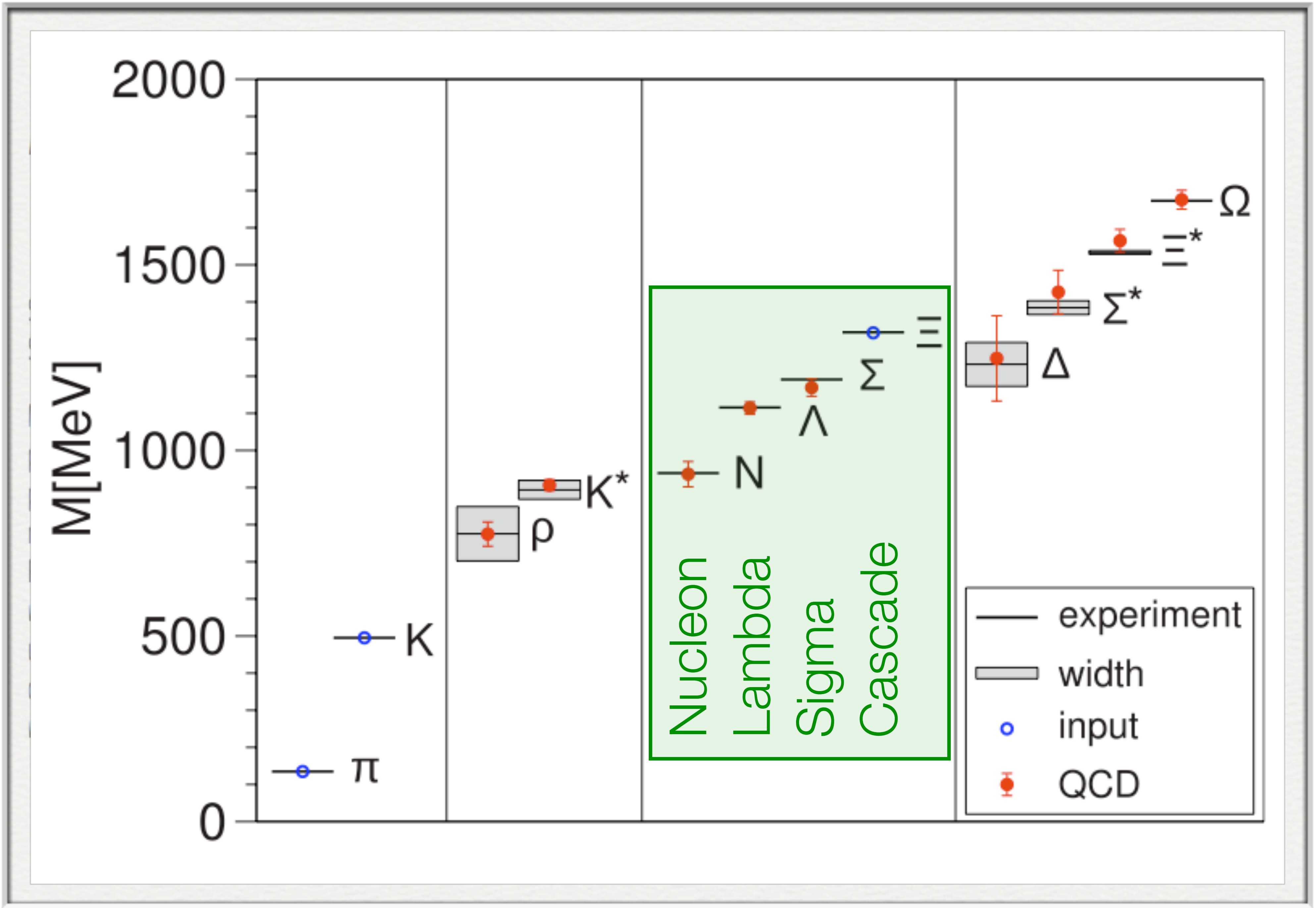
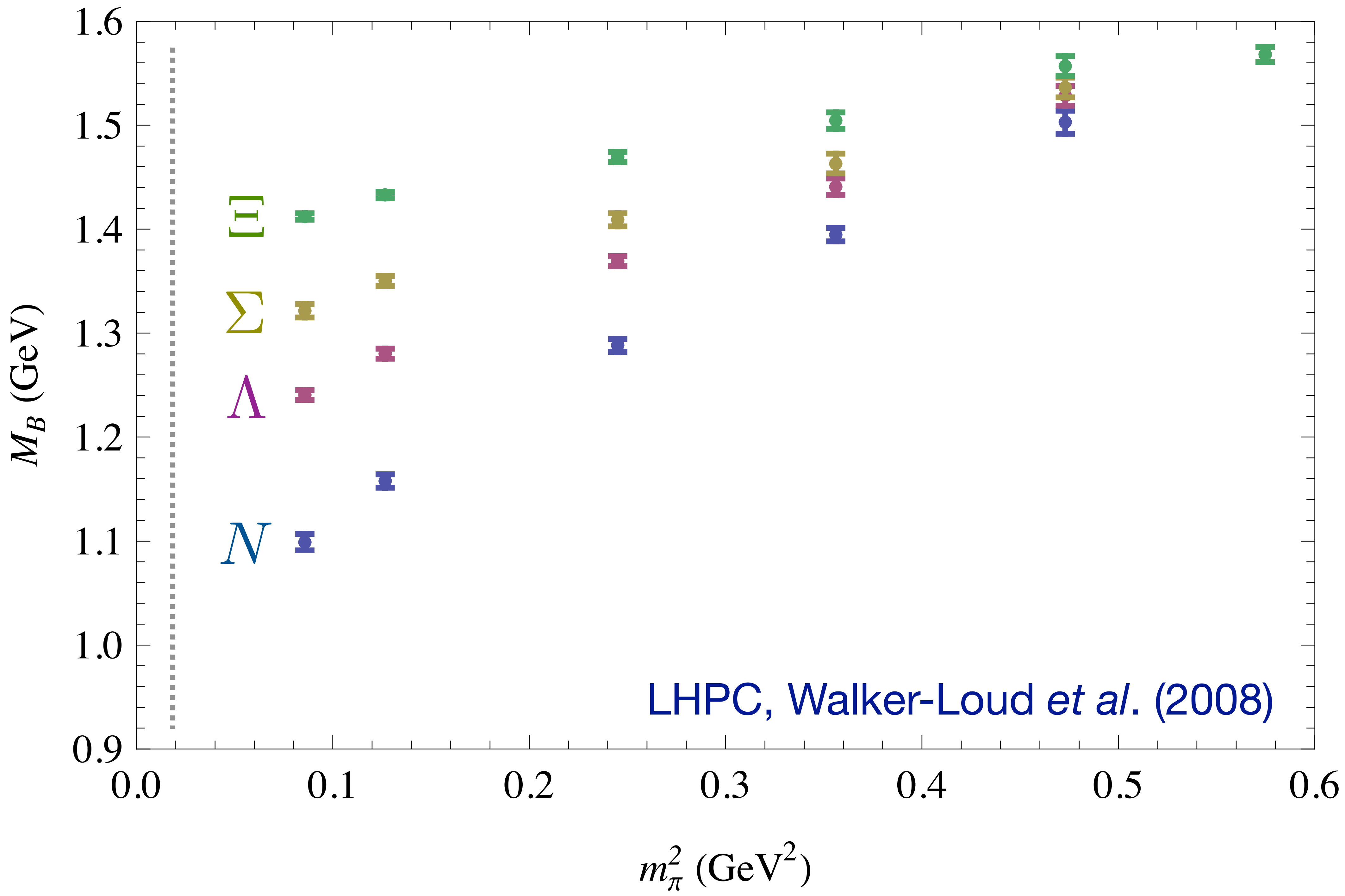


21 NOVEMBER 2008 VOL 322 **SCIENCE** www.sciencemag.org

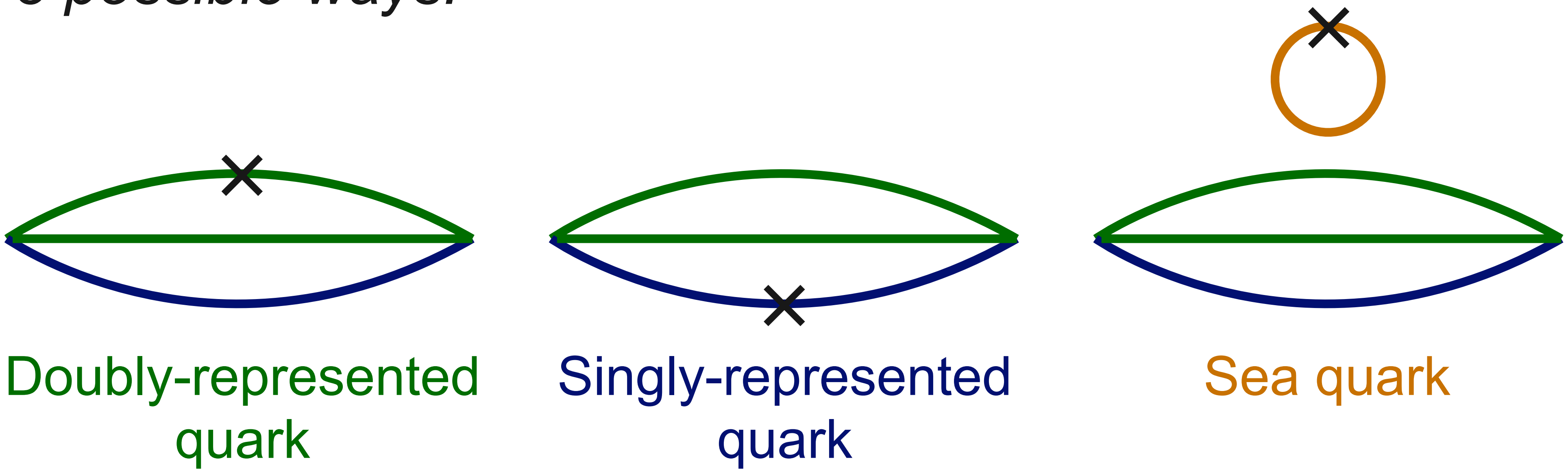
New (public) lattice results



Octet-baryon masses

N, Λ, Σ, Ξ SU(2) symmetry

- SU(3) chiral limit: $m_u = m_d = m_s = 0$
  - Octet are degenerate (one mass): M_0
- Chiral EFT perturbs about this limit
  - Leading term is a single insertion of quark mass operator
    - *3 possible ways:*



