NN potential from Lorentz-invariant χEFT

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Outline

- Motivation: one-baryon χPT
- structure of the R-TPE
- comparison with the HB approach: expressions
- comparison with the HB approach: observables
  - ⋆ peripheral waves
  - ⋆ renormalization of the NN interaction: deuteron properties and lower waves
- Summary

(one) baryon χPT

- Gasser, Sainio e Švarc: loops with dimensional regularization
  $\rightarrow$ violation of power counting: $\Delta\mathcal{L}_N = \mathcal{L}_N^{(2)} + \mathcal{L}_N^{(3)} + \Delta\mathcal{L}_N^{(0)} + \Delta\mathcal{L}_N^{(1)}$
- Jenkins and Manohar: heavy baryon expansion (HB-χPT)
  recovers power counting
- Ellis and Tang, Becher and Leutwyler: relativistic formalism
  (see also Fuchs *et al.*, Schindler *et al.*, ...)

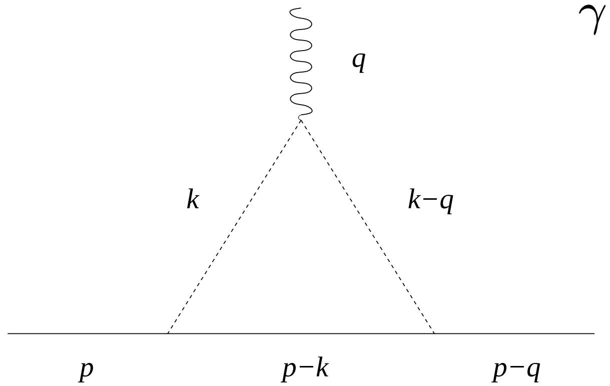
  ⋆ I (low-energy contributions)
  ⋆ R (high momenta contributions)

  - R is analytic in all low-energy region $\rightarrow$ Taylor expansion (local operators)

  - I exhibit a power counting rule

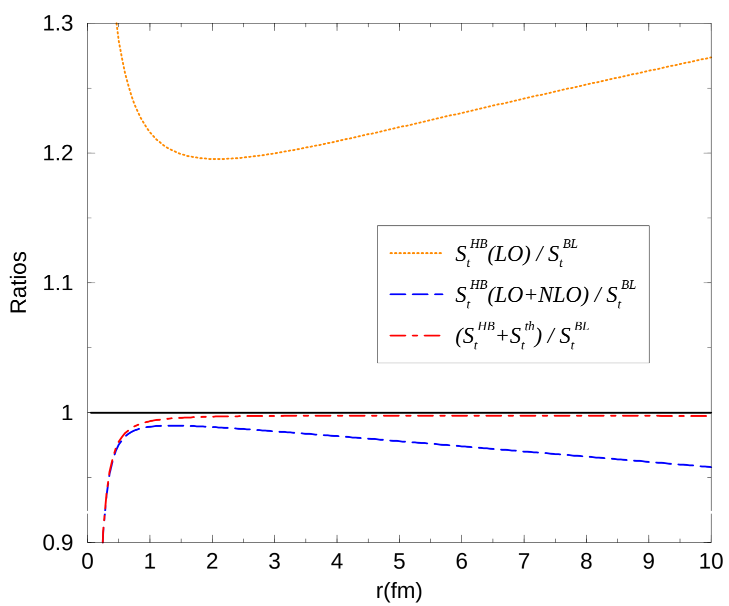
- q/m_N expansion of I $\rightarrow$ HB-χPT

- triangle integral: q/m_N expansion fails near $t = 4m_\pi^2$



$$\gamma(t) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty}\frac{dt'}{(t'-t)}\,\frac{1}{16\pi m_N\sqrt{t'}}\;\arctan\frac{2m_N\sqrt{t'-4m_\pi^2}}{t-2m_\pi^2}$$

$$\approx \frac{1}{\pi}\int_{4m_\pi^2}^{\infty}\frac{dt'}{(t'-t)}\,\frac{1}{16\pi m_N\sqrt{t'}}\left\{\left[\frac{\pi}{2}-\frac{(t'-2m_\pi^2)}{2m_N\sqrt{t'-4m_\pi^2}}\right]_{HB} + \left[\frac{m_\pi\sqrt{t'}}{2m_N\sqrt{t'-4m_\pi^2}} - \frac{\sqrt{t'}}{2m_\pi}\arctan\frac{m_\pi^2}{m_N\sqrt{t'-4m_\pi^2}}\right]_{th}\right\}.$$



◇ TPEP in coordinate space

(RH, AC da Rocha, MR Robilotta, Phys.Rev.C 69, 034009, 2004)

convergence of the HB series
breaks down at large distances!

