Spin Structure in the Resonance Region

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For the CLAS EG4 Collaboration

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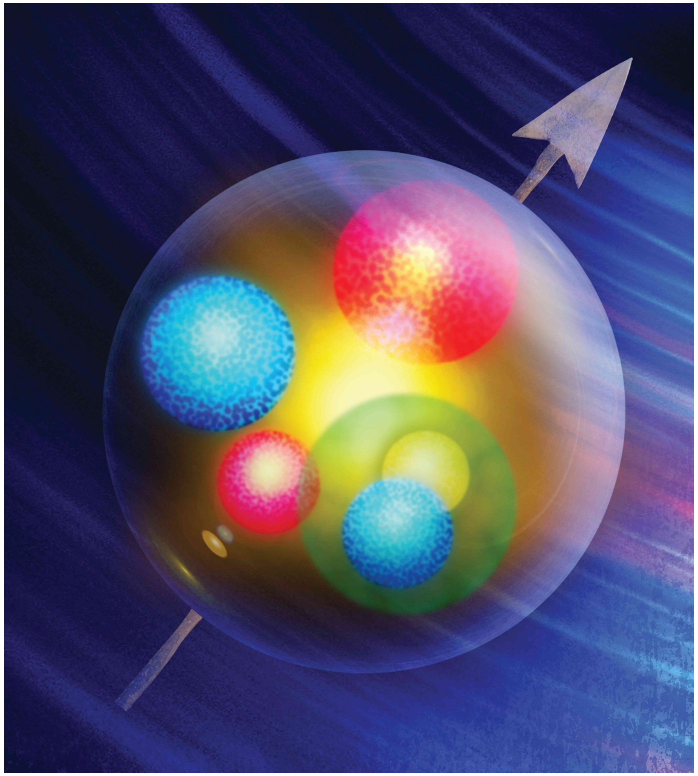
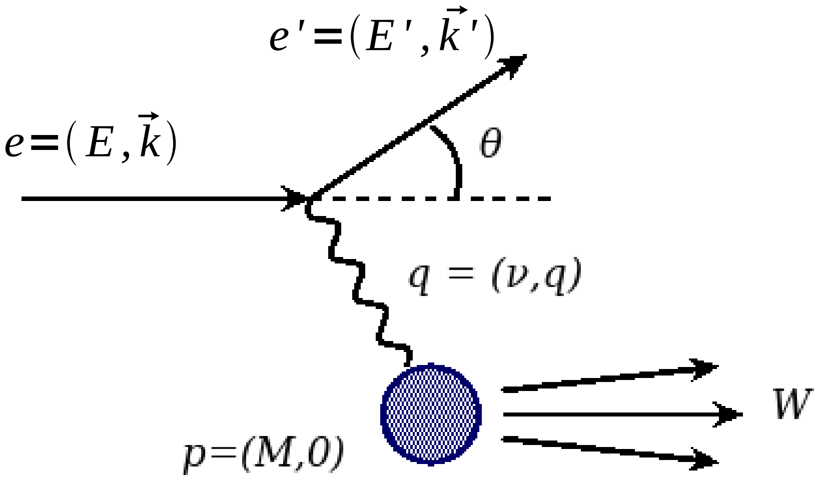
Nucleon Spin Structure in the Resonance Region



- Inclusive electron scattering
- GDH Sum Rule, moments, and spin polarizabilities
- Virtual photon asymmetries
- Jefferson Lab's Hall B
- CLAS EG4
    - Inclusive measurement
    - Exclusive measurement
- Future measurement: g_2^p
- Summary

Inclusive Electron Scattering



The usual definitions:

- Four-momentum transfer squared:

$$Q^2 = -q^2 = 4EE'\sin^2\frac{\theta}{2}$$

- Invariant mass squared:

