Pion Physics on the lattice

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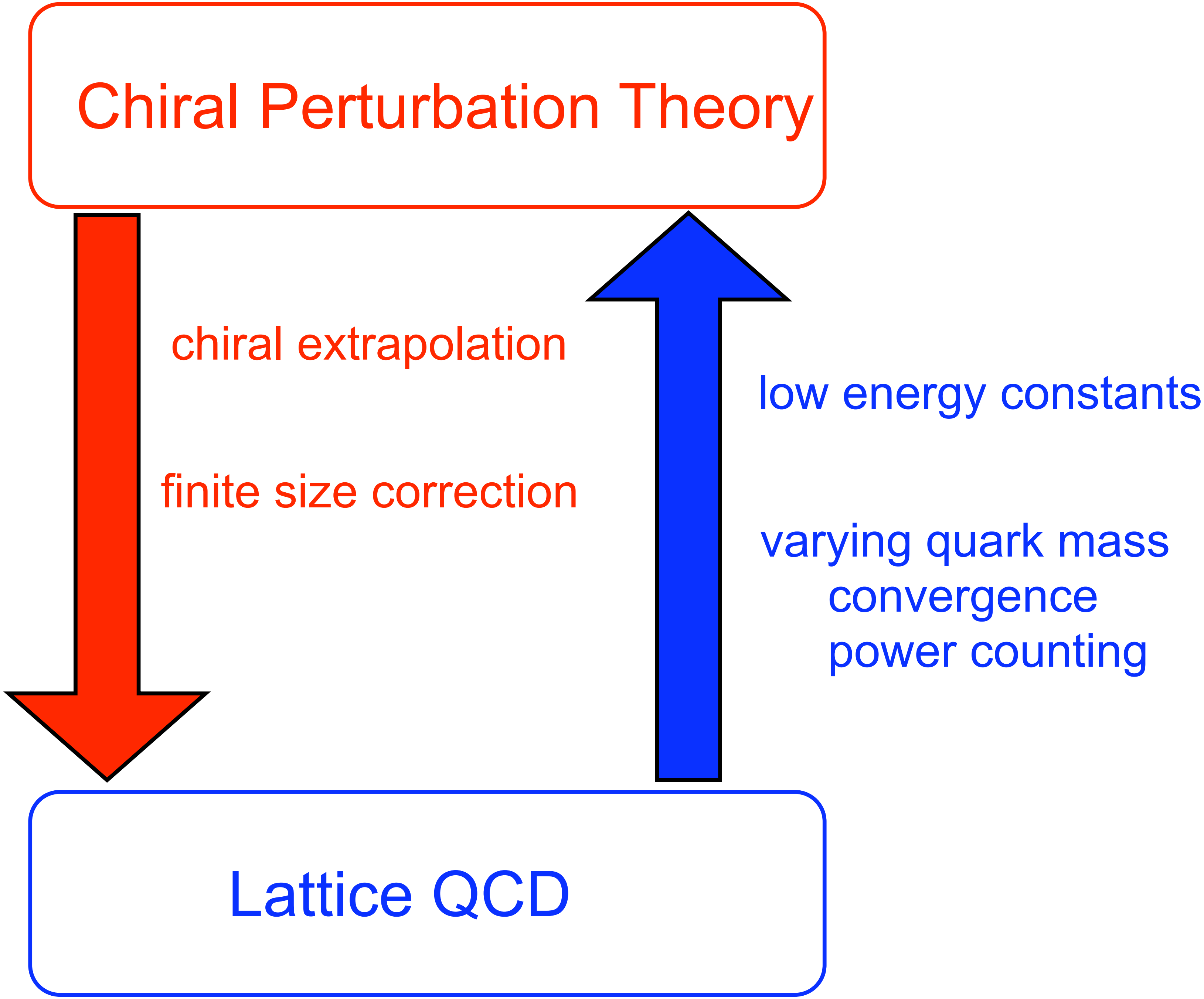

Chiral Perturbation Theory
chiral extrapolation
finite size correction
low energy constants
varying quark mass
convergence
power counting
Lattice QCD

Content

1. Recent full QCD simulations
2. Pion mass/decay constant and ChPT

   (1) 2-flavor QCD and ChPT

   (2) 2+1 flavor QCD and ChPT

3. Others

   (1) Pion form factors

   (2) epsilon-regime → S. Hashimoto's talk

   (3) S-parameter

   (4) topological susceptibility (from fixed topology)

4. Summary

No new results for pi-pi scattering.

Talk by A. Walker-Loud (July 7,17:30@WG3)

Recent Review: S. Necco@Confinement'08, arXiv:0901.4257

1. Recent full QCD simulations

Full QCD simulations near the physical quark mass becomes possible.

by improvements for both computers and algorithms

Systematic errors in lattice QCD

- finite size L
  - corrected by ChPT ?
- finite lattice spacing a
- heavier u,d quark mass
  - chiral extrapolation is needed. ChPT ?

(incomplete) lists of full QCD simulations

Group	flavors	a(fm)	L(fm)	$m_\pi^{\mathrm{min.}}$ (MeV)
PACS-CS	2+1	0.09	2.9	160
MILC	2+1	>0.06	3.3	240
BMW	2+1	>0.065	>4.2	190
JLab	2+1	0.12	1.5~2.9	385
CERN-ToV	2	>0.05	1.7~ 1.9	300
ETMC	2	>0.07	2.1	300
CLS	2	>0.06	2.5	260
QCDSF	2	>0.072	1.7~3.2	240
RBC-UKQCD	2+1	0.11	2.8	330
JLQCD	2+1	0.11	1.8	315
RBC	2	0.12	2.5	490
JLQCD	2	0.12	1.9	290

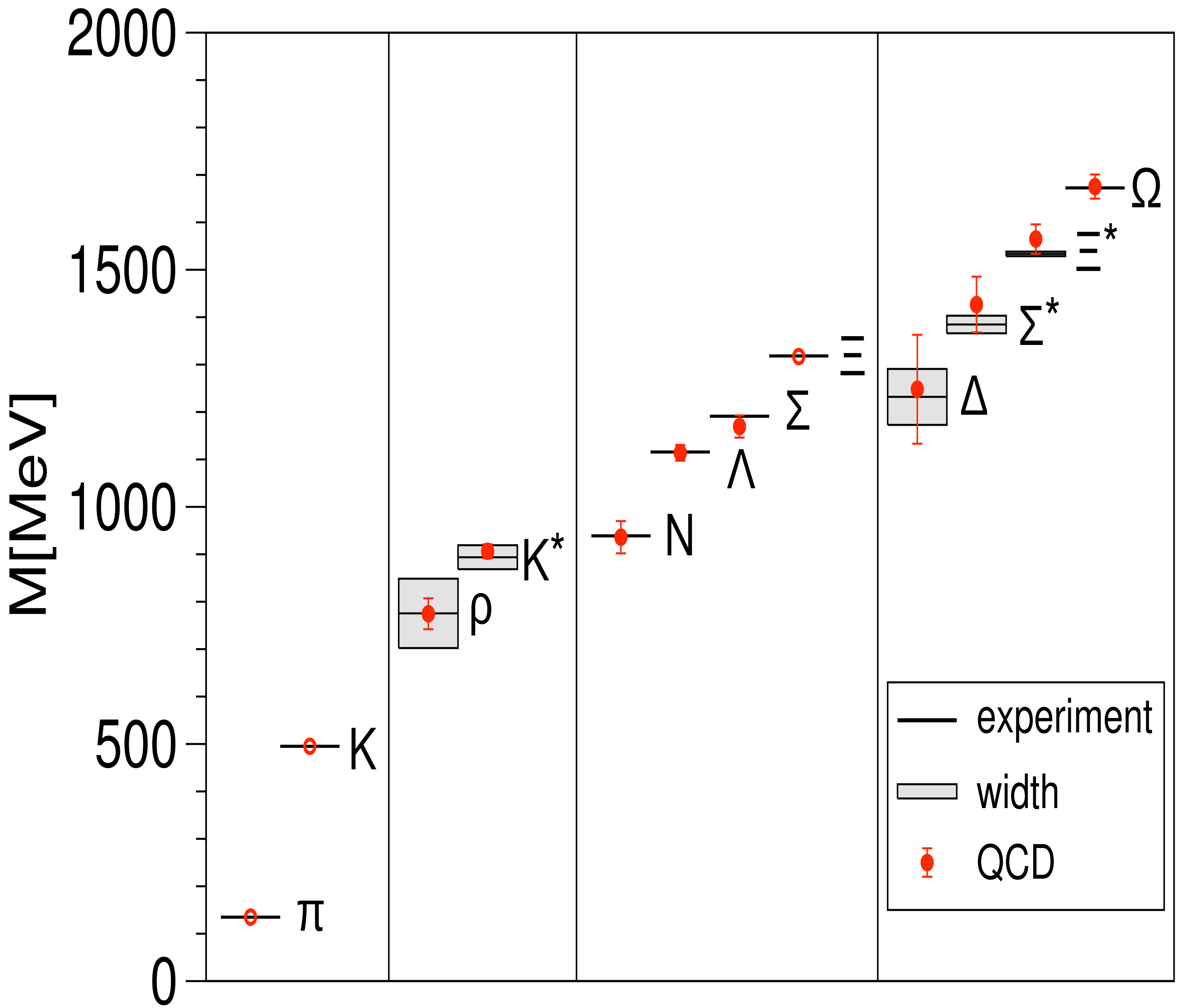
conventional quark action (Wilson/KS)

chirally symmetric quark action (Overlap/DW)

Hadron spectra

Science 322(2008)1224.

BMW collaboration



$a \to 0$

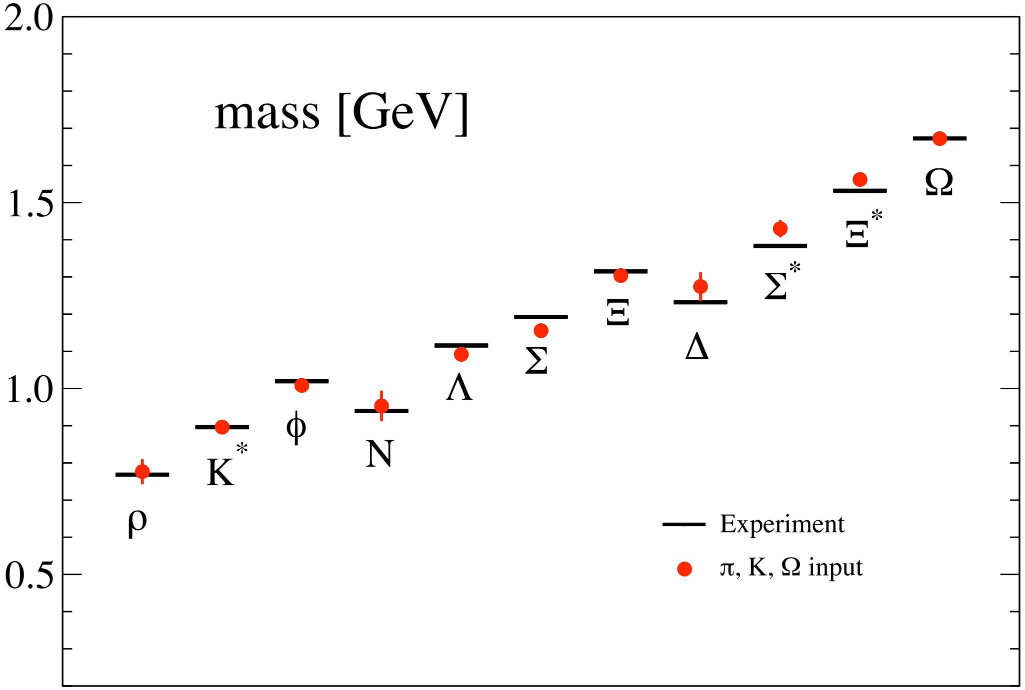
$m_\pi^{\mathrm{min.}} = 190\ \mathrm{MeV}$

$m_\pi L \geq 4$

Agreement between Lattice QCD and experiment is excellent !

PACS-CS Collaboration

Phys. Rev. D79(2009)034503.



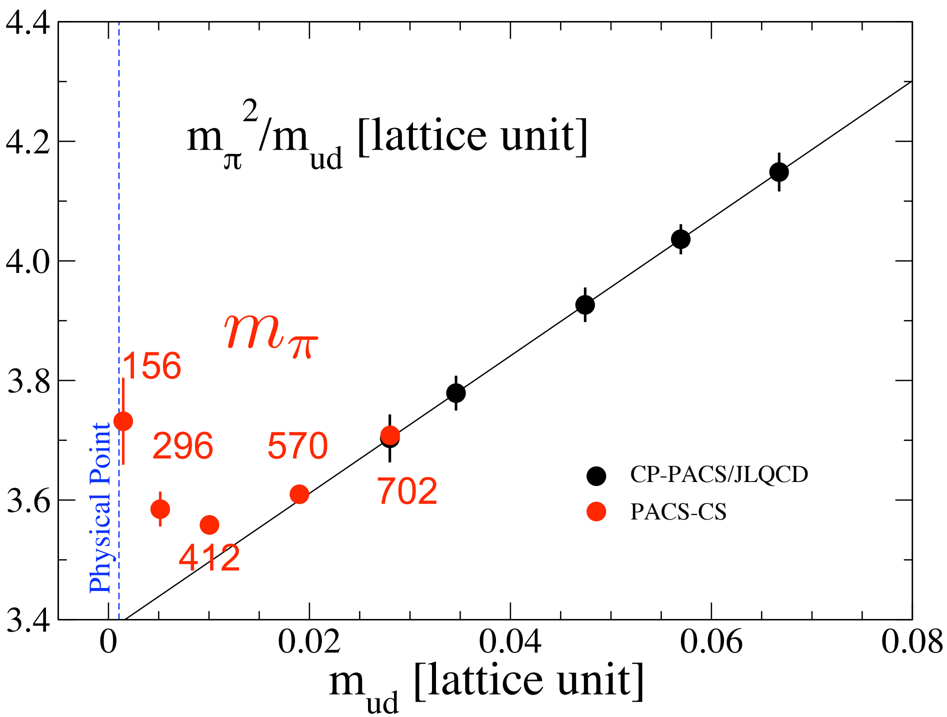
$a = 0.09$ fm

$L = 2.9$ fm

$m_\pi L = 2.3$

$m_\pi^{\text{min.}} = 156$ MeV

We are almost on the "physical point".



$m_\pi L > 4$

Calculations with L=5.8 fm and $m_\pi \simeq 140$ MeV are on-going.

"Real QCD"

2. Pion mass and decay constant

2-1. Nf=2 and Chiral Perturbation Theory

SU(2) ChPT formula

$f_\pi = 132 \text{ MeV}$

LO NLO log LOC

$$\frac{m_\pi^2}{m_q} = 2B\left\{1 + \frac{2Bm_q}{16\pi^2 f^2}\left[\ln\left(\frac{2Bm_q}{\mu^2}\right) - l_3(\mu)\right]\right\}$$

LO NLO log LOC

$$f_\pi = f\left\{1 - \frac{2Bm_q}{8\pi^2 f^2}\left[\ln\left(\frac{2Bm_q}{\mu^2}\right) - l_4(\mu)\right]\right\}$$

Gasser-Leutwyler, 1984

JLQCD/TWQCD Collaborations

Noaki, *et al.*, Phys.Rev.Lett. 101(2008)202004.

- Ovelap fermion (exact “chiral” symmetry), a=0.12fm
- fixed topology
  - 1/V correction by ChPT
- $m_\pi \geq 290\ \mathrm{MeV},\ m_\pi L \geq 2.9$
  - finite volume correction by ChPT (NLO LOCs are required)

destructive

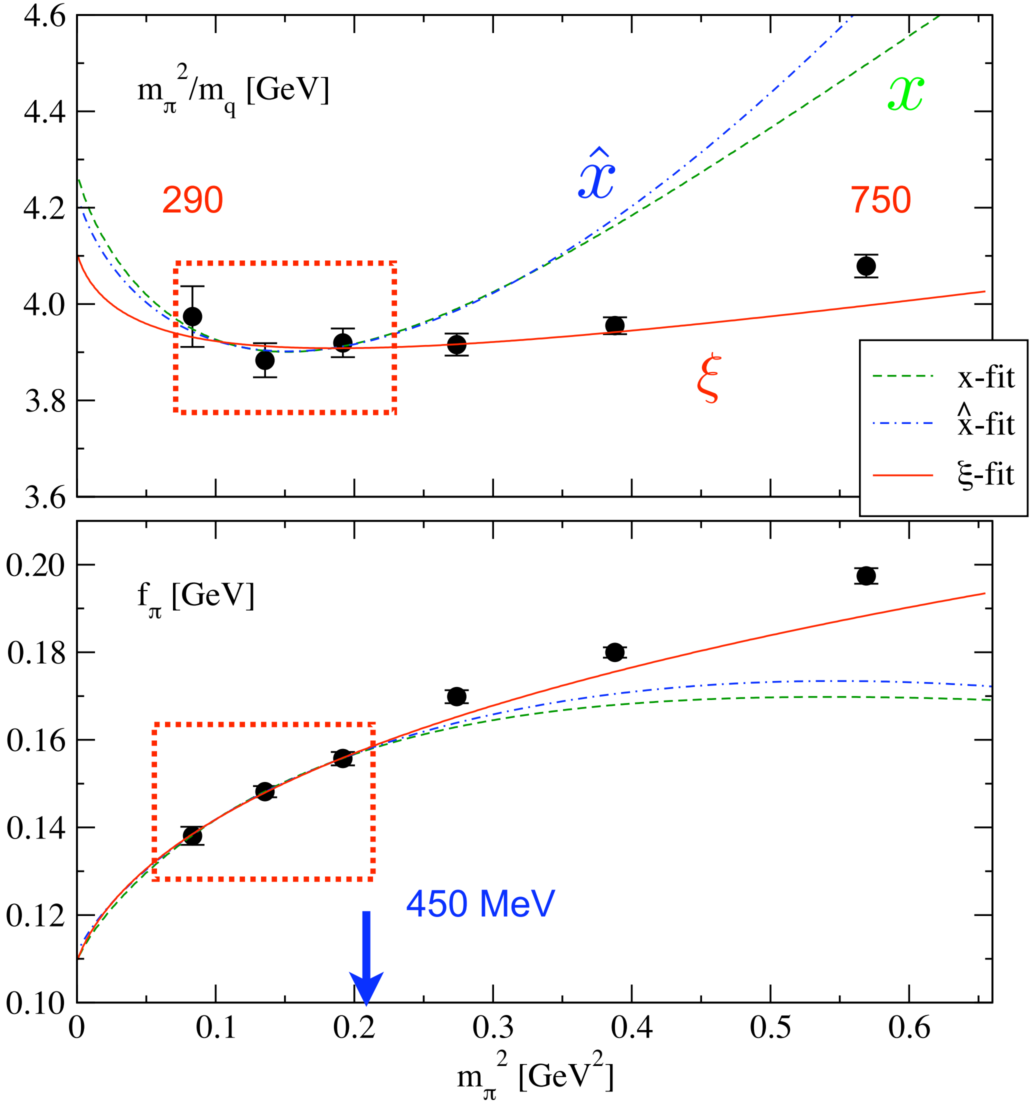
Colangero-Dürr-Haefeli’05

NLO Fits

$$\frac{m_\pi^2}{m_q} = 2B\left[1 + \frac{1}{2}x\ln x\right] + c_3 x \qquad f_\pi = f[1 - x\ln x] + c_4 x$$

choices of expansion parameter

$$x \equiv \frac{2Bm_q}{8\pi^2 f^2} \qquad \hat{x} \equiv \frac{m_\pi^2}{8\pi^2 f^2} \qquad \xi \equiv \frac{m_\pi^2}{8\pi^2 f_\pi^2}$$



