Determination of Low Energy Constants and Testing Chiral Perturbation Theory at Next to Next to Leading Order

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Outline

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Why are we looking for relations between observables?

Chiral Perturbation Theory $\rightarrow$ every observable can be written as a sum of terms of increasing importance in the Chiral expansion.

$$O = O^{(2)} + O^{(4)} + O^{(6)}$$

The p^6 part can be split as

$$O^{(6)} = O_{C_i(\text{tree level})} + O_{L_i(\text{one loop})} + O_{F_0(\text{two loops})}$$

We look for relations between observables such that the first contribution cancels out. Using these

- we can check how large is the loop contribution and test ChPT convergence in a C_i independent way
- we hoped to perform a fit of the L_i at NNLO not depending on the C_i. Unfortunately in most of the relations the NLO L_i contributions cancel too (the dependence on the L_i is only through the NNLO pieces)
- in this way we isolated combinations of the C_i

Overview of the processes considered and relations found

process	# observables	# relations
$\pi\pi$ scattering	11	5
πK scattering	14	5
πK and $\pi\pi$ scattering	no extra observables	2
$K_{\ell 4}$ (with πK scattering)	10	1
$\eta \to 3\pi$ (with πK)	6	2
scalar form factors $F_S^{\pi/K}(t)$	18	6
$F_S^{\pi/K}(t)$, $\pi\pi$ and πK scattering	no extra observables	2
$F_S^{\pi/K}(t)$,$K_{\ell 4}$,$\pi\pi$ and πK scattering	no extra observables	1
$F_S^{\pi/K}(t)$, masses and decay constants	6	4
Vector form factors $F_V^{\pi/K}$	11	7
Total	76	35

Outline

1. Relations in ChPT
2. Numerical analysis of some of the relations
3. Summary and Future Steps

Numerical analysis explanation

1. Evaluation of each side of the relation using experimental data and/or dispersive analysis:
   [CGL] G. Colangelo, J. Gasser and H. Leutwyler, *Nucl. Phys.* B **603** (2001) 125 ($\pi\pi$ scattering)
   [BDM] Büttiker, Descotes-Genon, Moussallam *Eur. Phys. J.* C 33 (2004) 409 (πK scattering)
   [NA48/2] NA48/2 coll., *Eur. Phys. J.* C 54 (2008) 411-423 ($K_{\ell 4}$)
   [E865] S. Pislak *et al.* , *Phys. Rev.* D **67** (2003) 072004 ($K_{\ell 4}$)
2. Evaluation using ChPT up to p^6 results; L_i =fit10 and $C_i = 0$.
   For references see J. Bijnens, Prog. Part. Nucl. Phys. **58** (2007) 521
3. We quote the difference of the two $\Rightarrow$ it contains only the p^6 piece coming from the C_i and higher order terms.
4. Errors obtained adding in quadrature the uncertainties from experiments/dispersive results. No theoretical uncertainty due to the values of L_i or to higher orders has been added

$\pi\pi$ scattering:generalities

- $A(\pi^a\pi^b \to \pi^c\pi^d) = \delta^{a,b}\delta^{c,d}A(s,t,u) + \delta^{cd}\delta^{bd}A(t,u,s) + \delta^{ad}\delta^{bc}A(u,t,s)$
- The isospin amplitudes $T^I(s,t)$ $(I = 0, 1, 2)$ are written in terms of the function $A(s,t,u)$ and then expanded in partial waves:

$$T^I(s,t) = 32\pi \sum_{\ell=0}^{+\infty} (2\ell+1) P_\ell(\cos\theta) t_\ell^I(s)$$

$$\text{Near threshold} \to t_\ell^I(s) = q^{2\ell}(a_\ell^I + b_\ell^I q^2 + \mathcal{O}(q^4))$$

$$q^2 = \frac{1}{4}(s - 4m_\pi^2) \qquad a_\ell^I, b_\ell^I \cdots = \text{scattering lengths, slopes, \ldots}$$

- We studied only those observables where a dependence on the C_i shows up $\to$ 11 threshold parameters

$\pi\pi$ scattering: relations

- $A(s,t,u)$ can be written in terms of 6 independent parameters

$$A(s,t,u) = b_1 + b_2 s + b_3 s^2 + b_4 (t-u)^2 + b_5 s^3 + b_6 s (t-u)^2 + \text{non polynomial part}$$