  - Lagrangian (3 parameters)

$$\mathcal{L}_{BBq} = 2\alpha_M(\bar{B}B\mathcal{M}) + 2\beta_M(\bar{B}\mathcal{M}B) + 2\sigma_M\bar{B}B\,\mathrm{Tr}(\mathcal{M})$$

Octet-baryon masses

- Leading-order expansion $O(1)$

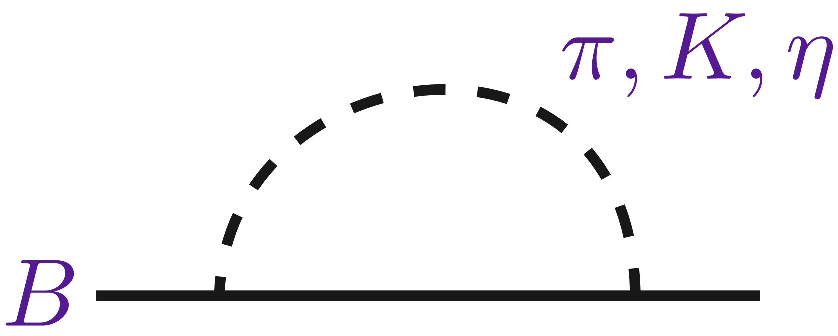
$$
\begin{aligned}
M_N &= M_0 + 2(\alpha_M + \beta_M)m_q + 2\sigma_M(2m_q + m_s) \\
M_\Lambda &= M_0 + (\alpha_M + 2\beta_M)m_q + \alpha_M m_s + 2\sigma_M(2m_q + m_s) \\
M_\Sigma &= M_0 + \frac{1}{3}(5\alpha_M + 2\beta_M)m_q + \frac{1}{3}(\alpha_M + 4\beta_M)m_s + 2\sigma_M(2m_q + m_s) \\
M_\Xi &= M_0 + \frac{1}{3}(\alpha_M + 4\beta_M)m_q + \frac{1}{3}(5\alpha_M + 2\beta_M)m_s + 2\sigma_M(2m_q + m_s)
\end{aligned}
$$

$$m_\pi^2 = 2Bm_q \quad m_K^2 = B(m_q + m_s)$$

$$m_q \to \frac{m_\pi^2}{2B}, \quad m_s \to \frac{2m_K^2 - m_\pi^2}{2B} \qquad \{\alpha, \beta, \sigma\} \to B\{\alpha', \beta', \sigma'\}$$

Beyond first derivative: Loop corrections

- At $O(3/2)$ contributions from meson dressing



- Up to overall factors, (HB) loop integral reduces to

$$\int_0^\infty dk \frac{k^4}{k^2 + m_\pi^2} = \int_0^\infty dk \frac{k^4 - m_\pi^4 + m_\pi^4}{k^2 + m_\pi^2}$$

$$= \int dk \left(k^2 - m_\pi^2\right) + \int dk \frac{m_\pi^4}{k^2 + m_\pi^2} = \frac{\pi}{2} m_\pi^3$$

- RENORMALIZATION:
  - Absorb divergences into LECs

What about Decuplet baryons?

- EFT integrates out all non-dynamical degrees of freedom
  - EFT can *only* be valid at energy scales below any physical threshold that has been integrated out
- Decuplet-*less* EFT cannot describe meson masses greater than Octet-Decuplet splitting
- Physical quark masses:

$$M_\Delta - M_N \sim 0.3\,\mathrm{GeV}$$

$$m_K\,,\, m_\eta \sim 0.5\,\mathrm{GeV}$$

  - For physical strange-quark mass, an EFT that includes all dynamical degrees of freedom must include Decuplet
- I don't know if there is a unique way to include Decuplet
  - Including Decuplet: perhaps model dependent
  - Not including Decuplet: no longer an EFT

My power-counting for including Decuplet

- Assume Octet and Decuplet-baryons degenerate
- Include all (Octet and Decuplet) loop contributions to a given order in the quark mass
- Evaluate Decuplet loop integrals with explicit mass-splitting in the relevant propagators
  - Ensures physical threshold and IR branch structure maintained
- Renormalize such that the Decuplet-*less* EFT is recovered in the limit $m_{PS} \to 0 \ll \Delta$

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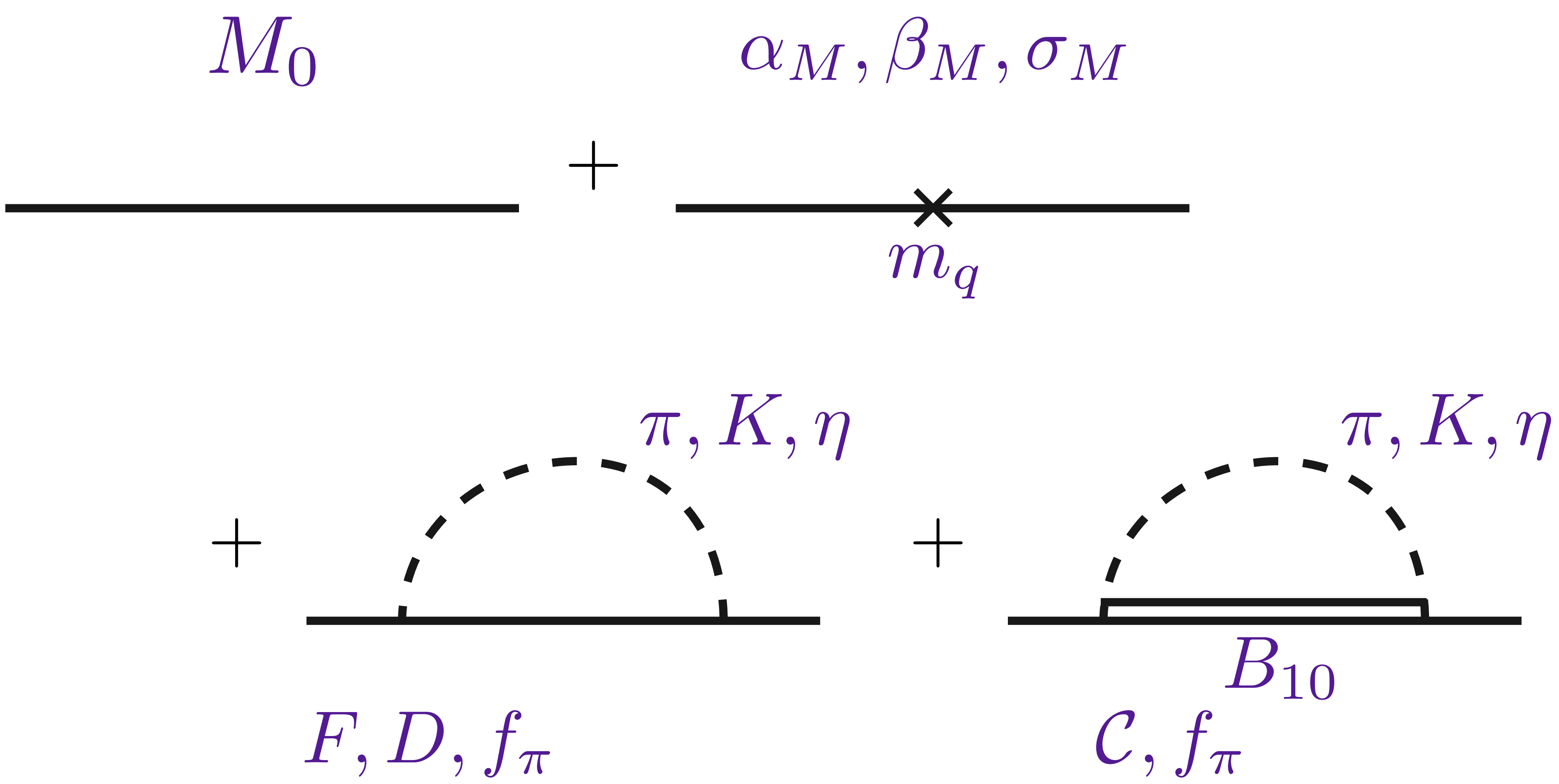
I am not claiming this is better than anyone else's method for including decuplet

Everything to O(3/2)



Inputs: $g_A = 1.267, D = \frac{3}{5} g_A, F + \frac{2}{5} g_A, \mathcal{C} = -2D, f_\pi = 0.087\,\mathrm{GeV}$

