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Isospin symmetry breaking

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Plan

- Separation of strong and electromagnetic interactions:

  "Purely strong" pion decay constant

  Quark masses in lattice QCD

- More on isospin breaking:

  Unitary cusp in three-particle decays
  K_{e4} *decays*
  Isospin breaking in πN *scattering*

- Conclusions

Separation of strong and electromagnetic interactions

The paradise world

- At low energy, Nature is described by the effective theory of QCD + QED at $m_u \neq m_d \neq m_s \cdots$ and $e \neq 0$
- Paradise world is described by the effective theory of pure QCD at $m_u = m_d = \hat{m}$ and $e = 0$

  Particle masses in the isospin multiplets equal

  Relations between amplitudes

  $\cdots$
- Most of theoretical predictions refer to the paradise world
- Comparison of the theory with experiment:

  Check the relations which are exact in the paradise world

  Purify the experimental data with respect to the isospin-breaking effects

$$H = H_S + \alpha H_{\text{em}} + (m_d - m_u) H_m + o(\alpha, m_d - m_u)$$

Is the splitting unambiguous?

Example: extracting pion decay constant

The Lagrangian:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{p^2} + \mathcal{L}_{p^4} + \cdots + \text{terms with leptons}$$

$$\mathcal{L}_{p^2} = \frac{F_0^2}{4}\langle d^\mu U d_\mu U^\dagger + \chi^\dagger U + \chi U^\dagger\rangle + e^2 F_0^4 Z\langle QUQU^\dagger\rangle - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \text{gauge fixing}$$

$$\mathcal{L}_{p^4} = L_1\langle d^\mu U d_\mu U^\dagger\rangle^2 + L_2\langle d^\mu U^\dagger d^\nu U\rangle\langle d_\mu U^\dagger d_\nu U\rangle + \cdots + e^2 F_0^2 K_1\langle Q^2\rangle\langle d^\mu U d_\mu U^\dagger\rangle + e^2 F_0^2 K_2\langle QUQU^\dagger\rangle\langle d^\mu U d_\mu U^\dagger\rangle + \cdots$$

$$Q = \text{diag}\,(2/3, -1/3, -1/3)\,, \qquad \Delta M_\pi^2 = 2e^2 F_0^2 Z + \cdots$$

see, e.g.: R. Urech, NPB 433 (1995) 234; M. Knecht and R. Urech, NPB 519 (1998) 329
U.-G. Meißner, G. Müller and S. Steininger, PLB 406 (1997) 154
M. Knecht, H. Neufeld, H. Rupertsberger and P. Talavera, EPJC 12 (2000) 469

Charged pion decay rate at one loop

see, e.g.: M. Knecht, H. Neufeld, H. Rupertsberger and P. Talavera, EPJC 12 (2000) 469
V. Cirigliano and I. Rosell, JHEP 0710 (2007) 005: $\longrightarrow$ two loops

$$\Gamma(\pi \to \ell\nu_\ell(\gamma)) = \frac{G_F^2|V_{ud}|^2 F_0^2 m_\ell^2 M_{\pi^+}}{4\pi}\left(1 - \frac{m_\ell^2}{M_{\pi^+}^2}\right)^2$$

$$\times\Bigg\{1 + \frac{8}{F_0^2}\left(L_4^r(M_\pi^2 + 2M_K^2) + L_5^r M_\pi^2\right)$$

$$-\frac{1}{32\pi^2 F_0^2}\left(2M_{\pi^+}^2 \ln\frac{M_{\pi^+}^2}{\mu^2} + 2M_{\pi^0}^2 \ln\frac{M_{\pi^0}^2}{\mu^2} + M_{K^+}^2 \ln\frac{M_{K^+}^2}{\mu^2} + M_{K^0}^2 \ln\frac{M_{K^0}^2}{\mu^2}\right)$$

$$+ \underbrace{e^2 E^r}_{\text{e.m. LECs}} + \underbrace{\frac{e^2}{16\pi^2}\left(3\ln\frac{M_\pi^2}{\mu^2} + H\left(\frac{m_\ell^2}{M_{\pi^+}^2}\right)\right)}_{\text{photon loops}}\Bigg\}$$

$$M_{\pi^0}^2 = M_\pi^2 = 2B_0\hat{m}\,, \quad M_{\pi^+}^2 = M_\pi^2 + 2e^2F_0^2 Z\,, \quad M_K^2 = B_0(m_s + \hat{m})$$

$$M_{K^+}^2 = M_K^2 - \frac{B_0}{2}(m_d - m_u) + 2e^2F_0^2 Z\,, \quad M_{K^0}^2 = M_K^2 + \frac{B_0}{2}(m_d - m_u)$$

Switching off electromagnetic contributions

In the paradise world, all LECs stay put, and

$$M_{\pi^+}^2,\ M_{\pi^0}^2 \to M_\pi^2 = 2B_0\hat{m} + O(m^2)$$
$$M_{K^+}^2,\ M_{K^0}^2 \to M_K^2 = B_0(m_s + \hat{m}) + O(m^2)$$

$$F_\pi = F_0\left\{1 + \frac{4}{F_0^2}\left(L_4^r(M_\pi^2 + 2M_K^2) + L_5^r M_\pi^2\right) - \frac{1}{32\pi^2 F_0^2}\left(2M_\pi^2 \ln\frac{M_\pi^2}{\mu^2} + M_K^2 \ln\frac{M_K^2}{\mu^2}\right)\right\}$$

Is it possible to extract the "purely strong" value of F_π from data after applying isospin-breaking corrections?

