Effective Theories for Magnetic Systems

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Outline

Cuprate Superconductors and Antiferromagnets

Correspondences between QCD and Antiferromagnetism

Effective Field Theories for Ferro- and Antiferromagnets

Quantum Heisenberg Model

Hubbard Model for Doped Antiferromagnets

Effective Field Theories for Magnons and Holes

Two-Hole States Bound by Magnon Exchange

Rotor Spectrum in the Single-Hole Sector and in QCD

Conclusions

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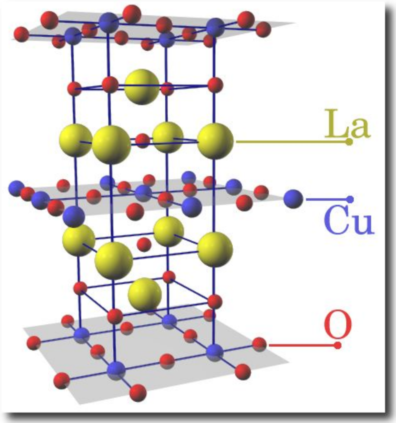
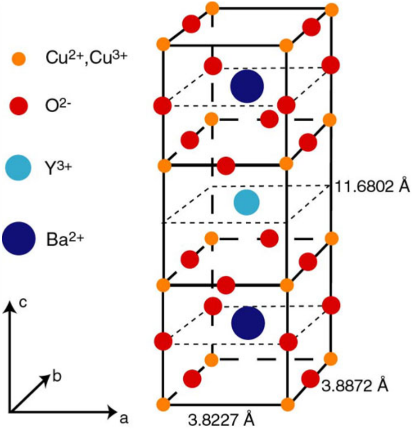
Antiferromagnetic precursors of high-T_c superconductors

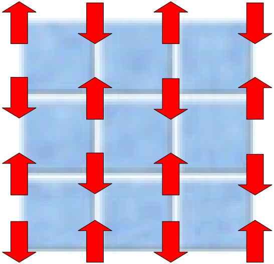


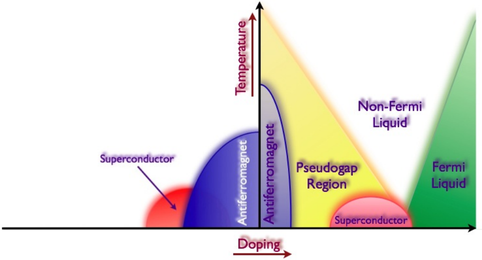
LaCuO

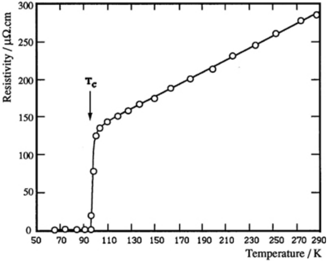


YBaCuO

Properties of cuprates

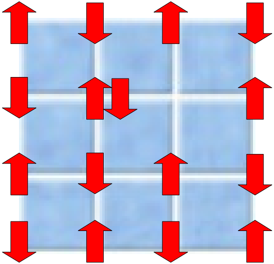


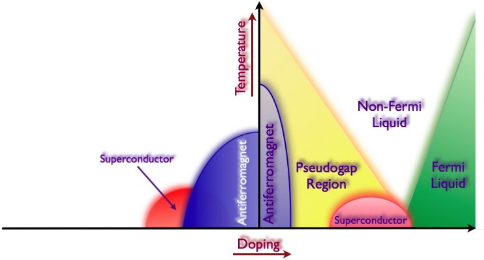


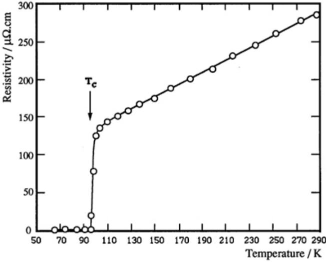


Temperature-dependence of resistivity

Properties of cuprates







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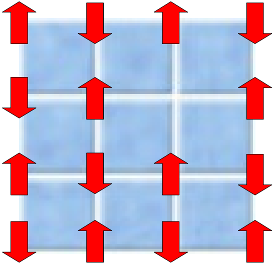

