Electromagnetic currents from chiral EFT

Stefan Kölling

Forschungszentrum Jülich, Institut für Kernphysik and Jülich Center for Hadron Physics
& Helmholtz-Institut für Strahlen- und Kernphysik (Theorie) and Bethe Center for Theoretical Physics

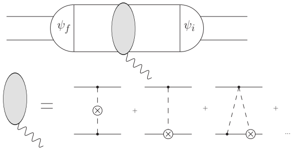
e-mail: s.koelling@fz-juelich.de

in collaboration with: E. Epelbaum and H. Krebs

Bern 09.07.2009

Motivation

- Successful derivation of nuclear potentials using method of unitary transformation & chiral perturbation theory in recent years[Epelbaum et al. '98]
- Consistent derivation of electromagnetic-current J^μ



- Treat em-interaction as perturbation
- Convolute between wave-functions.

- Define effective current with unitary transformation

$$\eta J^{\mu}_{\mathrm{eff}} \eta = \eta U'^{\dagger} \eta U^{\dagger} J^{\mu} U \eta U' \eta \quad U = \begin{pmatrix} \eta \left(1 + A^{\dagger} A\right)^{-\frac{1}{2}} & -A^{\dagger} \left(1 + A A^{\dagger}\right)^{-\frac{1}{2}} \\ A \left(1 + A^{\dagger} A\right)^{-\frac{1}{2}} & \lambda \left(1 + A A^{\dagger}\right)^{-\frac{1}{2}} \end{pmatrix},$$

with projectors η (λ) on the purely nucleonic (rest) subspace.

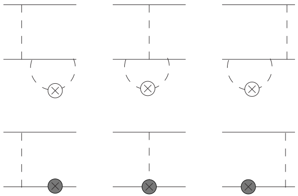
Additional transformations

- Expand A and J in chiral power-counting and compute J order by order.
- Renormalizability on the level of the $3N$-Hamiltonian $\rightarrow$ additional transformation U' [Epelbaum '07]
- In general, further unitary transformations are possible

$$U^{\dagger}(\mathcal{A}^{\mu}) H_{\pi N \gamma} U(\mathcal{A}^{\mu}),$$

- With $U(\mathcal{A}^{\mu})$ s.t. transformed Hamiltonian is block-diagonal (in η (λ) spaces)
- For $\mathcal{A}^{\mu} \rightarrow 0$, $U(\mathcal{A}^{\mu}) \rightarrow \mathbb{1}$
- We calculate the leading-loop order of the one-pion exchange current
- Check renormalizability

Additional transformations Ctd.



- These diagrams visualize the the topology, different meaning than in covariant/time-ordered perturbation theory.

- Nontrivial check, since β-functions were computed in a different formalism.
- For example, for the diagrams above

$$\hookrightarrow e\frac{g_A\, i}{4F_\pi^2}\left(d_{21}+\frac{11g_A^3}{96\pi^2(d-4)}\right)\to e\frac{g_A\, i}{4F_\pi^2}\left(-\frac{g_A^3}{16\pi^2(d-4)}+\frac{11g_A^3}{96\pi^2(d-4)}\right)$$

$$\xrightarrow{U(\mathcal{A}^\mu)} e\frac{g_A\, i}{4F_\pi^2}\left(-\frac{g_A^3}{16\pi^2(d-4)}+\frac{g_A^3}{16\pi^2(d-4)}+(1-\beta)\frac{5g_A^3}{96\pi^2(d-4)}\right)$$

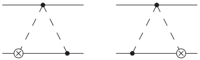
with β_{21} from Gasser et al. '02.

- Additional unitary transformations are needed to restore renormalizability.

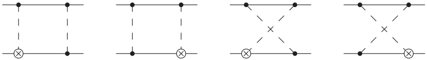
- However, this transformation does not lead to any changes in two-pion exchange sector.