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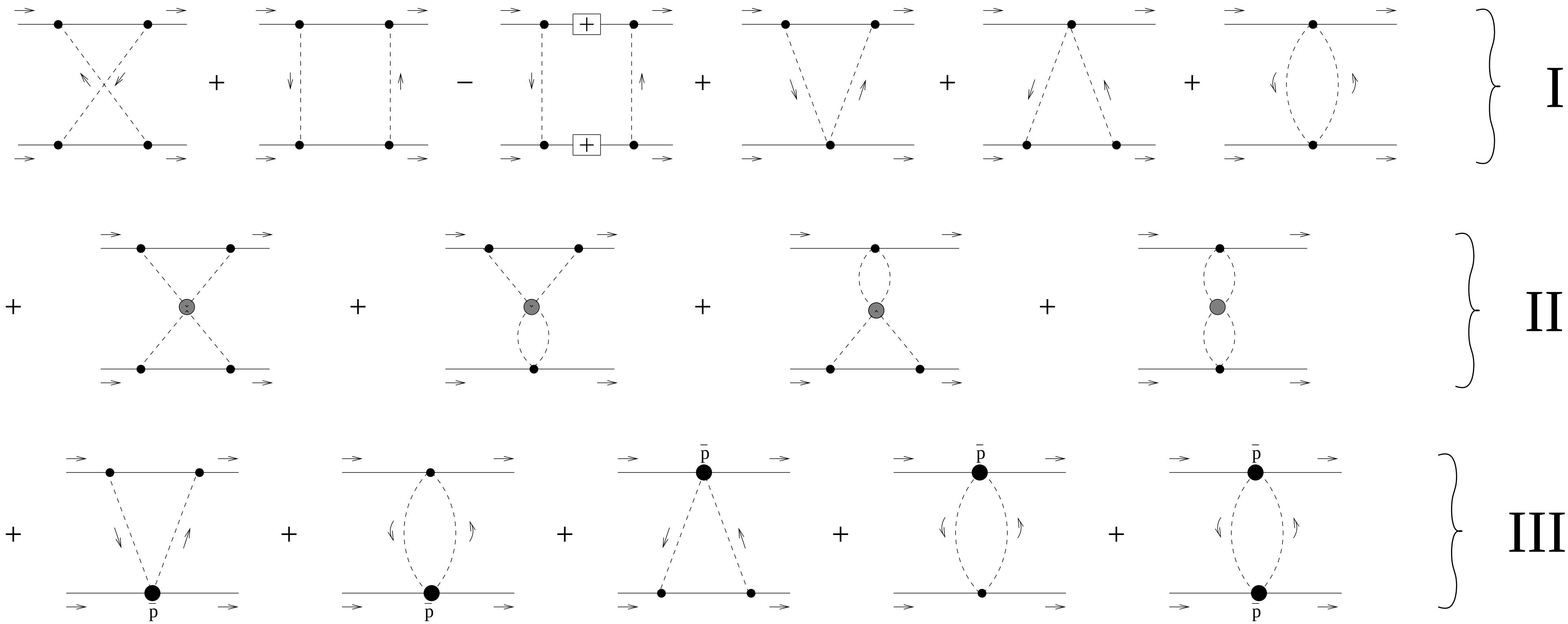
LI-TPE

RH and MR Robilotta, Phys.Rev. C 68, 024004 (2003)



Comparison with HB expressions

(Kaiser *et al.*, Kaiser, Entem and Machleidt, Epelbaum *et al.*)

- strategy: perform a formal $1/m_N$ expansion in our loop integrals
  (despite the fact that it doesn't converge!)

- relatively small number of differences

- origins RH, nucl-th/0411046
  - ⋆ dynamical equations (our choice: Blankenbecler and Sugar)
  - ⋆ two loop calculations (not completely understood)

$$V_C = V_C^+ = \frac{3g_A^2}{16\pi f_\pi^4}\left\{-\frac{g_A^2\,\mu^5}{16m(4\mu^2+\boldsymbol{q}^2)} + \left[2\mu^2(2c_1-c_3) - \boldsymbol{q}^2 c_3\right](2\mu^2+\boldsymbol{q}^2)\,A(q) + \frac{g_A^2(2\mu^2+\boldsymbol{q}^2)\,A(q)}{16m}\left[-3\boldsymbol{q}^2\ +(4\mu^2+\boldsymbol{q}^2)^\dagger\ \right]\right\}$$

$$+\frac{g_A^2\,L(q)}{32\pi^2 f_\pi^4 m}\left\{\frac{24\mu^6}{4\mu^2+\boldsymbol{q}^2}(2c_1+c_3) + 6\mu^4(c_2-2c_3) + 4\mu^2\boldsymbol{q}^2(6c_1+c_2-3c_3) + \boldsymbol{q}^4(c_2-6c_3)\right\}$$

$$-\frac{3L(q)}{16\pi^2 f_\pi^4}\left\{\left[-4\mu^2 c_1 + c_3(2\mu^2+\boldsymbol{q}^2) + c_2(4\mu^2+\boldsymbol{q}^2)/6\right]^2 + \frac{1}{45}(c_2)^2(4\mu^2+\boldsymbol{q}^2)^2\right\}$$