$$W^2 = M^2 + 2M\nu - Q^2$$

- Bjorken variable: $x = \dfrac{Q^2}{2M\nu}$

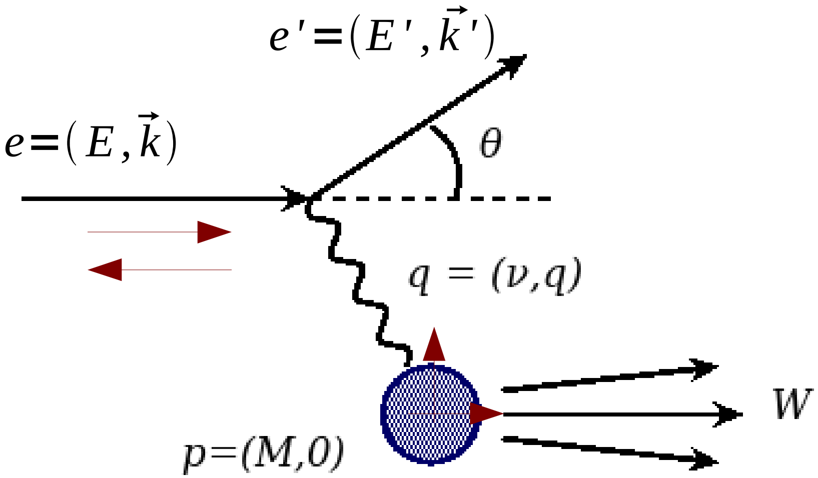
- Structure functions: $F_1(x,Q^2),\quad F_2(x,Q^2)$

Unpolarized Case

$$\frac{d^2\sigma}{d\Omega\, dE'} = \sigma_{Mott}\left[\frac{1}{\nu}F_2(x,Q^2) + \frac{2}{M}F_1(x,Q^2)\tan^2\frac{\theta}{2}\right]$$

Structure functions characterize deviation from point-like behavior

Inclusive Electron Scattering



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- Structure functions: $F_1(x,Q^2),\quad F_2(x,Q^2)$

- Spin-dependent structure functions: $g_1(x,Q^2),\quad g_2(x,Q^2)$

Polarized Case

$$\frac{d^2\sigma^{\uparrow\uparrow}}{d\Omega\, dE'} - \frac{d^2\sigma^{\downarrow\uparrow}}{d\Omega\, dE'} = \frac{4\alpha^2 E'}{\nu E Q^2}\left[(E+E'\cos\theta)\, g_1(x,Q^2) - 2Mx\, g_2(x,Q^2)\right]$$

$$\frac{d^2\sigma^{\uparrow\rightarrow}}{d\Omega\, dE'} - \frac{d^2\sigma^{\downarrow\rightarrow}}{d\Omega\, dE'} = \frac{4\alpha^2 E'}{\nu E Q^2}\sin\theta\left[g_1(x,Q^2) + \frac{2ME}{\nu} g_2(x,Q^2)\right]$$

All four (F_1, F_2, g_1, g_2) are needed for a complete description of nucleon structure!

The GDH Sum Rule

At $Q^2 = 0$ (real photon limit):

$$I_{GDH} = \frac{M^2}{8\alpha\pi^2}\int_{thr}^{\infty}\left(\sigma_{1/2}-\sigma_{3/2}\right)\frac{d\nu}{\nu} = -\frac{1}{4}\kappa^2$$

- The GDH Sum Rule relates the difference of the two photo-absorption cross sections to the anomalous magnetic moment of the nucleon κ.
- Circularly polarized photons incident on a longitudinally polarized target.
- $\sigma_{3/2}$ ($\sigma_{1/2}$) denotes the photo-absorption cross section with photon helicity parallel (anti-parallel) to the target spin.
- Sum rules are solid theoretical predictions based on general principles.
- Derived in the real photon limit, but can be generalized for virtual photons.

The Generalized GDH Sum Rule

For virtual photons,

- Rule can be expressed as the integral of $g_1(x,Q^2)$
- Can be linked to the forward spin-dependent Compton amplitude $S_1(0,Q^2)$ by the extended GDH sum rule

$$I_{GDH}(Q^2 \neq 0) = \frac{16\pi^2\alpha}{Q^2}\int_0^{x_{th}} g_1(x,Q^2)\,dx = \frac{16\pi^2\alpha}{Q^2}\Gamma_1 = 2\pi^2\alpha S_1(0,Q^2)$$

Ji and Osborne, J. Phys. G27, 127 (2001)

- At $Q^2 = 0$, the GDH sum rule is recovered.
- At $Q^2 \rightarrow \infty$, the Bjorken sum rule is recovered.

