Chiral low-energy couplings from lattice computations in the epsilon-regime

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Working Group 1 - Goldstone bosons: Light quark parameters

Outline

1. Introduction
   - Chiral effective theory in the ϵ-regime
2. Lattice QCD in the epsilon regime
3. Results
   - LECs from Dirac eigenvalues distribution
   - 2-point correlators
4. Conclusions

Introduction

Match lattice QCD with the chiral effective theory:

- Test if chiral symmetry is broken according to the expected pattern
- Extract the Low Energy Couplings (LECs) of the effective theory from first principles

- Ideally: $a \to 0$, $m \to 0$, $V \to \infty$
- Practically: simulations with $N_f = 2, 2+1$ light quarks

  $a \simeq 0.05 - 0.12$ fm, $M_\pi \simeq 200 - 500$ MeV , $L \sim 1.7 - 3$ fm, $M_\pi L \simeq 3 - 4$.

Need for good control over systematic effects:

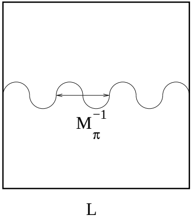
- lattice artefacts
- finite-volume effects
- higher order contributions in the effective theory

Introduction

Chiral regimes at finite volume V: *Gasser & Leutwyler ('87)*

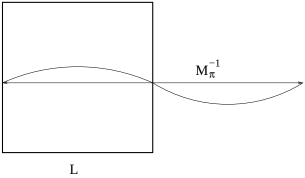
p-regime

- $(\lim_{m \to 0})_{M_\pi L \geq \mathcal{O}(1)}$
- $L \gg 1/(4\pi F)$
- $m \sim p^2, \frac{1}{L}, \frac{1}{T} \sim p$
- finite-volume effects exponentially suppressed



ϵ-regime

- $(\lim_{m \to 0})_{m\Sigma V = \mathcal{O}(1)}$
- $L \gg 1/(4\pi F)$; $(M_\pi L)^2 < 1$
- zero modes contribution non-perturbative
- $m \sim \epsilon^4, \frac{1}{L}, \frac{1}{T} \sim \epsilon$
- finite-volume effects sizeable (polynomial), mass effects suppressed



ϵ-regime

Reorganization of the power counting:
→ at a given order, less LECs appear with respect to p-regime
→ In principle: ideal regime to compute LO LECs (Σ, F)

Chiral symmetry breaking in a finite volume V

For $\mu = m\Sigma V \ll 1$

$$\mathcal{Z} = \int_{\mathrm{SU(N_f)}} dU_0 \exp\left[\mu \mathrm{Re}\mathrm{Tr} U_0\right]; \qquad U_0 \to \quad \textit{zero mode}$$

$$\Sigma(\mu) = \frac{\Sigma}{N_f}\frac{\partial}{\partial\mu} \ln \mathcal{Z} \propto \Sigma\mu$$

$\Sigma(\mu)_{\mu\to 0} \to 0$: no symmetry breaking at finite volume

but finite-size scaling gives informations about spontaneous symmetry breaking in infinite-volume

Hasenfratz & Leutwyler ('90)

Two-point functions in ϵ-regime

Example: pseudoscalar and axial correlator at NLO

$$C_P^{ab}(t) = \frac{1}{L^3}\int d^3x \langle P^a(x) P^b(0)\rangle = \delta^{ab}\Sigma^2\left[a_P + \frac{T}{F^2L^3} b_P h_1(t/T)\right]$$

$$C_A^{ab}(t) = \frac{1}{L^3}\int d^3x \langle A_0^a(x) A_0^b(0)\rangle = \delta^{ab}\frac{F^2}{V}\left[a_A + \frac{T}{F^2L^3}\left(b_A h_1(t/T) + c_A\right)\right]$$

- $h_1\left(\frac{t}{T}\right) = \frac{1}{2}\left[\left(\frac{t}{T} - \frac{1}{2}\right)^2 - \frac{1}{12}\right]$ parabolic time dependence
- a_P, b_P, a_A, b_A, c_A: functions of $\mu = m\Sigma V, L, T$

Hansen ('90)

$\rightarrow$ only LO LECs F and Σ appear

- in the ϵ-regime, topology plays a relevant role: predictions at fixed topology

$$\mathcal{Z}_\nu = \int_{\mathrm{U(N_f)}} dU_0 (\det U_0)^\nu \exp\left[\mu \mathbf{ReTr} U_0\right]$$

$\rightarrow \Sigma_\nu(\mu)$, $C_{P,\nu}^{ab}(t)$,$C_{A,\nu}^{ab}(t)$...