- $\Rightarrow$ 5 relations among the scattering lengths.
- They hold for $n_f = 2,\ 3$, at NLO and NNLO: not only the p^6 LECs cancel out, but also the tree level part involving the p^4 LECs does. Still there is L_i or l_i dependence through the non polynomial part

$$\left[5b_0^2 - 2b_0^0 - 27a_1^1 - 15a_0^2 + 6a_0^0\right]_{C_i} = -18\left[b_1^1\right]_{C_i} \quad (1)$$

$$\left[3a_1^1 + b_0^2\right]_{C_i} = 20\left[b_2^2 - b_2^0 - a_2^2 + a_2^0\right]_{C_i} \quad (2)$$

$$\left[b_0^0 + 5b_0^2 + 9a_1^1\right]_{C_i} = 90\left[a_2^0 - b_2^0\right]_{C_i} \quad (3)$$

$a_\ell^I(b_\ell^I)$ expressed in unit of $m_\pi^{2\ell}(m_\pi^{2\ell+2})$

	[CGL]	NLO 1-loop	NLO LECs	NNLO 2-loop	NNLO 1-loop	**remainder**
LHS (1)	0.009 ± 0.039	0.054	-0.044	-0.041	-0.002	0.041 ± 0.039
RHS (1)	-0.102 ± 0.002	-0.009	-0.044	-0.060	-0.008	0.018 ± 0.002
10 LHS (2)	0.334 ± 0.019	0.209	0.097	0.103	0.029	-0.105 ± 0.019
10 RHS (2)	0.322 ± 0.008	0.177	0.097	0.120	0.034	-0.107 ± 0.008
LHS (3)	0.216 ± 0.010	0.166	0.029	0.053	0.016	-0.047 ± 0.010
RHS (3)	0.189 ± 0.003	0.145	0.029	0.049	0.020	-0.054 ± 0.003

	[CGL]	two-flavour **[CGL]**	**remainder**
LHS (1)	0.009 ± 0.039	-0.003	0.007 ± 0.039
RHS (1)	-0.102 ± 0.002	-0.097	-0.005 ± 0.002
10 LHS (2)	0.334 ± 0.019	0.332	0.002 ± 0.019
10 RHS (2)	0.322 ± 0.008	0.318	0.004 ± 0.075
LHS (3)	0.216 ± 0.010	0.206	0.010 ± 0.010
RHS (3)	0.189 ± 0.003	0.189	0.000 ± 0.003

$$\left[3b_1^1 + 25a_2^2\right]_{C_i} = 10\left[a_2^0\right]_{C_i} \quad (4)$$

$$\left[-5b_2^2 + 2b_2^0\right]_{C_i} = 21\left[a_3^1\right]_{C_i} \quad (5)$$

$a_\ell^I(b_\ell^I)$ expressed in unit of $m_\pi^{2\ell}(m_\pi^{2\ell+2})$

	[CGL]	NLO 1-loop	NLO LECs	NNLO 2-loop	NNLO 1-loop	**remainder**
10 LHS (4)	0.213 ± 0.005	0.137	0.032	0.053	0.035	-0.043 ± 0.005
10 RHS (4)	0.175 ± 0.003	0.121	0.032	0.050	0.029	-0.057 ± 0.003
10^3 LHS (5)	0.92 ± 0.07	0.36	0.00	0.56	-0.01	0.00 ± 0.07
10^3 RHS (5)	1.18 ± 0.04	0.42	0.00	0.57	0.03	0.15 ± 0.04

	[CGL]	two-flavour **[CGL]**	**remainder**
10 LHS (4)	0.213 ± 0.005	0.204	0.009 ± 0.005
10 RHS (4)	0.175 ± 0.003	0.176	-0.001 ± 0.003
10^3 LHS (5)	0.92 ± 0.07	1.00	-0.08 ± 0.07
10^3 RHS (5)	1.18 ± 0.04	1.15	0.04 ± 0.04