$$+\frac{g_A^4}{32\pi^2 f_\pi^4 m^2}\left\{L(q)\left[\frac{2\mu^8}{(4\mu^2+\boldsymbol{q}^2)^2} + \frac{8\mu^6}{(4\mu^2+\boldsymbol{q}^2)} - 2\mu^4 - \boldsymbol{q}^4\right] + \frac{\mu^6/2}{(4\mu^2+\boldsymbol{q}^2)}\right\}$$

$$-\frac{3g_A^4[A(q)]^2}{1024\pi^2 f_\pi^6}(\mu^2+2\boldsymbol{q}^2)(2\mu^2+\boldsymbol{q}^2)^2 - \frac{3g_A^4(2\mu^2+\boldsymbol{q}^2)\,A(q)}{1024\pi^2 f_\pi^6}\left\{4\mu\,g_A^2\,(2\mu^2+\boldsymbol{q}^2) + 2\mu\,(\mu^2+2\boldsymbol{q}^2)]\right\}\ ,$$

$$V_T = -\frac{3\,V_T^+}{m^2} = \frac{3g_A^4\,L(q)}{64\pi^2 f_\pi^4} - \frac{g_A^4\,A(q)}{512\pi f_\pi^4 m}\left[9(2\mu^2+\boldsymbol{q}^2)\ +3(4\mu^2+\boldsymbol{q}^2)^\dagger\ \right]$$

$$-\frac{g_A^4\,L(q)}{32\pi^2 f_\pi^4 m^2}\left[\boldsymbol{z}^2/4 + 5\boldsymbol{q}^2/8 + \frac{\mu^4}{4\mu^2+\boldsymbol{q}^2}\right] + \frac{g_A^2\,(4\mu^2+\boldsymbol{q}^2)\,L(q)}{32\pi^2 f_\pi^4}\left[(\tilde{d}_{14}-\tilde{d}_{15})\ -(g_A^4/32\,\pi^2 f_\pi^2)^*\ \right]\ ,$$

$$V_{LS} = -\frac{V_{LS}^+}{m^2} = -\frac{3g_A^4\,A(q)}{32\pi f_\pi^4 m}\ \left[(2\mu^2+\boldsymbol{q}^2)\ +(\mu^2+3\boldsymbol{q}^2/8)^\dagger\ \right] - \frac{g_A^4\,L(q)}{4\pi^2 f_\pi^4 m^2}\left[\frac{\mu^4}{4\mu^2+\boldsymbol{q}^2} + \frac{11}{32}\boldsymbol{q}^2\right] - \frac{g_A^2\,c_2\,L(q)}{8\pi^2 f_\pi^4 m}(4\mu^2+\boldsymbol{q}^2)\ ,$$

$$V_{\sigma L} = \frac{4\,V_Q^+}{m^4} = -\frac{g_A^4\,L(q)}{32\pi^2 f_\pi^4 m^2}\ ,$$

$$W_C = V_C^- = \frac{L(q)}{384\pi^2 f_\pi^4}\left[4\mu^2\left(5g_A^4 - 4g_A^2 - 1\right) + \boldsymbol{q}^2\left(23g_A^4 - 10g_A^2 - 1\right) + \frac{48g_A^4\mu^4}{4\mu^2+\boldsymbol{q}^2}\right]$$

$$-\frac{g_A^2}{128\pi f_\pi^4 m}\left\{\frac{3g_A^2\mu^5}{4\mu^2+\boldsymbol{q}^2} + A(q)\,(2\mu^2+\boldsymbol{q}^2)\left[g_A^2\,(4\mu^2+3\boldsymbol{q}^2) - 2(2\mu^2+\boldsymbol{q}^2)\ \ +g_A^2(4\mu^2+\boldsymbol{q}^2)^\dagger\ \right]\right\} + \frac{\boldsymbol{q}^2\ c_4\ L(q)}{192\pi^2 f_\pi^4 m}\left[g_A^2(8\mu^2+5\boldsymbol{q}^2) + (4\mu^2+\boldsymbol{q}^2)\right]$$

$$+\frac{16g_A^4\mu^6}{768\pi^2 f_\pi^4 m^2}\ \frac{1}{4\mu^2+\boldsymbol{q}^2} - \frac{L(q)}{768\pi^2 f_\pi^4 m^2}\left\{(4\mu^2+\boldsymbol{q}^2)\boldsymbol{z}^2 + g_A^2\left[\frac{48\mu^6}{4\mu^2+\boldsymbol{q}^2} - 24\mu^4 - 12\,(2\mu^2+\boldsymbol{q}^2)\,\boldsymbol{q}^2 + (16\mu^2+10\boldsymbol{q}^2)\boldsymbol{z}^2\right]\right.$$