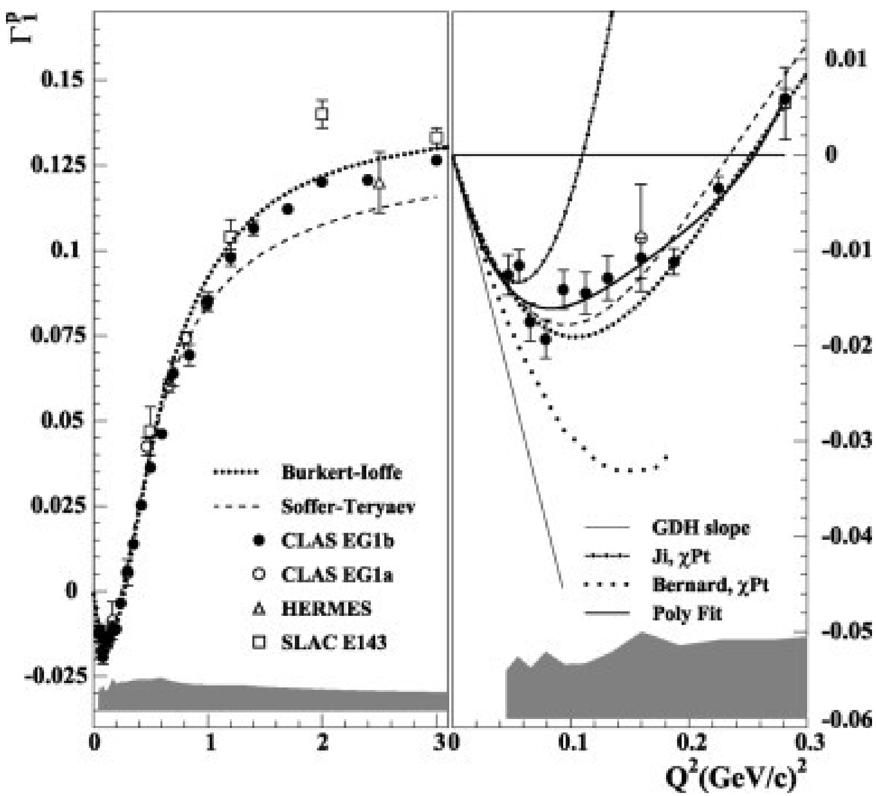
The first moment Γ_1

$$\Gamma_1(Q^2) = \int_0^1 g_1(x,Q^2)\,dx$$

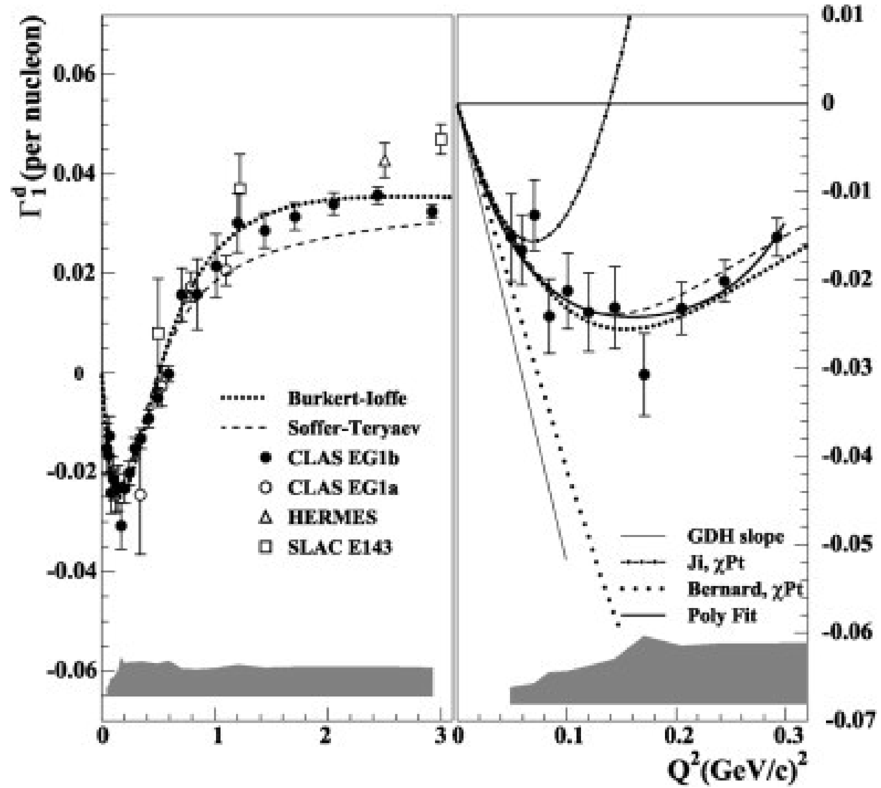
- Connected to the total spin carried by the quarks.