$$M_B \quad = M_0 + \delta M^{(1)} + \delta M^{(3/2)} + \mathcal{O}(2)$$

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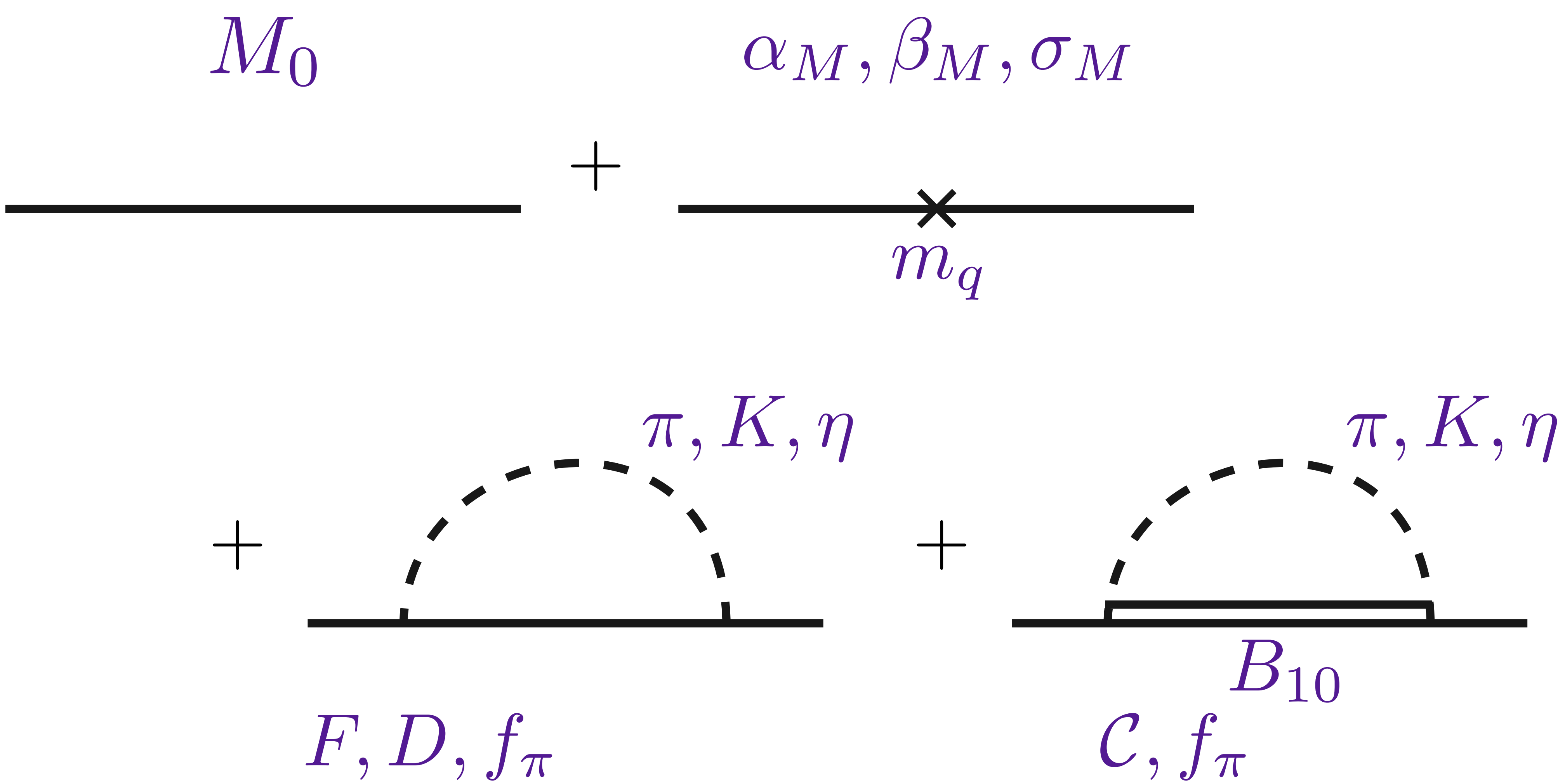

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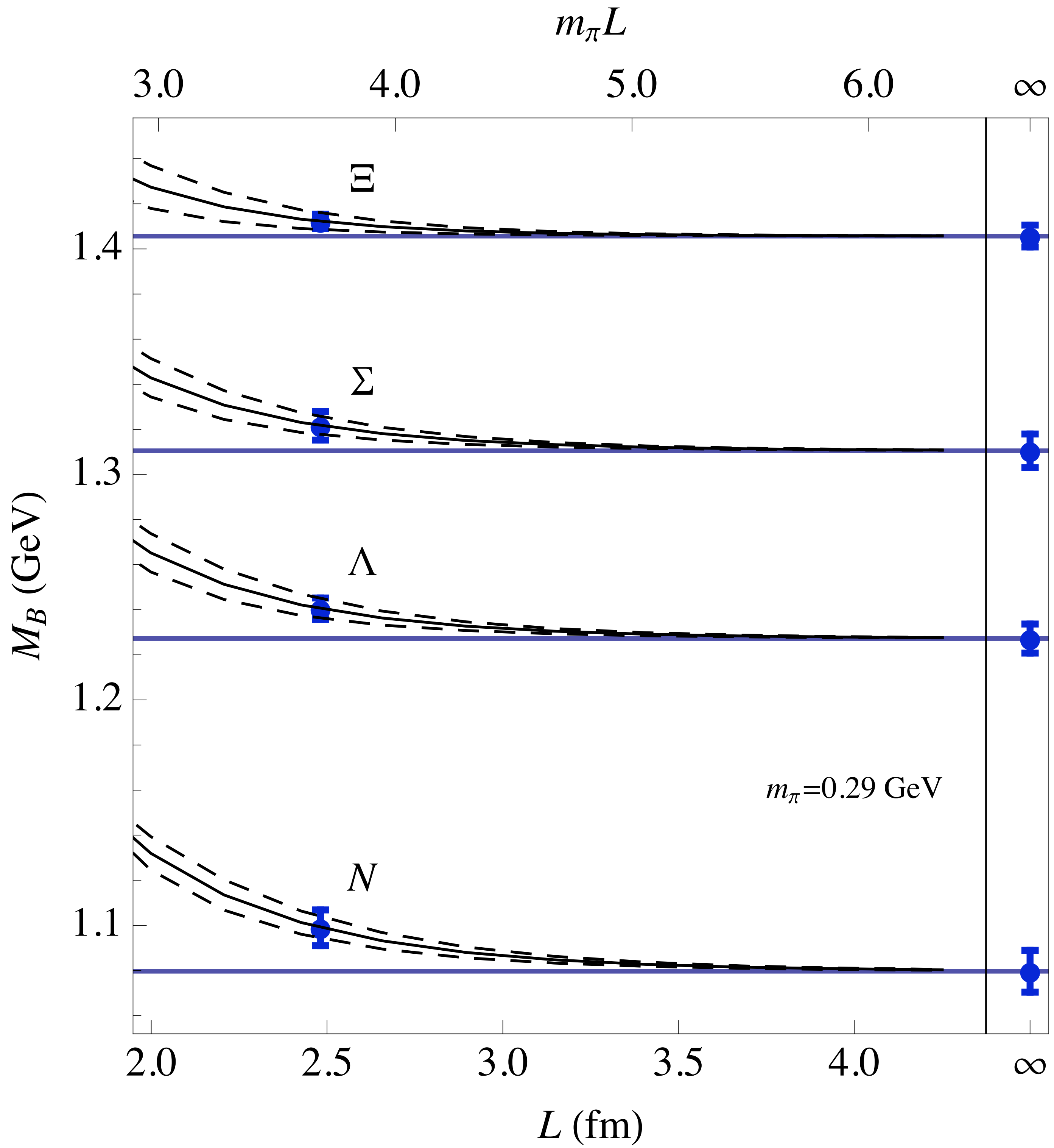
$\pm 15\%$ $\pm 15\%$ $\pm 15\%$ $\pm 5\%$

$$M_B = M_0 + \delta M^{(1)} + \delta M^{(3/2)} + \mathcal{O}(2)$$

Correct for lattice volume

- Same loop integrals describe leading finite-volume correction
- FV corrections purely infrared – should not be sensitive to UV regularisation
- Error estimate reflects FV corrections evaluated with and without a cutoff



Power counting estimate for O(2)

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- If we adopt conventional wisdom “4 pi fpi”
  - Physical point

$$\mathcal{O}(2) \sim \frac{m_\eta^4}{(4\pi f_\pi)^4} \sim 5\%$$

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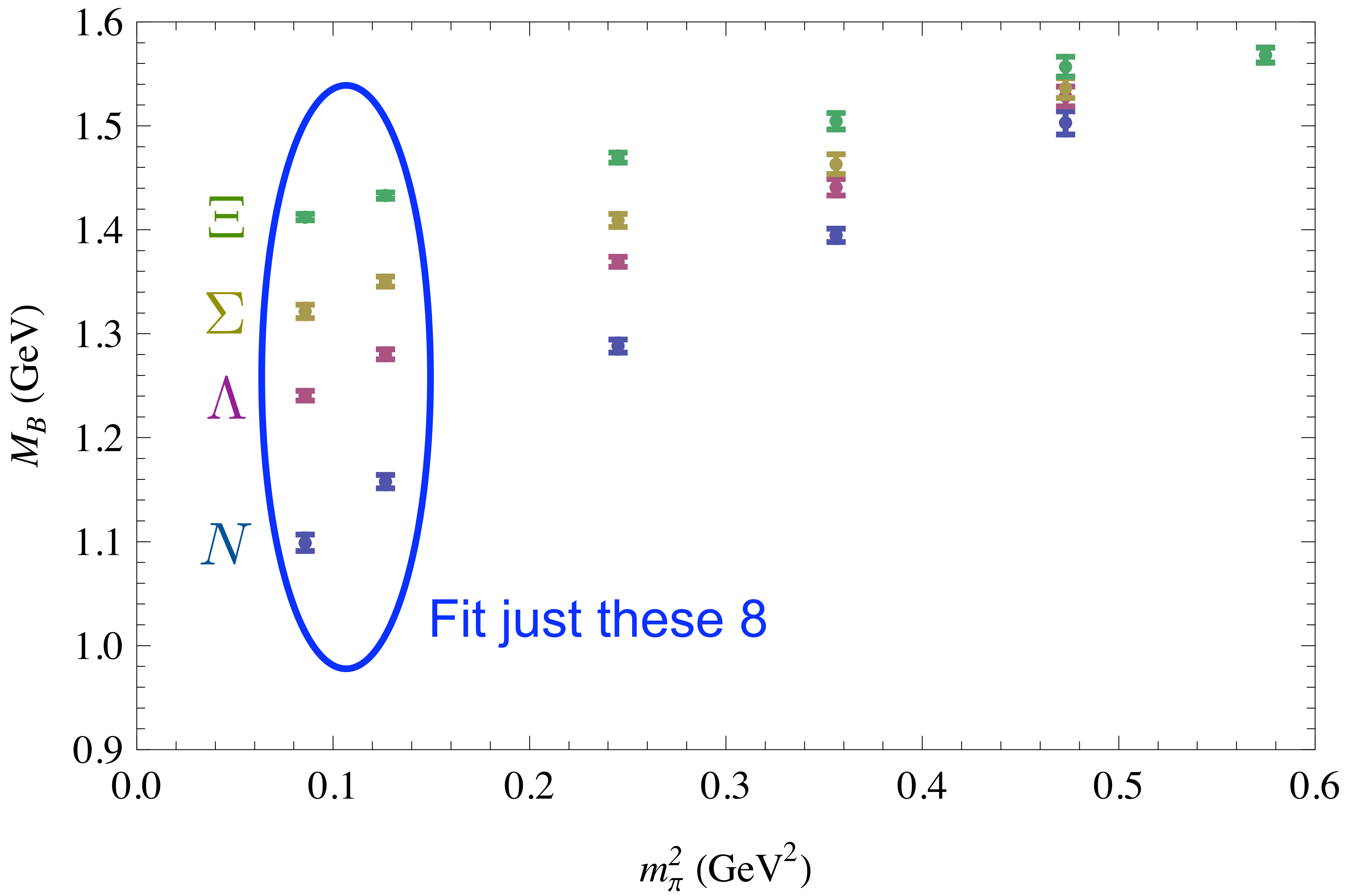
  - Lattice masses