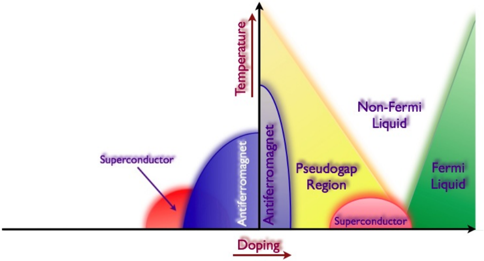

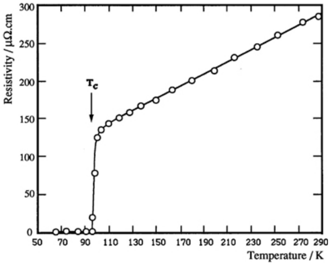

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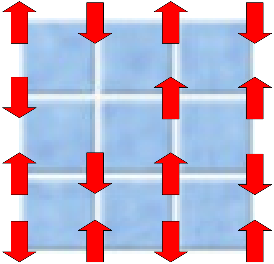

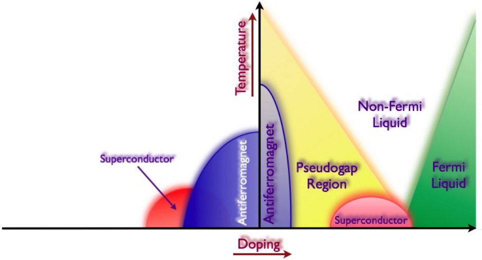

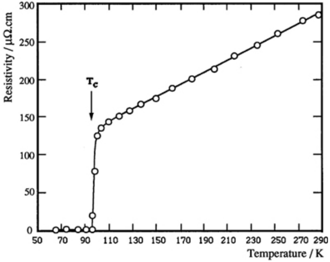

Temperature-dependence of resistivity

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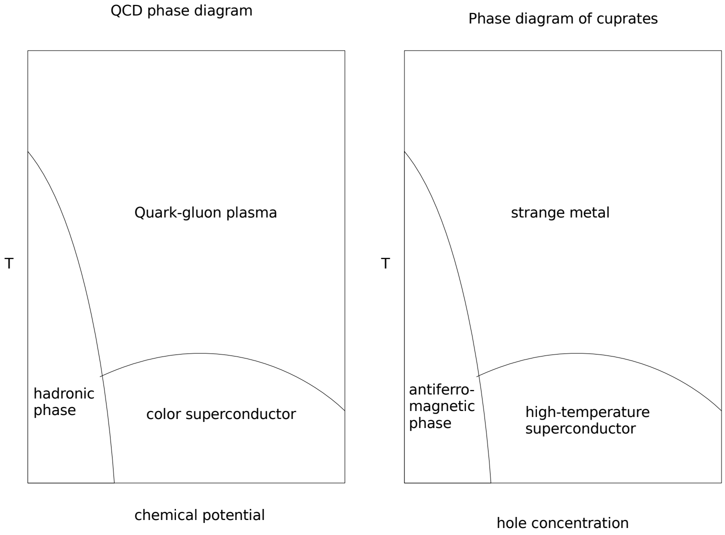
Phase diagrams of QCD and of doped antiferromagnets



Correspondences between QCD and Antiferromagnetism

	QCD	Antiferromagnetism
broken phase	hadronic vacuum	antiferromagnetic phase
global symmetry	chiral symmetry	spin rotations
symmetry group G	$SU(2)_L \otimes SU(2)_R$	$SU(2)_s$
unbroken subgroup H	$SU(2)_{L=R}$	$U(1)_s$
Goldstone boson	pion	magnon
Goldstone field in G/H	$U(x) \in SU(2)$	$\vec{e}(x) \in S^2$
order parameter	chiral condensate	staggered magnetization
coupling strength	pion decay constant F_π	spin stiffness ρ_s
propagation speed	velocity of light	spin-wave velocity c
conserved charge	baryon number $U(1)_B$	electric charge $U(1)_Q$
charged particle	nucleon or antinucleon	electron or hole
long-range force	pion exchange	magnon exchange
dense phase	nuclear or quark matter	high-T_c superconductor
microscopic description	lattice QCD	Hubbard or t-J model
effective description of Goldstone bosons	chiral perturbation theory	magnon effective theory
effective description of charged fields	baryon chiral perturbation theory	magnon-hole effective theory