- Rel (4) and (5) $\rightarrow$ ok at 2-sigma level

πK scattering: generalities

- $T^I(s,t,u)$ = scattering amplitude in isospin channel $I = \frac{1}{2}, \frac{3}{2}$
- As for the $\pi\pi$ scattering, it's possible to define scattering lengths a^I_ℓ, b^I_ℓ:

$$T^I(s,t,u) = 16\pi \sum_{\ell=0}^{+\infty} (2\ell+1) P_\ell(\cos\theta) t^I_\ell(s)$$

$$\text{Near threshold} \longrightarrow t^I_\ell = \frac{1}{2}\sqrt{s}\, q^{2\ell}_{\pi K}\left(a^I_\ell + b^I_\ell q^2_{\pi K} + \mathcal{O}(q^4_{\pi K})\right)$$

$$q^2_{\pi K} = \frac{s}{4}\left(1 - \frac{(m_K + m_\pi)^2}{s}\right)\left(1 - \frac{(m_K - m_\pi)^2}{s}\right)$$

$$t = -2q^2_{\pi K}(1-\cos\theta), \quad u = -s - t + 2m_K^2 + 2m_\pi^2$$

- Again we studied only those scattering lengths where a dependence on the C_i shows up $\longrightarrow$ 14 threshold parameters

πK scattering: relations

- On the other hand the isospin amplitudes $T^I(s,t,u)$ are written in terms of the crossing symmetric and antisymmetric amplitudes $T^{\pm}(s,t,u)$ which can be expanded around t=0, s=u $(\nu = \frac{s-u}{4m_K})$ (subthreshold expansion):

$$T^+(s,t,u) = \sum_{i,j=0}^{\infty} c_{ij}^+ t^i \nu^{2j} \qquad T^-(s,t,u) = \sum_{i,j=0}^{\infty} c_{ij}^- t^i \nu^{2j+1}$$

  where $16\rho^2 \left[c_{20}^-\right]_{C_i} = 3\left[c_{01}^-\right]_{C_i}$, $\rho = m_K/m_\pi$ and c_{ij}^- are expressed in unit of $m_\pi^{2i+2j+1}$

  $\rightarrow$ 9 independent subthreshold parameters.

- $\Rightarrow$ 5 relations between the scattering lengths holding both at p^6 and at p^4: no dependence of the L_i at NLO.

For simplicity we introduce the notation

$$a_\ell^- = a_\ell^{1/2} - a_\ell^{3/2} \qquad b_\ell^- = b_\ell^{1/2} - b_\ell^{3/2}$$
$$a_\ell^+ = a_\ell^{1/2} + 2a_\ell^{3/2} \qquad b_\ell^+ = b_\ell^{1/2} + 2b_\ell^{3/2}$$

$$\left(\rho^4 + 3\rho^3 + 3\rho + 1\right) \left[a_1^-\right]_{C_i} = 2\rho^2 (\rho+1)^2 \left[b_1^-\right]_{C_i} - \frac{2}{3}\rho \left(\rho^2+1\right) \left[b_0^-\right]_{C_i} + \frac{1}{2\rho} \left(\rho^2 + \frac{4}{3}\rho + 1\right) \left(\rho^2+1\right) \left[a_0^-\right]_{C_i} \tag{6}$$

$$5\left(\rho^2+1\right) \left[a_2^-\right]_{C_i} = \left[a_1^-\right]_{C_i} + 2\rho \left[b_1^-\right]_{C_i} \tag{7}$$

$$5(\rho+1)^2 \left[b_2^-\right]_{C_i} = \frac{(\rho-1)^2}{\rho^2} \left[a_1^-\right]_{C_i} - \frac{\rho^4 + \frac{2}{3}\rho^2 + 1}{4\rho^4} \left[a_0^-\right]_{C_i} + \frac{\rho^2 - \frac{2}{3}\rho + 1}{2\rho^2} \left[b_0^-\right]_{C_i} \tag{8}$$

All quantities are in the units of powers of m_{π^+}

	[BDM]	NLO 1-loop	NLO LECs	NNLO 2-loop	NNLO 1-loop	**remainder**
LHS (6)	5.4 ± 0.3	0.16	0.97	0.77	-0.11	0.6 ± 0.3
RHS (6)	6.9 ± 0.6	0.42	0.97	0.77	-0.03	1.8 ± 0.6
10 LHS (7)	0.32 ± 0.01	0.03	0.12	0.11	0.00	0.07 ± 0.01
10 RHS (7)	0.37 ± 0.01	0.02	0.12	0.10	-0.01	0.14 ± 0.01
100 LHS (8)	-0.49 ± 0.02	0.08	-0.25	-0.17	0.05	-0.21 ± 0.02
100 RHS (8)	-0.85 ± 0.60	0.03	-0.25	0.11	-0.03	-0.71 ± 0.60