$$\left.+g_A^4\left[\boldsymbol{z}^2\left(\frac{16\mu^4}{4\mu^2+\boldsymbol{q}^2} - 7\boldsymbol{q}^2 - 20\mu^2\right) - \frac{64\mu^8}{(4\mu^2+\boldsymbol{q}^2)^2} - \frac{48\mu^6}{4\mu^2+\boldsymbol{q}^2} + \frac{16\mu^4\boldsymbol{q}^2}{4\mu^2+\boldsymbol{q}^2} + 20\boldsymbol{q}^4 + 24\mu^2\boldsymbol{q}^2 + 24\mu^4\right]\right\}$$

$$-\frac{L(q)}{18432\pi^4 f_\pi^6}\left\{\left[192\pi^2 f_\pi^2\tilde{d}_3\ \ -\frac{(15+7g_A^4)^*}{5}\ \right](4\mu^2+\boldsymbol{q}^2)\left[2g_A^2(2\mu^2+\boldsymbol{q}^2) - 3/5(g_A^2-1)(4\mu^2+\boldsymbol{q}^2)\right]\right.$$

$$+\left[6g_A^2(2\mu^2+\boldsymbol{q}^2) - (g_A^2-1)\,(4\mu^2+\boldsymbol{q}^2)\right]\left[384\pi^2 f_\pi^2\left((2\mu^2+\boldsymbol{q}^2)\,(\tilde{d}_1+\tilde{d}_2) + 4\mu^2\,\tilde{d}_5\right) + L(q)\,\left(4\mu^2(1+2g_A^2) + \boldsymbol{q}^2(1+5g_A^2)\right)\right.$$

$$\left.\left.-\left(\frac{\boldsymbol{q}^2}{3}\,(5+13g_A^2) + 8\mu^2(1+2g_A^2)\right)\ \ +\left(2g_A^4(2\mu^2+\boldsymbol{q}^2) + \frac{2}{3}\,\boldsymbol{q}^2(1+2g_A^2)\right)^*\ \right]\right\},$$

$$W_T = -\frac{3}{m^2}\ V_T^- = \frac{g_A^2 A(q)}{32\pi f_\pi^4}\left\{\left(c_4 + \frac{1}{4m}\right)(4\mu^2+\boldsymbol{q}^2) - \frac{g_A^2}{8m}\left[10\mu^2 + 3\boldsymbol{q}^2\ \ -(4\mu^2+\boldsymbol{q}^2)^\dagger\ \right]\right\} - \frac{c_4^2\ L(q)}{96\pi^2 f_\pi^4}(4\mu^2+\boldsymbol{q}^2)$$

$$+\frac{c_4\ L(q)}{192\pi^2 f_\pi^4 m}\left[g_A^2(16\mu^2+7\boldsymbol{q}^2) - (4\mu^2+\boldsymbol{q}^2)\right] - \frac{L(q)}{1536\pi^2 f_\pi^4 m^2}\left[g_A^4\left(28\mu^2 + 17\boldsymbol{q}^2 + \frac{16\ \mu^4}{4\mu^2+\boldsymbol{q}^2}\right) - g_A^2(32\mu^2+14\boldsymbol{q}^2) + (4\mu^2+\boldsymbol{q}^2)\right]$$

$$-\frac{[A(q)]^2 g_A^4\ (4\mu^2+\boldsymbol{q}^2)^2}{2048\pi^2 f_\pi^6} - \frac{A(q) g_A^4\ (4\mu^2+\boldsymbol{q}^2)}{1024\pi^2 f_\pi^6}\ \mu(1+2g_A^2)\ ,$$

$$W_{LS} = -\frac{1}{m^2}\ V_{LS}^- = \frac{A(q)}{32\pi f_\pi^4 m}\left[g_A^2\,(g_A^2-1)\,(4\mu^2+\boldsymbol{q}^2)\ \ +g_A^4\ (2\mu^2+3\boldsymbol{q}^2/4)^\dagger\ \right] + \frac{c_4\ L(q)}{48\pi^2 m f_\pi^4}\left[g_A^2(8\mu^2+5\boldsymbol{q}^2) + (4\mu^2+\boldsymbol{q}^2)\right]$$

$$+\frac{L(q)}{256\pi^2 m^2 f_\pi^4}\left[(4\mu^2+\boldsymbol{q}^2) - 16g_A^2\,(\mu^2+3\boldsymbol{q}^2/8) + \frac{4g_A^4}{3}\left(9\mu^2 + 11\boldsymbol{q}^2/4 - \frac{4\mu^4}{4\mu^2+\boldsymbol{q}^2}\right)\right],$$

$$W_{\sigma L} \simeq 0\ ,$$

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Comparison with HB expressions