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Effective magnon field in $SU(2)/U(1) = S^2$

$$\vec{e}(x) = (e_1(x), e_2(x), e_3(x)), \quad \vec{e}(x)^2 = 1$$

Low-energy effective action for antiferromagnetic magnons

$$S[\vec{e}] = \int d^2x \ dt \ \frac{\rho_s}{2} \left(\partial_i \vec{e} \cdot \partial_i \vec{e} + \frac{1}{c^2} \partial_t \vec{e} \cdot \partial_t \vec{e} \right)$$

Neuberger, Ziemann (1989); Hasenfratz, Leutwyler (1990); Hasenfratz, Niedermayer (1993)

Low-energy effective action for ferromagnetic magnons

$$S[\vec{e}] = \int d^2x \left[\int_{S^1} dt \ \frac{\rho_s}{2} \partial_i \vec{e} \cdot \partial_i \vec{e} - im \int_{H^2} dt d\tau \ \vec{e} \cdot (\partial_t \vec{e} \times \partial_\tau \vec{e}) \right]$$

The prefactor of the Wess-Zumino term — the total spin $M = \int d^2x \ m$, i.e. the magnetization of the ferromagnet — is quantized in half-integer units.

Leutwyler (1994); Hofmann (1999); Bär, Imboden, Wiese (2004)

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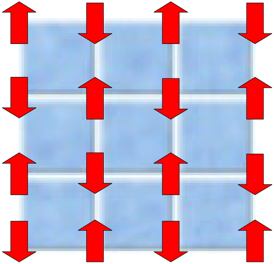
The spin 1/2 quantum Heisenberg model



Quantum spins $[S_x^a, S_y^b] = i\delta_{xy}\varepsilon_{abc}S_x^c$ and their Hamiltonian

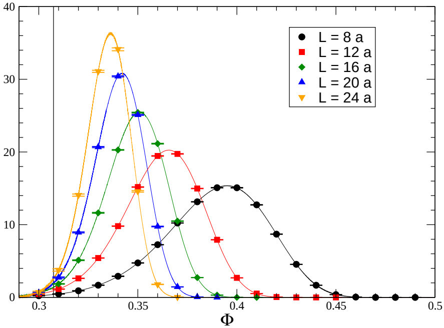
$$H = J \sum_{\langle xy \rangle} \vec{S}_x \cdot \vec{S}_y$$

Partition function at inverse temperature $\beta = 1/T$

$$Z = \mathrm{Tr}\exp(-\beta H)$$

Spin chain with periodic boundary conditions

Excellent agreement of Monte Carlo simulations with effective field theory predictions for the constraint effective potential
Göckeler, Leutwyler (1991)



Very accurate determination of low-energy parameters:

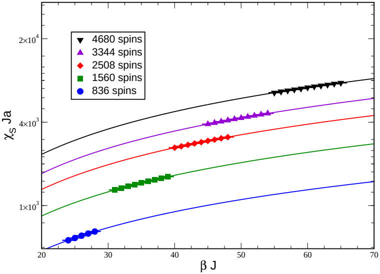
$$\mathcal{M}_s = 0.30743(1)/a^2, \quad \rho_s = 0.1808(4)J, \quad c = 1.6585(10)Ja$$

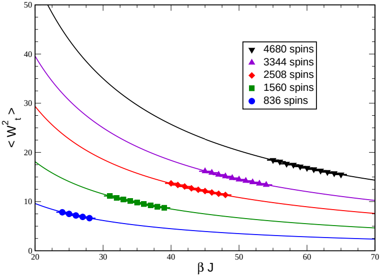
Gerber, Hofmann, Jiang, Nyfeler, Wiese (2009)

Fit to ε-regime predictions of magnon chiral perturbation theory for the Heisenberg model on a honeycomb lattice

$$\chi_s = \frac{\mathcal{M}_s^2 L^2 \beta}{3}\left\{1 + 2\frac{c}{\rho_s L l}\beta_1(l) + \left(\frac{c}{\rho_s L l}\right)^2 \left[\beta_1(l)^2 + 3\beta_2(l)\right]\right\}$$

$$\chi_u = \frac{2\rho_s}{3c^2}\left\{1 + \frac{1}{3}\frac{c}{\rho_s L l}\widetilde{\beta}_1(l) + \frac{1}{3}\left(\frac{c}{\rho_s L l}\right)^2 \left[\widetilde{\beta}_2(l) - \frac{1}{3}\widetilde{\beta}_1(l)^2 - 6\psi(l)\right]\right\}$$





$$\widetilde{\mathcal{M}}_s = 0.2689(4), \quad \rho_s = 0.102(2)J, \quad c = 1.297(16)Ja$$