- Rel (6) $\rightarrow$ ok at 2 sigma
- Rel (7) $\rightarrow$ ok with a theoretical error about half the NNLO contribution
- Rel (8) $\rightarrow$ ok but large uncertainty

$$7\left(\rho^2+1\right)\left[a_3^-\right]_{C_i} = \left[a_2^-\right]_{C_i} + 2\rho\left[b_2^-\right]_{C_i} \quad (9)$$

$$7\left[a_3^+\right]_{C_i} = \frac{1}{2\rho}\left[a_2^+\right]_{C_i} - \left[b_2^+\right]_{C_i} + \frac{1}{5\rho}\left[b_1^+\right]_{C_i} - \frac{1}{60\rho^3}\left[a_0^+\right]_{C_i} - \frac{1}{30\rho^2}\left[b_0^+\right]_{C_i} \quad (10)$$

All quantities are in the units of powers of m_{π^+}

	[BDM]	NLO 1-loop	NLO LECs	NNLO 2-loop	NNLO 1-loop	**remainder**
100 LHS (9)	0.13 ± 0.01	0.04	0.00	0.01	0.03	0.05 ± 0.01
100 RHS (9)	0.01 ± 0.01	0.01	0.00	0.00	0.00	-0.01 ± 0.01
10^3 LHS (10)	0.29 ± 0.05	0.09	0.00	0.06	0.01	0.13 ± 0.03
10^3 RHS (10)	0.31 ± 0.07	0.03	0.00	0.06	0.05	0.17 ± 0.07

- Rel (9) $\rightarrow$ ChPT seems to underestimate a_3^-

$\pi\pi$ scattering and πK scattering

Considering $\pi\pi$ and πK scattering together two more relations appear. These are due to the following identities:

$$[b_5]_{C_i} = [c_{30}^{+}]_{C_i} + \frac{3}{4\rho}[c_{20}^{-}]_{C_i}\,, \qquad [b_6]_{C_i} = \frac{1}{4\rho}[c_{20}^{-}]_{C_i} + \frac{1}{16\rho^2}[c_{11}^{+}]_{C_i}$$

which in terms of the threshold parameters read

$$6\left[a_3^1\right]_{C_i} = (1+\rho)\left[a_3^+ + 3a_3^-\right]_{C_i} \qquad (11)$$

$$3\left[(1+\rho)^2\left[b_2^2\right]_{C_i} + 7(1-\rho)^2\left[a_3^1\right]_{C_i}\right] = (1+\rho)\left[7\left(1-4\rho+\rho^2\right)\left[a_3^-\right]_{C_i} + \left[a_2^+ + 2\rho b_2^+\right]_{C_i}\right] \qquad (12)$$

All quantities in units of powers of m_{π^+}

	[CGL] [BDM]	NLO 1-loop	NLO LECs	NNLO 2-loop	NNLO 1-loop	**remainder**
10^3 LHS (11)	0.34 ± 0.01	0.12	0.00	0.16	0.00	0.05 ± 0.01
10^3 RHS (11)	0.38 ± 0.03	0.12	0.00	0.05	0.04	0.16 ± 0.03
10 LHS (12)	-0.13 ± 0.01	-0.12	0.00	-0.05	0.02	0.01 ± 0.01
10 RHS (12)	-0.09 ± 0.02	-0.05	0.00	-0.02	-0.01	-0.01 ± 0.02

- Rel (11) $\rightarrow a_3^-$ appears
  similar discrepancy seen in Kampf, Moussallam Eur. Phys. J. C. 47 (2006) 723 (see talk by Bijnens)

$K_{\ell 4}$: generalities and relation

- In the transition amplitude 4 form factors appear: F, G, H, R (R in K_{e4} is suppressed $\rightarrow$ only in $K_{\mu 4}$)
- Using partial wave expansion and neglecting d wave terms (10 observables):

$$\begin{aligned} F &= f_s + f_s' q^2 + f_s'' q^4 + f_e' s_e/4m_\pi^2 + f_t \sigma_\pi X \cos\theta + \ldots\,, \\ G_p &= g_p + g_p' q^2 + g_g'' q^4 + g_e' s_e/4m_\pi^2 + g_t \sigma_\pi X \cos\theta + \ldots \end{aligned}$$

$s_\pi (s_e)$ =invariant mass of dipion (dilepton) $\quad q^2 = (s_\pi/(4m_\pi^2) - 1)$