Mapping $SU(3) \to SU(2)$:
$$F = F_0\left\{1 + \frac{B_0 m_s}{F_0^2}\left(8L_4^r - \frac{1}{32\pi^2}\ln\frac{B_0 m_s}{\mu^2}\right)\right\}$$

Linear σ-model with photons

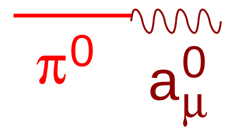
J. Gasser, AR and I. Scimemi, EPJC 32 (2003) 97

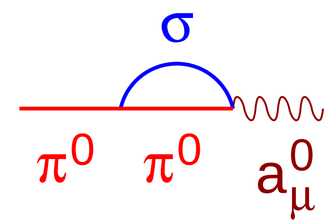
$$
\begin{aligned}
\mathcal{L}_\sigma &= \mathcal{L}^0 + \mathcal{L}_{\text{ct}} \\
\mathcal{L}_0 &= \frac{1}{2}(d_\mu\phi)^T d^\mu\phi + \frac{m^2}{2}\phi^T\phi - \frac{g}{4}(\phi^T\phi)^2 + c\phi^0 + \frac{\delta m^2}{2}(Q\phi)^T(Q\phi) \\
&- \frac{\delta g}{2}(Q\phi)^T(Q\phi)(\phi^T\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 \\
\mathcal{L}_{\text{ct}} &= \sum_{i=1}^{8}\beta_i\mathcal{O}_i \\
d^\mu\phi &= \partial^\mu\phi + (F^\mu + eQA_\mu)\phi\,, \qquad F_\mu^{0i} = a_\mu^i\,, \quad F_\mu^{ij} = -\epsilon^{ijk}v_\mu^k
\end{aligned}
$$

Power counting:

$$\delta m = O(e^2)\,, \quad \delta g = O(e^2)\,, \qquad c,\ e^2 = O(p^2)$$

Neutral pion decay constant

$F_{\pi^0} = Z_{\pi^0}^{1/2} \{$ 

$+$ 

$\}$

ChPT:
$$F_{\pi^0} = F\left\{1 - \frac{M_{\pi^+}^2}{16\pi^2 F^2} \ln \frac{M_{\pi^+}^2}{\mu^2} + \frac{M_{\pi^0}^2}{F^2} l_4^r - e^2 \sum_i c_i k_i^r \right\}$$

$$\hookrightarrow \qquad F(1 - e^2 \sum_i c_i k_i^r) = \frac{m}{\sqrt{g}} \left\{1 - \frac{3g}{16\pi^2} \ln \frac{2m^2}{\mu^2} + \frac{7g}{32\pi^2}\right\}$$

RG equations: running of g and m^2 is e-dependent

$$\mu \frac{dm^2}{d\mu} = \frac{1}{4\pi^2} ((3g + \delta g)m^2 + g\,\delta m^2)$$

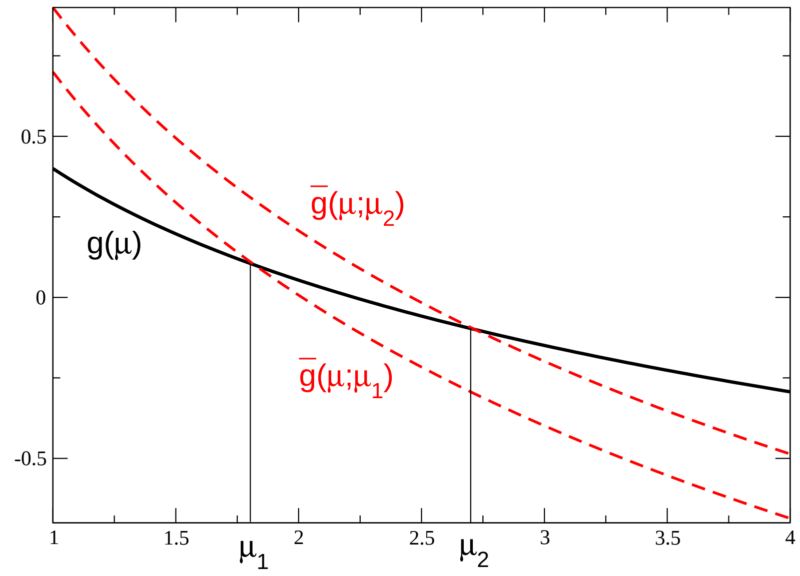
$$\mu \frac{dg}{d\mu} = \frac{1}{2\pi^2} (3g^2 + g\,\delta g)$$

Switching off electromagnetic contributions

Paradise world:

$$\mu \frac{d\bar{m}^2}{d\mu} = \frac{3}{4\pi^2}\,\bar{g}\bar{m}^2$$

$$\mu \frac{d\bar{g}}{d\mu} = \frac{3}{2\pi^2}\,\bar{g}^2$$



$$\hookrightarrow \quad m^2(\mu) = \bar{m}^2(\mu;\mu_1) + \frac{1}{4\pi^2}\,\ln\frac{\mu}{\mu_1}\,(\delta g\, m^2 + g\,\delta m^2)$$

$$g(\mu) = \bar{g}(\mu;\mu_1) + \frac{1}{2\pi^2}\,\ln\frac{\mu}{\mu_1}\,g\,\delta g$$

μ_1 is the matching point: $\quad m^2(\mu_1) = \bar{m}^2(\mu_1;\mu_1)\,, \quad g(\mu_1) = \bar{g}(\mu_1;\mu_1)$

Pion decay constant in the paradise world

F appears in the Lagrangian of ChPT, stays put as $e \to 0$

$$F = \frac{\bar{m}}{\sqrt{\bar{g}}} \left\{ 1 - \frac{3\bar{g}}{16\pi^2} \ln \frac{2\bar{m}^2}{\mu^2} + \frac{7\bar{g}}{32\pi^2} \right\}$$

Scale dependence:

$$\mu \frac{dF}{d\mu} = 0 \qquad \text{but} \qquad \mu_1 \frac{dF}{d\mu_1} \neq 0$$

$F(\mu_1 = 1\ \text{GeV}) - F(\mu_1 = 0.5\ \text{GeV}) \simeq 0.1\ \text{MeV}$ (in LσM)

Conclusions:

- The pion decay constant in the chiral limit F in the paradise world is convention-dependent
- F_π is convention dependent as well

Quark masses

$\longrightarrow$ Leutwyler

Non-lattice determinations:

pure QCD:

$$
\begin{aligned}
M_{\pi^+}^2 &= M_{\pi^0}^2 = (m_u + m_d)B_0 + O(m^2) \\
M_{K^+}^2 &= (m_u + m_s)B_0 + O(m^2) \\
M_{\bar{K}^0}^2 &= (m_d + m_s)B_0 + O(m^2)
\end{aligned}
$$