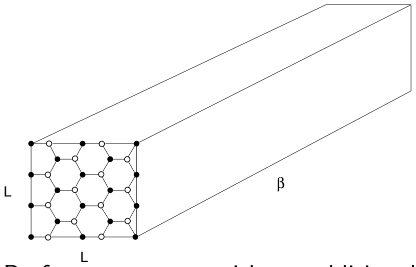
Hasenfratz, Niedermayer (1993); Jiang, Kämpfer, Nyfeler, Wiese (2008)

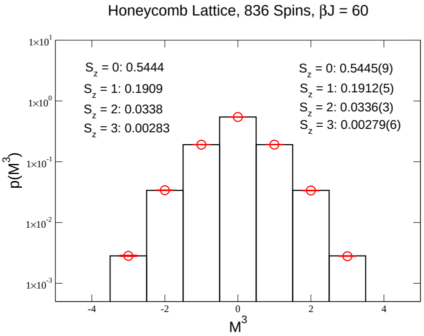
Effective quantum mechanical rotor in the δ-regime

$$\mathcal{L} = \int d^2x \, \frac{\rho_s}{2} \left(\partial_i \vec{e} \cdot \partial_i \vec{e} + \frac{1}{c^2} \partial_t \vec{e} \cdot \partial_t \vec{e} \right) = \frac{\Theta}{2} \partial_t \vec{e} \cdot \partial_t \vec{e}$$

Probability distribution of magnetization $M^3 = S^3$

$$p(M^3) = \frac{1}{Z} \sum_{S \geq |M^3|} \exp(-\beta E_S), \quad E_S = \frac{S(S+1)}{2\Theta}, \quad \Theta = \frac{\rho_s L^2}{2c^2}$$





Perfect agreement without additional adjustable parameters

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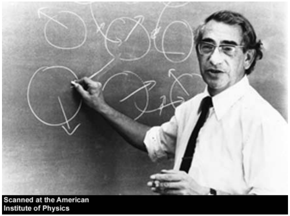
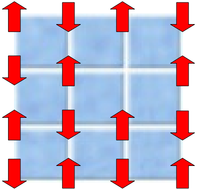
The Hubbard Model





$$H = -t \sum_{\langle xy \rangle} (c_x^\dagger c_y + c_y^\dagger c_x) + U \sum_x (c_x^\dagger c_x - 1)^2, \quad c_x = \begin{pmatrix} c_{x\uparrow} \\ c_{x\downarrow} \end{pmatrix}$$

For large repulsion U it reduces to the t-J model

$$H = P \left\{ -t \sum_{\langle xy \rangle} (c_x^\dagger c_y + c_y^\dagger c_x) + J \sum_{\langle xy \rangle} \vec{S}_x \cdot \vec{S}_y \right\} P$$

which further reduces to the Heisenberg model at half-filling

$$H = J \sum_{\langle xy \rangle} \vec{S}_x \cdot \vec{S}_y$$

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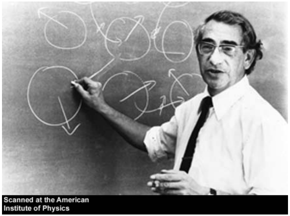

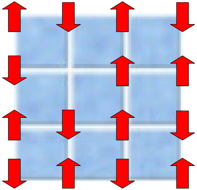

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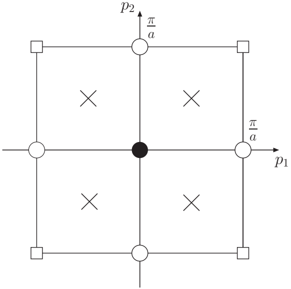
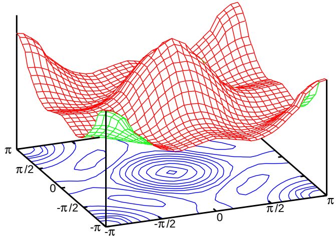
Hole dispersion in the t-J model