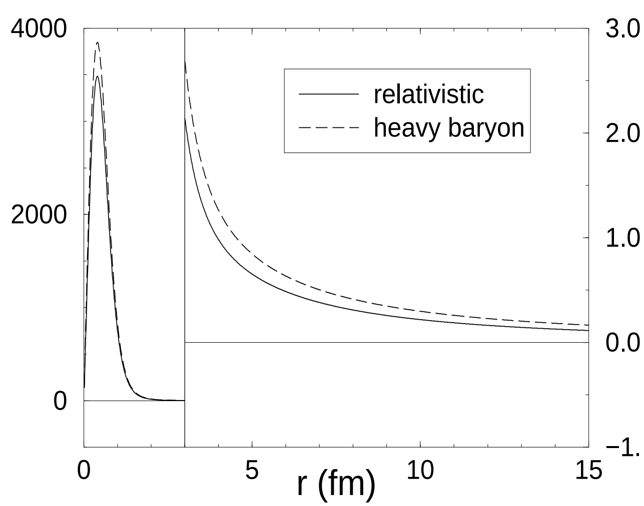
(Kaiser *et al.*, Kaiser, Entem and Machleidt, Epelbaum *et al.*)

- strategy: perform a formal $1/m_N$ expansion in our loop integrals (despite the fact that it doesn't converge!)
- relatively small number of differences
- origins RH, nucl-th/0411046
  - ⋆ dynamical equations (our choice: Blankenbecler and Sugar) (6)
  - ⋆ two loop calculations (not completely understood) (3)
- to isolate the $1/m_N$ expansion effect: ignore the above differences
- cutoff: $\left[1 - \exp(-cr^2)\right]^4 \times$TPEP

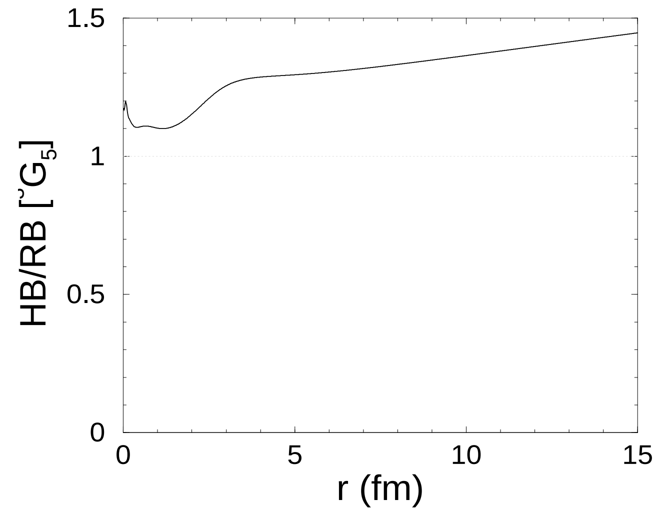
LECs

LEC	BM 00	Nij 03	EM 02
c_1	-0.81	-0.76	-0.81
c_2	8.43	3.20	3.28
c_3	-4.70	-4.78	-3.40
c_4	3.40	3.96	3.40
$\bar{d}_1 + \bar{d}_2$	3.06	3.06	3.06
$\bar{d}_3$	-3.27	-3.27	-3.27
$\bar{d}_5$	0.45	0.45	0.45
$\bar{d}_{14} - \bar{d}_{15}$	-5.65	-5.65	-5.65

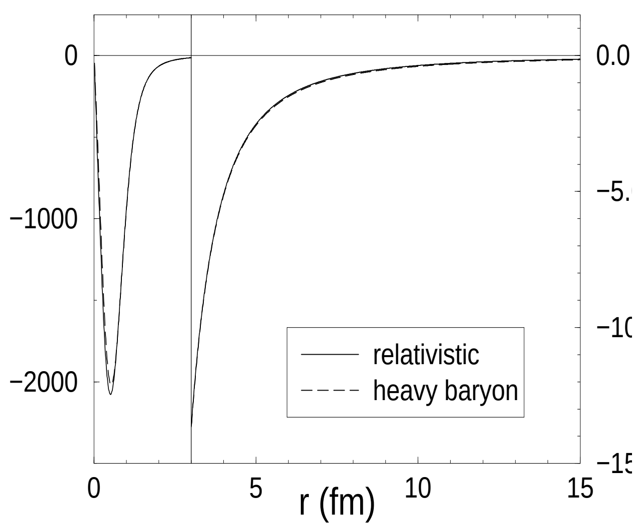
d_i's from Fettes *et al.*, Nucl.Phys. A 640, 199 (1998)