$$\mathcal{O}(2) \sim \frac{m_\eta^4}{(4\pi f_\pi)^4} \sim 11\%$$

Lattice Simulation Results: LHPC



Best fit to lightest 2 quark masses

- Poor fit

$$\chi^2/\mathrm{dof} \sim 40$$

$$m_\pi \lesssim 0.35\,\mathrm{GeV}$$
$$m_K \lesssim 0.6\,\mathrm{GeV}$$

- "Best" fit $M_0 \sim 0.27\,\mathrm{GeV}$
- *Empirical suggestion*

$$\mathcal{O}(2) \sim \left(\frac{m_\eta}{\Lambda_B}\right)^4 \sim 300\%$$

$$\Lambda_B \sim 0.6\,\mathrm{GeV}$$

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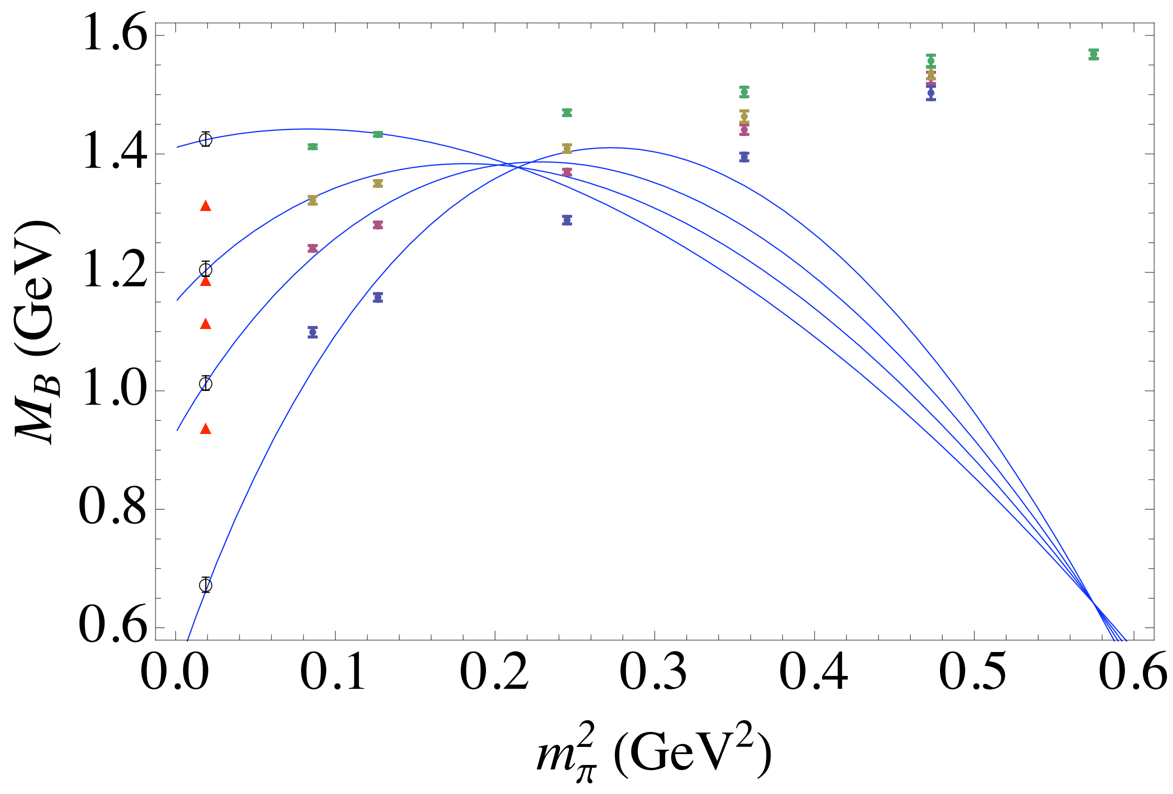

What about Finite-Range Regularisation (FRR)?

- Introduce a resummation of higher-order terms with a single parameter
- Chiral loop integrals modified to cut off divergences

$$\int_0^\infty dk \, \frac{k^4}{k^2+m^2} \left(\frac{\Lambda^2}{\Lambda^2+k^2} \right)^4$$

- Upon renormalisation gives identical expansion to *O*(3/2)

Text book: $M_B^{(3/2)} = M_0 + \delta M^{(1)} + \delta M^{(3/2)} + 0$

FRR: $M_B^{(3/2)} = M_0 + \delta M^{(1)} + \delta M^{(3/2)} + \mathcal{O}(\frac{m_{PS}^4}{\Lambda})$

Regularisation parameter?

- Model-indepence of EFT only exists if results independent of this cutoff
- Can the lattice results select a preferred scale to regularise the EFT?