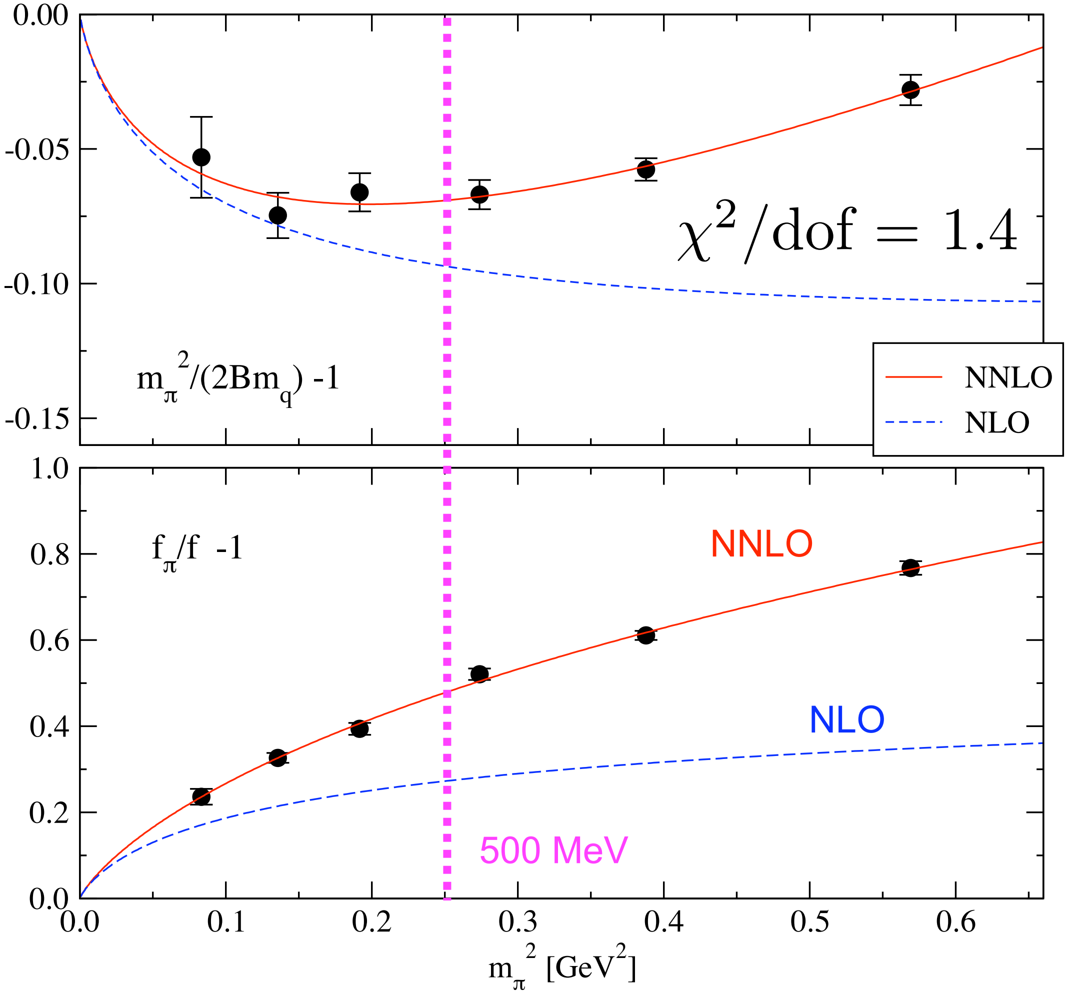
- NLO fits work for the lightest 3 data.
  - all 3 choices
  - establish the validity of NLO ChPT fits
- Xi-fit describes data beyond the fitted region.

NNLO Fits

Colangero-Gasserr-Leutwyler'01

$$m_\pi^2/m_q = 2B\Big[1 + \tfrac{1}{2}\xi\ln\xi + \tfrac{7}{8}(\xi\ln\xi)^2 + \Big(\tfrac{c_4}{f} - \tfrac{1}{3}(\tilde{l}^{\,\rm phys} + 16)\Big)\xi^2\ln\xi\Big] + c_3\,\xi(1 - \tfrac{9}{2}\xi\ln\xi) + \alpha\,\xi^2,$$

$$f_\pi = f\Big[1 - \xi\ln\xi + \tfrac{5}{4}(\xi\ln\xi)^2 + \tfrac{1}{6}(\tilde{l}^{\,\rm phys} + \tfrac{53}{2})\xi^2\ln\xi\Big] + c_4\,\xi(1 - 5\xi\ln\xi) + \beta\,\xi^2.$$



$$\bar{l}^{\rm phys} \equiv 7\bar{l}_1 + 8\bar{l}_2 - 15\ln(2\sqrt{2}\pi f_\pi/m_\pi)^2 = -32.0 \pm 4.3$$

Input

Fit is reasonable, but c3 and c4 are different from NLO values, and

$$\frac{|\text{NNLO}-\text{NLO}|}{|\text{NLO}-\text{LO}|} = 0.3(m_\pi) \text{ or } 0.7(f_\pi) \text{ at } m_\pi = 500 \text{ MeV}$$

Determination of LOCs

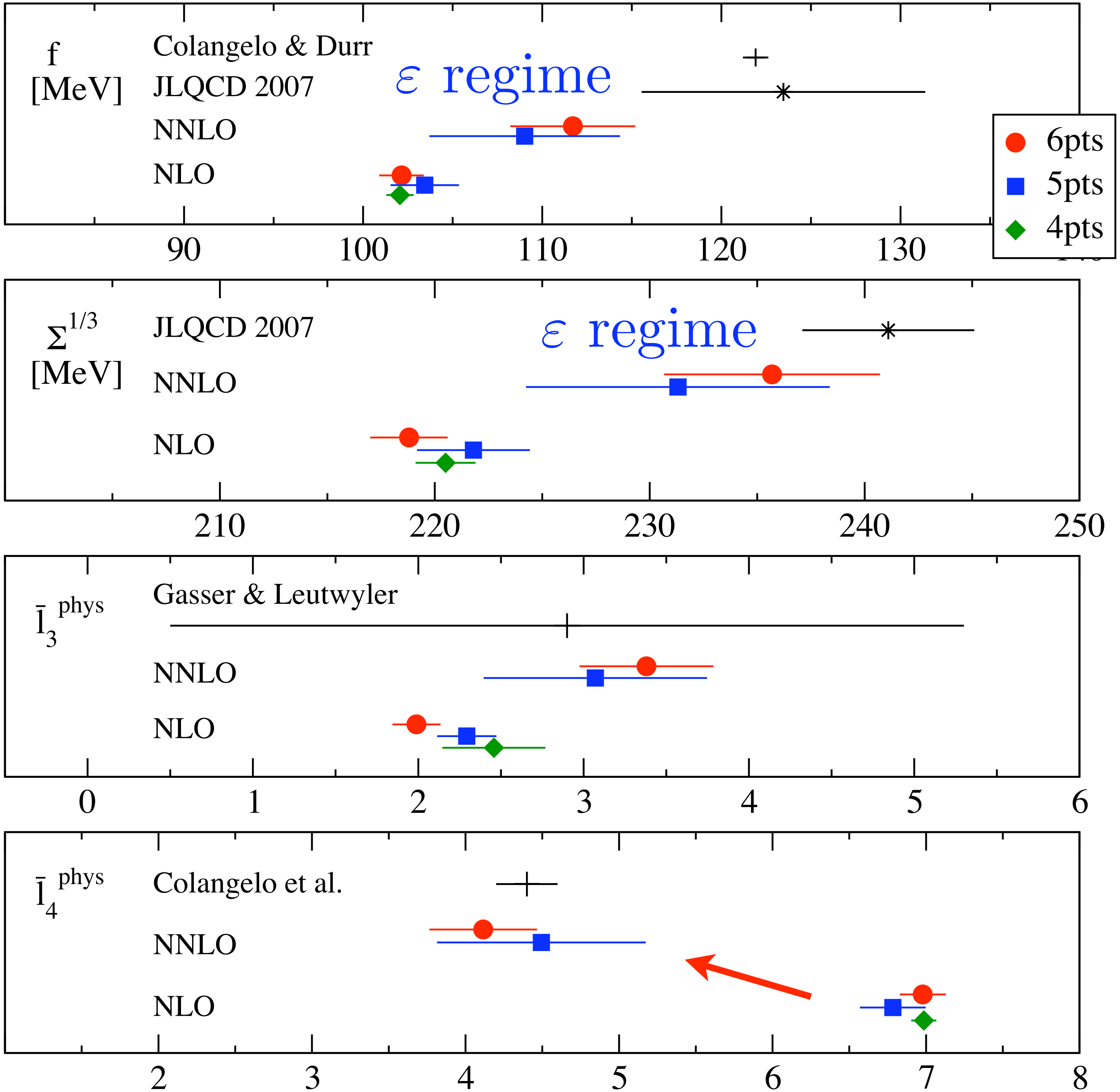
Variations of some LOCs from NLO to NNLO are significant.

LO — f

LO — $\Sigma^{1/3}_{\overline{MS}}(2\text{GeV})$

NLO — $l_3(m_\pi)$

NLO — $l_4(m_\pi)$



European Twisted Mass Collaborations

Boucaud, *et al.*, Phys.Lett. B650(2007)304
Boucaud, *et al.*,arXiv:0810.2873[hep-lat]
C. Urbach, arXiv:0710.1517[hep-lat]

Talk by G. Herdoizo (July 6,14:40@WG1)

- twisted mass QCD $L_{\rm tm}^{\rm cont.} = m_q \bar{q} e^{i\theta\gamma_5\tau_3} q, \quad q = \begin{pmatrix} u \\ d \end{pmatrix}$
  - $\theta = \pi/2$ (maximal twist) ⟶ O(a) lattice artifact is absent.
- a=0.087, 0.067 fm, L=2.1 fm $m_\pi \geq 310\ {\rm MeV},\ m_\pi L \geq 3.3\ (4.3)$
  - finite volume correction by ChPT(GL, CDH) Gasser-Luetwyler'87

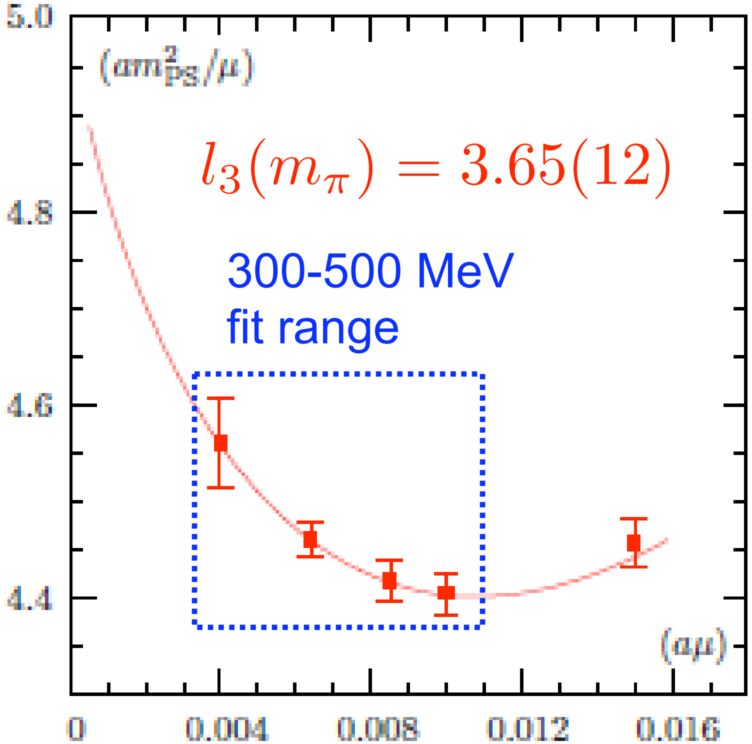
$$R_O = \frac{O(L) - O(\infty)}{O(\infty)} = c_O \xi \tilde{g}_1(\lambda), \quad \xi = \frac{m_\pi^{\rm LO}}{(4\pi f)^2},\ \lambda = m_\pi^{\rm LO} L$$

$$\tilde{g}_1(x) = \sum_{n=1}^{\infty} \frac{4m(n)}{\sqrt{n}x} K_1(\sqrt{n}x) \qquad c_{m_\pi} = \frac{1}{N_f},\ c_{f_\pi} = -N_f$$

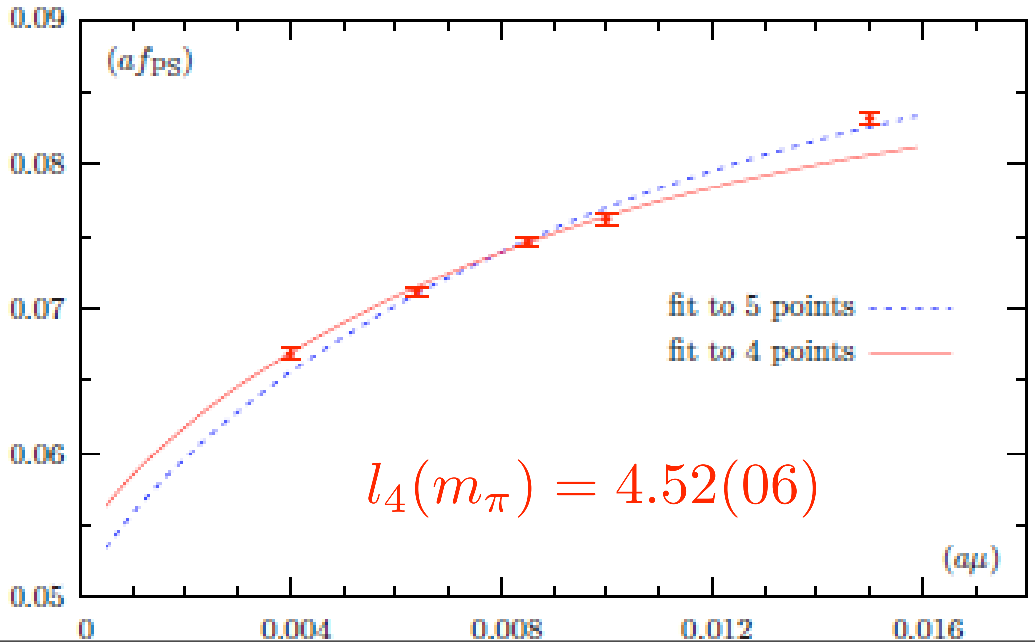
NLO Fits

a=0.087 fm,
with finite volume correction

Boucaud, *et al.*, Phys.Lett. B650(200)304



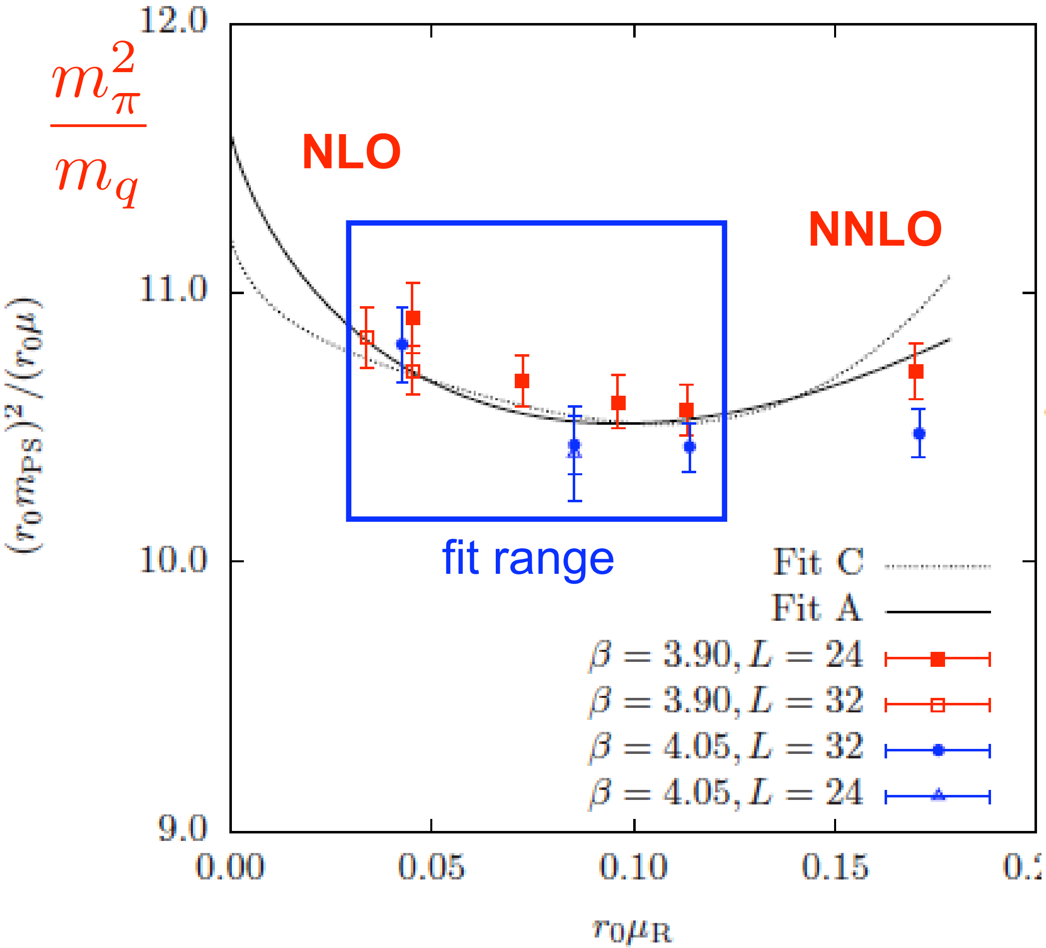
NLO fits work at 500 MeV or below.