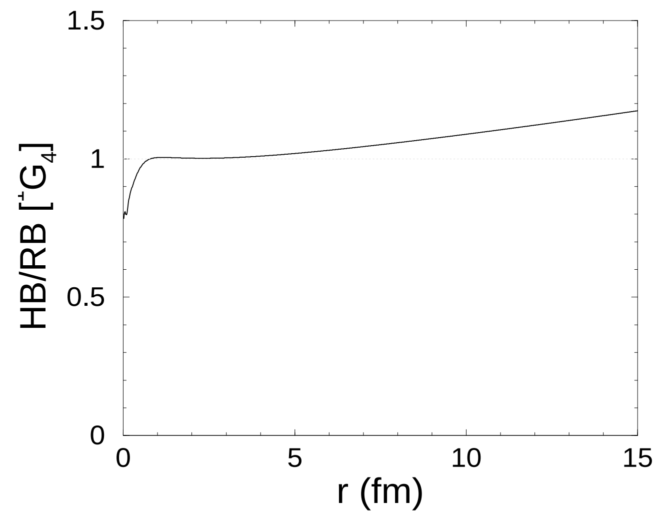
4000
2000
0
3.0
2.0
1.0
0.0
relativistic
heavy baryon
0
5
10
15
r (fm)



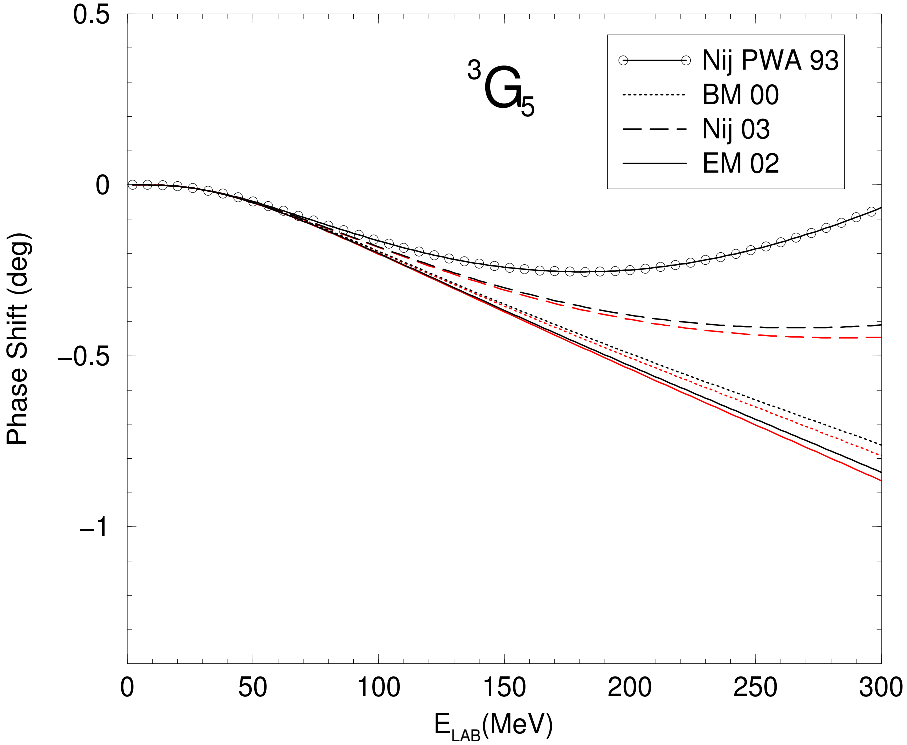
1.5
1
0.5
0
HB/RB
0
5
10
15
r (fm)



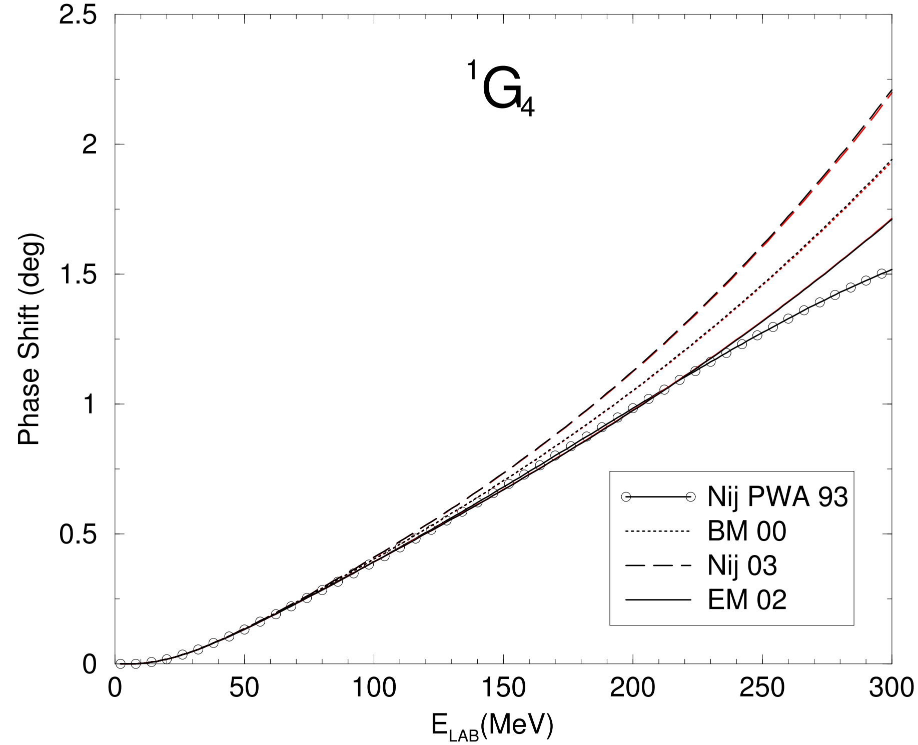
0
−1000
−2000
0.0
relativistic
heavy baryon
0
5
10
15
r (fm)