Using Dashen's theorem $(M_{K^+}^2 - M_{K^0}^2)^{\text{em}} = (M_{\pi^+}^2 - M_{\pi^0}^2)^{\text{em}}$

$$
\hookrightarrow \quad \frac{m_u}{m_d} \simeq \left.\frac{M_{K^+}^2 - M_{K^0}^2 + M_{\pi^+}^2}{M_{K^0}^2 - M_{K^+}^2 + M_{\pi^+}^2}\right|_{\text{QCD}} \simeq 0.56
$$

$$
\frac{m_s}{m_d} \simeq \left.\frac{M_{K^0}^2 + M_{K^+}^2 - M_{\pi^+}^2}{M_{K^0}^2 - M_{K^+}^2 + M_{\pi^+}^2}\right|_{\text{QCD}} \simeq 20.1
$$

Leutwyler's ellipse

H. Leutwyler, PLB 378 (1996) 313

$$Q^2 = \frac{M_K^2}{M_\pi^2} \frac{M_K^2 - M_\pi^2}{M_{K^0}^2 - M_{K^+}^2}\bigg|_{\text{QCD}} = \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}\left(1 + O(\hat{m}^2)\right) \simeq 24.2$$

$$\hookrightarrow \quad \left(\frac{m_u}{m_d}\right)^2 + \frac{1}{Q^2}\left(\frac{m_s}{m_d}\right)^2 = 1$$

Information on Q coming from $\eta \to 3\pi$ decays:

Dispersion relations:

J. Kambor, C. Wiesendanger and D. Wyler, NPB 465 (1996) 215: $Q = 22.4 \pm 0.9$

A.V. Anisovich and H. Leutwyler, PLB 375 (1996) 335

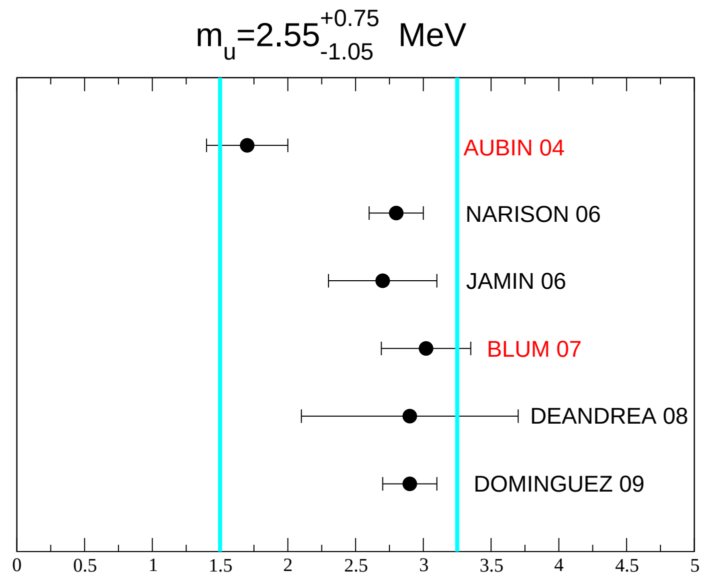
M. Walker, Diploma Thesis, Uni. Bern (1998): $Q = 22.8 \pm 0.8$

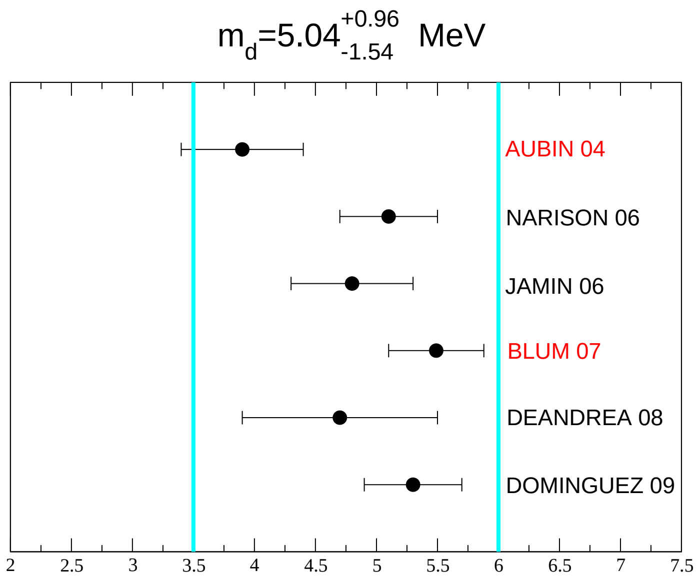
Two-loop ChPT:

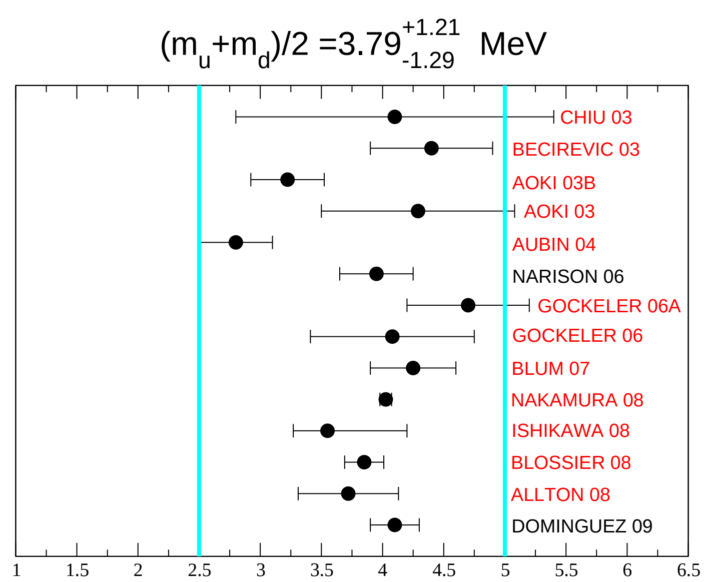
J. Bijnens and K. Ghorbani, JHEP 0711 (2007) 030: $Q = 23.2$