Measurements of Γ_1

Proton



Deuteron



- Measurements from EG1 (a and b), SLAC, Hermes
- EG4 will push to lower Q^2
- Other low Q^2 data from EG1b and Hall A's E97-110 and E94-010 (on polarized ^{3}He)

Y. Prok et al. Phys. Lett. B672 12, 2009

Generalized Forward Spin Polarizabilities

Higher moments of spin structure functions are interesting too!

- Additional x-weighting emphasizes the kinematic region measured at JLab.

$$\gamma_0(Q^2) = \frac{16\alpha M^2}{Q^6}\int_0^{x_0} x^2\left[g_1(x,Q^2) - \frac{4M}{Q^2}x^2 g_2(x,Q^2)\right]dx$$

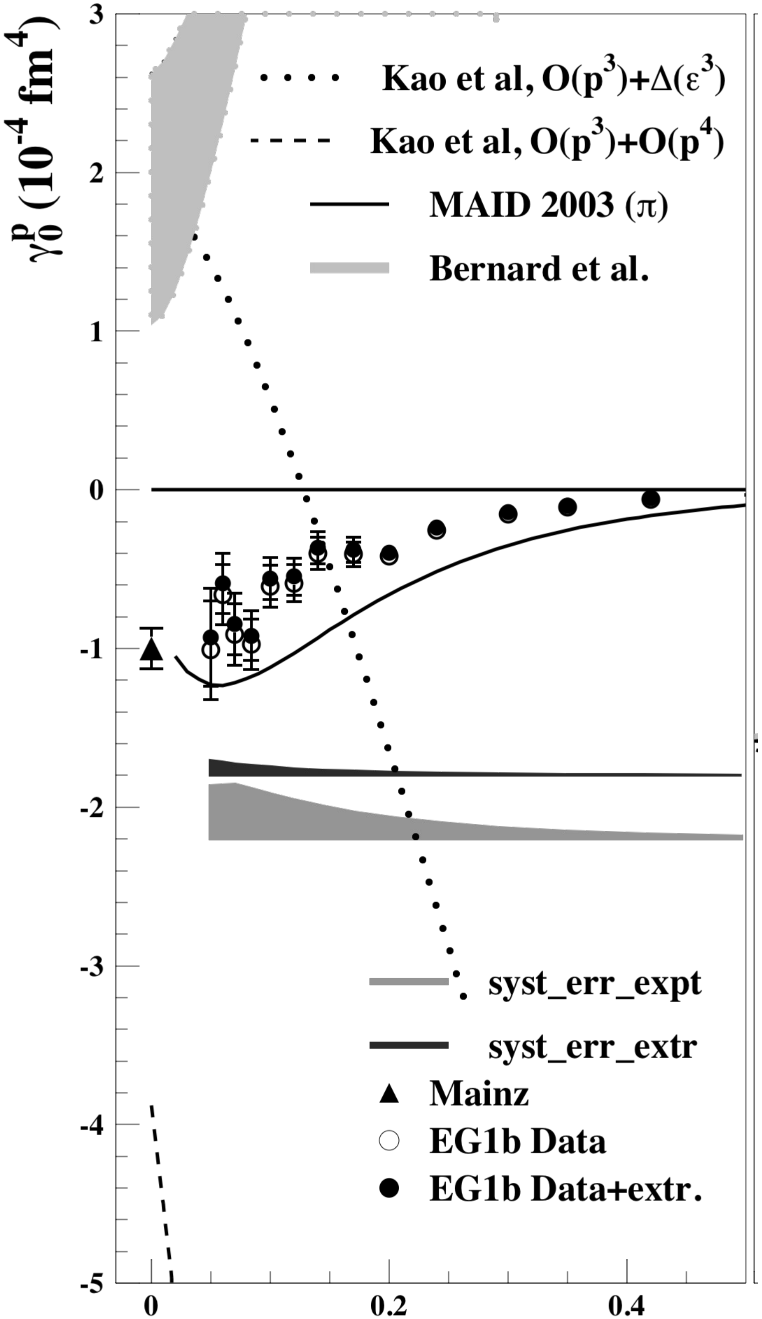
$$\delta_{LT}(Q^2) = \frac{16\alpha M^2}{Q^6}\int_0^{x_0} x^2\left[g_1(x,Q^2) + g_2(x,Q^2)\right]dx$$

D. Drechsel et al. Phys. Rep. 378 (2003) 99

Ideal quantities to test calculations of χPT at low Q^2!