1.5
1
0.5
0
HB/RB
0
5
10
15
r (fm)



$^3G_5$
Nij PWA 93
BM 00
Nij 03
EM 02
Phase Shift (deg)
E_{LAB}(MeV)
0.5
0
−0.5
−1
0
50
100
150
200
250
300



$^1G_4$
Nij PWA 93
BM 00
Nij 03
EM 02
Phase Shift (deg)
E_{LAB}(MeV)
2.5
2
1.5
1
0.5
0
0
50
100
150
200
250
300

Comparison with HB expressions

(Kaiser *et al.*, Kaiser, Entem and Machleidt, Epelbaum *et al.*)

- strategy: perform a formal $1/m_N$ expansion in our loop integrals
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- to isolate the $1/m_N$ expansion effect: ignore the above differences
- renormalization of the singular OPE+TPE

Singular potentials with boudary conditions

- look at the divergence of the potential as $r \to 0$
  - ⋆ repulsive $\longrightarrow$ prediction
  - ⋆ attractive $\longrightarrow$ requires physical input
- multi-channels: behavior of the eigen-potentials
  - ⋆ orthogonality of the wave functions near the origin

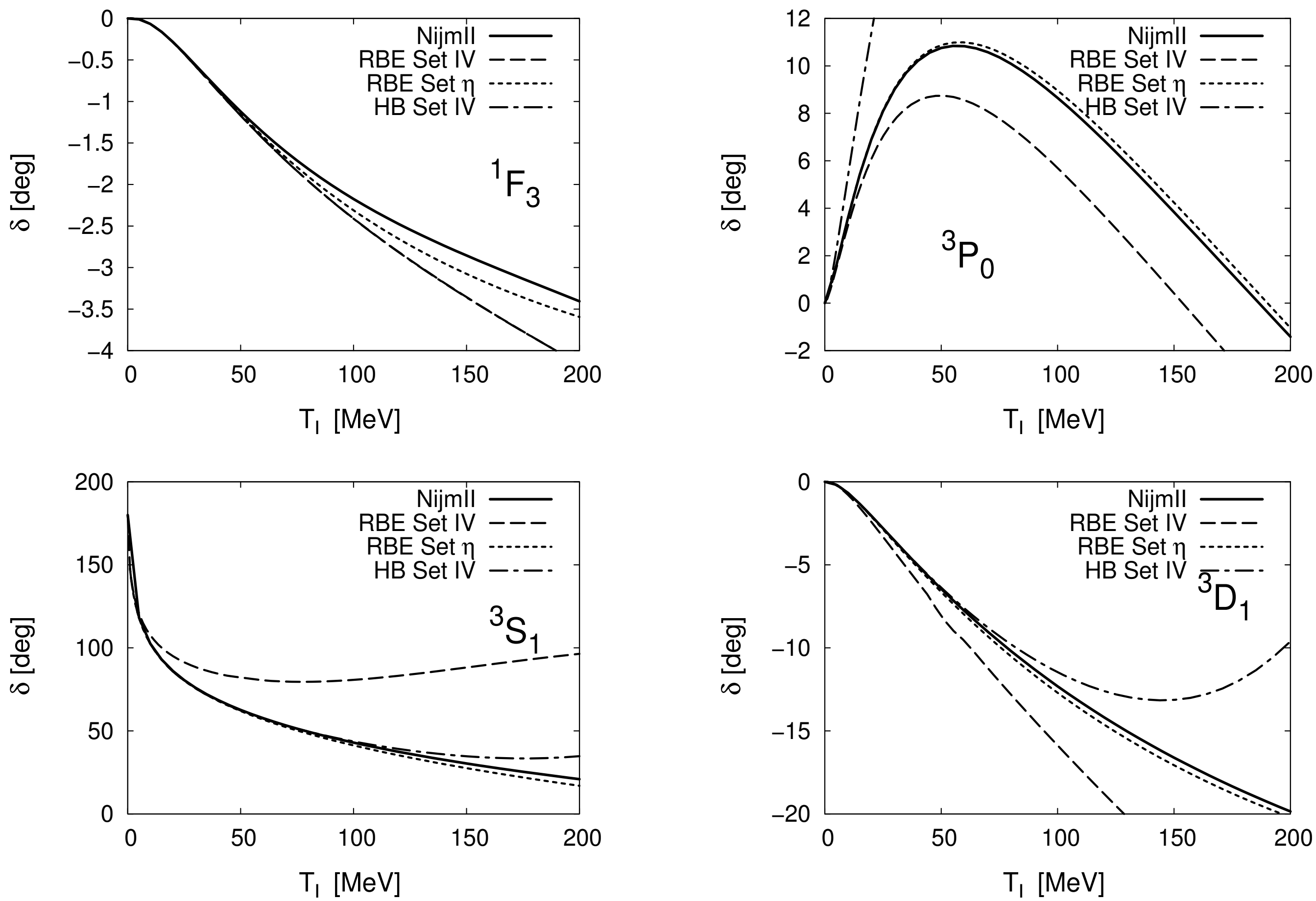
Set	Source	$c_1(\mathrm{GeV}^{-1})$	$c_3(\mathrm{GeV}^{-1})$	$c_4(\mathrm{GeV}^{-1})$
Set IV	EM 03	-0.81	-3.20	5.40
Set η	RH, Valderrama, Arriola	-0.81	-3.80	4.50