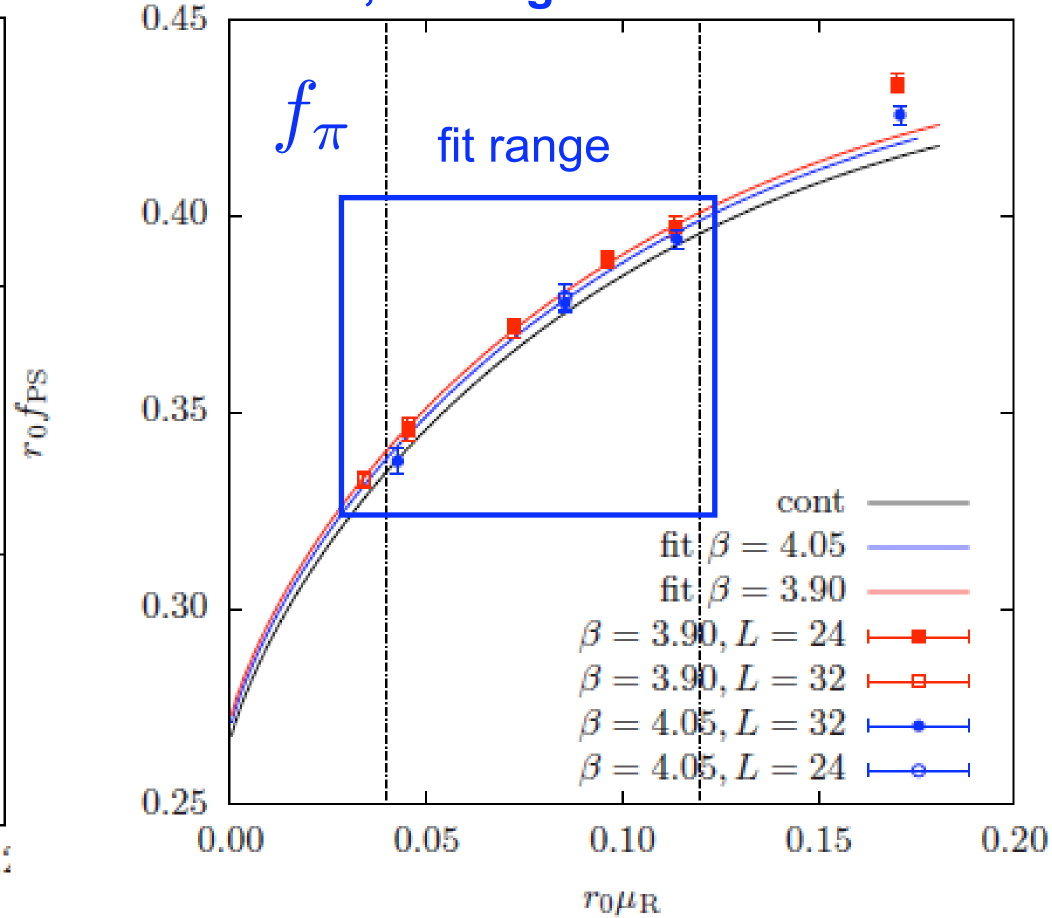
NNLO vs. NLO

a=0.087, 0.067 fm,
with finite volume correction

Boucaud, *et al.*,arXiv:0810.2873[hep-lat]



NLO, scaling violation is included



variation is significant

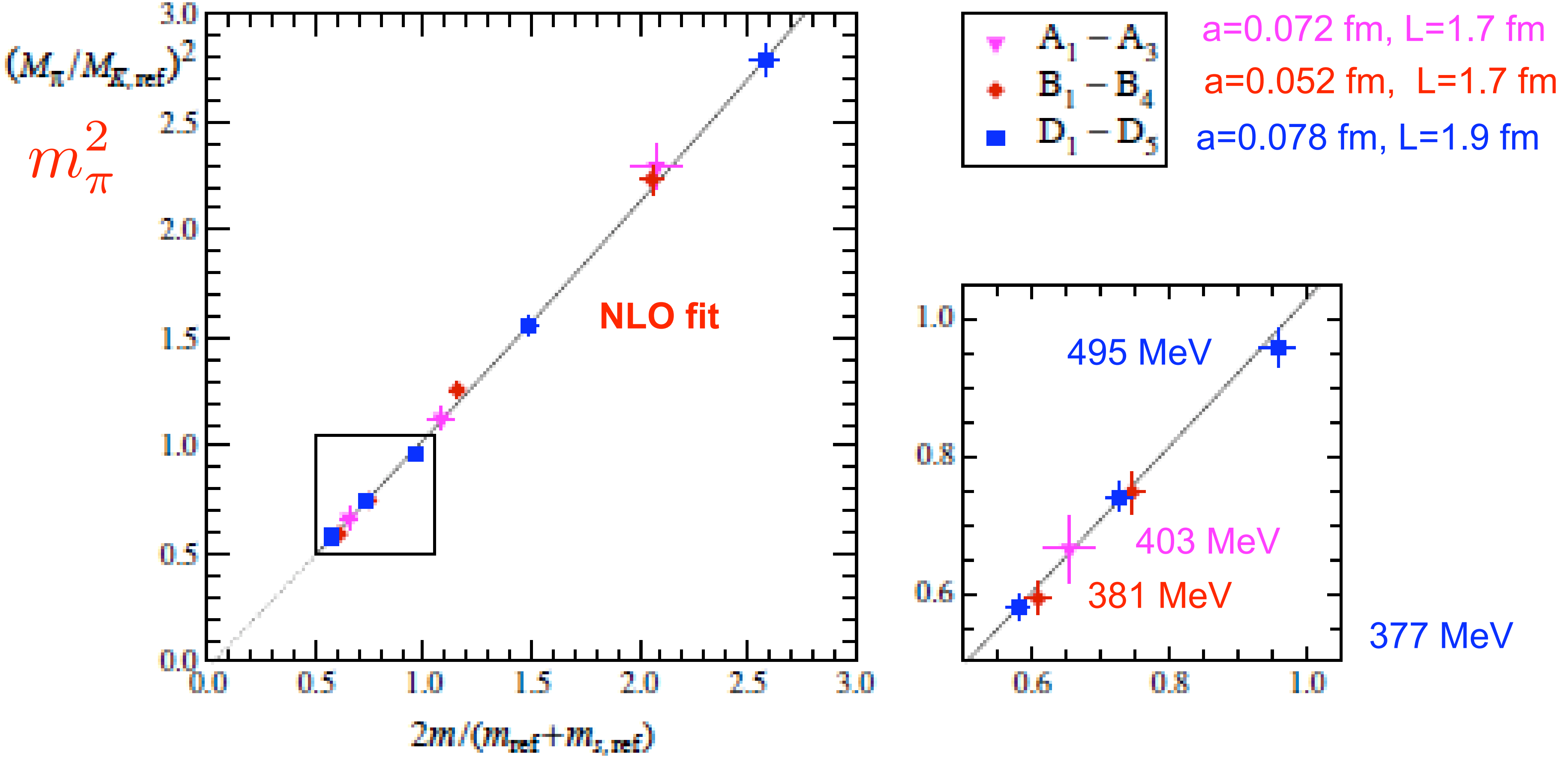
	$\Sigma^{1/3}$	f	l_3	l_4
NLO	267(2)	121.7(1)	3.42(8)	4.59(4)
NNLO	263(2)	121.7(3)	3.15(9)	4.72(12)

CERN-TorVergata

Del Debbio, *et al.*, JHEP 02(2007)056

- (O(a)-improved) Wilson, a= 0.052,0.072,0.078 fm

m_π^2



$$l_3(m_\pi) = 3.0(5)$$

Summary: Nf=2 QCD and SU(2) ChPT

Now

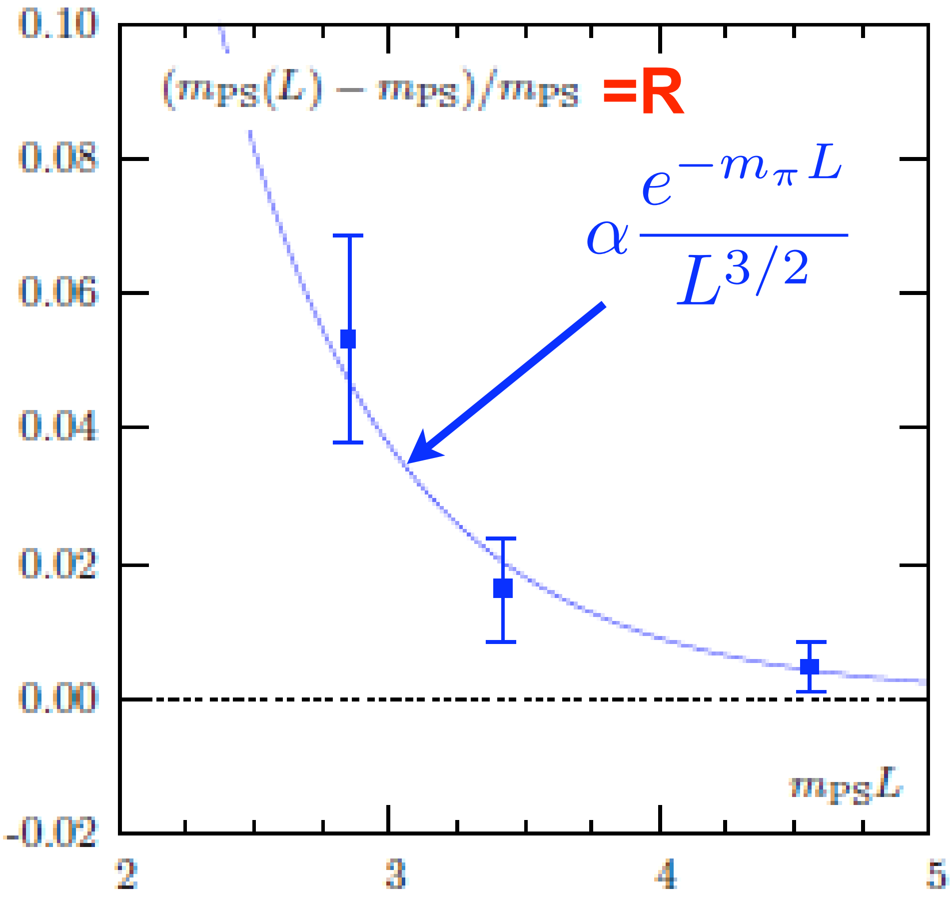
- NLO ChPT works at $m_\pi \leq 500$ MeV
  - "Chiral-log" is unambiguously observed on the lattice.(first time).
- NNLO may describe data beyond that region.
  - some LOCs are fixed to phenomenological values.
  - NNLO corrections seem large, in particular for f_π
  - some NLO LOCs are significantly affected.

Future

- Finite size correction: ChPT formula should be checked by Lattice
- NNLO fits without using phenomenological inputs.
  - simultaneous fits to various quantities are needed.
  - Finite Size correction should be also included.
- inclusion of the lattice artifact in ChPT(Wilson, tmQCD, KS)

Finite size correction

ETMC, C. Urbach, arXiv:0710.1517[hep-lat]



R

	$m_\pi L$	data	GL	CDH
m_π	3.0	+6.2%	+1.8%	+4.7%
	3.3	+1.8	+0.62	+1.0
	3.5	+1.1	+0.8	+1.3
f_π	3.0	-10.7	-7.3	-8.9
	3.3	-2.5	-2.5	-2.4
	3.5	-1.8	-3.2	-2.9

The resummed Lüscher formula is roughly consistent with lattice data, but more detailed studies are needed for definite conclusions.

2-2. Nf=2+1 QCD and ChPT

Problem of Nf=2+1 QCD

- K meson mass is too heavy for NLO ChPT to work ?
  - SU(2) vs. SU(3) ChPT

SU(3)

$$\frac{m_\pi^2}{m_l} = 2B_0\left\{1+\mu_\pi-\frac{1}{3}\mu_\eta+\frac{2B_0}{f_0^2}\left(16m_l(2L_8-L_5)+16(2m_l+m_s)(2L_6-L_4)\right)\right\}$$

$$f_\pi = f_0\left\{1-2\mu_\pi-\mu_K+\frac{2B_0}{f_0^2}\left(8m_lL_5+8(2m_l+m_s)L_4\right)\right\}$$

$$\mu_{\rm PS} = \frac{\tilde{m}_{\rm PS}^2}{16\pi^2 f_0^2}\ln\left(\frac{\tilde{m}_{\rm PS}^2}{\mu^2}\right)$$

$$\tilde{m}_\pi^2 = 2m_lB_0,\ \tilde{m}_K^2 = (m_l+m_s)B_0,\ \tilde{m}_\eta^2 = \frac{2}{3}(m_l+2m_s)B_0$$

SU(2)

$$\frac{m_\pi^2}{m_q} = 2B\left\{1+\frac{2Bm_q}{16\pi^2f^2}\left[\ln\left(\frac{2Bm_q}{\mu^2}\right)-l_3(\mu)\right]\right\}$$

$$f_\pi = f\left\{1-\frac{2Bm_q}{8\pi^2f^2}\left[\ln\left(\frac{2Bm_q}{\mu^2}\right)-l_4(\mu)\right]\right\}$$

$B,\ f,\ l_{3,4}$: m_s dependent

PACS-CS Collaboration

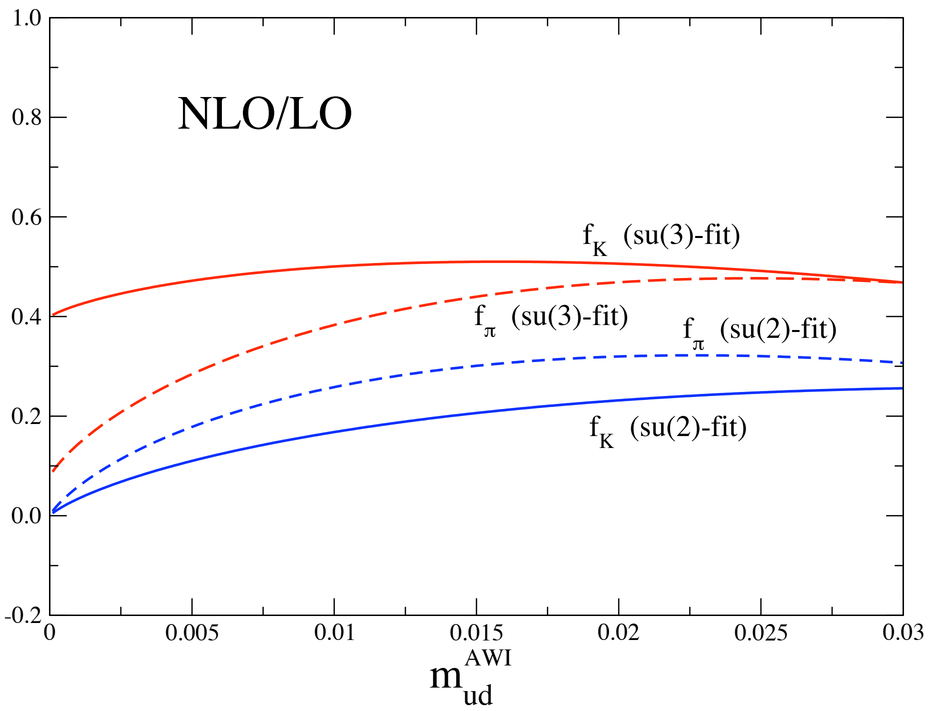
Aoki *et al.*, Phys. Rev. D79(2009)034503.
Kadoh *et al.*, arXiv:0810.0351[hep-lat]
Y. Kuramashi, arXiv: 0811.2630[hep-lat]

- O(a) improved Wilson, a=0.9 fm, L=2.9 fm

$$m_\pi \geq 160 \text{ MeV}, \; m_\pi L \geq 2.3$$

- perturbative renormalization

- Wilson ChPT +O(a) improvement → continuum ChPT at NLO



$$\chi^2/\text{dof}(\text{SU}(3)) \simeq 4$$

$$\chi^2/\text{dof}(\text{SU}(2)) \simeq 0.4$$

NLO/LO(SU(3)) > NLO/LO(SU(2))

SU(2) ChPT works much better than SU(3) ChPT at NLO

RBC-UKQCD Collaborations

Allton *et al.*, Phys. Rev. D78(2008)114509.
E. Scholz, arXiv: 0809.3251[hep-lat]

- a=0.11 fm, L=2.7 fm / a=0.08 fm, L=2.6 fm

$m_\pi \geq 330$ MeV, $m_\pi L \geq 4.6$ / $m_\pi \geq 310$ MeV, $m_\pi L \geq 4.1$