Leutwyler, Smilga ('92), *Damgaard et al* ('01,'02)

Lattice QCD in the ϵ-regime

Simulate lattice QCD with $L \gg 1/(4\pi F)$ and $m\Sigma V \lesssim 1$

$\rightarrow$ need small quark masses

Which discretization?

Ginsparg-Wilson fermions

Ginsparg, Wilson ('82)

Neuberger, Fixed-point actions, Domain Wall with $L_5 \rightarrow \infty$

- ☺ Chiral Symmetry preserved at finite lattice spacing
- ☺ Dirac spectrum bounded from below $\rightarrow$ small quark masses accessible
- ☺ Topology naturally defined through index theorem
- ☹ Dynamical simulations numerically challenging

$\rightarrow$ many quenched simulations in the ϵ-regime

P. Hernández et al ('99), *T. DeGrand* ('01), *W. Bietenholz et al* ('04), *L. Giusti et all* ('04,'07)

$\rightarrow$ still few dynamical simulations

JLQCD, JLQCD + TWQCD ('07,'08), *P. Hasenfratz et al* ('07), *DeGrand and Schaefer* ('07)

Lattice QCD in the ϵ-regime

Simulate lattice QCD with $L \gg 1/(4\pi F)$ and $m\Sigma V \lesssim 1$

$\rightarrow$ need small quark masses

Which discretization?

Wilson fermions

- ☹ Chiral symmetry explicitly broken at finite lattice spacing
- ☹ Small quark masses problematic: Dirac spectrum not bounded from below.
  Empirical stability bound: $m > m_{min}$, $m_{min} \propto a/\sqrt{V}$

  Del Debbio et al ('06)
- No natural definition of topological charge
- ☺ Dynamical simulations feasible

Lattice QCD in the ϵ-regime

Simulate lattice QCD with $L \gg 1/(4\pi F)$ and $m\Sigma V \lesssim 1$

$\rightarrow$ need small quark masses

Which discretization?

Wilson fermions

- ☹ Chiral symmetry explicitly broken at finite lattice spacing
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  Possible solution: reweighting $\rightarrow$ first simulations in the ϵ-regime

A. Hasenfratz et al ('08)

For reweighting see also *Lüscher and Palombi* ('08)

Wilson Twisted-Mass fermions

- Dirac spectrum bounded from below $\rightarrow$ ϵ-regime can be reached

K. Jansen et al (ETM) ('07,'08)

Matching lattice QCD with chiral effective theory

- Ideally: first perform continuum extrapolation
  $\rightarrow$ then match with chiral effective theory
- Alternative: incorporate discretization effects in the chiral effective theory $\rightarrow$ Symanzik approach

Symanzik ('75)

(Disadvantage: new unknown LECs)

- GW fermions: described by continuum ChPT; LECs(a^2)
- Wilson-like fermions: WChPT, TMChpt

Sharpe & Singleton ('98), *Rupak & Shoresh* ('02),
Sharpe & Wu ('04), *Aoki & Bär* ('04)...