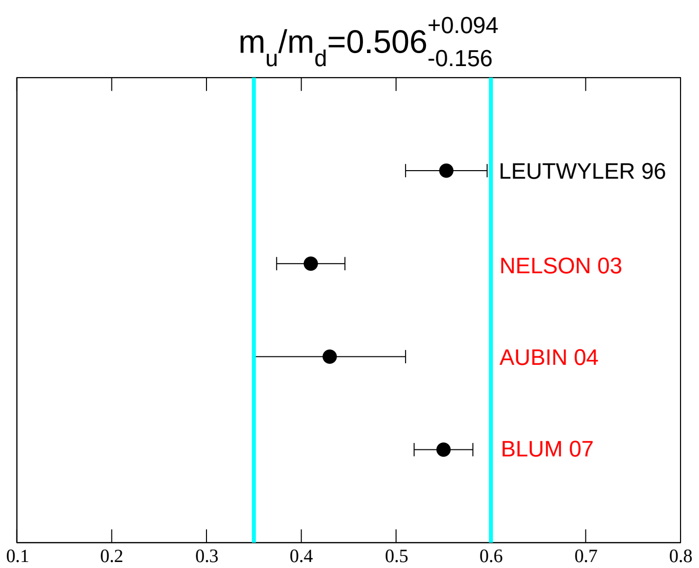
Light quark masses at $\mu = 2$ GeV (PDG)

$\longrightarrow$ Leutwyler









Isospin breaking in lattice QCD

A. Duncan, E. Eichten, and A. Thacker, hep-lat/9608143; hep-lat/9609015
Y. Namekawa and Y. Kikukawa, hep-lat/0509120; T. Blum *et al*, PRD 76 (2007) 114508;
S. Basak *et al*, arXiv:0812.4468; C. Aubin *et al*, PRD 70 (2004) 114501
S. R. Beane *et al*, NPB 768 (2007) 38, T. Ishikawa et al, PRD 78 (2008) 011502...

Gauge group: $SU(3) \times U(1)$ [non-compact QED]

$$
\begin{aligned}
m_{PS}^2 &= e^2(Q_q + Q_{\bar{q}})^2 A_{PS}(Q_q, Q_{\bar{q}}) + (m_q + m_{\bar{q}}) B_{PS}(Q_q, Q_{\bar{q}}) + O(m^2) \\
m_V^2 &= A_V(Q_q, Q_{\bar{q}}) + (m_q + m_{\bar{q}}) B_V(Q_q, Q_{\bar{q}}) + O(m^2) \\
& \cdots
\end{aligned}
$$

Bare mass: $m_q = \frac{1}{2}\left(\kappa_q^{-1} - \kappa_{qc}^{-1}\right)$

- Perform simulations for different values of e and of the hopping parameter κ_q
- Assume, A, B polynomials in $Q_q, Q_{\bar{q}}$, fit A, B and the critical hopping parameter κ_{qc} to the measured spectrum

Continuum limit

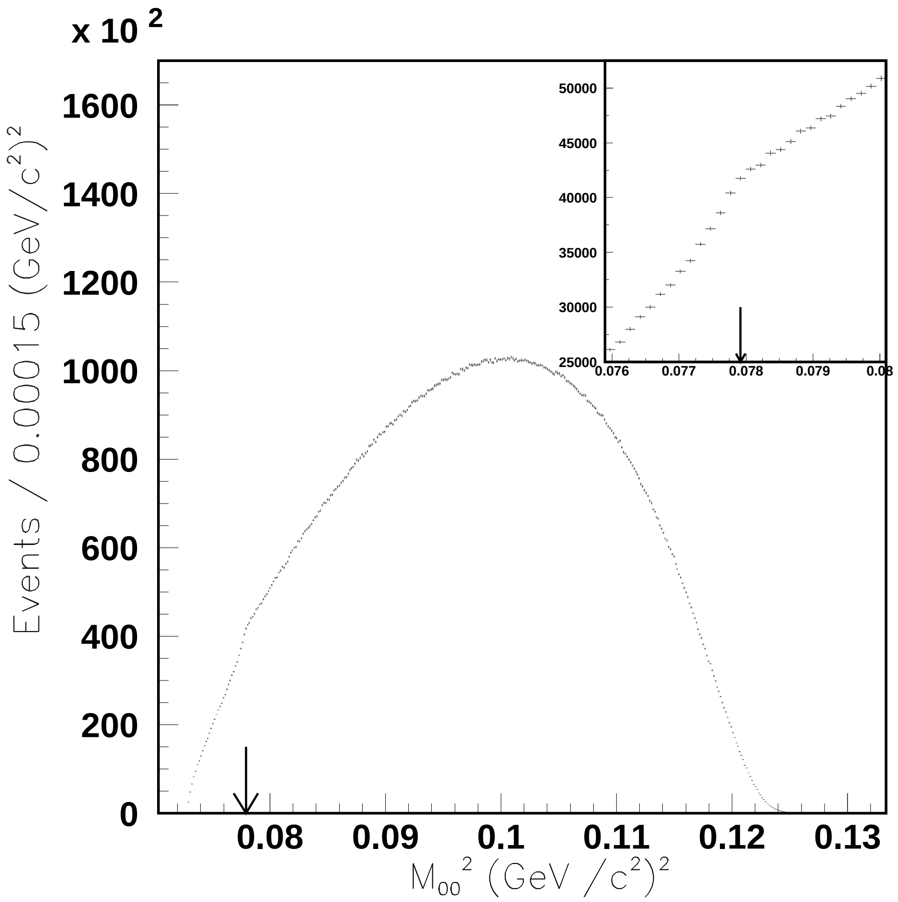
Running masses: a smooth continuum limit as $a \to 0$

$$m_q(\mu) = Z_m(a\mu)\, m_q$$

- $m_q(\mu)$ are quark masses in QCD + QED
- Switching off electromagnetic interactions $e \to 0$ while keeping all other parameters fixed is a perfectly legitimate procedure *at a given a as $a \neq 0$*
- Uncertainty in the definition of the quark mass in the paradise world resurfaces in the continuum limit. Fixing the values of the bare parameters in physical units is equivalent to the choice of μ_1

More on isospin breaking

The cusp in the $\pi^0\pi^0$ invariant mass distribution (NA48/2)