New

- Domain-Wall quarks (almost “chiral”) $m_{\rm res} a = 0.003 \;/\; 0.0007$

additive mass renormalization

- DW-ChPT ⟶ continuum ChPT at NLO with $\tilde{m}_f = m_f + m_{\rm res}$

  - SU(2) / SU(3) Partially Quenched ChPT at NLO

- one strange quark mass

SU(2) PQChPT fits with $m_x + m_y \leq 0.02$

a=0.11 fm

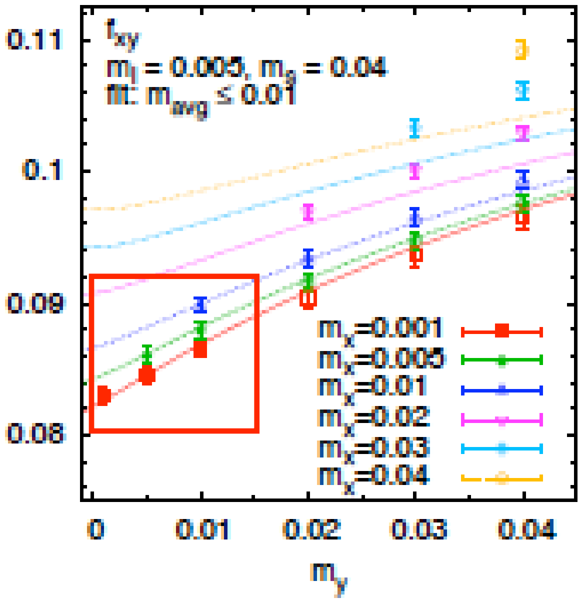
$m_s = 0.04$

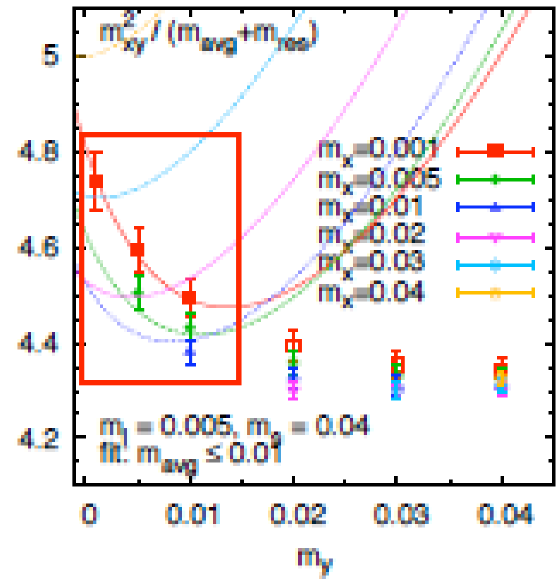
f_π

$\frac{m_\pi^2}{m_q}$

$m_\pi^{\rm sea}$

331 MeV

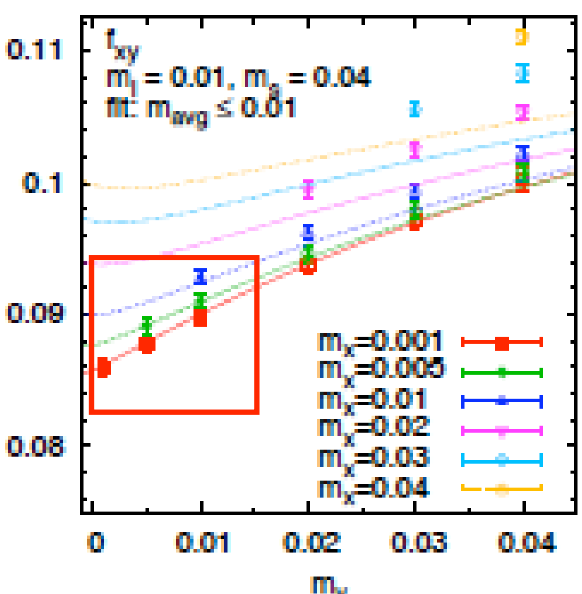


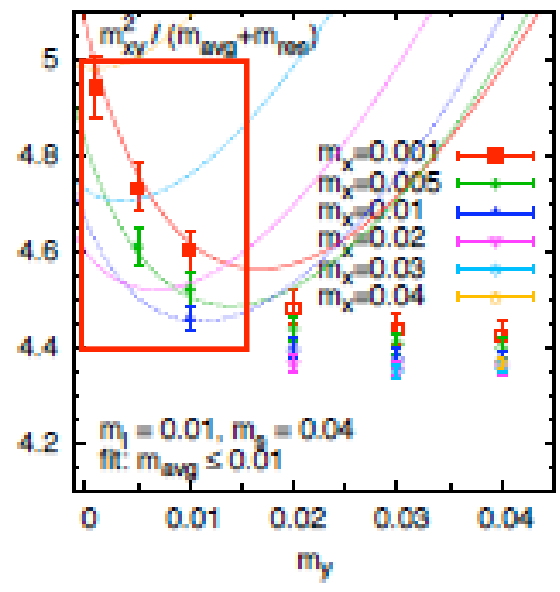


NLO SU(2) PQChPT works.

$\chi^2/{\rm dof} \simeq 0.3$

419 MeV





$l_3(m_\pi) = 3.13(33)$

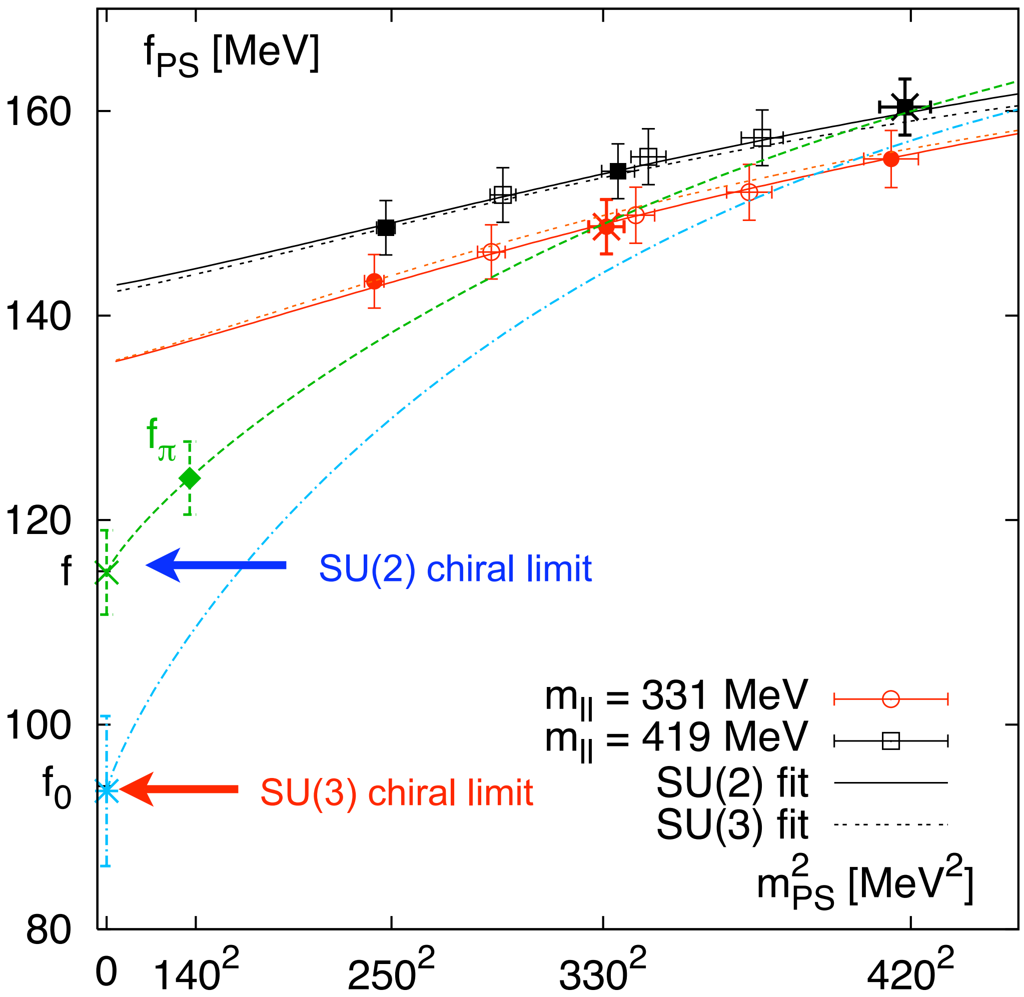
$l_4(m_\pi) = 4.43(14)$

SU(2) vs. SU(3)

NLO SU(3) PQChPT works only at $m_x + m_y \leq 0.02$ $\chi^2/\mathrm{dof} \simeq 0.7$

This can not cover $m_s = 0.04$

Partially quenched data



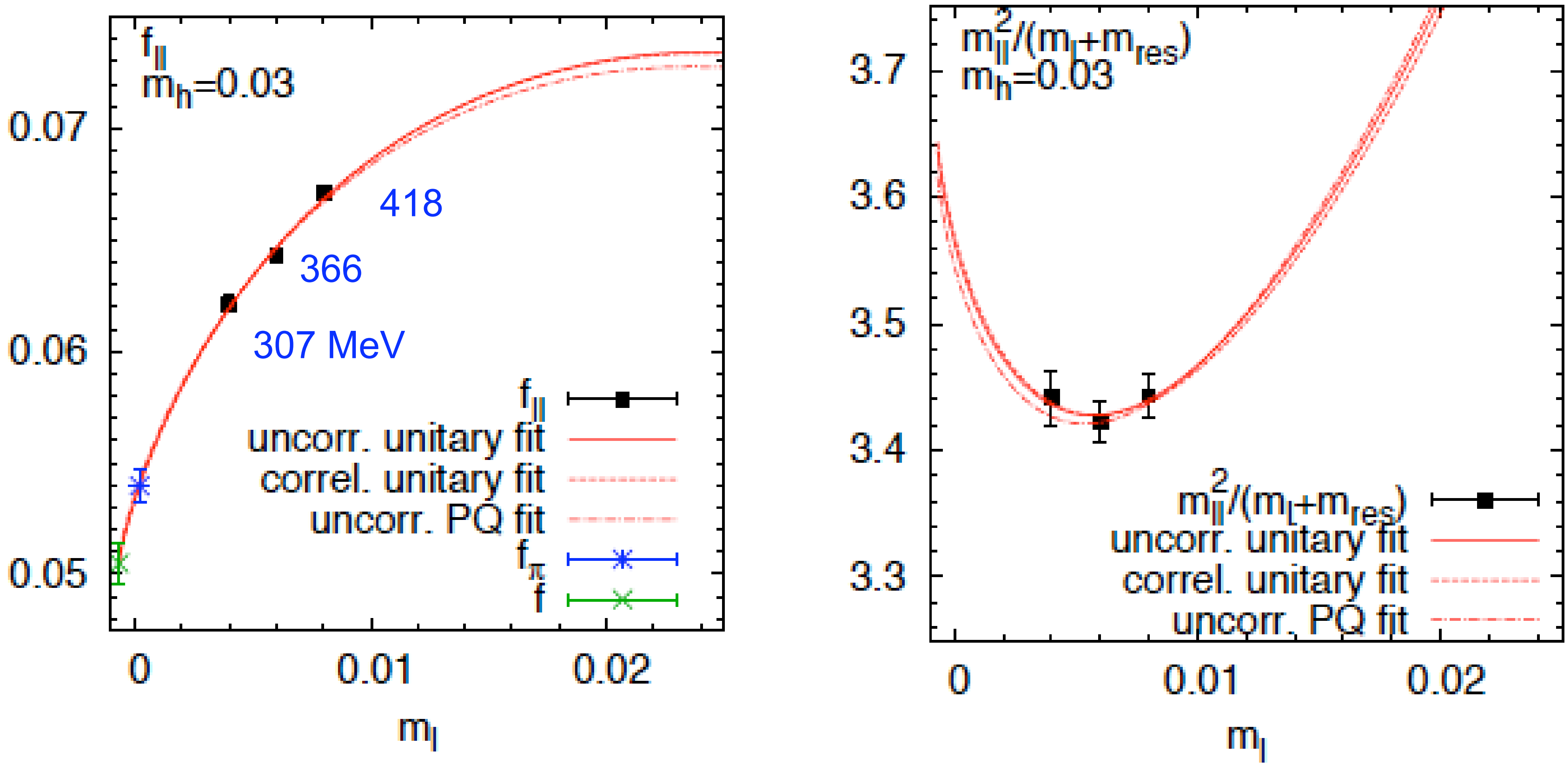
NLO correction : 30-40% for SU(2)
60-70% for SU(3)

NLO SU(2) PQChPT behaves better than SU(3).

NLO SU(3) ChPT is not sufficient for the strange quark.

results at a=0.08 fm

E. Scholz, arXiv: 0809.3251[hep-lat]



NLO SU(2) ChPT is consistent with NLO SU(2) PQChPT.

$$m_x + m_y \leq 0.016$$

MILC Collaborations

Aubin *et al.*, Phys. Rev. D70(2004)114501.
Bernard *et al.*, arXiv: 0710.1118[hep-lat]

- a=0.06,0.09,0.12,0.15 fm, L > 2.4 fm

$$m_\pi L \geq 3.4$$

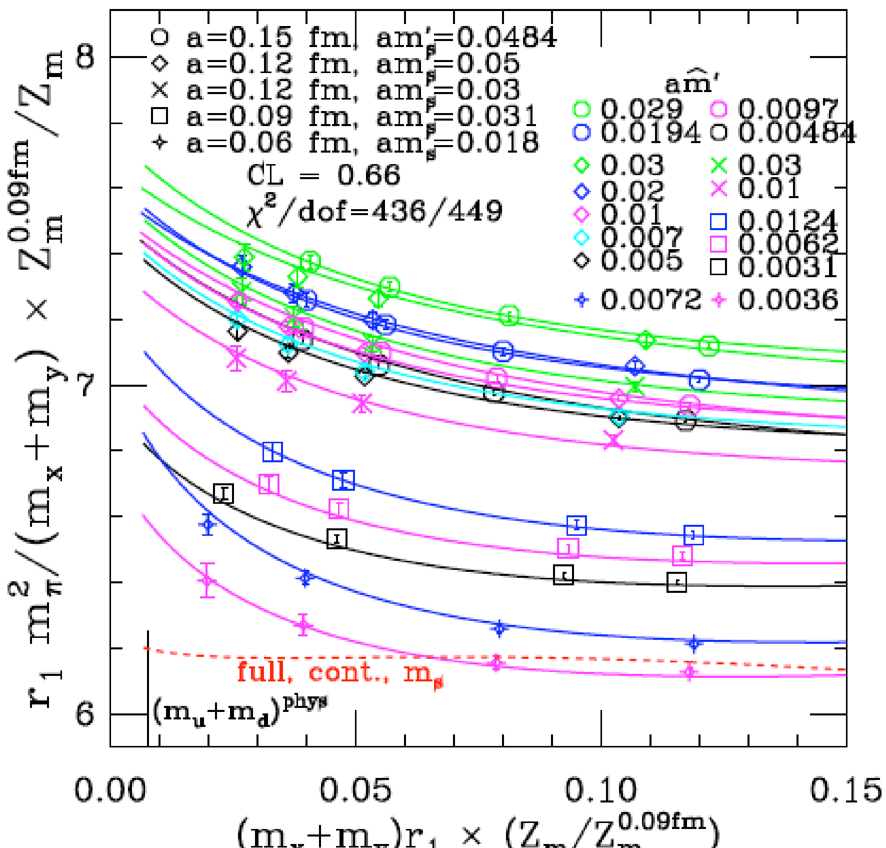
- rooted staggered quarks
- rooted staggered SU(3) PQChPT fits(includes lattice artifacts)

$$m_x + m_y \leq (0.39 \sim 0.56) m_s$$

  - need NNLO analytic terms to fit data

failure of NLO SU(3) PQChPT ?
need NLO SU(2) fits?

Large lattice artifact



Talk by U. Heller (July 6,15:05@WG1)

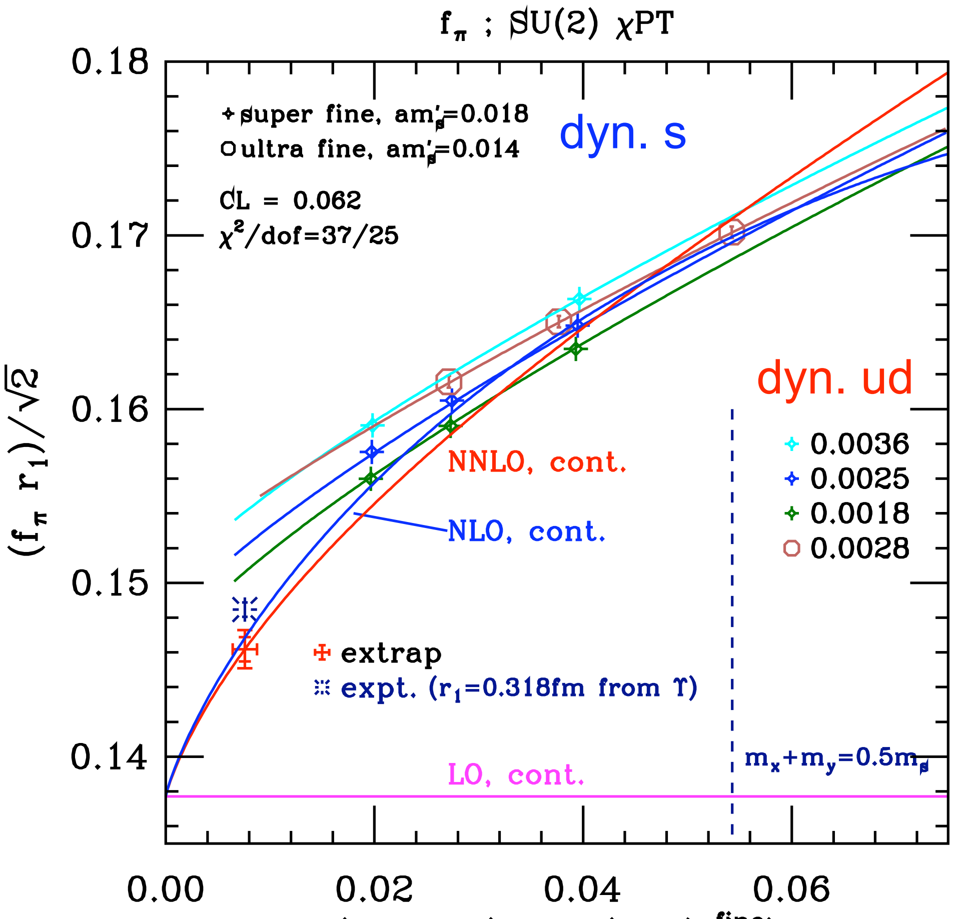
Latest analysis

Talk by U. Heller (July 6,15:05@WG1)