- γ_0 is sensitve to resonances, but δ_{LT} is insensitive to the Δ resonance

Generalized Forward Spin Polarizabilities

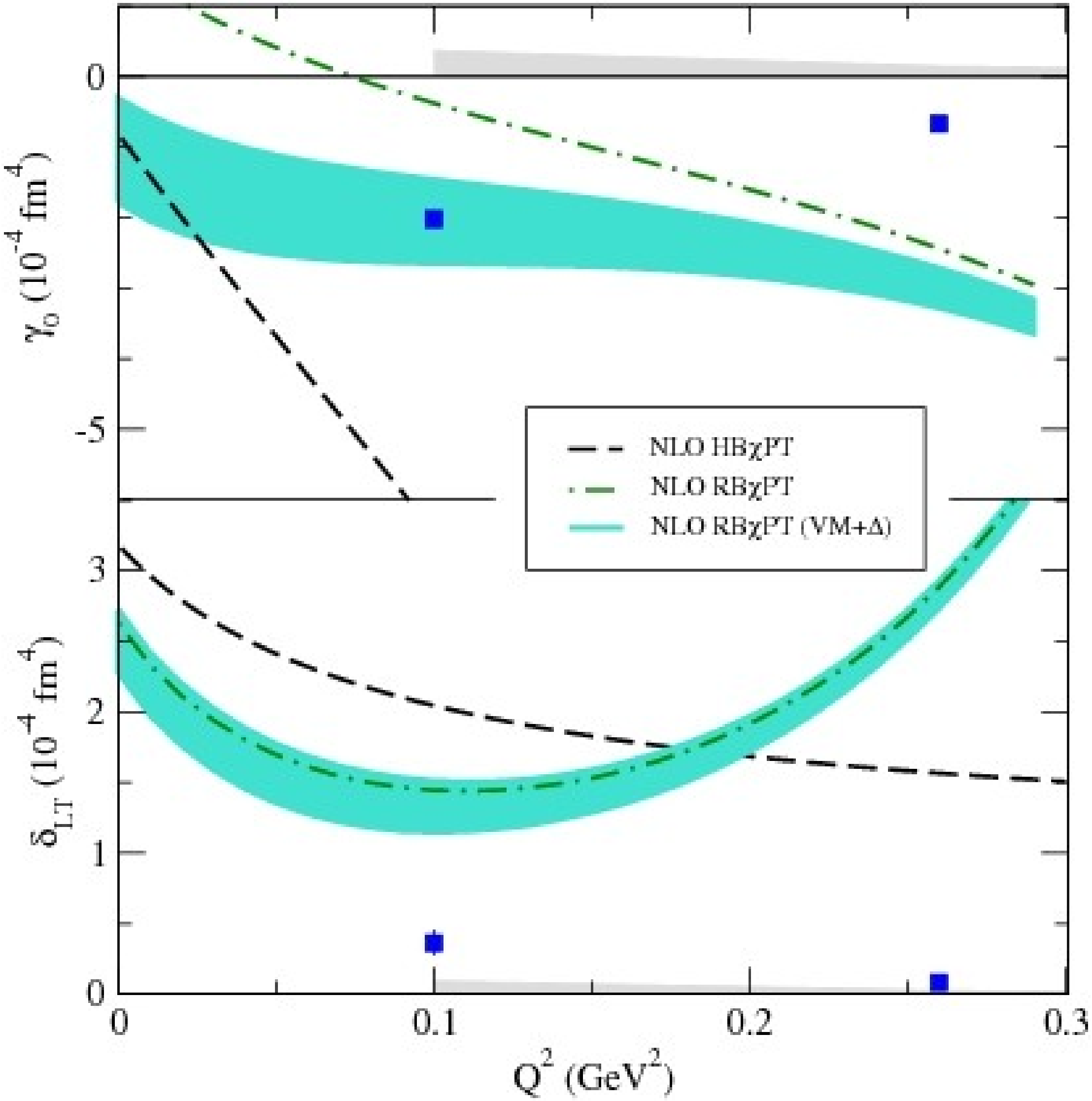


$$\gamma_0(Q^2) = \frac{16\alpha M^2}{Q^6}\int_0^{x_0} x^2\left[g_1(x,Q^2) - \frac{4M^2}{Q^2}x^2 g_2(x,Q^2)\right]dx$$