Two-Pion exchange currents

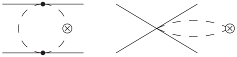
Class 1:



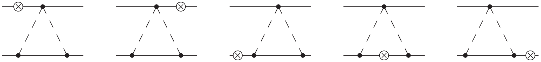
Class 2:



Class 3:

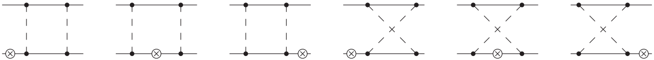


Class 4:



Class 5:

Class 6:



Class 7:

Two-Pion exchange currents in configuration-space

$$
\begin{aligned}
\vec{J}_{c1}(\vec{r}_{10},\vec{r}_{20}) &= e\,\frac{g_A^2\,M_\pi^7}{128\pi^3 F_\pi^4}\left[\vec{\nabla}_{10}\,[\vec{\tau}_1\times\vec{\tau}_2]^3 + 2\left[\vec{\nabla}_{10}\times\vec{\sigma}_2\right]\tau_1^3\right]\,\delta(\vec{x}_{20})\,\frac{K_1(2x_{10})}{x_{10}^2} + (1\leftrightarrow 2)\,,\\
\vec{J}_{c2}(\vec{r}_{10},\vec{r}_{20}) &= -\,e\,\frac{g_A^4\,M_\pi^7}{256\pi^3 F_\pi^4}\left(3\nabla_{10}^2-8\right)\left[\vec{\nabla}_{10}\,[\vec{\tau}_1\times\vec{\tau}_2]^3 + 2\left[\vec{\nabla}_{10}\times\vec{\sigma}_2\right]\tau_1^3\right]\delta(\vec{x}_{20})\,\frac{K_0(2x_{10})}{x_{10}}\\
&\quad+\,e\,\frac{g_A^4\,M_\pi^7}{32\pi^3 F_\pi^4}\left[\vec{\nabla}_{10}\times\vec{\sigma}_1\right]\tau_2^3\,\delta(\vec{x}_{20})\,\frac{K_1(2x_{10})}{x_{10}^2} + (1\leftrightarrow 2)\,,\\
\vec{J}_{c3}(\vec{r}_{10},\vec{r}_{20}) &= -\,e\,\frac{M_\pi^7}{512\pi^4 F_\pi^4}\,[\vec{\tau}_1\times\vec{\tau}_2]^3\,(\vec{\nabla}_{10}-\vec{\nabla}_{20})\,\frac{K_2(x_{10}+x_{20}+x_{12})}{(x_{10}\,x_{20}\,x_{12})(x_{10}+x_{20}+x_{12})} + (1\leftrightarrow 2)\,,\\
\vec{J}_{c5}(\vec{r}_{10},\vec{r}_{20}) &= -\,e\,\frac{g_A^2\,M_\pi^7}{256\pi^4 F_\pi^4}\left(\vec{\nabla}_{10}-\vec{\nabla}_{20}\right)\left[[\vec{\tau}_1\times\vec{\tau}_2]^3\,\vec{\nabla}_{12}\cdot\vec{\nabla}_{20} - 2\tau_1^3\vec{\sigma}_2\cdot\left[\vec{\nabla}_{12}\times\vec{\nabla}_{20}\right]\right]\\
&\quad\times\,\frac{K_1(x_{10}+x_{20}+x_{12})}{(x_{10}\,x_{20}\,x_{12})} + (1\leftrightarrow 2)\,,\\
\vec{J}_{c7}(\vec{r}_{10},\vec{r}_{20}) &= e\,\frac{g_A^4\,M_\pi^7}{512\pi^4 F_\pi^4}\left(\vec{\nabla}_{10}-\vec{\nabla}_{20}\right)\Big[[\vec{\tau}_1\times\vec{\tau}_2]^3\,\vec{\nabla}_{12}\cdot\vec{\nabla}_{10}\,\vec{\nabla}_{12}\cdot\vec{\nabla}_{20} + 4\tau_2^3\vec{\sigma}_1\cdot\left[\vec{\nabla}_{12}\times\vec{\nabla}_{10}\right]\\
&\quad\times\,\vec{\nabla}_{12}\cdot\vec{\nabla}_{20}\Big]\,\frac{x_{10}+x_{20}+x_{12}}{x_{10}\,x_{20}\,x_{12}}\,K_0(x_{10}+x_{20}+x_{12}) + (1\leftrightarrow 2)\\
\vec{J}_4(\vec{r}_{10},\vec{r}_{20}) &= \vec{J}_{c6}(\vec{r}_{10},\vec{r}_{20}) = 0\,,
\end{aligned}
$$

with $\vec{r}_{1/2/0}$ the positions of the first/second nucleon/the photon, and $\vec{x}_{10} = M_\pi\,(\vec{r}_1-\vec{r}_0)$, $\vec{x}_{20} = M_\pi\,(\vec{r}_2-\vec{r}_0)$, $\vec{x}_{12} = M_\pi\,(\vec{r}_1-\vec{r}_2)$ and $\vec{\nabla}_{ij} \equiv \partial/\partial x_{ij}$ and $x_{ij} \equiv |\vec{x}_{ij}|$.
All derivatives have to be evaluated as if the variables were independent.