NLO rooted staggered PQChPT+ NNLO continuum PQChPT

SU(2) fit

dynamical $m_s \simeq m_s^{\text{phys.}}$



valence quark

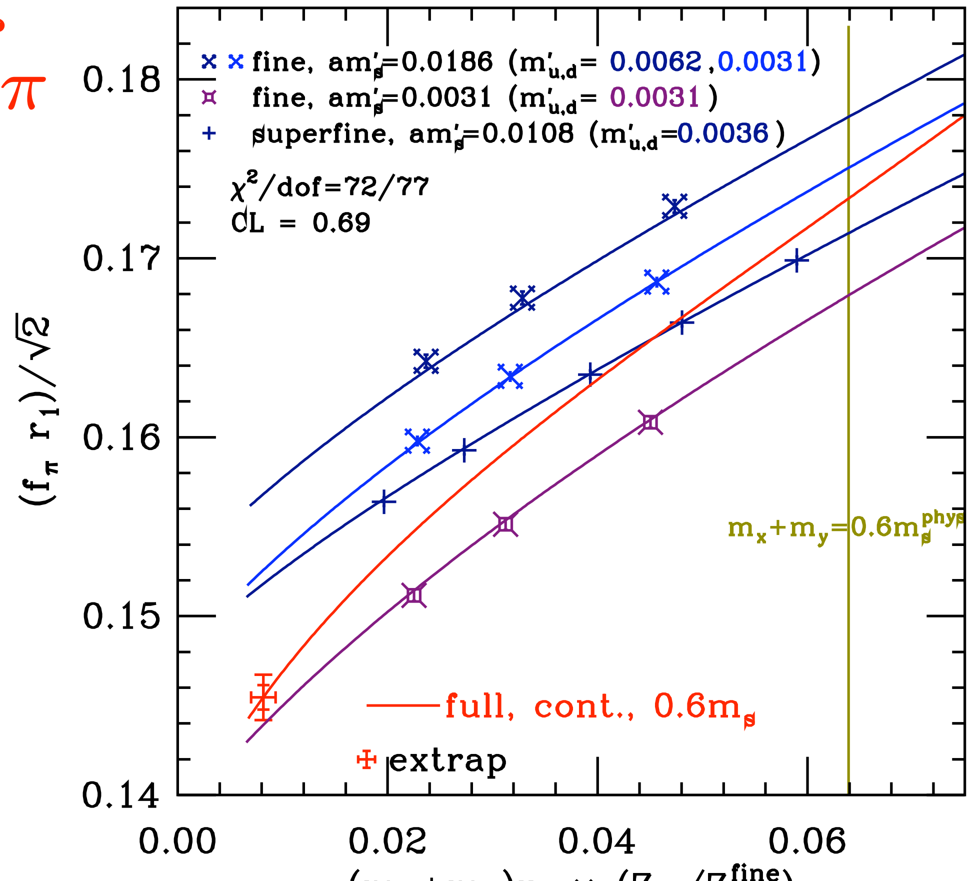
SU(3) fit

dynamical $m_s \leq 0.6 m_s^{\text{phys.}}$

valence: $m_x + m_y \leq 0.6\ m_s^{\text{phys.}}$

dyn. s (dyn. ud)

f_π



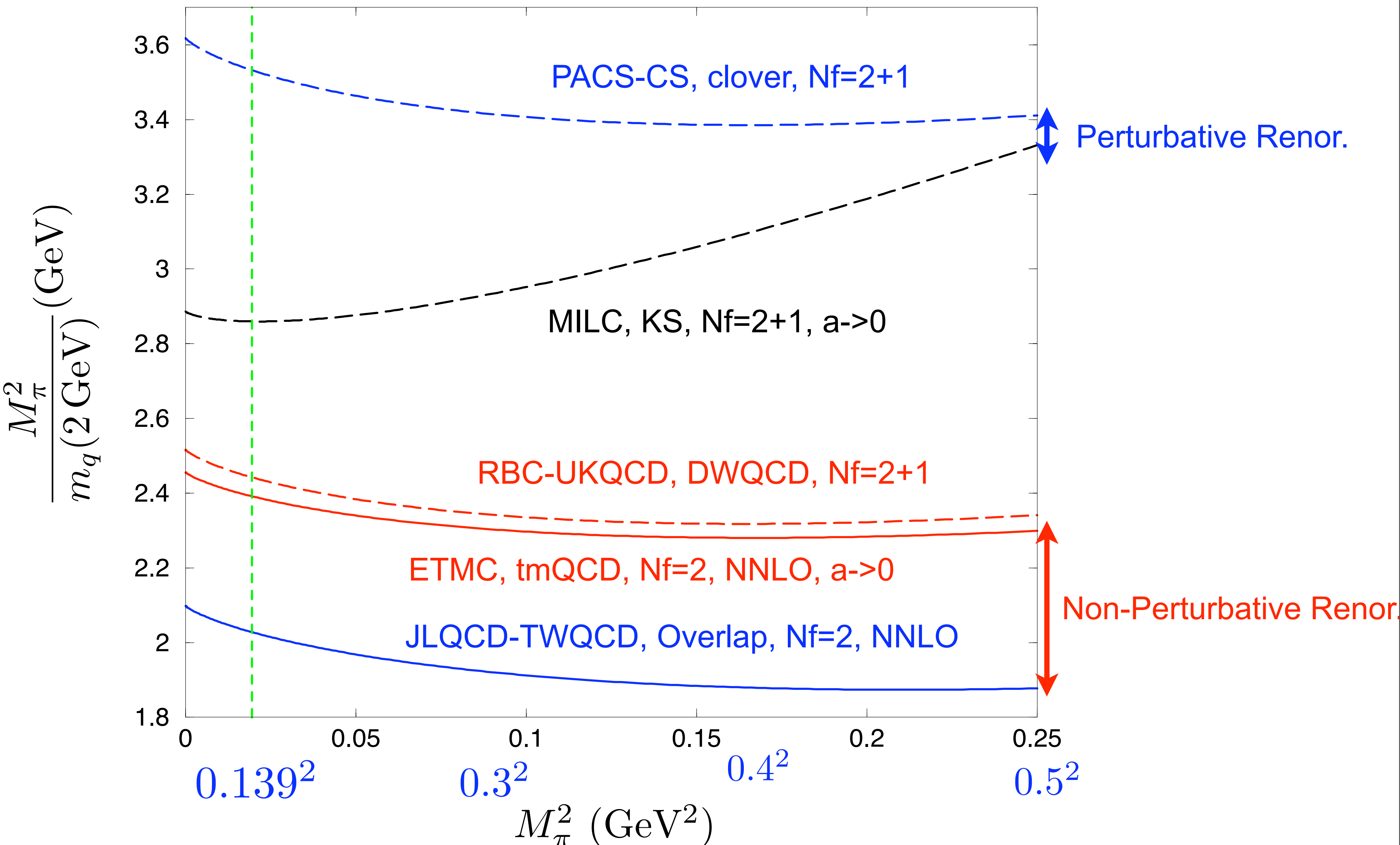
valence quark

Summary: Nf=2+1 QCD and ChPT

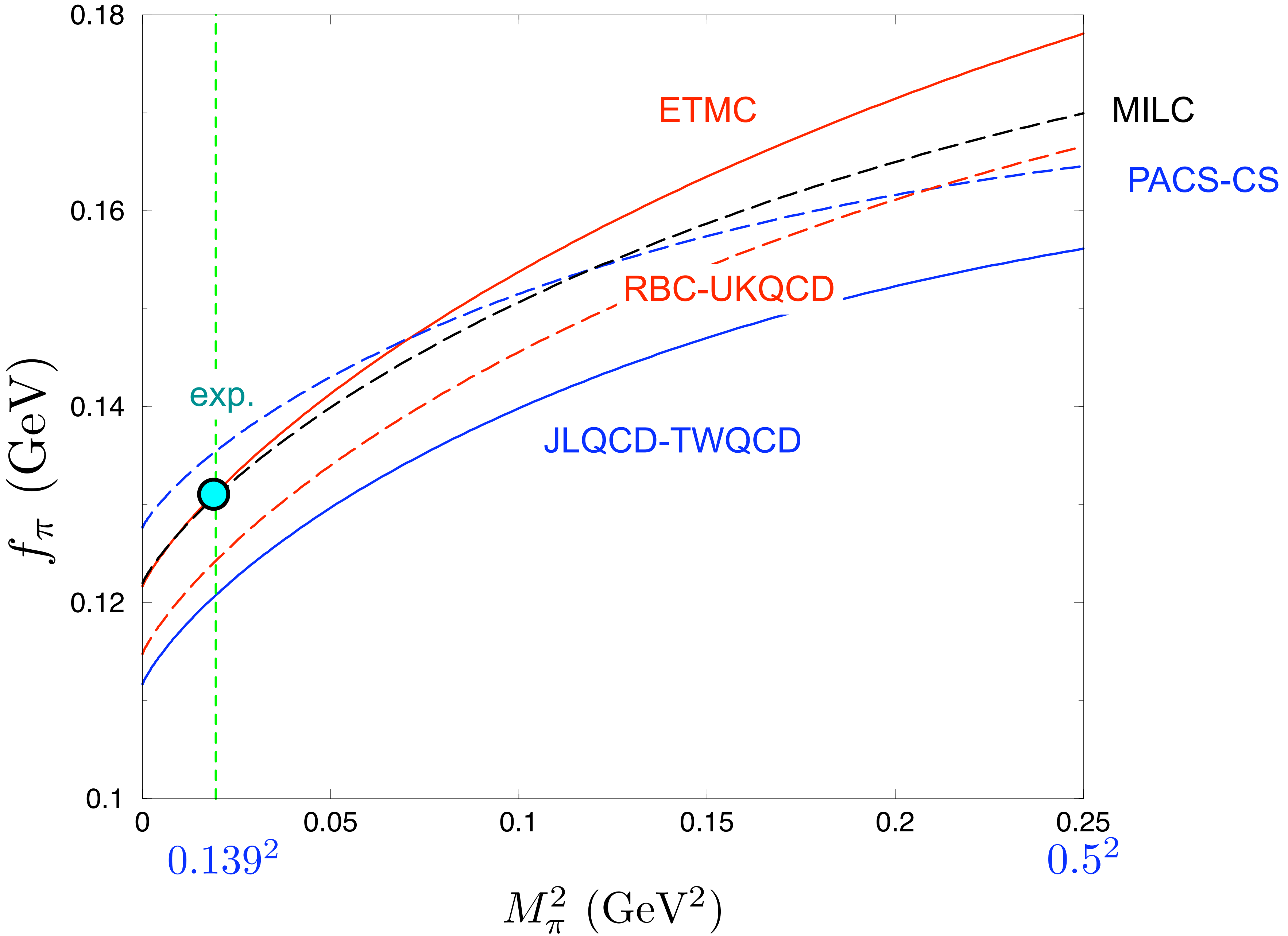
- NLO SU(3) (PQ)ChPT seems to fail at strange quark mass
- NLO SU(2) (PQ)ChPT seems to work at $m_\pi \le 500\ \mathrm{MeV}$
  - strange quark mass dependence needed to be interpolated.
  - SU(2) LOCs may be extracted.
- Nf=3 QCD simulation may be required to determine SU(3) LOCs

  Talk by U. Heller (July 6,15:05@WG1)
  - NLO SU(3) (PQ)ChPT at $m_\pi \le 500\ \mathrm{MeV}$
  - exact “chiral” symmetry is preferable.

$$\frac{M_\pi^2}{m_q(2\,\mathrm{GeV})} = 2B(2\,\mathrm{GeV})\left\{1 + \frac{M_\pi^2}{16\pi^2 f^2}\left[\ln\left(\frac{M_\pi^2}{m_\pi^2}\right) - l_3(m_\pi)\right]\right\}$$



$$f_\pi = f\left\{1 - \frac{M_\pi^2}{8\pi^2 f^2}\left[\ln\left(\frac{M_\pi^2}{m_\pi^2}\right) - l_4(m_\pi)\right]\right\}$$



3. Others

3-1. Pion Form Factors

Vector From Factor

$$\langle \pi(p')|V_\mu|\pi(p)\rangle = (p+p')_\mu F_V(q^2),\ \ q^2 = (p-p')^2$$

$$\langle r^2 \rangle_V = 6\frac{\partial F_V(q^2)}{\partial q^2}\bigg|_{q^2=0}$$ charge radius

$$c_V = \frac{\partial^2 F_V(q^2)}{\partial (q^2)^2}\bigg|_{q^2=0}$$ curvature

Scalar From Factor

$$\langle \pi(p')|S|\pi(p)\rangle = F_S(q^2)$$

$$\langle r^2 \rangle_S = 6\frac{\partial F_S(q^2)}{\partial q^2}\bigg|_{q^2=0}$$ scalar radius

Recent full QCD calculations

$Q^2 = -q^2$

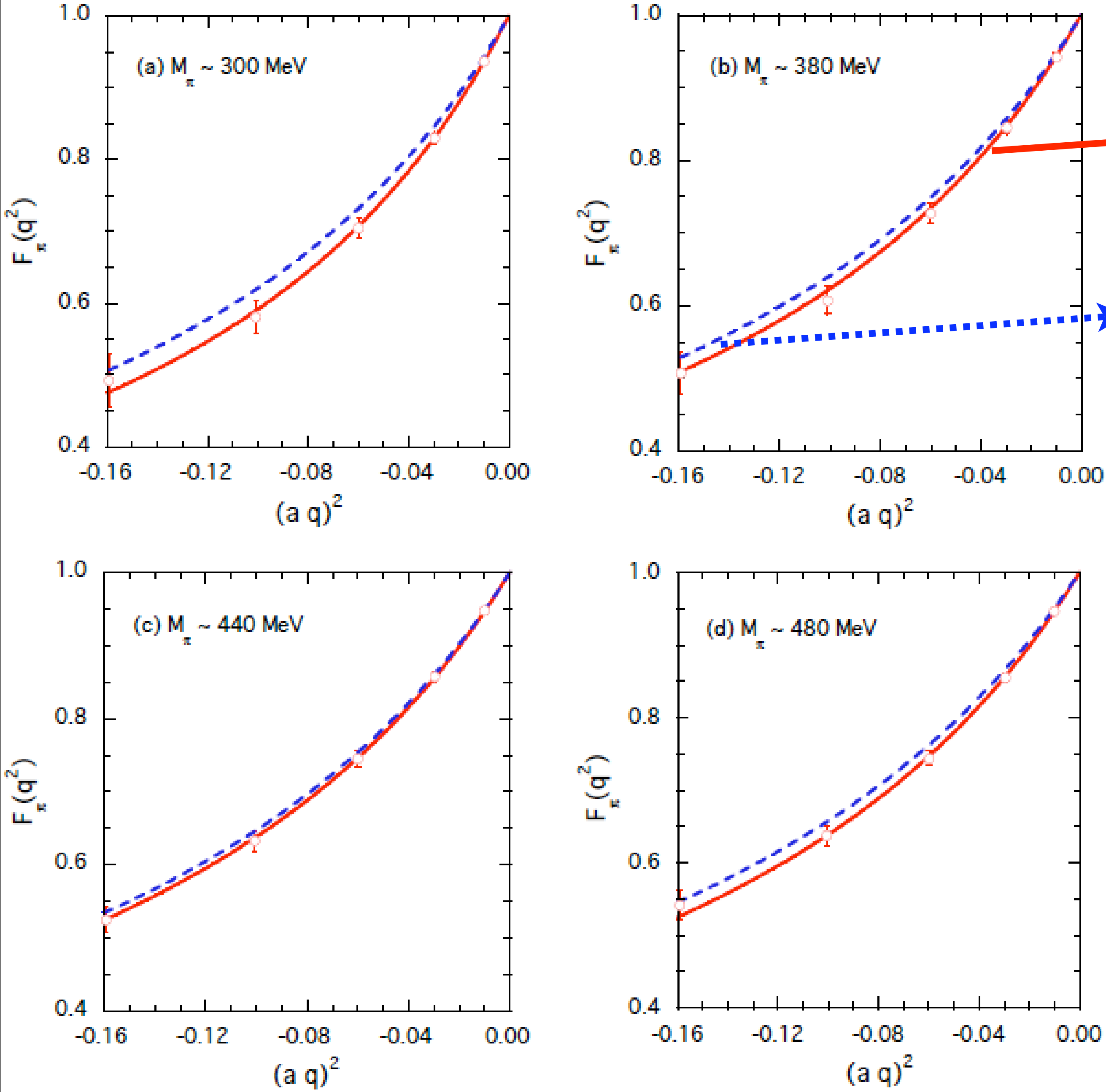
QCDSF-UKQCD, Eur. Phys. J. C51(2007)335.
RBC-UKQCD, JHEP07(2008)112.
ETMC, arXiv:0812.4042[hep-lat].
JLQCD-TWQCD, arXiv:0905.2465[hep-lat]

		quarks	a(fm)	L(fm)	$Q^2_{\rm min}$ $({\rm GeV}^2)$	$Q^2_{\rm max}$ $({\rm GeV}^2)$	m_π $({\rm MeV})$	F_X
QCDSF-UKQCD	2	O(a) Wilson	0.07~ 0.12	1.4~ 2.0	0.31	4.3	400~ 1011	V
RBC-UKQCD	2+1	Domain Wall	0.11	2.8	0.013	0.258	330	V
ETMC	2	twisted mass	0.07~ 0.09	2.2~ 2.9	0.05	0.8	260~ 580	V
JLQCD-TWQCD	2	Overlap	0.12	1.9	0.252	1.7	290~ 750	V,S