- However, agreement is not so great between EG1b data and χPT calculations!
- Same problem exists for the proton and neutron.

Y. Prok et al. Phys. Lett. B672 12, 2009

Generalized Forward Spin Polarizabilities



M. Amarian et al. Phys. Rev. Lett. 93, 152301 (2004)

$$\gamma_0(Q^2) = \frac{16\alpha M^2}{Q^6}\int_0^{x_0} x^2\left[g_1(x,Q^2) - \frac{4M}{Q^2}x^2 g_2(x,Q^2)\right]dx$$

- Same problem exists for the E94-010 neutron data and χPT calculations!

$$\delta_{LT}(Q^2) = \frac{16\alpha M^2}{Q^6}\int_0^{x_0} x^2\left[g_1(x,Q^2) + g_2(x,Q^2)\right]dx$$

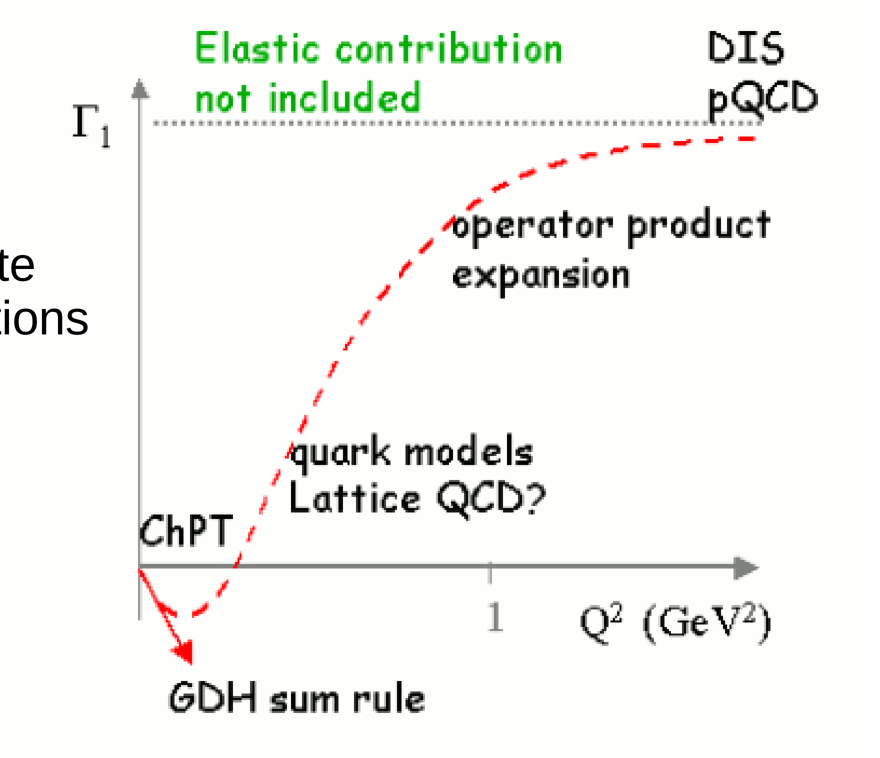
Kao, Spitzenberg, and Vanderhaeghen, Phys.Rev.D67:016001 (2003)

Bernard, Hemmert, Meissner, Phys.Rev.D67:076008 (2003)

Bernard, Hemmert, Meissner with Δ resonance and vector meson contributions

Importance of Spin Structure Measurements at Low Q^2

- At low Q^2, the behaviour of the GDH integral and Γ_1 is predicted by chiral perturbation theories
    - Data at very low Q^2 can give an accurate test of chiral perturbation theory predictions
- Sheds light on questions like
    - At what distance scale are these calculations valid?
    - Where do resonances give important contributions to the first moment?