$K^+ \to \pi^+\pi^0\pi^0$

Partial sample of $\sim 2.3 \cdot 10^7$ decays

J. R. Batley *et al.* [NA48/2 Collaboration], PLB 633 (2006) 173

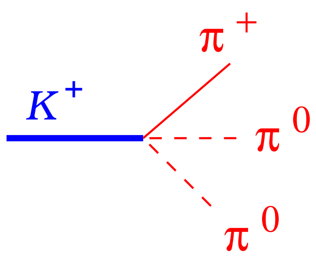
The cusp: $M_{\pi^+} \neq M_{\pi^0}$

Interference of tree + 1 loop *(N. Cabibbo, PRL 93 (2004) 121801)*

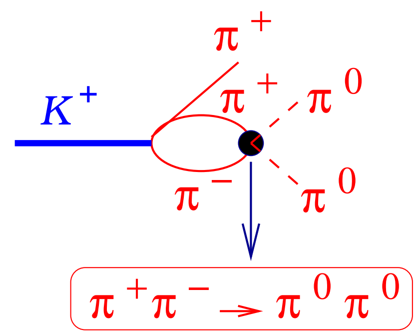
Prophecy: P. Budini and L. Fonda, PRL 6 (1961) 419

also: U.-G. Meißner et al, PLB 406 (1997) 154 for the $\pi\pi$ scattering amplitudes

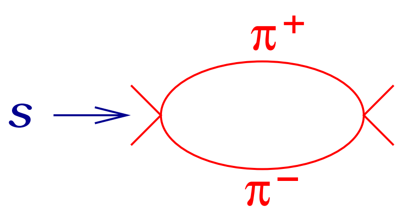
M_{00+} =



+



$$\frac{d\Gamma_{00+}}{ds_{\pi\pi}} \propto \int |\mathcal{M}_{00+}|^2$$



$$= \text{“smooth”} + \frac{i}{16\pi} \underbrace{\left(1 - \frac{4M_\pi^2}{s}\right)^{1/2}}_{=\sigma_c(s)}$$

Parameters of the cusp $\Rightarrow$ S-wave $\pi\pi$ scattering lengths a_0, a_2

At present experimental precision, a simple parameterization of the cusp does not suffice. A systematic theoretical framework is needed that describes $K \to 3\pi$ in the whole Dalitz plot region

$K \to 3\pi$ decays: theory

- N. Cabibbo and G. Isidori, JHEP 0503 (2005) 021:
  Parameterization of the decay amplitudes up to and including two loops, using analyticity and unitarity

- Also: E. Gamiz, J. Prades, and I. Scimemi, EPJC 50 (2007) 405:
  A supplementary approach, merger to ChPT at one loop
- Also: M. Zdrahal, K.Kampf, M. Knecht and J. Novotny, arXiv:0905.4868:
  An approach based on dispersion relations

$\Rightarrow$ *Not a full dynamical scheme (photons?)*

$\Rightarrow$ *The analytic ansatz for the amplitudes, which has been assumed, is not valid beyond one loop*

One needs a systematic theory of $K \to 3\pi$, which would provide a reliable control on the accuracy!

Non-relativistic EFT: essentials

Bern-Bonn coll. (M. Bissegger, G. Colangelo, A. Fuhrer, J. Gasser, B. Kubis, AR):
PLB 638 (2006) 187, PLB 659 (2008) 576, NPB 806 (2008) 178

⇒ *Include distant singularities, emerging in relativistic QFT, into the effective couplings of non-relativistic Lagrangian*

$$\frac{1}{M_\pi^2 - p^2} = \underbrace{\frac{1}{2w(\mathbf{p})}\,\frac{1}{w(\mathbf{p}) - p^0}}_{\text{particles}} + \underbrace{\frac{1}{2w(\mathbf{p})}\,\frac{1}{w(\mathbf{p}) + p^0}}_{\text{antiparticles}}$$

$$w(\mathbf{p}) = M_\pi + \frac{\mathbf{p}^2}{2M_\pi} - \frac{\mathbf{p}^4}{8M_\pi^3} + \cdots \qquad \text{at} \quad |\mathbf{p}| \ll M_\pi$$

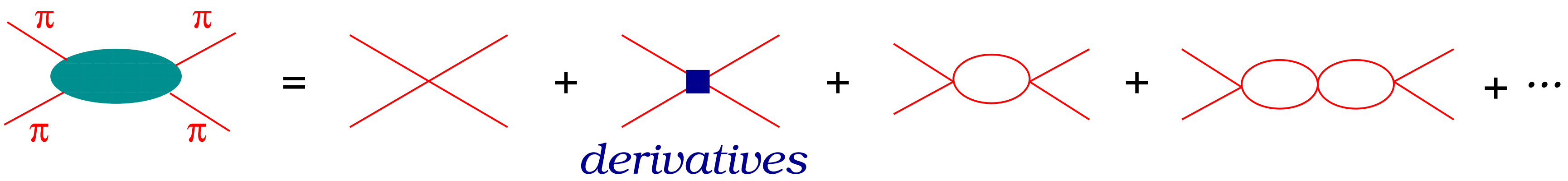
- *Two-particle sector: Lagrangian*

$$\mathcal{L}_{NR} = \Phi^\dagger(2W)(i\partial_t - W)\Phi + C_0\Phi^\dagger\Phi^\dagger\Phi\Phi + \text{deriv. couplings}$$

⇒ *Do loops with this Lagrangian in dim. reg. + threshold expansion*

Why non-relativistic theory?

- *It is a full dynamical scheme based on a Lagrangian (photons!)*
- *Analyticity + unitarity automatically taken into account*
- *Isospin breaking through loops (cusp!) + couplings*



$\Rightarrow$ *Each loop $\propto i|\mathbf{p}|$, vanishes at threshold, $O(\epsilon)$ suppressed*

$$\begin{array}{ccccccccc} \mathrm{Re}\,T_{NR} & = & a & + & b\mathbf{p}^2 & + & c\mathbf{p}^4 & + & \cdots \\ & & \mathrm{tree} & & \mathrm{tree}+\mathrm{two}-\mathrm{loop} & & \cdots & & \end{array}$$