ETMC: Talk by A. Juettner (July 6,16:40@WG1)
JLQCD-TWQCD: Talk by T. Kaneko (July 6, 17:05@WG1)

q^2 dependence of F_V

ETMC



$$F_V(q^2) = \frac{1}{1 - q^2/M_{\rm pole}^2}$$

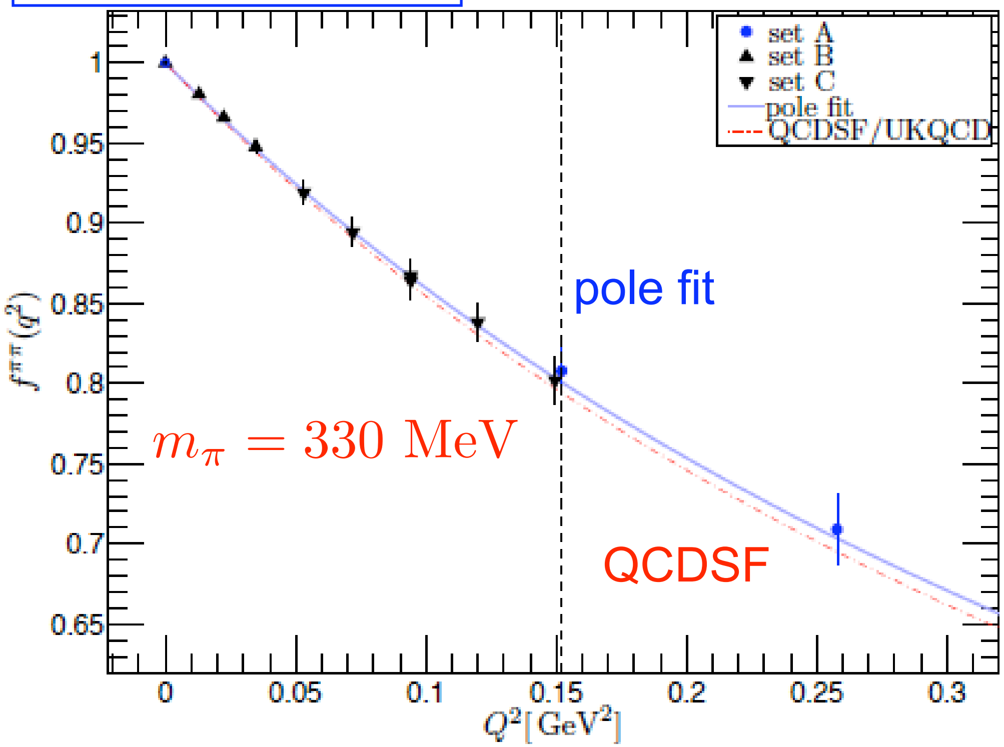
Vector Meson Dominace

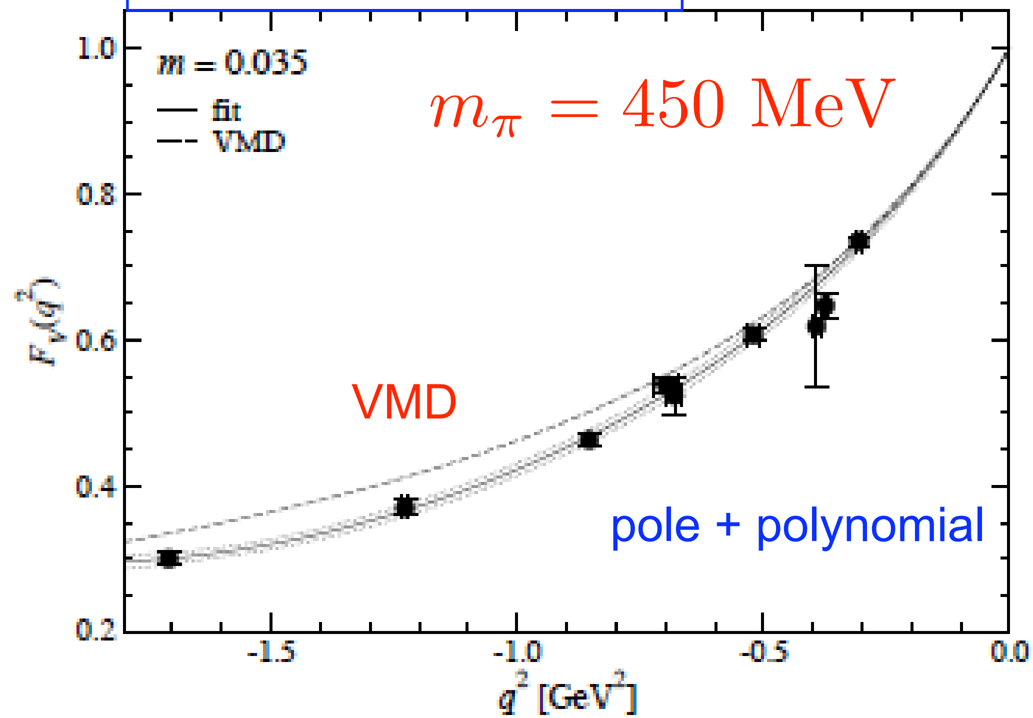
$$M_{\rm pole} = M_\rho$$

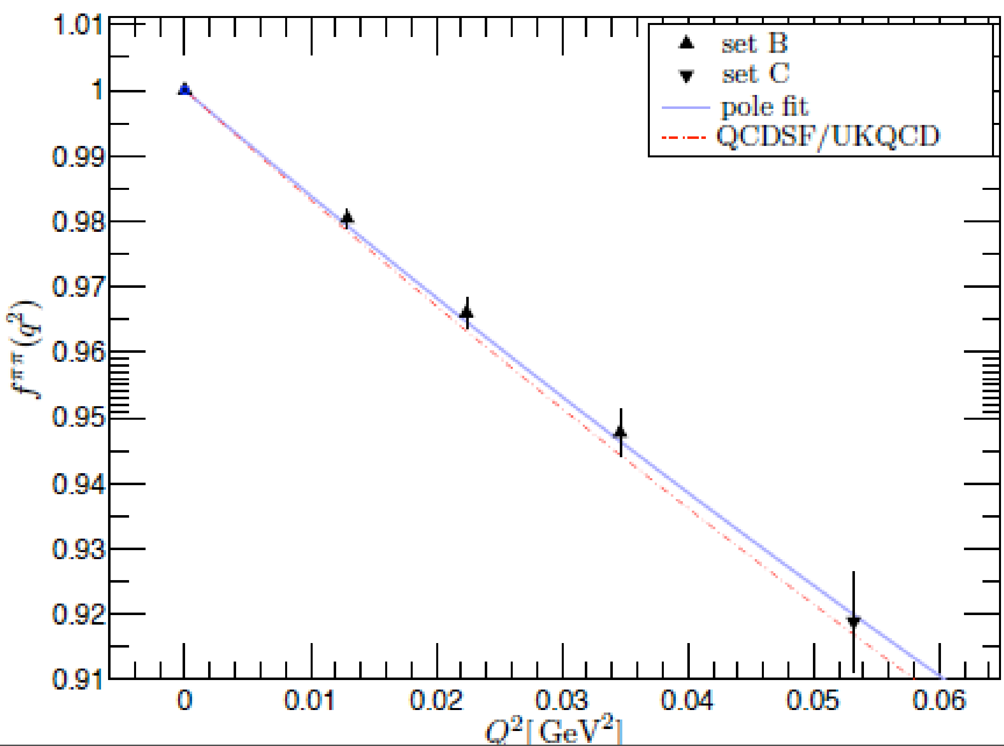
a single pole ansatz works rather well.

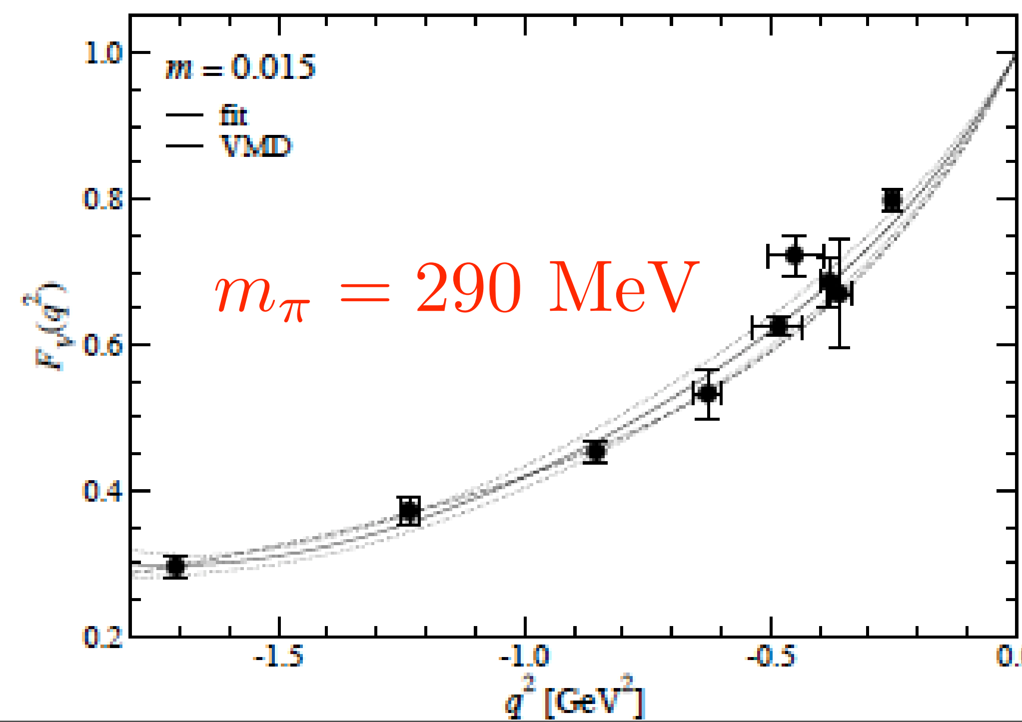
RBC-UKQCD

JLQCD-TWQCD









NLO ChPT and a problem

A single pole ansatz works rather well in all lattice simulations at small q^2

$$\langle r^2 \rangle_V \simeq \frac{6}{M_{\rm pole}^2}, \quad c_V \simeq \frac{1}{M_{\rm pole}^4} \simeq \left(\frac{\langle r^2\rangle_V}{6}\right)^2$$

However the above relation is NOT built in NLO ChPT.

$$\langle r^2 \rangle_V^{\rm NLO} = -\frac{2}{Nf^2}\left(1 + 6Nl_6^r + \ln\left[\frac{m_\pi^2}{\mu^2}\right]\right)$$

$$c_V^{\rm NLO} = \frac{1}{30Nf^2m_\pi^2}$$

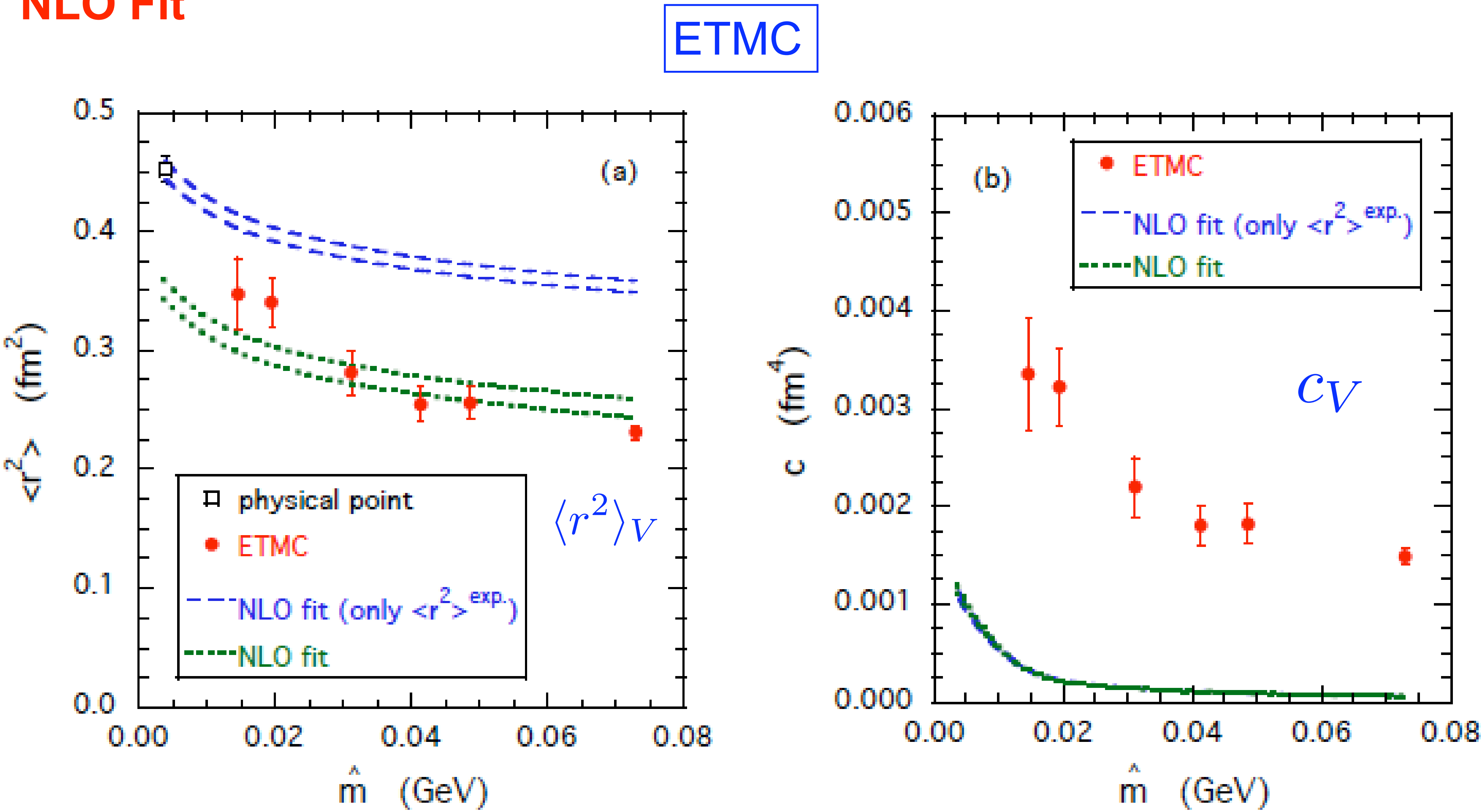
$$N = (4\pi)^2$$

The above relation implies

$$\langle r^2 \rangle_V \simeq \sqrt{\frac{6}{5}}\frac{1}{4\pi f m_\pi} \simeq 0.22\,{\rm fm}^2$$

$$\langle r^2 \rangle_V^{\rm exp,PDG} = 0.452(11)\,{\rm fm}^2$$

NLO Fit



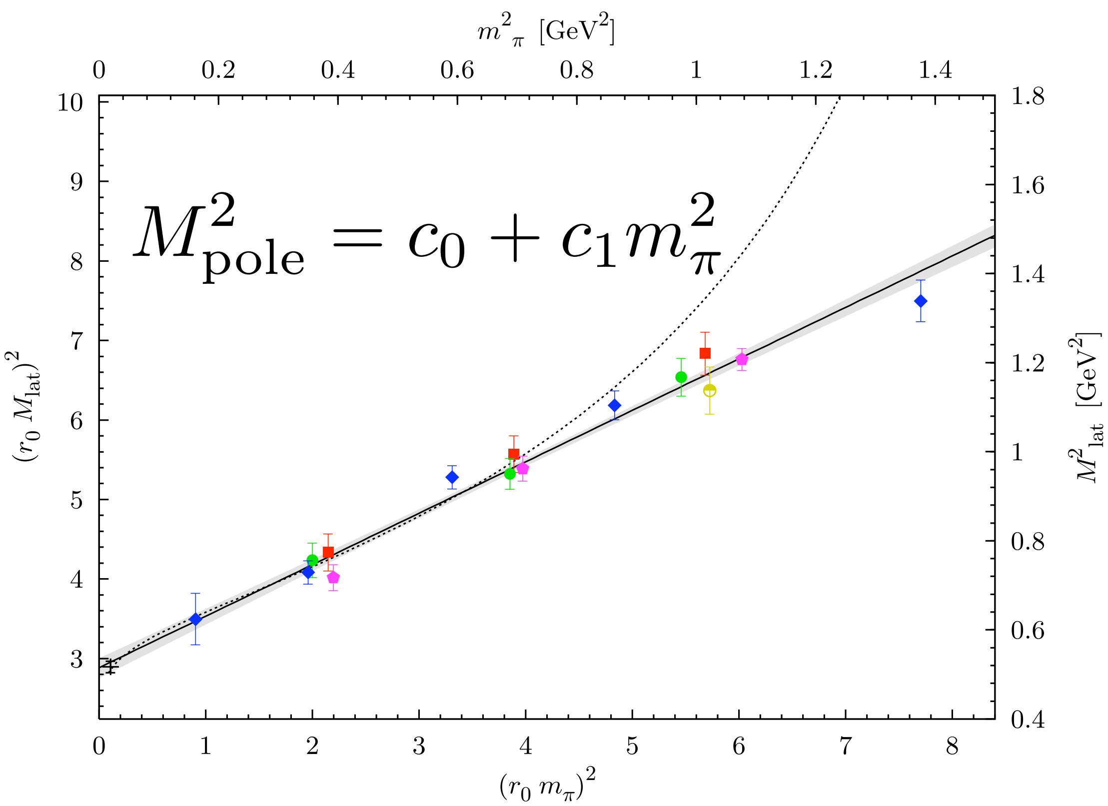
NLO ChPT does not reproduce lattice data and a single pole ansatz.
A similar conclusion is obtained by JLQCD-TWQCD.

Possible "solutions"

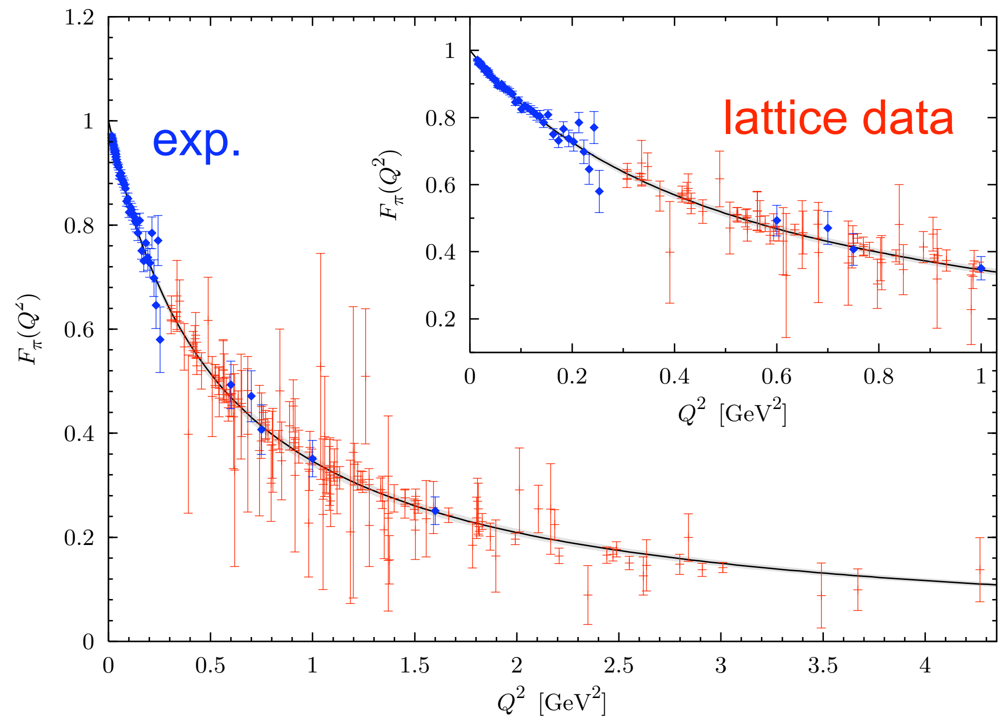
QCDSF-UKQCD

ChPT is NOT used for the chiral extrapolation.

extrapolate the "pole" mass



$F_V(q^2)$ at physical pion mass



$$\langle r^2 \rangle_V = 0.441(19)(56)(-29)\ \mathrm{fm}^2$$

$$\langle r^2 \rangle_V^{\rm exp,PDG} = 0.452(11)\,\mathrm{fm}^2$$

RBC-UKQCD

NLO SU(2)/SU(3) ChPT for the form factor very small momentum transfer only

$$F_V^{\mathrm{SU}(2)}(q^2) = 1 + \frac{1}{f^2}\left[-2l_6^r q^2 + 4\bar{\mathcal{H}}(m_\pi^2, q^2, \mu^2)\right]$$

$$F_V^{\mathrm{SU}(3)}(q^2) = 1 + \frac{1}{f_0^2}\left[4L_9^r q^2 + 4\bar{\mathcal{H}}(m_\pi^2, q^2, \mu^2) + 2\bar{\mathcal{H}}(m_K^2, q^2, \mu^2)\right]$$