Can be extented to the ϵ-regime

O. Bär, S.N., S. Schaefer ('09), *A. Shindler* ('09)

for $m \sim a\Lambda_{QCD}^2$ (GSM regime) : explicit breaking of chiral symmetry still dominated by quark mass $\rightarrow$ lattice spacing effects suppressed (NNLO)

Results: LECs from Dirac eigenvalues distribution

χPT at LO in the ϵ-expansion $\leftrightarrow$ Random Matrix Theory

$\downarrow$

probability distribution of single eigenvalues $p_{k,\nu}(\zeta_k, \mu)$; $\zeta_k = \lambda_k \Sigma V, \mu = m\Sigma V$

$\downarrow$

match low-lying spectrum of Dirac operator:

$\langle \zeta_k \rangle^{RMT} = \langle \lambda_k \rangle^{QCD} \Sigma V \rightarrow$ extract Σ (F)

Shuryak and Verbaarschot ('93), *Damgaard* ('98), *Akemann and Damgaard* ('98), *Wilke et al* ('98), *Damgaard and Nishigaki* ('98), *Osborn et al* ('99), *Akemann and Damgaard* ('08)

Possible issue on systematic errors:

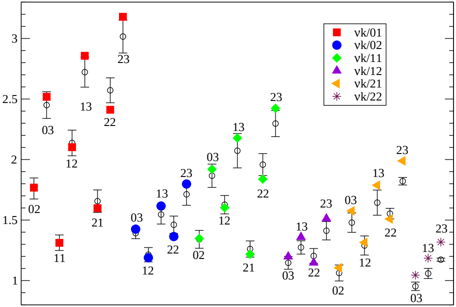
for the distribution of single eigenvalues, higher order corrections are not known

Recent unquenched computations with GW fermions:

JLQCD+TWQCD ('07), DeGrand and Schaefer ('07), Lang et al ('06), P. Hasenfratz et al ('07)

Results: LECs from Dirac eigenvalues distribution

Eigenvalue ratios can be compared directly with RMT

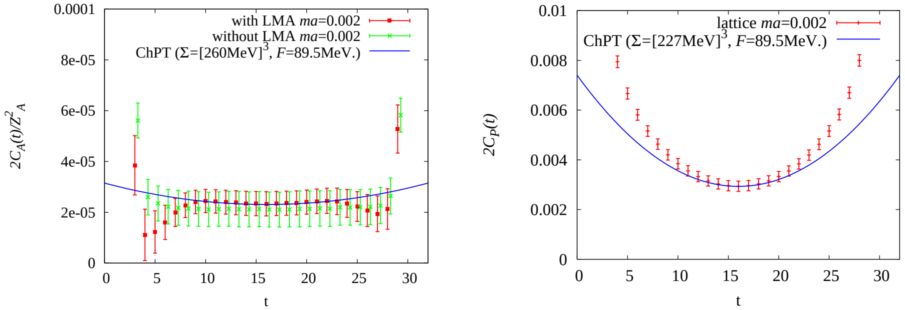


P. Hasenfratz et al ('07-'09): Fixed-point action

$N_f = 2+1$, a=0.129(5) fm, $L \simeq 1.6$ fm, $\mu_{u,d} = m_{u,d}\Sigma V \simeq 1.4$, $\mu_s = m_s \Sigma V \simeq 12.3$

$$\Sigma^{\overline{MS}}(\mu = 2\ GeV) = (239(11)\ \ MeV)^3$$

Results: LECs from 2-point correlators



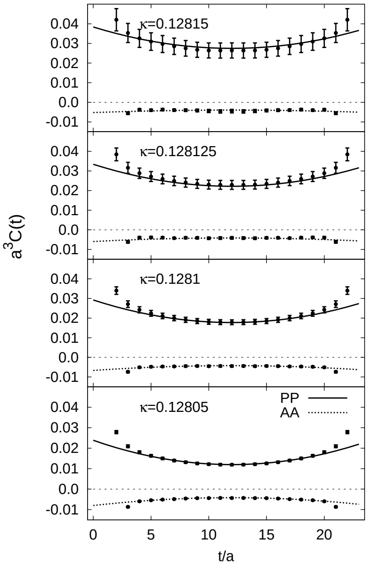
JLQCD ('08): Neuberger Fermions

$N_f = 2$, a=0.1111(24) fm, $L \simeq 1.78$ fm, $\mu = m\Sigma V \simeq 0.556$, $\nu = 0$