Two-Pion exchange currents in configuration-space Ctd.

$$\rho_{c1}(\vec{r}_{10},\vec{r}_{20}) = \rho_{c2}(\vec{r}_{10},\vec{r}_{20}) = \rho_{c3}(\vec{r}_{10},\vec{r}_{20}) = 0\,,$$

$$\rho_{c4}(\vec{r}_{10},\vec{r}_{20}) = e\,\frac{g_A^2 M_\pi^7}{256\pi^2 F_\pi^4}\,\tau_1^3\,\delta(\vec{x}_{20})\left(\nabla_{10}^2 - 2\right)\frac{e^{-2x_{10}}}{x_{10}^2} + (1\leftrightarrow 2)\,,$$

$$\rho_{c5}(\vec{r}_{10},\vec{r}_{20}) = -\,e\,\frac{g_A^2 M_\pi^7}{256\pi^2 F_\pi^4}\,\tau_2^3\,\delta(\vec{x}_{20})\left(\nabla_{10}^2 - 2\right)\frac{e^{-2x_{10}}}{x_{10}^2} + (1\leftrightarrow 2)\,,$$

$$\begin{aligned}\rho_{c6}(\vec{r}_{10},\vec{r}_{20}) = &-\,e\,\frac{g_A^4 M_\pi^7}{256\pi^2 F_\pi^4}\,\delta(\vec{x}_{20})\left[\tau_1^3\left(2\nabla_{10}^2 - 4\right) + \tau_2^3\,\vec{\sigma}_1\cdot\vec{\nabla}_{10}\,\vec{\sigma}_2\cdot\vec{\nabla}_{10} - \tau_2^3\vec{\sigma}_1\cdot\vec{\sigma}_2\right]\frac{e^{-2x_{10}}}{x_{10}^2}\\ &-\,e\,\frac{g_A^4 M_\pi^7}{128\pi^2 F_\pi^4}\,\delta(\vec{x}_{20})\,\tau_1^3\left(3\nabla_{10}^2 - 11\right)\frac{e^{-2x_{10}}}{x_{10}} + (1\leftrightarrow 2)\,,\end{aligned}$$

$$\begin{aligned}\rho_{c7}(\vec{r}_{10},\vec{r}_{20}) = &-\,e\,\frac{g_A^4 M_\pi^7}{512\pi^3 F_\pi^4}\Big[(\tau_1^3+\tau_2^3)\Big(\vec{\nabla}_{12}\cdot\vec{\nabla}_{10}\vec{\nabla}_{12}\cdot\vec{\nabla}_{20} + \vec{\nabla}_{12}\cdot\left[\vec{\nabla}_{10}\times\vec{\sigma}_1\right]\vec{\nabla}_{12}\cdot\left[\vec{\nabla}_{20}\times\vec{\sigma}_2\right]\Big)\\ &+[\vec{\tau}_1\times\vec{\tau}_2]^3\,\vec{\nabla}_{12}\cdot\vec{\nabla}_{10}\,\vec{\nabla}_{12}\cdot\left[\vec{\nabla}_{20}\times\vec{\sigma}_2\right]\Big]\frac{e^{-x_{10}}}{x_{10}}\,\frac{e^{-x_{20}}}{x_{20}}\,\frac{e^{-x_{12}}}{x_{12}} + (1\leftrightarrow 2)\,.\end{aligned}$$

- Results also available in momentum-space, expressed in standard loop-function $L(q)$, $A(q)$ and three-point functions.
- Can be easily treated numerically.
- Continuity-equation is fulfilled $\rightarrow$ Current is consistent with potential obtained within the method of unitary transformation

Conclusion and outlook

Conclusion

- We derived the long-range part of the em-current and charge-density in the method of unitary transformation at leading loop-order.
- An explicit check of renormalizability of the one-pion exchange contributions was performed.
- Expressions are given in momentum-space in terms of loop-functions $L(q)$, $A(q)$ and three-point functions.
- We analytically carried out the Fourier-transform to arrive at very compact expressions in configuration-space.
- The current fulfills the continuity-equation, i.e. is consistent with the potential.
- We checked with the corrected results of Pastore et al. '09.

Outlook

- Full treatment of short-range effects.
- Inclusion of Δ-degrees of freedom.
- Going to the sub-leading loop-order.
- ...