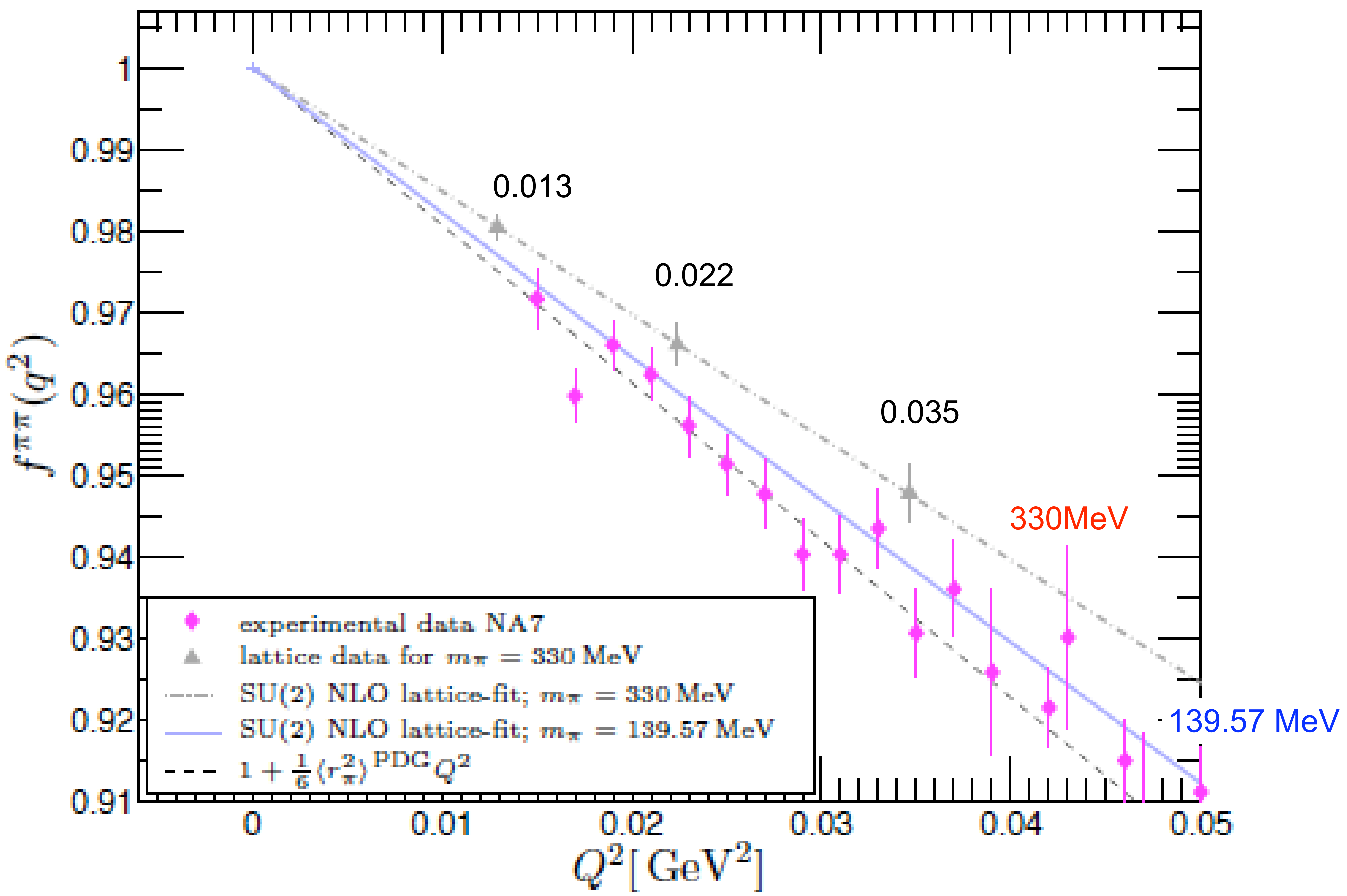
$$\bar{\mathcal{H}}(m^2, q^2, \mu^2) = \frac{m^2 H(q^2/m^2)}{32\pi^2} - \frac{q^2}{192\pi^2}\ln\frac{m^2}{\mu^2}$$

$$H(x) = -\frac{4}{3} + \frac{5x}{18} - \frac{x-4}{6}\sqrt{\frac{x-4}{x}}\ln\left(\frac{\sqrt{(x-4)/x}+1}{\sqrt{(x-4)/x}-1}\right)$$

$af = 0.0665(47)$, $af_0 = 0.0541(40)$: input

$l_6^r(m_\rho) = -0.0093(10)$, $\langle r^2\rangle_V^{330\mathrm{MeV}} = 0.354(31)\ \mathrm{fm}^2$, $\langle r^2\rangle_V^{139\mathrm{MeV}} = 0.418(38)\ \mathrm{fm}^2$

SU(2) ChPT $\quad Q^2_{\mathrm{max}} = 0.013$ (1 point)



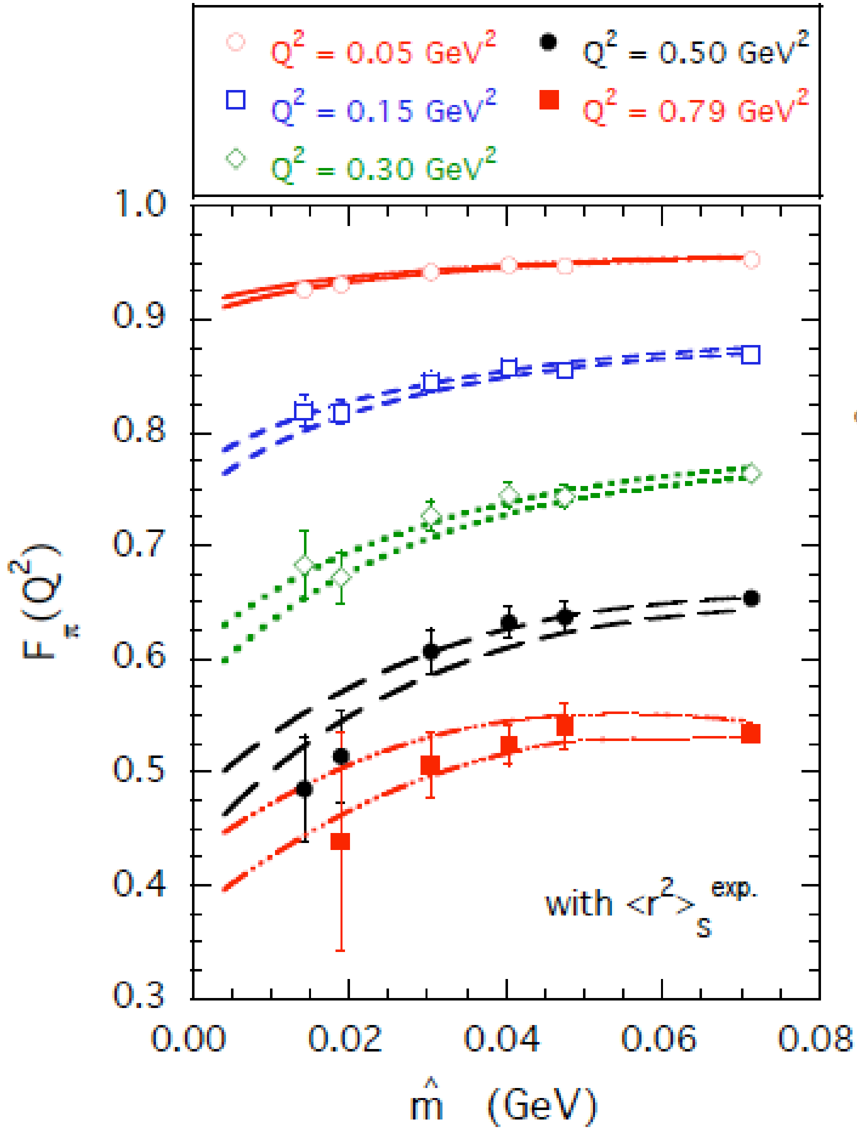
0.013
0.022
0.035
330MeV
139.57 MeV
experimental data NA7
lattice data for m_π = 330 MeV
SU(2) NLO lattice-fit; m_π = 330 MeV
SU(2) NLO lattice-fit; m_π = 139.57 MeV
$1 + \frac{1}{6}\langle r_\pi^2 \rangle^{PDG} Q^2$
$f^{\pi\pi}(q^2)$
Q^2[GeV^2]
1
0.99
0.98
0.97
0.96
0.95
0.94
0.93
0.92
0.91
0
0.01
0.02
0.03
0.04
0.05

ETMC

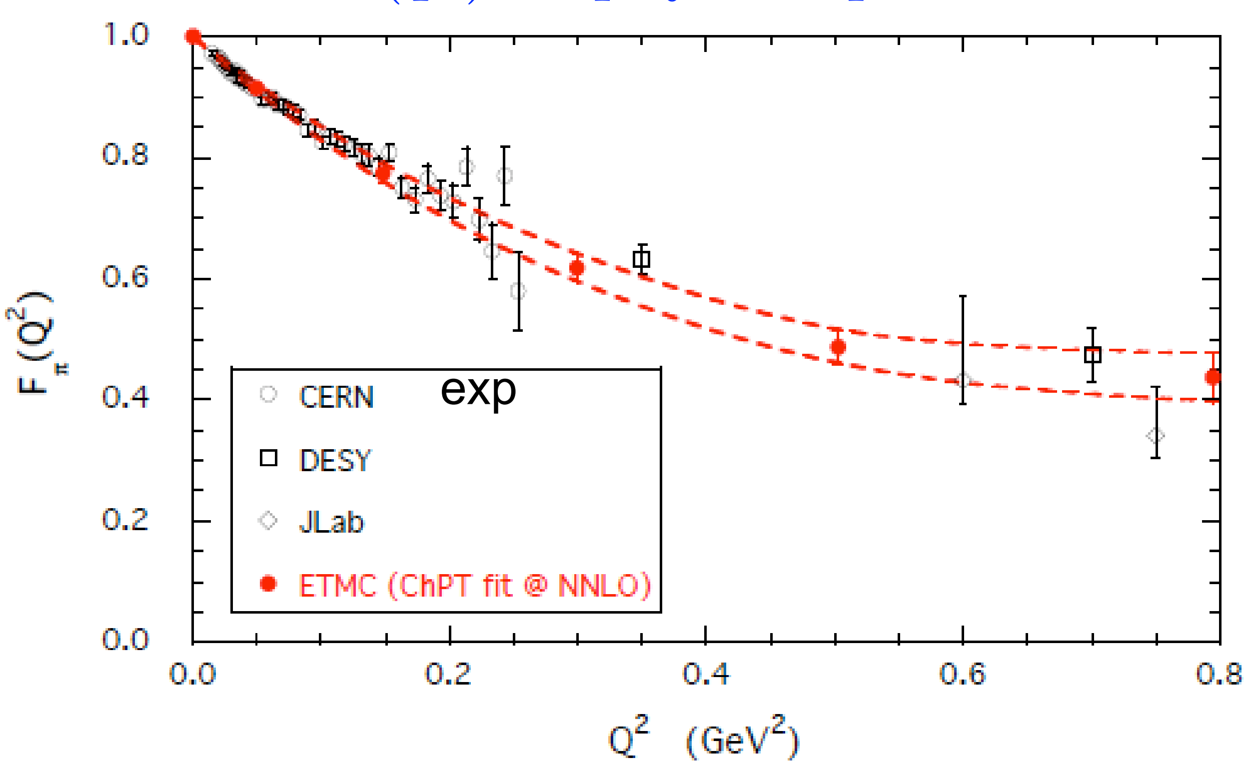
NNLO SU(2) ChPT for the form factor

Simultaneous fit for m_π, f_π and $F_V(q^2)$

experimental values of $\langle r^2 \rangle_S = 0.61(4)\ \mathrm{fm}^2$ → fix some LECs through NNLO formula



$F_V(q^2)$ at physical pion mass

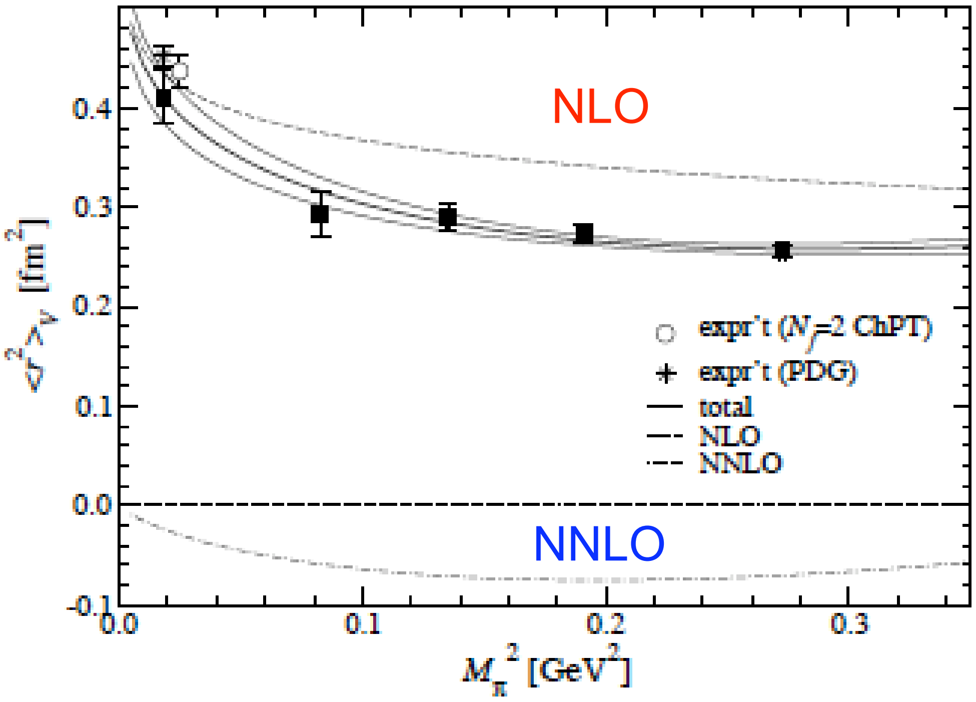


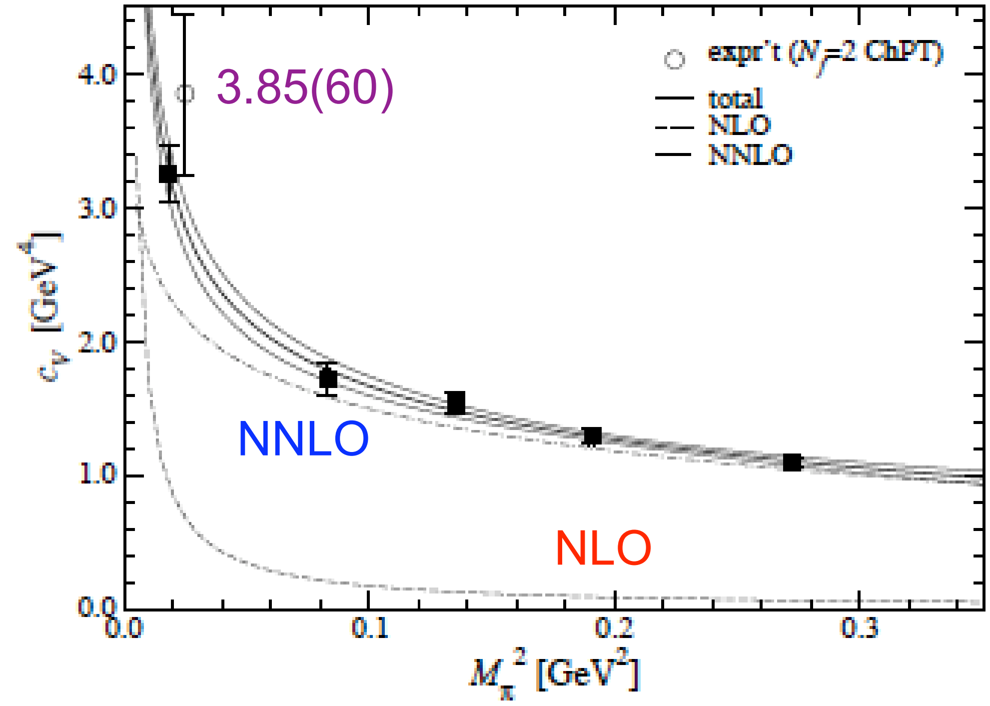
$$\langle r^2 \rangle_V = 0.438(29)\ \mathrm{fm}^2$$