Wave	α NijmII (Reid93)	OPE	HB-NLO	HB-NNLO Set I, II & III	HB-NNLO SetIV	RB-TPE
$^1S_0$	-23.727(-23.735)	Input	Input	Input	Input	Input
$^3P_0$	-2.468(-2.469)	Input	—	Input	—	—(*)
$^1P_1$	2.797(2.736)	—	—	Input	—	—
$^3P_1$	1.529(1.530)	—	Input	Input	Input	Input
$^3S_1$	5.418(5.422)	Input	—	Input	Input	Input
$^3D_1$	6.505(6.453)	—	—	Input	Input	—
E_1	1.647(1.645)	—	—	Input	Input	—
$^1D_2$	-1.389(-1.377)	—	Input	Input	Input	Input
$^3D_2$	-7.405(-7.411)	Input	Input	Input	Input	Input
$^3P_2$	-0.2844(-0.2892)	Input	Input	Input	Input	Input
$^3F_2$	-0.9763(-0.9698)	—	—	Input	—	—(*)
E_2	1.609(1.600)	—	—	Input	—	—(*)
$^1F_3$	8.383(8.365)	—	—	Input	—	Input
$^3F_3$	2.703(2.686)	—	Input	Input	Input	Input
$^3D_3$	-0.1449(-0.1770)	Input	—	Input	Input	Input
$^3G_3$	4.880(4.874)	—	—	Input	Input	—
E_3	-9.695(-9.683)	—	—	Input	Input	—
$^1G_4$	-3.229(-3.210)	—	Input	Input	Input	Input
$^3G_4$	-19.17(-19.14)	Input	Input	Input	Input	Input
$^3F_4$	-0.01045(-0.01053)	Input	Input	Input	Input	Input
$^3H_4$	-1.250(-1.240)	—	—	Input	—	— (*)
E_4	3.609(3.586)	—	—	Input	—	— (*)
$^1H_5$	28.61(28.57)	—	—	Input	—	Input
$^3H_5$	6.128(6.082)	—	Input	Input	Input	Input
$^3G_5$	-0.0090(-0.010)	Input	—	Input	Input	Input
$^3I_5$	10.68(10.66)	—	—	Input	Input	—
E_5	-31.34(-31.29)	—	—	Input	Input	—

Singular potentials with boudary conditions

Set	$\gamma(\mathrm{fm}^{-1})$	η	$A_S(\mathrm{fm}^{-1/2})$	$r_d(\mathrm{fm})$	$Q_d(\mathrm{fm}^2)$	P_D
OPE	Input	0.02634	0.8681(1)	1.9351(5)	0.2762(1)	7.88(1)%
HB Set IV	Input	Input	0.884(4)	1.967(6)	0.276(3)	8(1)%
RBE Set IV	Input	0.03198(3)	0.8226(5)	1.8526(10)	0.3087(2)	22.99(13) %
RBE Set η	Input	0.02566(1)	0.88426(2)	1.96776(1)	0.2749(1)	5.59(1) %
NijmII	0.231605	0.02521	0.8845(8)	1.9675	0.2707	5.635%
Reid93	0.231605	0.02514	0.8845(8)	1.9686	0.2703	5.699%
Exp.	0.231605	0.0256(4)	0.8846(9)	1.971(6)	0.2859(3)	5.67(4)%

Singular potentials with boudary conditions



Summary

- derivation of R-TPE to $O(q^4)$ (NN-N^{3}LO) based on BL studies
- problem with the HB formalism: shows up in NN at large distances
- $1/m_N$ expansion of R-TPEP reproduce most of the HB results
  (discrepancy: 2-loop results)
- peripheral waves: R vs. HB is small, dependence on the πN LECs is large
- renormalization of the singular R-TPE: good description of observables with less counterterms ($\sim$ half) than HB-TPE
- improvement of codes, determination of short-distance parameters

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