Hole pockets centered at lattice momenta

$$k^{\alpha} = \left(\frac{\pi}{2a}, \frac{\pi}{2a}\right), \quad k^{\alpha\prime} = -k^{\alpha}, \quad k^{\beta} = \left(\frac{\pi}{2a}, -\frac{\pi}{2a}\right), \quad k^{\beta\prime} = -k^{\beta}$$

Hole fields

$$\psi_{+}^{f}(x) = \frac{1}{\sqrt{2}}\left[\psi_{+}^{k^f}(x) - \psi_{+}^{k^{f\prime}}(x)\right], \quad \psi_{-}^{f}(x) = \frac{1}{\sqrt{2}}\left[\psi_{-}^{k^f}(x) + \psi_{-}^{k^{f\prime}}(x)\right]$$

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Nonlinear realization of the $SU(2)_s$ symmetry

$$u(x)\vec{e}(x)\cdot\vec{\sigma}u(x)^\dagger = \sigma_3, \quad u_{11}(x) \geq 0$$

Under $SU(2)_s$ the diagonalizing field $u(x)$ transforms as

$$u(x)' = h(x)u(x)g^\dagger, \quad u_{11}(x)' \geq 0,$$

$$h(x) = \exp(i\alpha(x)\sigma_3) = \begin{pmatrix} \exp(i\alpha(x)) & 0 \\ 0 & \exp(-i\alpha(x)) \end{pmatrix} \in U(1)_s$$

The composite vector field

$$v_\mu(x) = u(x)\partial_\mu u(x)^\dagger = iv_\mu^a(x)\sigma_a, \quad v_\mu^\pm(x) = v_\mu^1(x) \mp iv_\mu^2(x)$$

transforms as

$$v_\mu^3(x)' = v_\mu^3(x) - \partial_\mu\alpha(x), \quad v_\mu^\pm(x)' = \exp(\pm 2i\alpha(x))v_\mu^\pm(x)$$

Transformation rules of fermion fields

$$\begin{aligned}
SU(2)_s: &\quad \psi^f_\pm(x)' = \exp(\pm i\alpha(x))\psi^f_\pm(x), \\
U(1)_Q: &\quad {}^Q\psi^f_\pm(x) = \exp(i\omega)\psi^f_\pm(x), \\
D_i: &\quad {}^{D_i}\psi^f_\pm(x) = \mp\exp(ik^f_i a)\exp(\mp i\varphi(x))\psi^f_\mp(x), \\
O: &\quad {}^O\psi^\alpha_\pm(x) = \mp\psi^\beta_\pm(Ox), \quad {}^O\psi^\beta_\pm(x) = \psi^\alpha_\pm(Ox), \\
R: &\quad {}^R\psi^\alpha_\pm(x) = \psi^\beta_\pm(Rx), \quad {}^R\psi^\beta_\pm(x) = \psi^\alpha_\pm(Rx)
\end{aligned}$$

Leading terms in the effective Lagrangian for holes

$$\begin{aligned}
\mathcal{L} = & \sum_{\substack{f=\alpha,\beta \\ s=+,-}} \Big[M\psi^{f\dagger}_s\psi^f_s + \psi^{f\dagger}_s D_t\psi^f_s + \Lambda\big(\psi^{f\dagger}_s v^s_1\psi^f_{-s} + \sigma_f\psi^{f\dagger}_s v^s_2\psi^f_{-s}\big) \\
+ & \frac{1}{2M'}D_i\psi^{f\dagger}_s D_i\psi^f_s + \sigma_f\frac{1}{2M''}\big(D_1\psi^{f\dagger}_s D_2\psi^f_s + D_2\psi^{f\dagger}_s D_1\psi^f_s\big)\Big]
\end{aligned}$$

Covariant derivative coupling to composite magnon gauge field

$$D_\mu\psi^f_\pm(x) = \left[\partial_\mu \pm iv^3_\mu(x)\right]\psi^f_\pm(x)$$

Brügger, Kämpfer, Moser, Pepe, Wiese (2006)

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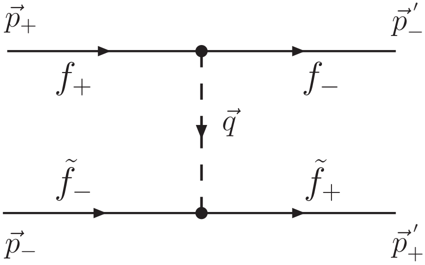
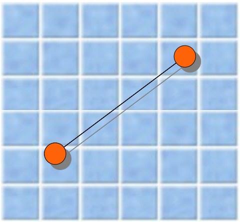
Magnon exchange