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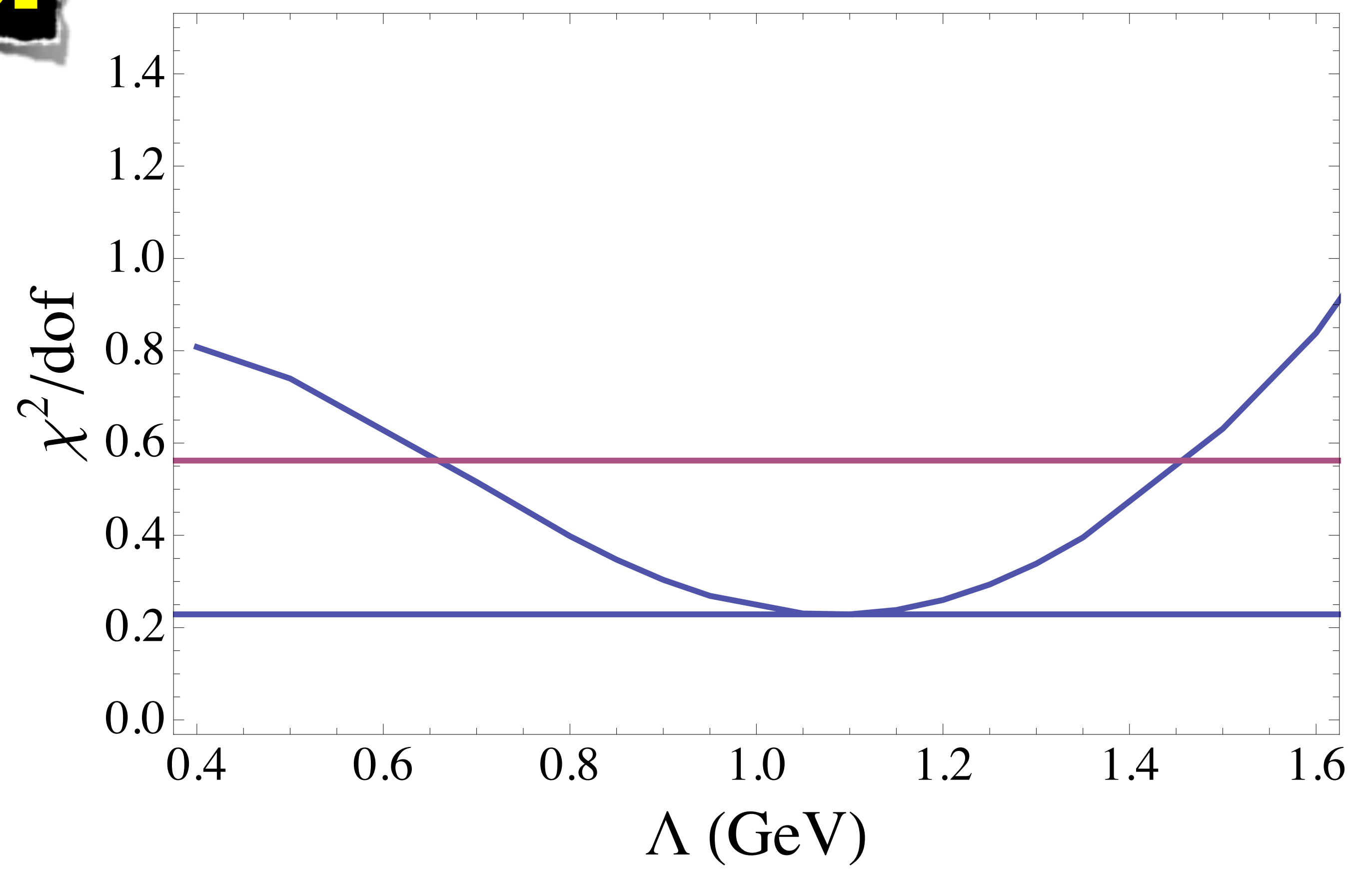
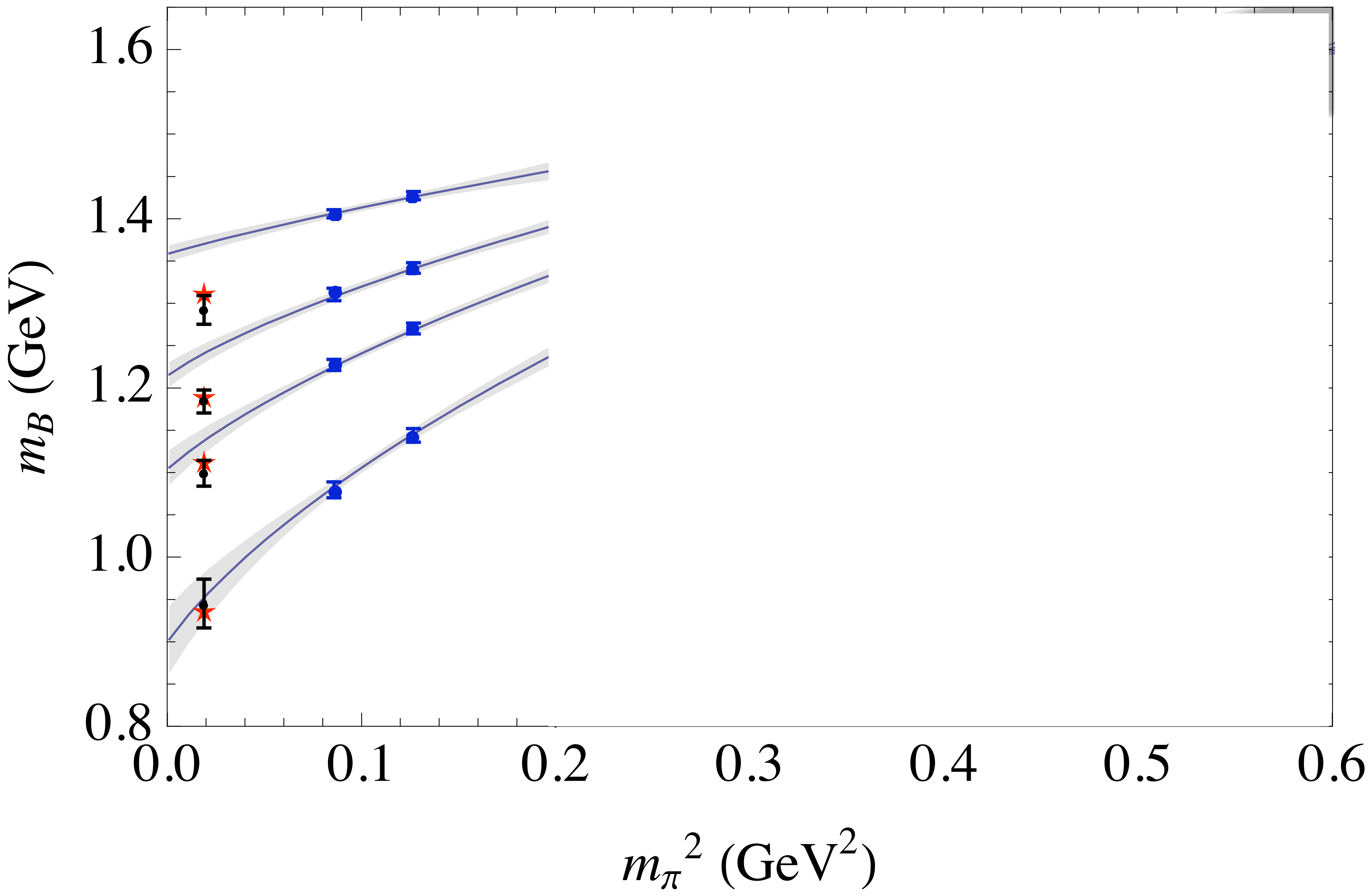
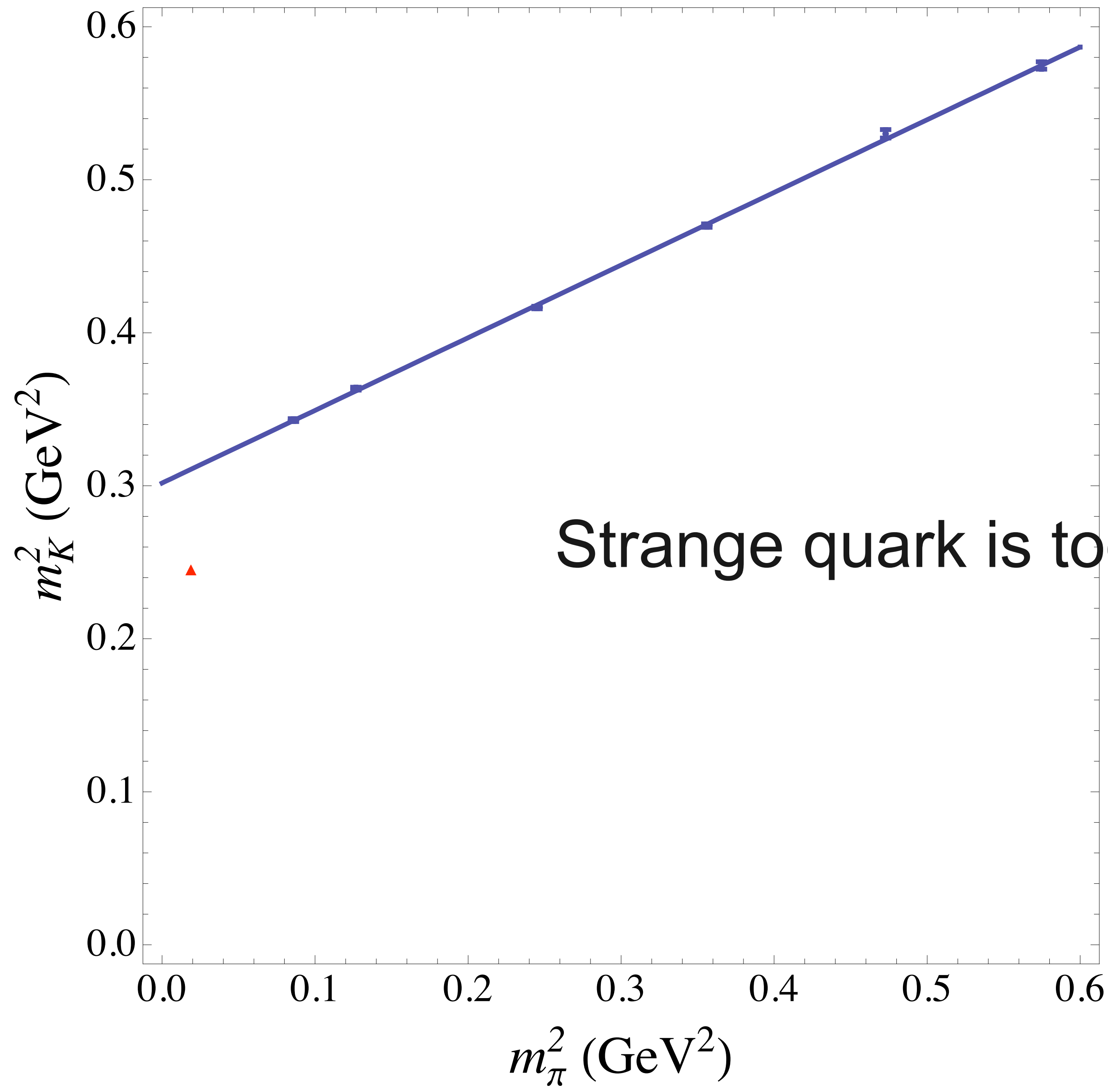
YES!



Fits to 2 lightest LHPC points

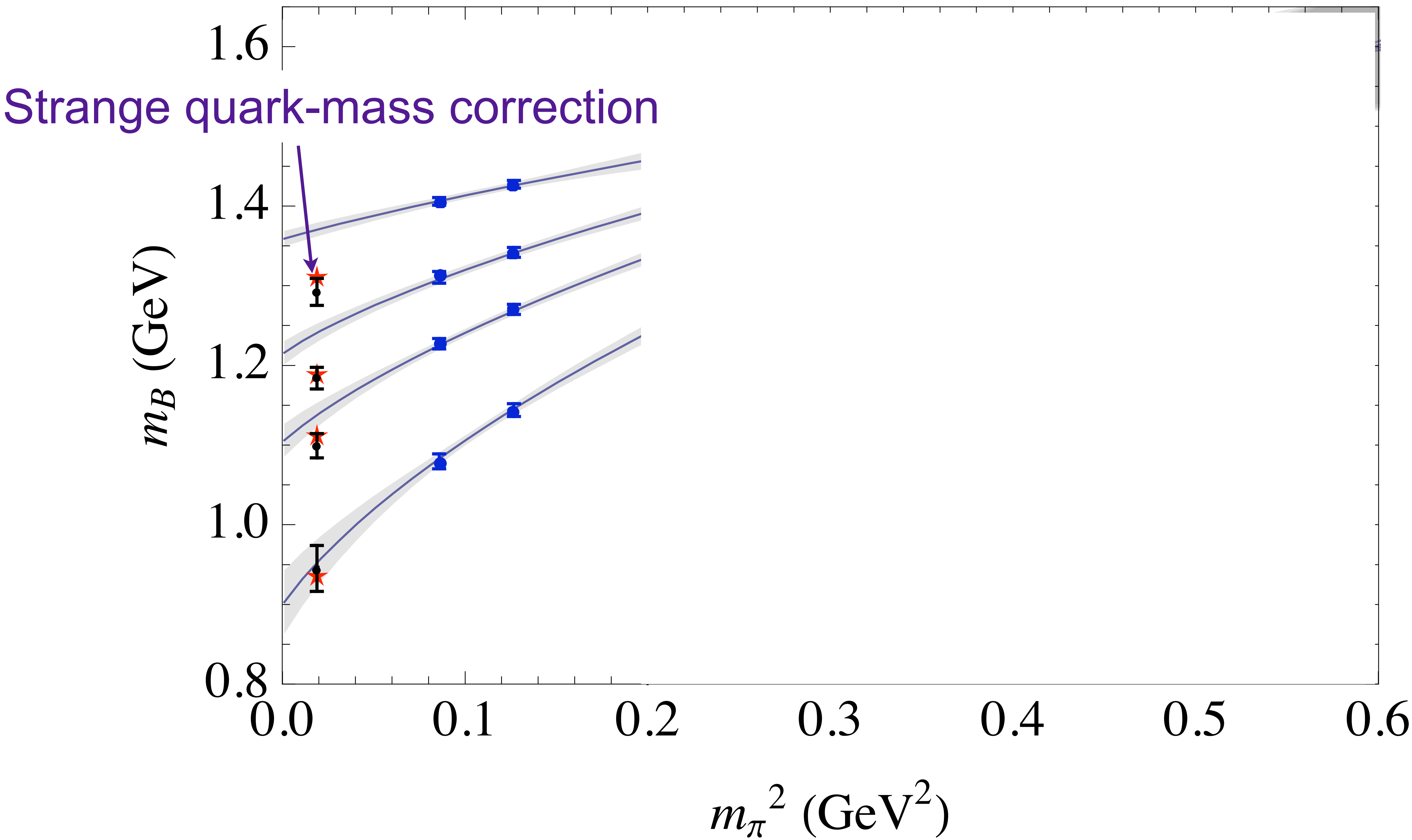


Meson masses - LHPC



Strange quark is too heavy!