Non-relativistic couplings = scattering lengths a, $\cdots$

On the contrary, in ChPT: $a = O(M_\pi^2) + O(M_\pi^4) + O(M_\pi^6) + \cdots$

Non-relativistic approach for $K \to 3\pi$ decays

$$\mathcal{L}_{\pi\pi} = C_x(\pi_-^\dagger \pi_+^\dagger \pi_0 \pi_0 + \text{h.c.}) + \cdots\,, \qquad C_x = (a_0 - a_2) + \text{isospin br.}$$

$$\mathcal{L}_{K^+ \to \pi^0\pi^0\pi^+} = \frac{G_0}{2}\,(K^\dagger \pi_+ \pi_0^2 + \text{h.c.}) + \cdots$$

$$\mathcal{L}_{K^+ \to \pi^+\pi^+\pi^-} = \frac{H_0}{2}\,(K^\dagger \pi_- \pi_+^2 + \text{h.c.}) + \cdots$$

$$\cdots$$

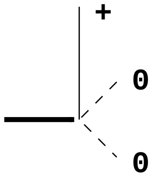
- Non-relativistic region = whole decay region, and slightly beyond
- Double expansion in:

  a (scattering lengths, effective ranges...) and ϵ (small momenta)

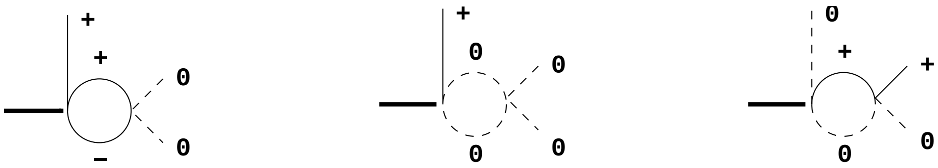
- Expansion in a and ϵ are correlated: adding one pion loop increases powers of both a and ϵ by one
- One expects that the expansion in a is convergent, as $a \ll 1$

The graphs $K^+ \to \pi^0\pi^0\pi^+$

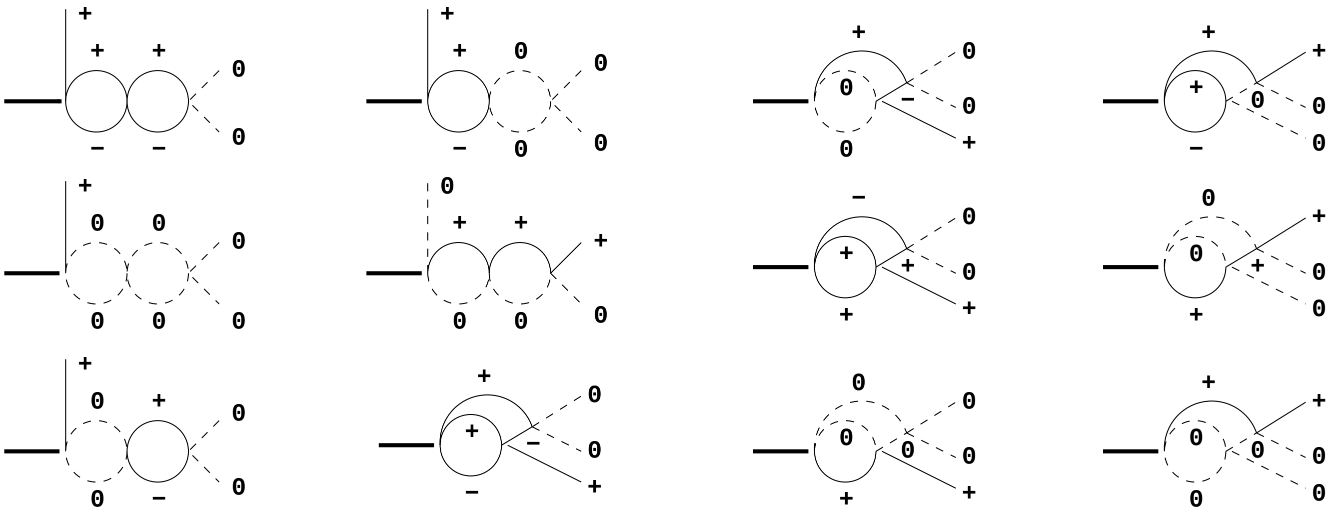
Tree:



1-loop:



2-loops:



Including photons

Minimal substitution:

Bern-Bonn coll., NPB 806 (2008) 178

$$\partial_\mu \Phi_\pm \to (\partial_\mu \mp ieA_\mu)\Phi_\pm \,, \qquad \partial_\mu K_+ \to (\partial_\mu - ieA_\mu)K_+$$

+ all possible non-minimal gauge-invariant terms

$$\frac{d\Gamma}{ds_3}\bigg|_{E_\gamma<E_{max}} = \frac{d\Gamma(K\to 3\pi)}{ds_3} + \frac{d\Gamma(K\to 3\pi\gamma)}{ds_3}\bigg|_{E_\gamma<E_{max}} + O(\alpha^2)$$

- Electromagnetic corrections are perturbative everywhere except a very small region around the cusp

  $\longrightarrow$ exclude this region in the fit

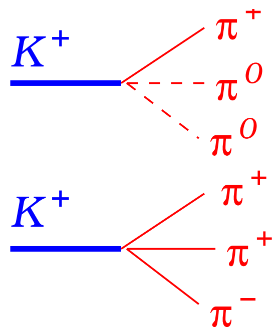
  also: S.R. Gevorkyan *et al*, PLB 649 (2007) 159, Phys.Part.Nucl.Lett.5 (2008) 85

$\hookrightarrow$ A systematic parameterization of decay amplitudes, including real and virtual photons

The strategy for determining scattering lengths

$K^+ \to \pi^0\pi^0\pi^+$ and $K^+ \to \pi^+\pi^+\pi^-$ decay amplitudes depend on:

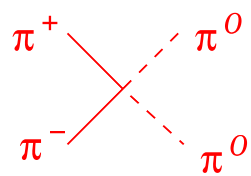
$K \to 3\pi$ decay couplings



$G_0, G_1, G_2, G_3, \ldots$

$H_0, H_1, H_2, H_3, \ldots$

$\pi\pi$ effective range parameters



C_i, D_i, E_i, F_i, ...