- 1 relation between πK scattering and K_{e4} observables:

$$\sqrt{2}\,[f_s'']_{C_i} = 64\rho F_\pi \left[c_{30}^+\right]_{C_i}$$

in terms of the πK threshold parameters reads

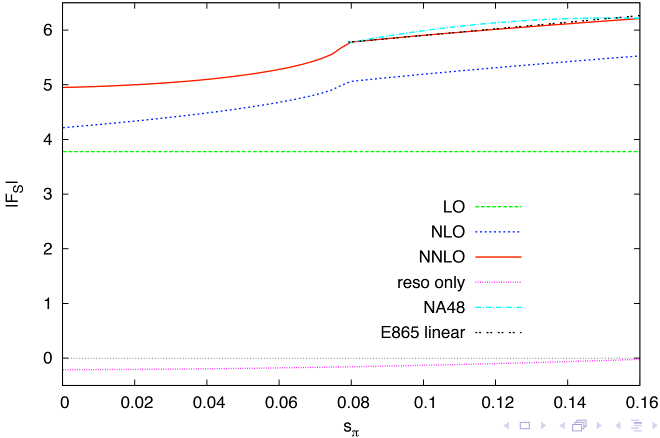
$$\sqrt{2}\,[f_s'']_{C_i} = 32\pi \frac{\rho}{1+\rho} F_\pi \left[\frac{35}{6}\left(2+\rho+2\rho^2\right)\left[a_3^+\right]_{C_i} - \frac{5}{4}\left[a_2^+ + 2\rho b_2^+\right]_{C_i}\right]$$

All quantities are in units of powers of m_{π^+}

$K_{\ell 4}$: numerics

(see talk by Bijnens)

	[BDM], [E865], [NA48/2]	NLO 1-loop	NLO LECs	NNLO 2-loop	NNLO 1-loop	**remainder**
LHS (17)	-0.73 ± 0.10	-0.23	0.00	-0.15	-0.05	-0.29 ± 0.10
RHS (17)	0.50 ± 0.07	0.19	0.00	0.10	0.03	0.18 ± 0.07



Outline

Summary and Future Steps

- many observables at NNLO, depending on many correlated LECs
- we found relations among observables not depending on the NNLO constants (and most of them not depending on the NLO either) $\rightarrow$ a way to check ChPT
- although many relations work well, results for $K_{\ell 4}$ and πK scattering show discrepancies: further investigation needed
- arXiv:0906.3118 [hep-ph]

Future steps:

- new fit of the L_i with a better treatment of the C_i and using new exp data available, dispersive analysis and lattice results (masses, scalar form factors)
- include corrections (e.g. isospin breaking)

Status of the L_i fit PRELIMINARY!!!!

(see talk by Bijnens)

- Program for fitting (Minuit) almost ready (pion scalar radius, $\pi\pi$ and πK scattering observables now included)
- C_i obtained through Resonance Estimates (Vector, Scalar, PseudoScalar) (see Amoros, Bijnens, Talavera, Nucl. Phys. B 602 (2001) 87)

	fit 10 iso	NA48	F_K/F_π	All
$10^3 L_1^r$	0.40 ± 0.12	0.98	0.97	0.99 ± 0.13
$10^3 L_2^r$	0.76 ± 0.12	0.78	0.79	0.60 ± 0.22
$10^3 L_3^r$	-2.40 ± 0.37	-3.14	-3.12	-3.07 ± 0.59
$10^3 L_4^r$	$\equiv 0$	$\equiv 0$	$\equiv 0$	0.65 ± 0.64
$10^3 L_5^r$	0.97 ± 0.11	0.93	0.72	0.53 ± 0.10
$10^3 L_6^r$	$\equiv 0$	$\equiv 0$	$\equiv 0$	0.07 ± 0.65
$10^3 L_7^r$	-0.30 ± 0.15	-0.30	-0.26	-0.21 ± 0.15
$10^3 L_8^r$	0.61 ± 0.20	0.59	0.48	0.37 ± 0.17
χ^2 (dof)	0.25 (1)	0.17 (1)	0.19 (1)	0.78 (4)

- NA48 $\longrightarrow$ NA48 exp data. No change in the fit including curvature: $f_s'' = -0.90$ (exp value: $f_s'' = -1.58 \pm 0.064$)
- $F_K/F_\pi = 1.19$ (value of L_5^r changes)
- All $\longrightarrow$ add a_0^0, a_0^2, $a_0^{1/2}$, $a_0^{3/2}$, scalar pion radius