Fits to 2 lightest LHPC points



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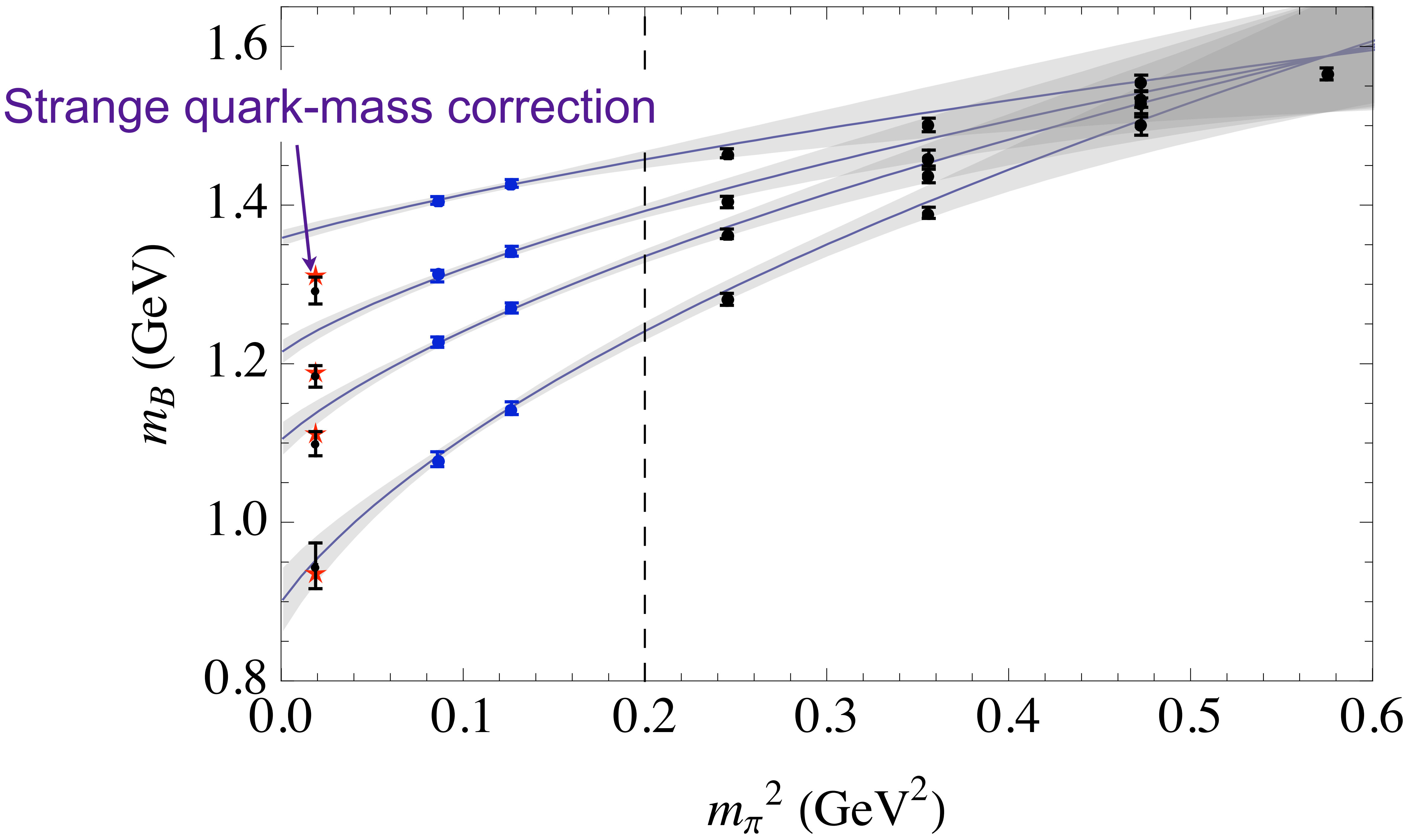
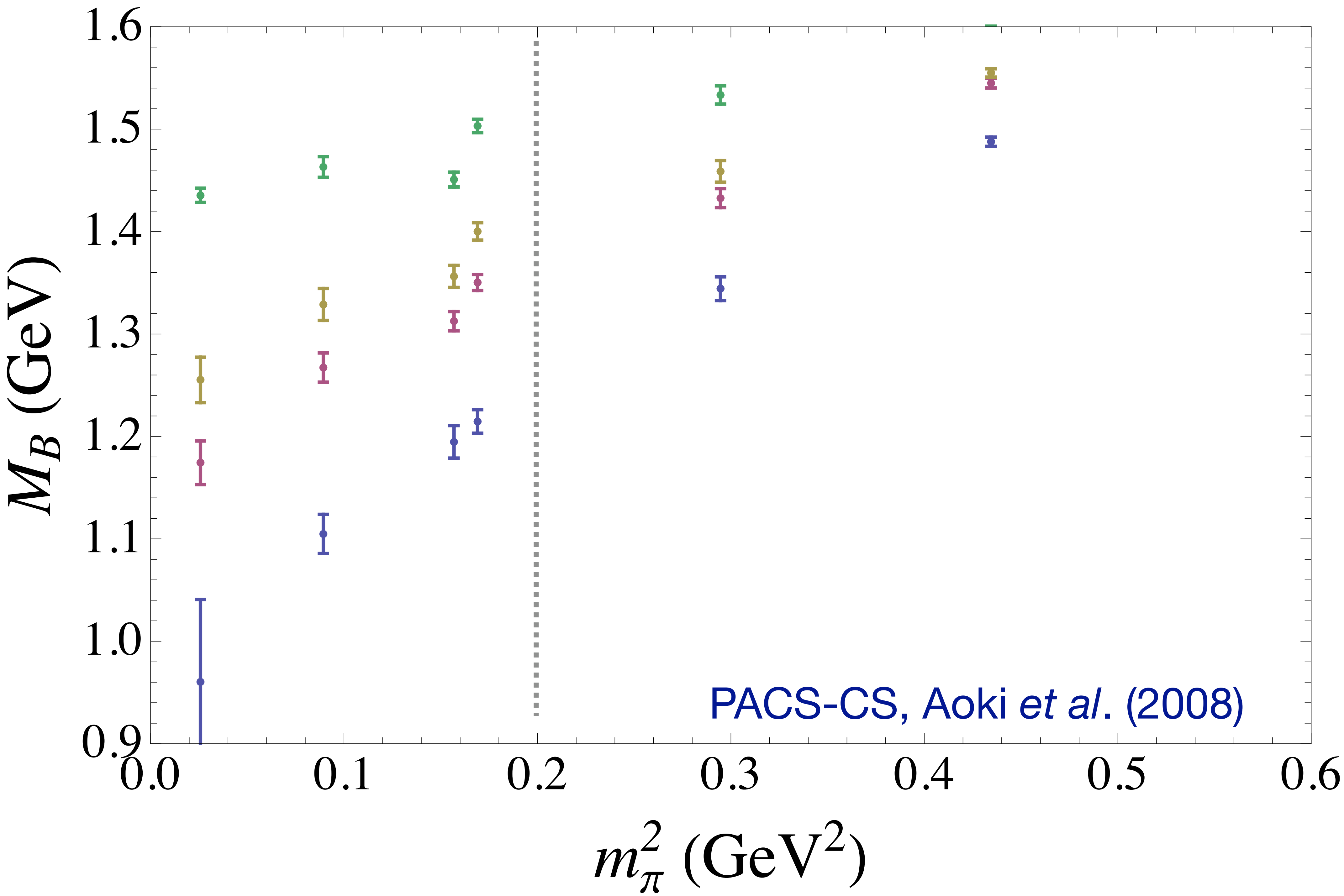
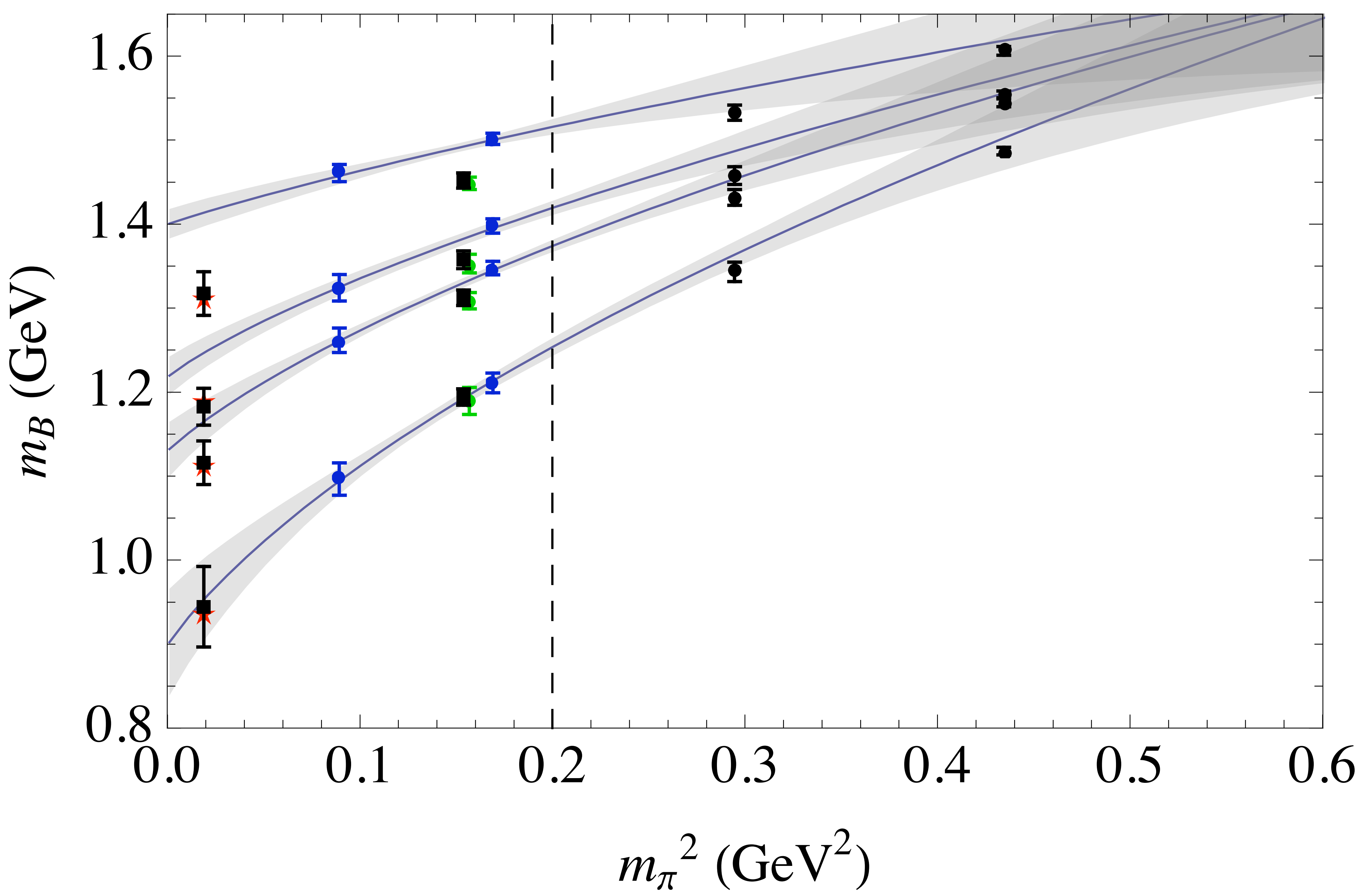