$O(1)$ | $O(p^2)$ | $O(p^4)$

Fit $G_i, H_i, C_i, \cdots$ *to the decay data;* $C_i \Rightarrow \pi\pi$ *scattering lengths*

Bern-Bonn approach with chiral constraint + radiative corrections

$$a_0 - a_2 = 0.263 \pm 0.0024\,(stat) \pm 0.0014\,(syst) \pm 0.0019\,(ext)$$

S. Giudici, talk at this conference

Cuspology

$K_L \to 3\pi^0$

- The size of the cusp is suppressed $\longrightarrow$ Giudici

$\eta \to 3\pi$

Ch. Ditsche, B. Kubis and U.-G. Meißner, EPJC 60 (2009) 83 $\longrightarrow$ Ditsche
C.-O. Gullström, A. Kupsc and AR, PRC 79 (2009) 028201 $\longrightarrow$ Kupsc

- Allows one to extract precise value of $Q^2 = \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}$. Corrections at $O(e^2\hat{m}), O(e^2(m_d - m_u))$ turn out to be small
- The size of the cusp is suppressed; Sign??? $\longrightarrow$ Prakhov
- The presence of the cusp may affect a precise extraction of the Dalitz plot parameters

$\eta' \to \eta\, 2\pi$

B. Kubis and S. Schneider, arXiv:0904.1320

- More than 8% cusp effect in the decay spectrum $\longrightarrow$ Schneider

K_{e4} decays

$\longrightarrow$ Bloch-Devaux

Kinematics:

$s_\pi = (p_1+p_2)^2,\ t=(p_1-p_2)^2,\ u=(p_2-p)^2,\ s_l=(p_e+p_\nu)^2$

$$\mathcal{M} = \frac{G_F}{\sqrt{2}}\, V_{us}^*\, \langle\pi^+\pi^-|V^\mu - A^\mu|K^+\rangle\, \bar u(p_\nu)\gamma_\mu(1-\gamma_5)v(p_e)$$

$$\langle\pi^+\pi^-|V^\mu - A^\mu|K^+\rangle = \frac{-i}{M_K}\,(F(p_1+p_2)^\mu + G(p_1-p_2)^\mu) + \cdots$$

Partial-wave expansion:

$$F_1 = F + \frac{(M_K^2 - s_\pi - s_l)\sigma}{\lambda^{1/2}(M_K^2, s_\pi, s_l)}\cos\theta_\pi\, G\,,\quad F_1 = \sum_k P_k(\cos\theta_\pi) f_k(s_\pi, s_l)$$

Watson theorem (isospin symmetric world):

$$f_k(s_\pi + i\varepsilon, s_l) = \mathrm{e}^{2i\delta_k} f_k(s_\pi - i\varepsilon, s_l)\,,\qquad \left\{\begin{array}{l}\delta_0 = \delta_0^0\\ \delta_1 = \delta_1^1\end{array}\right. \quad\Rightarrow$$

measure

$\delta_0^0 - \delta_1^1$

The fate of Watson theorem in case of isospin breaking

G. Colangelo, J. Gasser and AR, EPJC 59 (2009) 777

Scalar formfactor:

$$-F_c(s) = \langle 0|\mathcal{O}(0)|\pi^+(p_1)\pi^-(p_2);\text{in}\rangle \qquad , \qquad F = \begin{pmatrix} F_c \\ F_0 \end{pmatrix}$$
$$F_0(s) = \langle 0|\mathcal{O}(0)|\pi^0(p_1)\pi^0(p_2);\text{in}\rangle$$

Isospin symmetry limit $F_c = F_0$:

$$\text{Im}\, F_c = t_0^0\, \sigma_c\, F_c^* \quad \Rightarrow \quad F_c = \text{e}^{i\delta_0^0}\, |F_c|$$

Isospin broken:

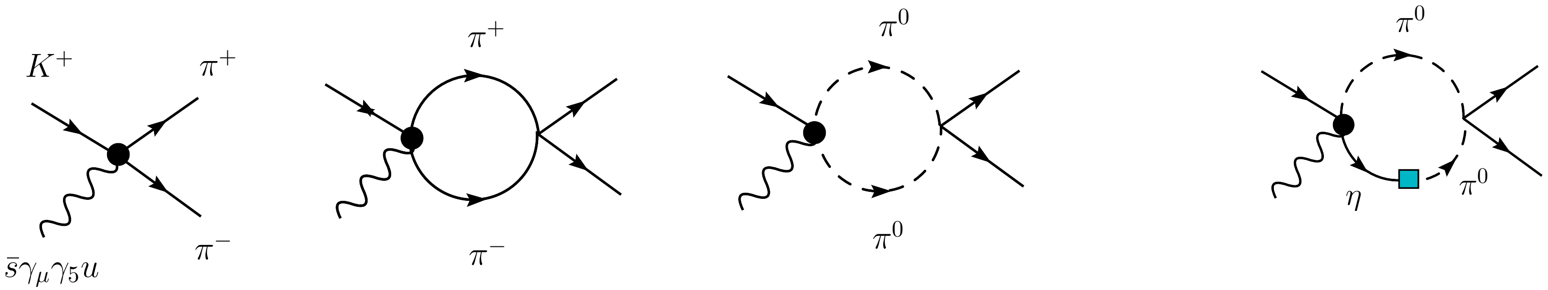
- Phases δ_c, δ_0 are not determined by the $\pi\pi$ amplitude alone
- $\delta_c \neq 0$ at $s = 4M_\pi^2$
- $F_c = \text{e}^{i\delta_c} \hat{F}_c$ with $\hat{F}_c \propto \sigma_c \sigma_0 + \cdots$ *non-analytic* at $s = 4M_\pi^2$

see also: S.R. Gevorkyan et al, hep-ph 0704.2675, 0711.4618

One-loop result in ChPT

⇒ Use ChPT with no photons, calculate isospin-breaking corrections, subtract from the measured phase shifts

$$\mathcal{L}_2 \to \mathcal{L}_2 + C\langle QUQU^\dagger\rangle\,,\cdots \qquad Q = \frac{e}{3}\,\text{diag}\,(2,-1,-1)$$



Scalar FF: $m_d - m_u$ effect:

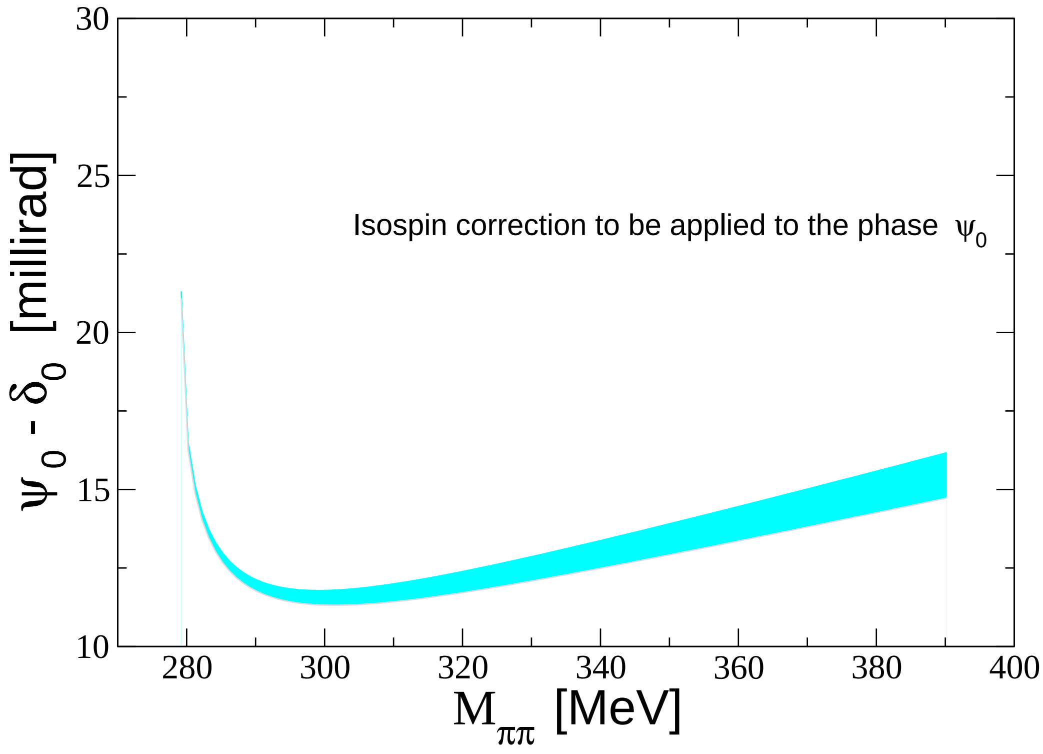
$$\delta_c = \frac{1}{32\pi F_0^2}\left\{4(M_\pi^2 - M_{\pi^0}^2 + s)\sigma_c(s) + (s - M_{\pi^0}^2)\left(1 + \frac{3}{2R}\right)\sigma_0(s)\right\}$$

$$R = \frac{m_s - \hat{m}}{m_d - m_u} \simeq 37 \pm 5$$

also: A. Nehme, PRD 69 (2004) 094012; EPJC 40 (2005) 367; V. Cuplov and A. Nehme, hep-ph/0311274

S. Descotes-Genon and M. Knecht, in progress

Including isospin-breaking correction



Applying isospin-breaking corrections. . .

$$a_0 \quad = \quad 0.2206 \pm 0.0049\,(stat) \pm 0.0018\,(syst) \pm 0.0064\,(th)$$

B.Bloch-Devaux, talk at KAON 09,June 09

Isospin breaking in πN scattering

Dispersion relations: B. Tromborg *et al*, PRD 15 (1977) 725 . . .

Potential models: E. Matsinos *et al*, NPA 778 (2006) 95 . . .

EFT: U.-G. Meißner *et al*, PLB 419 (1998) 403; NPA 693 (2001) 693
J. Gasser *et al*, EPJC 26 (2002) 13 . . .

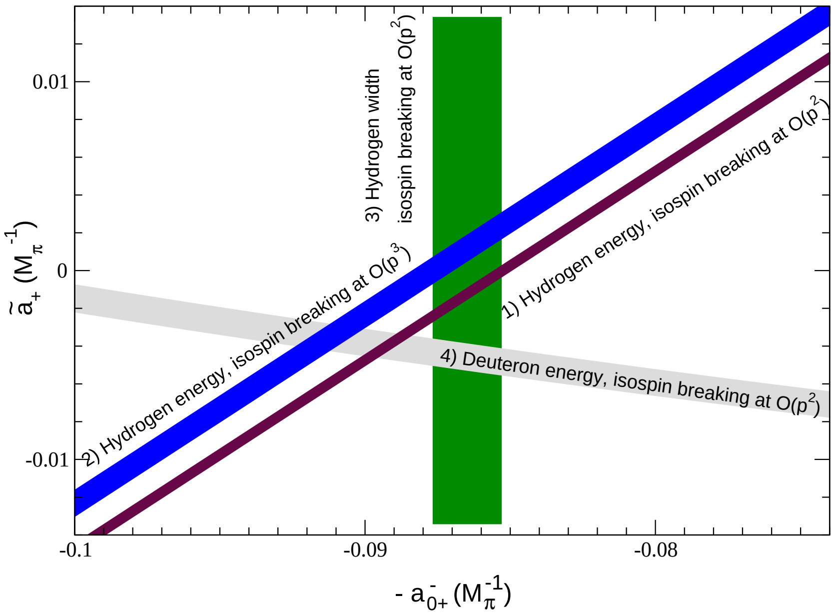
$\longrightarrow$ Isospin-breaking corrections to the threshold amplitudes in all channels at order p^3 in relativistic baryon ChPT

M. Hoferichter, B. Kubis and U.-G. Meißner, PLB 678 (2009) 65 $\longrightarrow$ Hoferichter

Experimental accuracy:

Pionic Hydrogen coll., PSI

Energy shift: 0.2%
Width: 2%



Conclusions

- Using field-theoretical framework enables one to systematically separate isospin-breaking effects from the "purely strong" quantities defined in the paradise world – up to the uncertainties related to the choice of the parameters of the paradise world
- Recent developments, in particular, include:
  - Cusps in three-particle decays $\longrightarrow$ Giudici, Ditsche, Prakhov, Kupsc, Schneider, Lanz, Kampf, Zdrahal
  - Isospin breaking in $K_{e2}/K_{\mu 2}$ and $\pi_{e2}/\pi_{\mu 2}$ $\longrightarrow$ Rosell
  - Isospin breaking in kaon decays $\longrightarrow$ Neufeld
  - K_{e4} decays $\longrightarrow$ Bloch-Devaux
  - Isospin breaking in πN scattering $\longrightarrow$ Hoferichter
  - Light quark masses in lattice QCD $\longrightarrow$ Leutwyler
  - Isospin breaking in τ decays
  - Isospin breaking in few-nucleon forces