One-magnon exchange potentials

$$V^{\alpha\alpha}(\vec{r}) = \gamma\frac{\sin(2\varphi)}{r^2}, \quad V^{\beta\beta}(\vec{r}) = -\gamma\frac{\sin(2\varphi)}{r^2},$$

$$V^{\alpha\beta}(\vec{r}) = V^{\beta\alpha}(\vec{r}) = \gamma\frac{\cos(2\varphi)}{r^2}, \quad \gamma = \frac{\Lambda^2}{2\pi\rho_s}$$

Two-hole Schrödinger equation for an $\alpha\beta$ pair

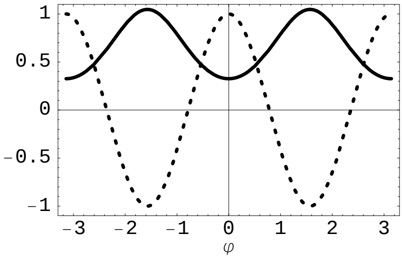
$$\begin{pmatrix} -\frac{1}{M'}\Delta & V^{\alpha\beta}(\vec{r}) \\ V^{\alpha\beta}(\vec{r}) & -\frac{1}{M'}\Delta \end{pmatrix} \begin{pmatrix} \Psi_1(\vec{r}) \\ \Psi_2(\vec{r}) \end{pmatrix} = E \begin{pmatrix} \Psi_1(\vec{r}) \\ \Psi_2(\vec{r}) \end{pmatrix}$$

Making the ansatz

$$\Psi_1(\vec{r}) \pm \Psi_2(\vec{r}) = R(r)\chi_\pm(\varphi)$$

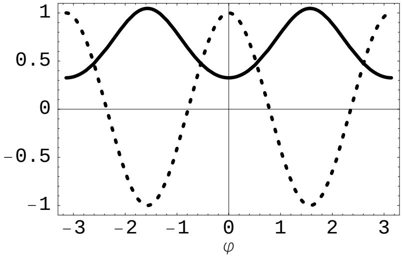
for the angular part of the wave function one obtains

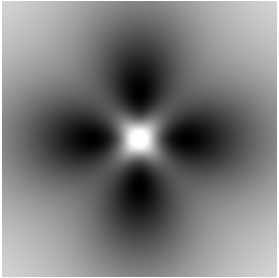
$$-\frac{d^2\chi_\pm(\varphi)}{d\varphi^2} \pm M'\gamma\cos(2\varphi)\chi_\pm(\varphi) = -\lambda\chi_\pm(\varphi)$$



looks like s-wave,
but turns out to be p-wave

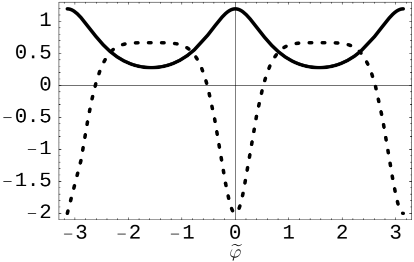
Two-hole bound states of $\alpha\beta$ and $\alpha\alpha$ pairs

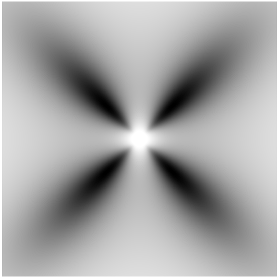




Angular wave function

Probability density





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Effective Lagrange function for quantum mechanical rotor

$$\mathcal{L} = \frac{\Theta}{2}\partial_t\vec{e}\cdot\partial_t\vec{e} + \sum_{f=\alpha,\beta} \Psi^{f\dagger}\left[E(\vec{p}) - i\partial_t + v_t^3\sigma_3\right]\Psi^f, \Psi(t) = \begin{pmatrix} \psi_+^f(t) \\ \psi_-^f(t) \end{pmatrix}$$

Spherical coordinates for the staggered magnetization

$$\vec{e} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta) \;\Rightarrow\; v_t^3 = \sin^2\frac{\theta}{2}\partial_t\varphi$$