More new lattice results: PACS-CS



Fit to 2 (blue) PACS-CS points - fixed strange mass



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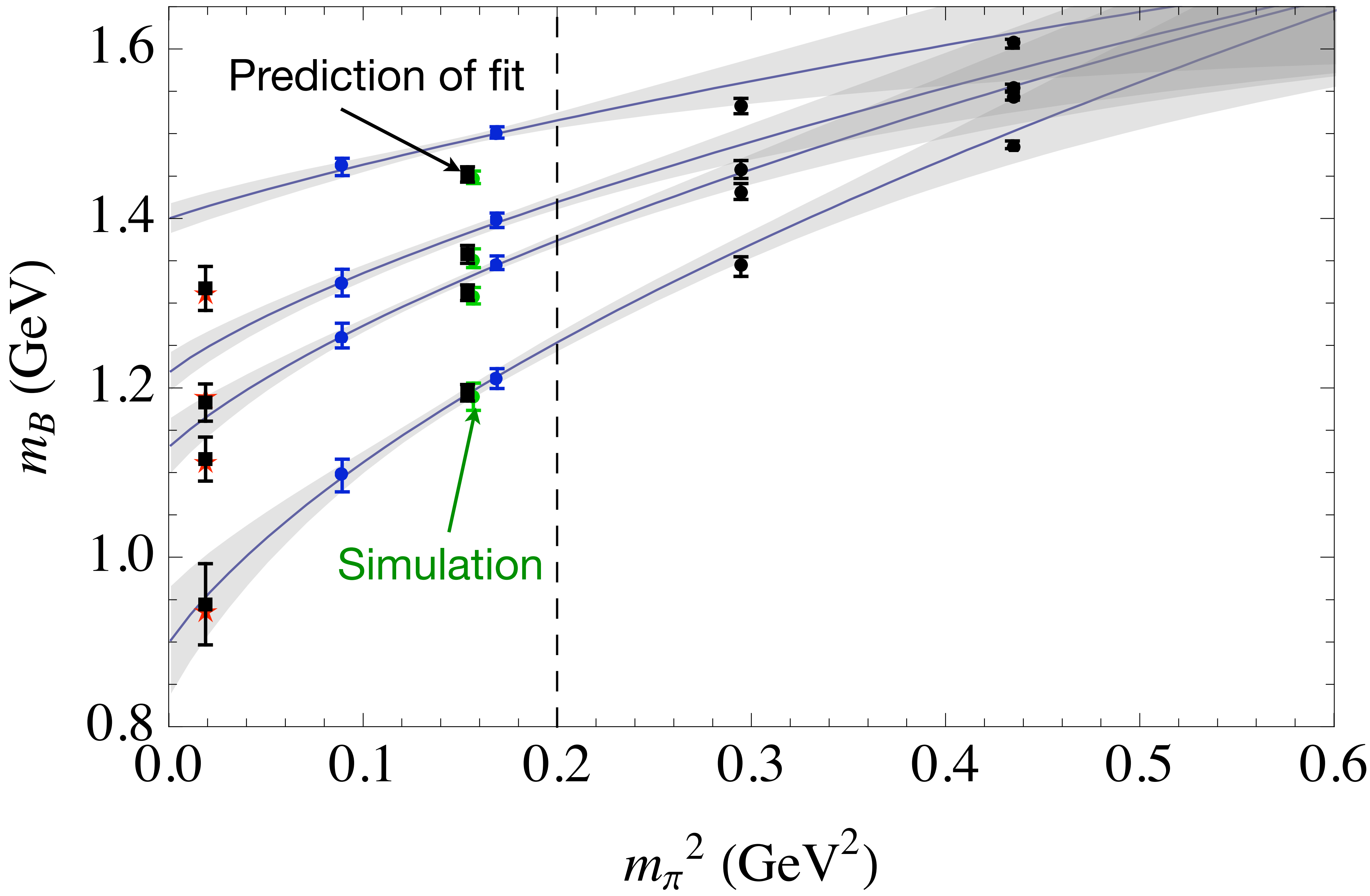

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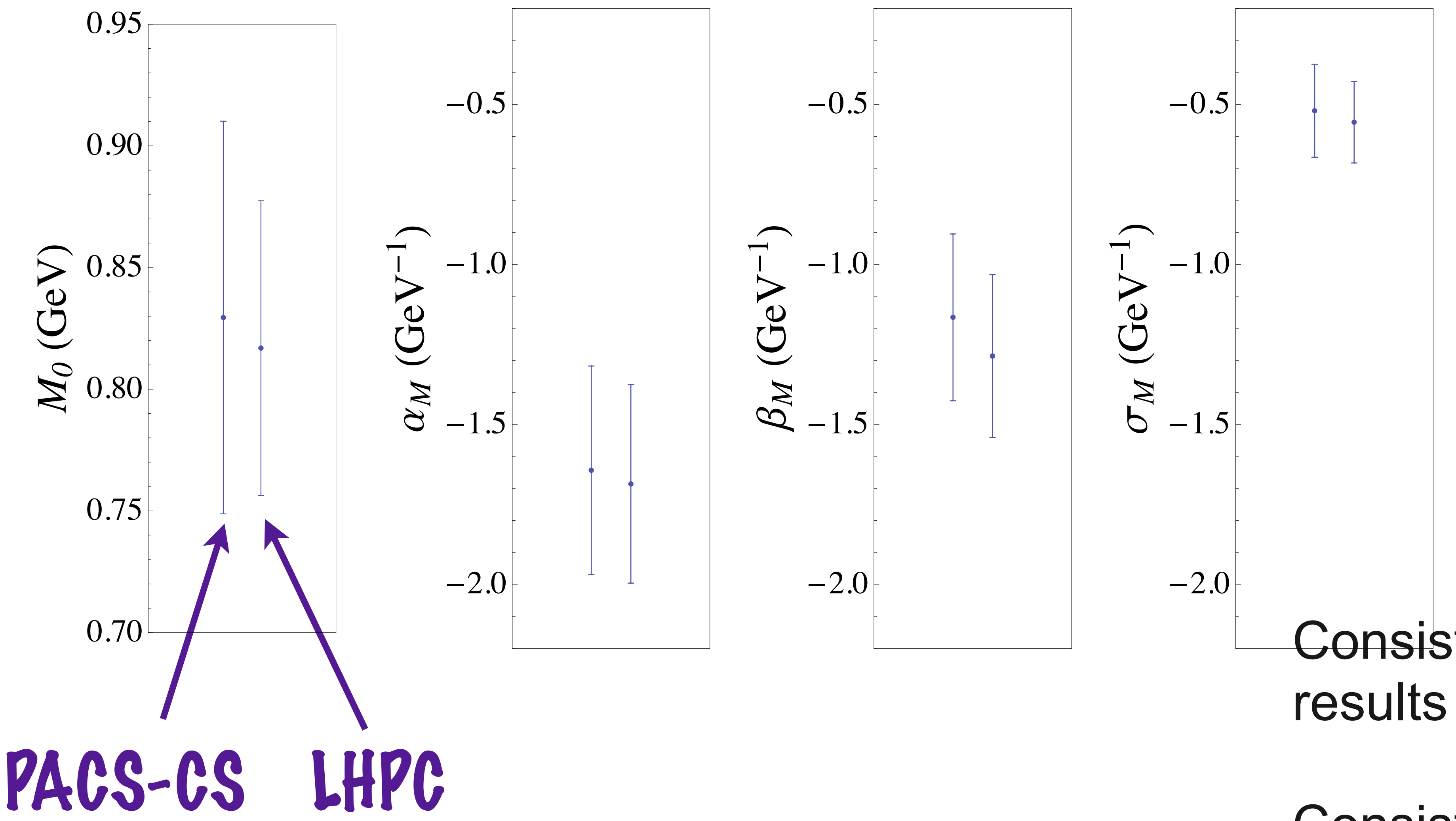
Consistency in LECs?