$$0.05 \le Q^2 \le 0.8\ [\mathrm{GeV}^2]$$

JLQCD-TWQCD

NNLO SU(2) ChPT for $\langle r^2 \rangle_V$, c_V and $\langle r^2 \rangle_S$



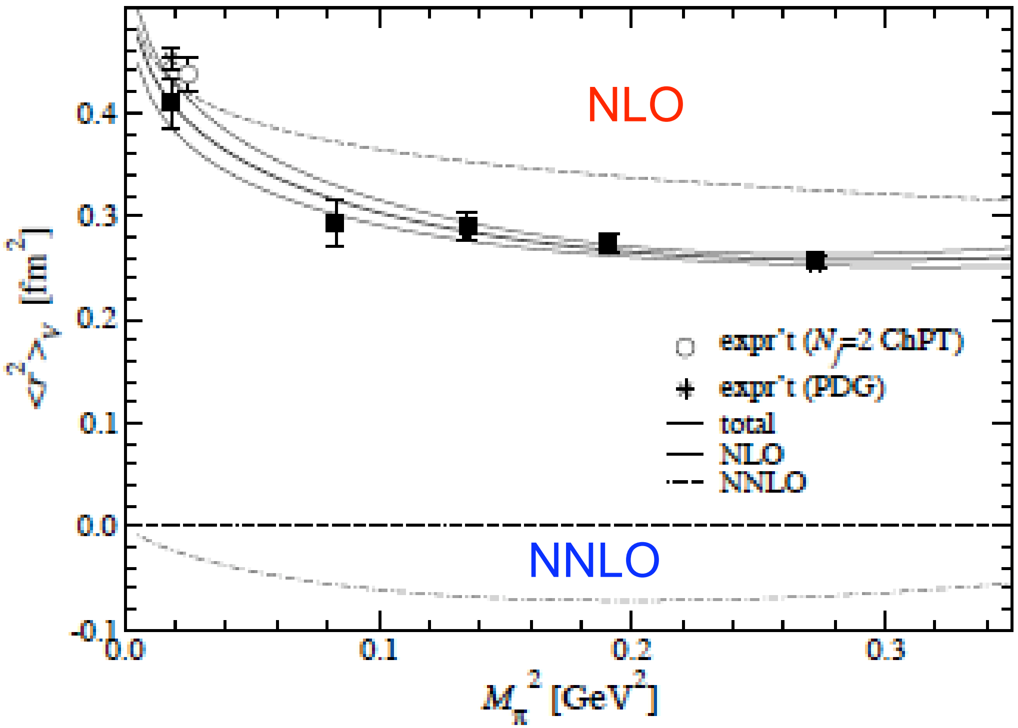


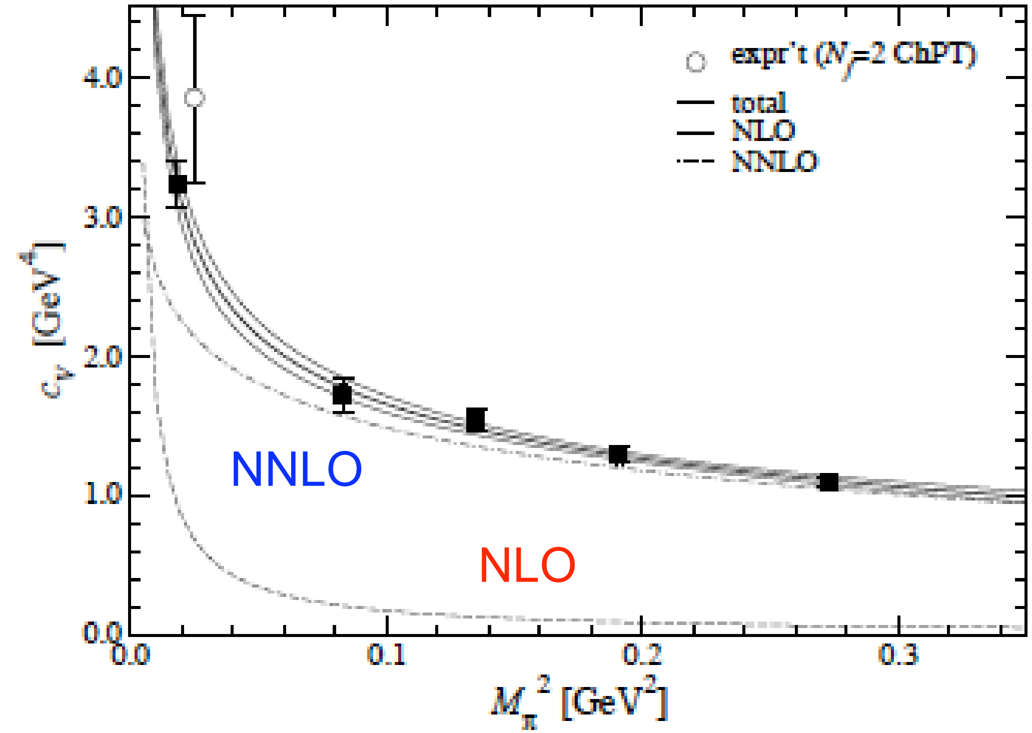
$$\langle r^2 \rangle_V = 0.411(26)\ \mathrm{fm}^2,\ c_V = 3.26(21)\ \mathrm{GeV}^{-4}$$

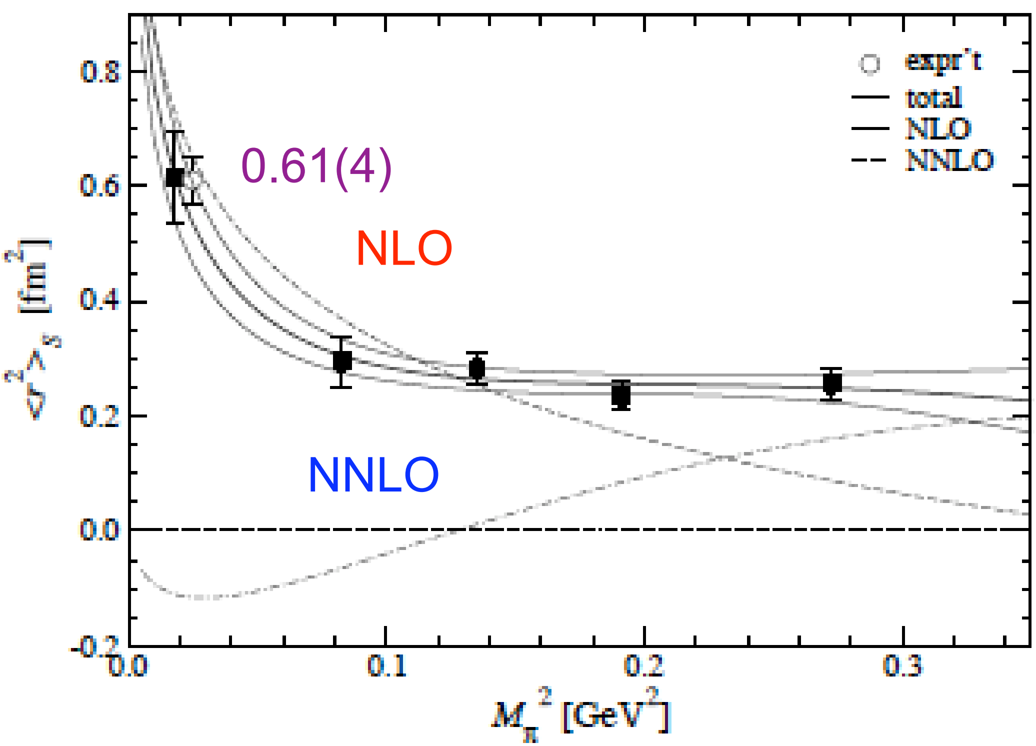
$$= 0.00488\ \mathrm{fm}^4$$

$$c_V^{\mathrm{pole}} = 0.00469\ \mathrm{fm}^4$$

$c_V^{\mathrm{NLO}} < c_V^{\mathrm{NNLO}}$ convergence ?







$$\langle r^2 \rangle_V = 0.409(23)\ \mathrm{fm}^2$$

$$c_V = 3.22(17)\ \mathrm{GeV}^{-4}$$

$$\langle r^2 \rangle_S = 0.617(79)\ \mathrm{fm}^2$$

3-3. S-parameter

Vacuum polarization functions

$$\Pi_{V_{\mu\nu}}(q) - \Pi_{A_{\mu\nu}}(q) = (q^2\delta_{\mu\nu} - q_\mu q_\nu)\Pi^{(1)}_{V-A}(q^2) - q_\mu q_\nu \Pi^{(0)}_{V-A}(q^2)$$

$$\Pi_{J_{\mu\nu}}(q) = \int d^4x\, e^{iqx} \langle 0|T[J^{ud}_\mu(x) J^{du}_\nu(0)]|0\rangle$$

NLO ChPT(Gasser-Leutwyler'85)

$$\Pi^{(1)}_{V-A}(q^2) = -\frac{f_\pi^2}{q^2} - 8L^r_{10}(\mu) - \frac{1}{24\pi^2}\left[\ln\frac{m_\pi^2}{\mu^2} + \frac{1}{3} - H(4m_\pi^2/q^2)\right]$$

$$H(x) = (1+x)\left[\sqrt{1+x}\,\ln\left(\frac{\sqrt{1+x}-1}{\sqrt{1+x}-1}\right) + 2\right]$$

$$S = -16\pi\left[L^r_{10}(\mu) - \frac{1}{192\pi^2}\left\{\ln\frac{\mu^2}{m_H^2} - \frac{1}{6}\right\}\right]$$

Higgs mass

$\Pi^{(i)}_{V-A} = 0$ if the chiral symmetry is exact.

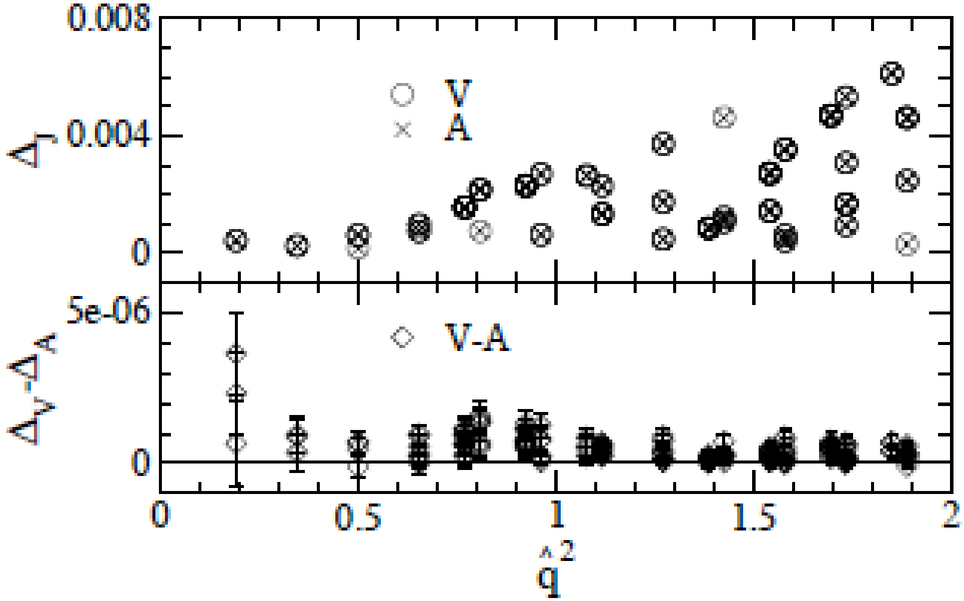
JLQCD-TWQCD

Shintani *et al.*, Phys. Rev. Lett. 101(2008)242001.

- 2 flavor, Overlap quarks, fixed topology, a=0.12 fm, L=1.9 fm

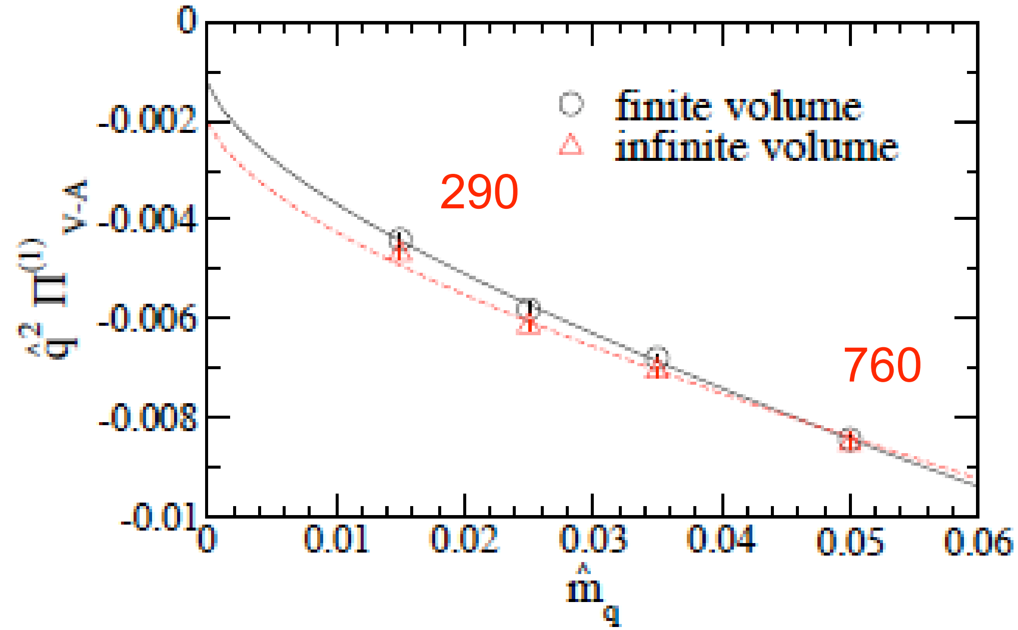
$$m_\pi \geq 290 \text{ MeV}$$

"Lorentz" symmetry violation



Significant in each channel.
Small in the difference.

Chiral fit



$$q^2 \leq (0.32)^2 \text{ GeV}^{-2} \ \text{(1 point)}$$

$$L_{10}^r(m_\rho) = -5.2(2)(^{+0}_{-3})(^{+5}_{-0}) \times 10^{-3}$$

$$\text{exp:} \ -5.09(47) \times 10^{-3}$$

3-4. Topological susceptibility from fixed topology

Full QCD simulations

Changing topological charges becomes difficult
at lighter quark mass and/or near the continuum limit.

Topological susceptibility from QCD with fixed topology.

Basic formula at fixed Q

Aoki-Fukaya-Hashimoto-Onogi, PRD76(2007)065608

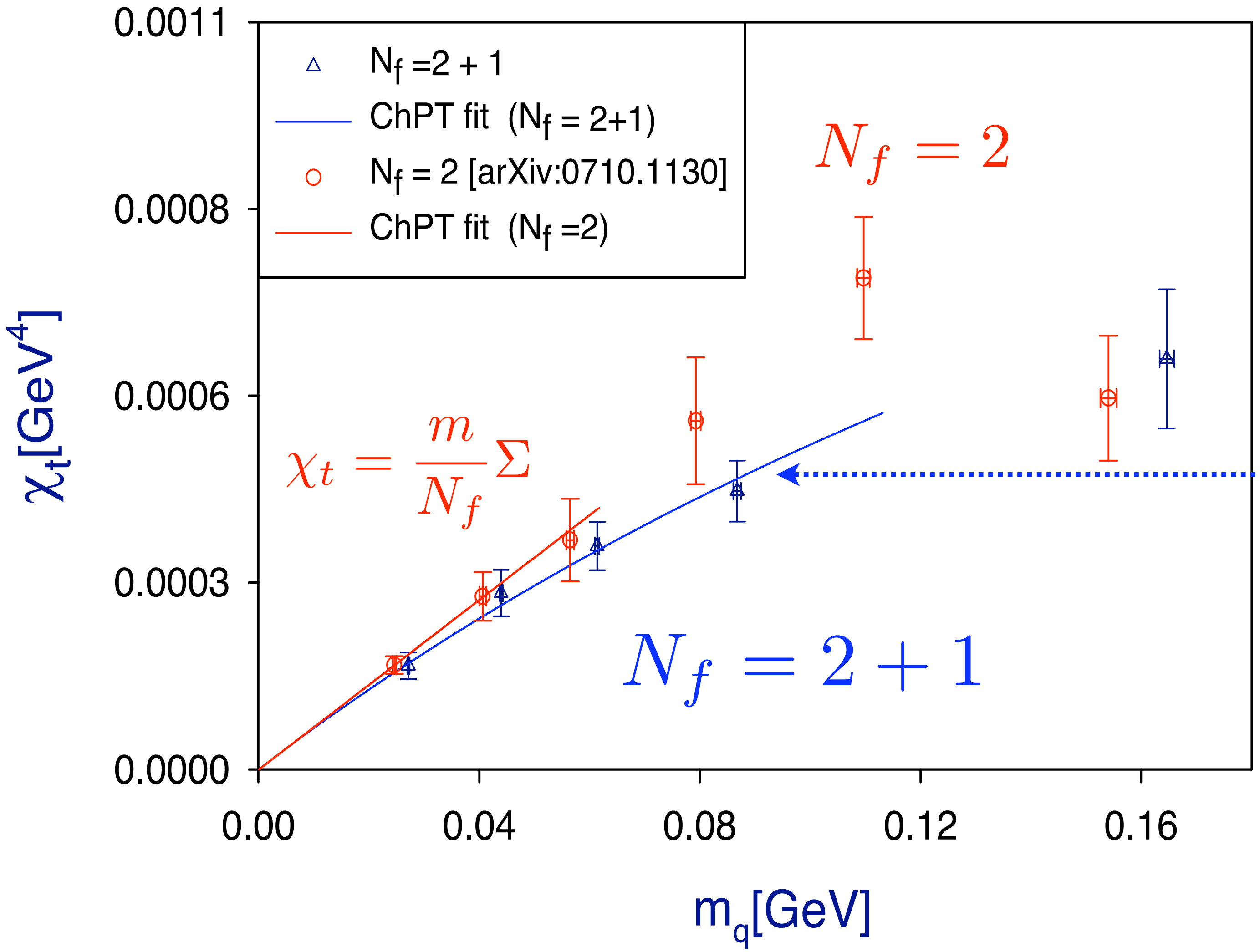
$$\lim_{|x|\to\infty} \langle mP^0(x) mP^0(0)\rangle_Q = -\frac{\chi_t}{V} + \frac{1}{V^2}\left(Q^2 - \frac{c_4}{2\chi_t}\right) + O\left(\frac{1}{V^3}\right)$$

$P^0(x)$: singlet pseudo-scalar ensity
Q: fixed topological charge
m: quark mass

$\chi_t = \frac{1}{V}\langle Q^2\rangle$: topological susceptibility at $\theta = 0$
$c_4 = \frac{1}{V}\langle Q^4\rangle_c$: 4-th cumulant

JLQCD-TWQCD

Aoki *et al.*, Phys. Lett. B655(2008)294.
Chiu *et al.*, arXiv:0810.0085[hep-lat]



LO ChPT fits

$$\chi_t = \left(\frac{1}{m_u} + \frac{1}{m_d} + \frac{1}{m_s}\right)^{-1}\Sigma$$

$$\Sigma^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = (251 \pm 7 \pm 11\ \mathrm{MeV})^3 \quad (N_f = 2)$$

$$\Sigma^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = (249 \pm 4 \pm 2\ \mathrm{MeV})^3 \quad (N_f = 2+1)$$

4. Summary

- pion mass and decay constant
  - chiral log is clearly seen.
  - NLO SU(2) ChPT works at pion mass less than 500 MeV.
  - NLO SU(3) ChPT fails to work for the dynamical strange quark
    - NLO SU(2) ChPT even for 2+1 flavor QCD
- pion form factors: need more investigations
  - data vs. ChPT, convergence of ChPT
- New quantities and ChPT
  - S-parameter: need more investigations
  - topological susceptibility: try a fit with NLO ChPT
- (future) direct calculation of pi-pi scattering and ChPT

Talk by A. Walker-Loud (July 7,17:30@WG3)