Canonically conjugate momenta

$$\Theta\;\partial_t\theta = p_\theta, \quad \Theta\;\partial_t\varphi = \frac{1}{\sin^2\theta}(p_\varphi + iA_\varphi)$$

Abelian monopole Berry gauge field

$$A_\theta = 0, \quad A_\varphi = i\sin^2\frac{\theta}{2}\sigma_3, \quad F_{\theta\varphi} = \partial_\theta A_\varphi - \partial_\varphi A_\theta = \frac{i}{2}\sin\theta\;\sigma_3$$

Rotor Hamiltonian in the single-hole sector

$$
\begin{aligned}
H &= -\frac{1}{2\Theta}\left\{\frac{1}{\sin\theta}\partial_\theta[\,\sin\theta\partial_\theta] + \frac{1}{\sin^2\theta}(\partial_\varphi - A_\varphi)^2\right\} + E(\vec{p}) \\
&= \frac{1}{2\Theta}\left(\vec{J}^2 - \frac{1}{4}\right) + E(\vec{p})
\end{aligned}
$$

Angular momentum operators

$$
J_\pm = \exp(\pm i\varphi)\left(\pm\,\partial_\theta + i\cot\theta\;\partial_\varphi - \frac{1}{2}\tan\frac{\theta}{2}\sigma_3\right), \quad J_3 = -i\partial_\varphi - \frac{\sigma_3}{2}
$$

Energy spectrum

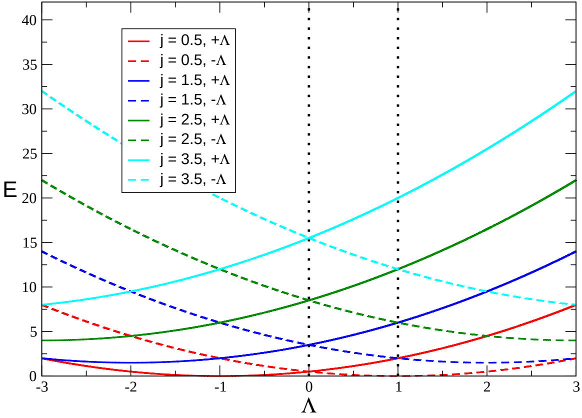
$$
E_j = \frac{1}{2\Theta}\left[j(j+1) - \frac{1}{4}\right] + E(\vec{p}), \quad j \in \{\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots\}
$$

Wave functions are monopole harmonics

$$
Y^\pm_{\frac{1}{2},\pm\frac{1}{2}}(\theta,\varphi) = \frac{1}{\sqrt{2\pi}}\sin\frac{\theta}{2}\exp(\pm i\varphi), \quad Y^\pm_{\frac{1}{2},\mp\frac{1}{2}}(\theta,\varphi) = \frac{1}{\sqrt{2\pi}}\cos\frac{\theta}{2}
$$

Rotor spectrum in the single-nucleon sector ($\Lambda = g_A|\vec{p}|/M$)

$$E_j = \frac{1}{2\Theta}\left[j'(j'+2) + \frac{\Lambda^2 - 1}{2}\right] + E(\vec{p}), \quad j' = j \pm \frac{\Lambda}{2}$$



Leutwyler (1987); Chandrasekharan, Jiang, Pepe, Wiese (2008)

Outline

Cuprate Superconductors and Antiferromagnets

Correspondences between QCD and Antiferromagnetism

Effective Field Theories for Ferro- and Antiferromagnets

Quantum Heisenberg Model

Hubbard Model for Doped Antiferromagnets

Effective Field Theories for Magnons and Holes

Two-Hole States Bound by Magnon Exchange

Rotor Spectrum in the Single-Hole Sector and in QCD

Conclusions

Conclusions

• There are intriguing analogies between antiferromagnets and QCD.

• Doped antiferromagnets are described quantitatively by a systematic low-energy effective field theory analogous to chiral perturbation theory.

• Magnon exchange binds hole pairs in analogy to the deuteron.

• Spirals phases are analogous to pion condensation in nuclear matter.

• Systems on the honeycomb lattice as well as electron-doped systems have been investigated with the same techniques.

• Fermions have characteristic effects on the rotor spectrum, caused by Abelian or non-Abelian monopole Berry gauge fields.

• The rotor problem tests the effective theory nonperturbatively.

• Perturbative matching of Λ to the infinite volume effective theory is necessary before g_A could be extracted from the rotor level splitting.