Consistent extrapolation of lattice results to SU(3) chiral limit

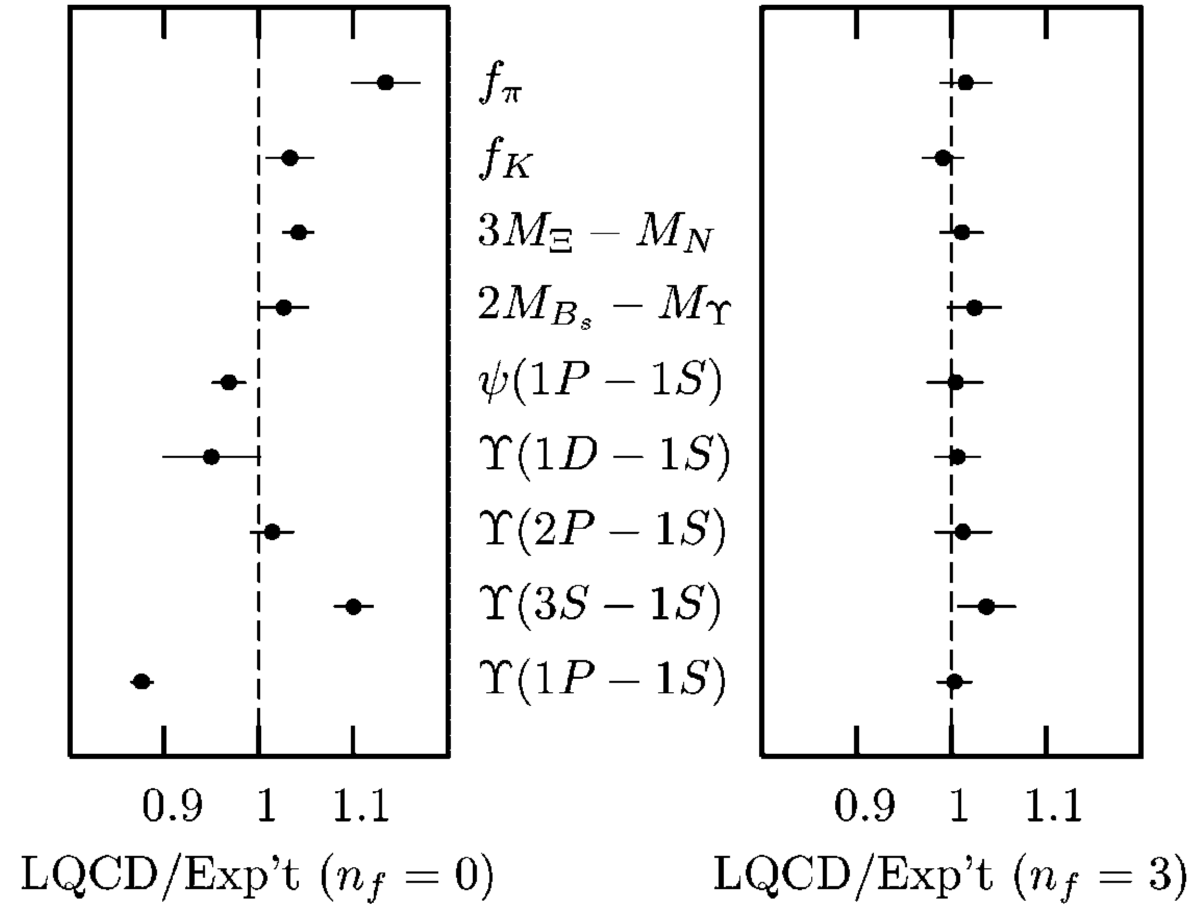
Consistency of lattice at finite "*a*"

	M_0	α_M	β_M	σ_M
LHPC	0.82(6)	1.69(31)	1.29(26)	0.56(13)
PACS-CS	0.83(8)	1.64(33)	1.17(26)	0.52(15)

Test of scale determination

B	Mass (GeV)	Expt.
N	0.945(24)(4)(3)	0.939
Λ	1.103(13)(9)(3)	1.116
Σ	1.182(11)(2)(6)	1.193
Ξ	1.301(12)(9)(1)	1.318

Statistics   Discretisation   “Model”



HPQCD / UKQCD / MILC / Fermilab, PRL92,022001(2004)

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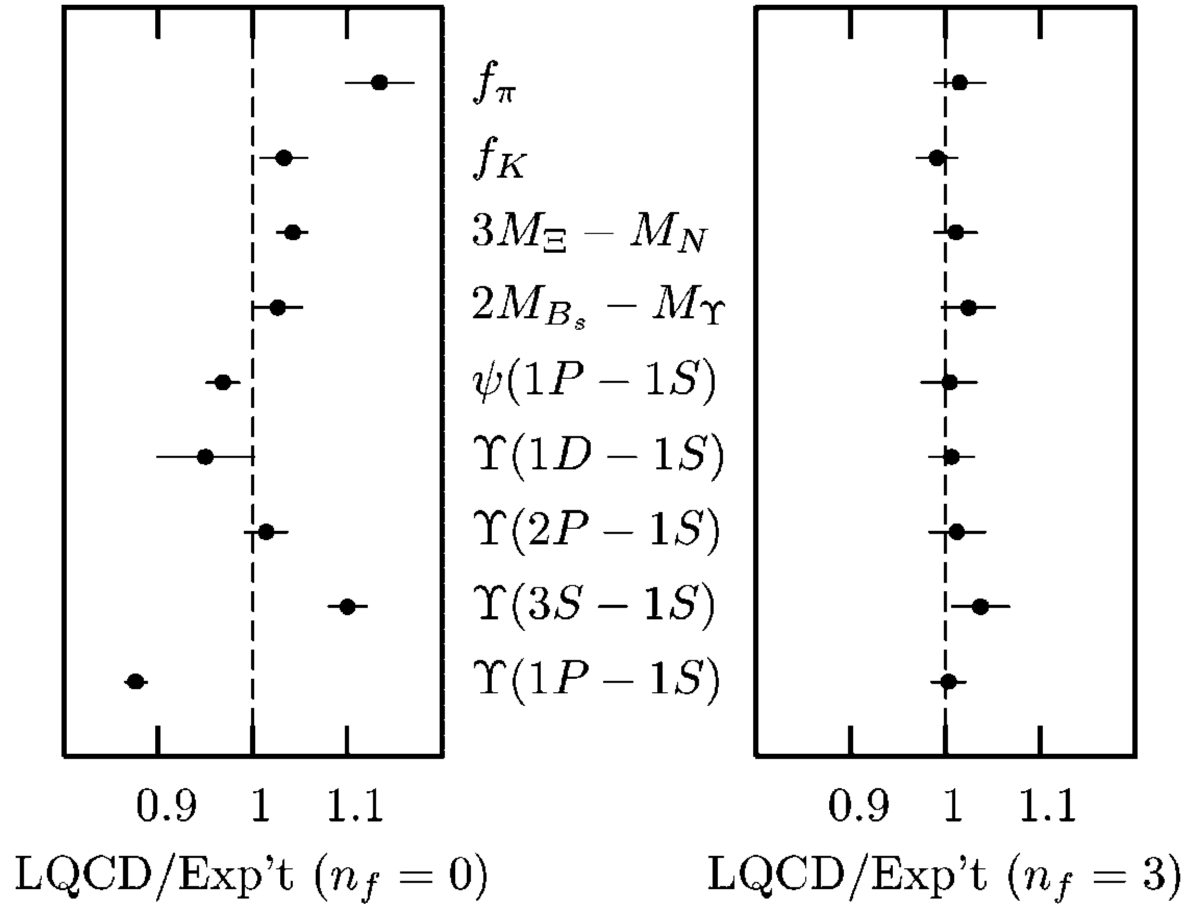

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Excellent confirmation of consistent scale determination

Strangeness content of the nucleon

- Gell-Mann–Okubo Relation and Pion-Nucleon sigma term $M_N, M_\Lambda, M_\Sigma, M_\Xi$

$$m_s\langle N|\bar{s}s|N\rangle \simeq 335 \pm 132\,\mathrm{MeV}$$

Nelson & Kaplan PLB(1987)

$$\sim M_N^{phys} - M_N^{SU(3)chiral\ limit}$$

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QCD Lagrangian $\sim \ldots \bar{s}(\not{D} + m_s)s$

$$m_s\langle N|\bar{s}s|N\rangle = m_s \frac{\partial M_N}{\partial m_s}$$

evaluated at *physical* point!

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Improved Effective Field Theory estimate

$$m_s \frac{\partial M_N}{\partial m_s} = 113 \pm 108\,\mathrm{MeV}$$

Borasoy & Meissner (1997)

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Improved Effective Field Theory estimate

Lattice? $m_s\frac{\partial M_N}{\partial m_s} = 113 \pm 108\,\text{MeV}$

Borasoy & Meissner (1997)

Beyond the masses

- Absolute masses competitive precision with recent Science article
- Can determine the sensitivity of observables to strange-quark mass
  - Important for lattice QCD: fine-tuning the strange-quark mass is computationally expensive
- Can extract strangeness nucleon sigma term

$$\bar{\sigma}_s = \frac{m_s}{M_N}\frac{\partial M_N}{\partial m_s} = 0.033(16)(4)(2)$$

Statistics — Lattice discretisation — Model-dependence

Strangeness scalar term is small

see also Toussaint & Freeman (arXiv:0905.2432): $0.063(6)(9)$

Important for dark matter searches

Direct dark matter detection

- "Hadronic uncertainties in the elastic scattering of supersymmetric dark matter", Ellis *et al*. PRD77(2008)
- Spin-independent neutralino scattering cross section

$$\sigma_{\rm SI} = \frac{4m_r^2}{\pi}[Zf_p + (A-Z)f_n]^2,$$

$$\frac{f_N}{m_N} = \sum_{q=\rm u,d,s} f_{T_q}^{(N)} \frac{\alpha_{3q}}{m_q} + \frac{2}{27} f_{TG}^{(N)} \sum_{q=\rm c,b,t} \frac{\alpha_{3q}}{m_q}$$

- Light quarks: $\alpha_{3q} \sim c m_q$

$$\bar{\sigma}_{\{u,d,s\}} = f_{T\{u,d,s\}}^{(p)} \sim \{0.027, 0.039, 0.363\}$$

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New result: ~0.033

please wait: Giedt et al.

Summary

- Robust chiral extrapolation
  - Excellent precision in absolute mass determination
  - Confirmation of scale determination efforts by HPQCD/MILC etc.
  - Moderates the fine-tuning problem of the strange quark mass

- Strangeness sigma term is small

- New opportunities for precision baryon studies in lattice QCD

Finite-volume correction (estimated in EFT)

- Lightest LHPC simulation