NLO Fit:

$$\Sigma^{\overline{MS}}(\mu = 2\ GeV) = (239.8(4.0)\ \ MeV)^3\,; \qquad F = 87.3(5.6)\ \ MeV$$

Results: LECs from 2-point correlators



A. Hasenfratz, Hoffmann, Schaefer ('08)

NHYP Wilson fermions

$N_f = 2$, $a \simeq 0.115$ fm, $L \simeq 1.84, 2.8$ fm,

$\mu = m\Sigma V \simeq 0.7 - 5$, ν not fixed

- Continuum NLO fit: ($L = 2.8$ fm)

$F = 90(4)$ MeV,

$\Sigma^{\overline{MS}}(2\ GeV) = (248(6)\ \ MeV)^3$

- GSM* NLO fit: ($L = 2.8$ fm)
  Include leading ($O(a^2)$)
  correction at NLO

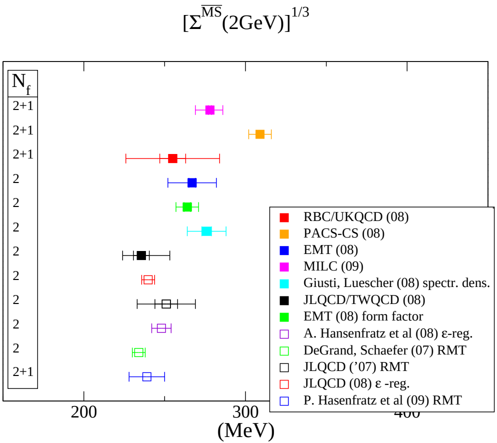
O. Bär, S.N., S. Schaefer('09)

$F = 88(3)$ MeV

$\Sigma^{\overline{MS}}(2\ GeV) = (249(4)\ \ MeV)^3$

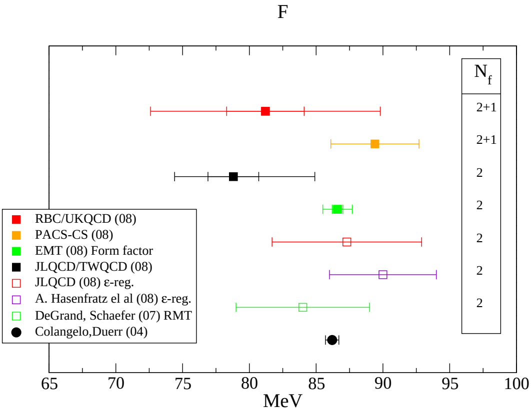
$c_2 = 0.02(8)\ GeV^4$

Summary of lattice results for Σ



see *FLAG report*, in preparation

Summary of lattice results for *F*



see *FLAG report*, in preparation

Conclusions

- Lattice simulations in the ϵ-regime give important informations on low-energy QCD $\rightarrow$ LECs

  systematic errors are different with respect to infinite-volume determinations

- Still numerically challenging
  $\rightarrow$ but simulations with Wilson-like fermions seem feasible

  Lattice effects due to explicit breaking of chiral symmetry are under control, at least for mesonic 2-point correlators

- More studies needed $\rightarrow$ volumes, lattice spacings
  - check if the predicted NLO scaling is verified
  - systematic errors under control?

Conclusions

- Other applications:
  - Mixed regimes:
    "heavy" quarks in the p-regime, "light" quarks in the ϵ-regime

    Bernardoni & Hernández ('07), *Fukaya & Damgaard* ('07)
  - Spectral density: connect p- and ϵ-regime

    Fukaya & Damgaard ('09), see also *Giusti & Lüscher* ('09)

    $\longrightarrow$ See plenary talk by Hashimoto
  - Heavy Meson chiral perturbation theory for heavy-light systems

    Burdman & Donoghue ('92), *Wise* ('92)
  - LECs of effective weak Hamiltonian $\rightarrow K \rightarrow \pi\pi$ amplitutes
    quenched investigations:

    Giusti et al ('04)