- Measurements are important for calculations of hydrogen hyperfine structure

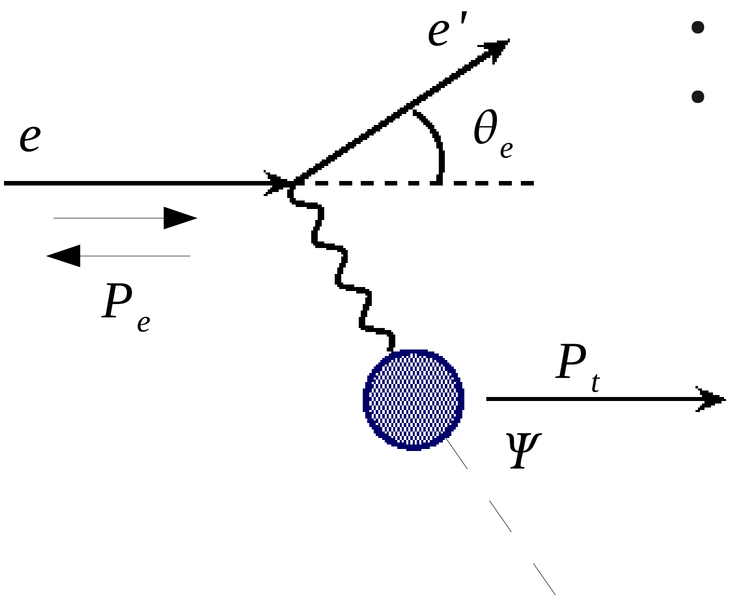
How can we measure this?

Extract helicity-dependent inclusive cross sections, then extract the structure function g_1.

Virtual Photon Asymmetries

Inclusive doubly polarized cross section:

$$\frac{d\sigma}{dE'\,d\Omega} = \Gamma_\nu\left[\sigma_T + \epsilon\sigma_L + P_e P_t\left(\sqrt{1-\epsilon^2}A_1\sigma_T\cos\psi + \sqrt{2\epsilon(1-\epsilon)}A_2\sigma_T\sin\psi\right)\right]$$



- A_1, A_2 are the spin-dependent asymmetries
- σ_T, σ_L are the total absorption cross sections for transverse and longitudinal cross sections

The measured asymmetries are defined as

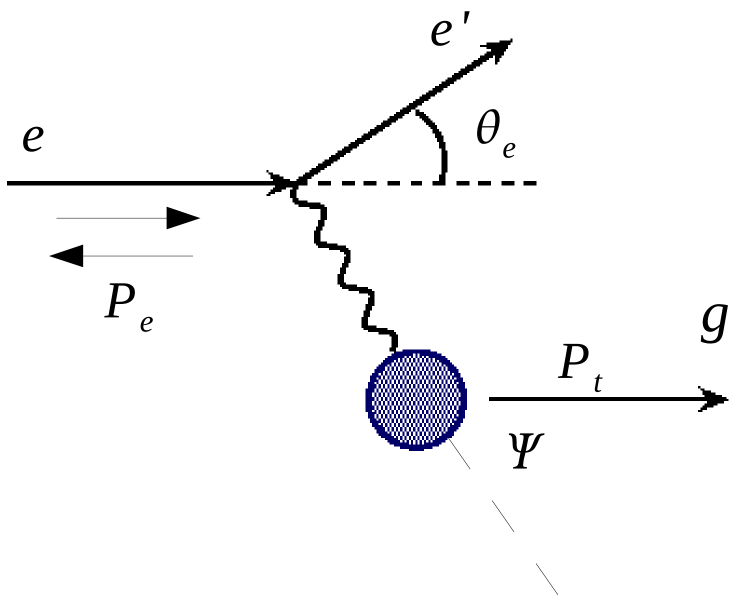
$$A = \frac{1}{f\cdot P_t\cdot P_b}\left(\frac{N_+ - N_-}{N_+ + N_-}\right)$$

- $A_\parallel$ - target polarization held parallel to the longitudinally polarized electrons
- $A_\perp$ - target polarization held perpendicular

Virtual Photon Asymmetries

Form the polarized cross section differences:

$$\Delta\sigma_{\parallel,\perp} = 2A_{\parallel,\perp}\cdot\sigma_{raw}$$



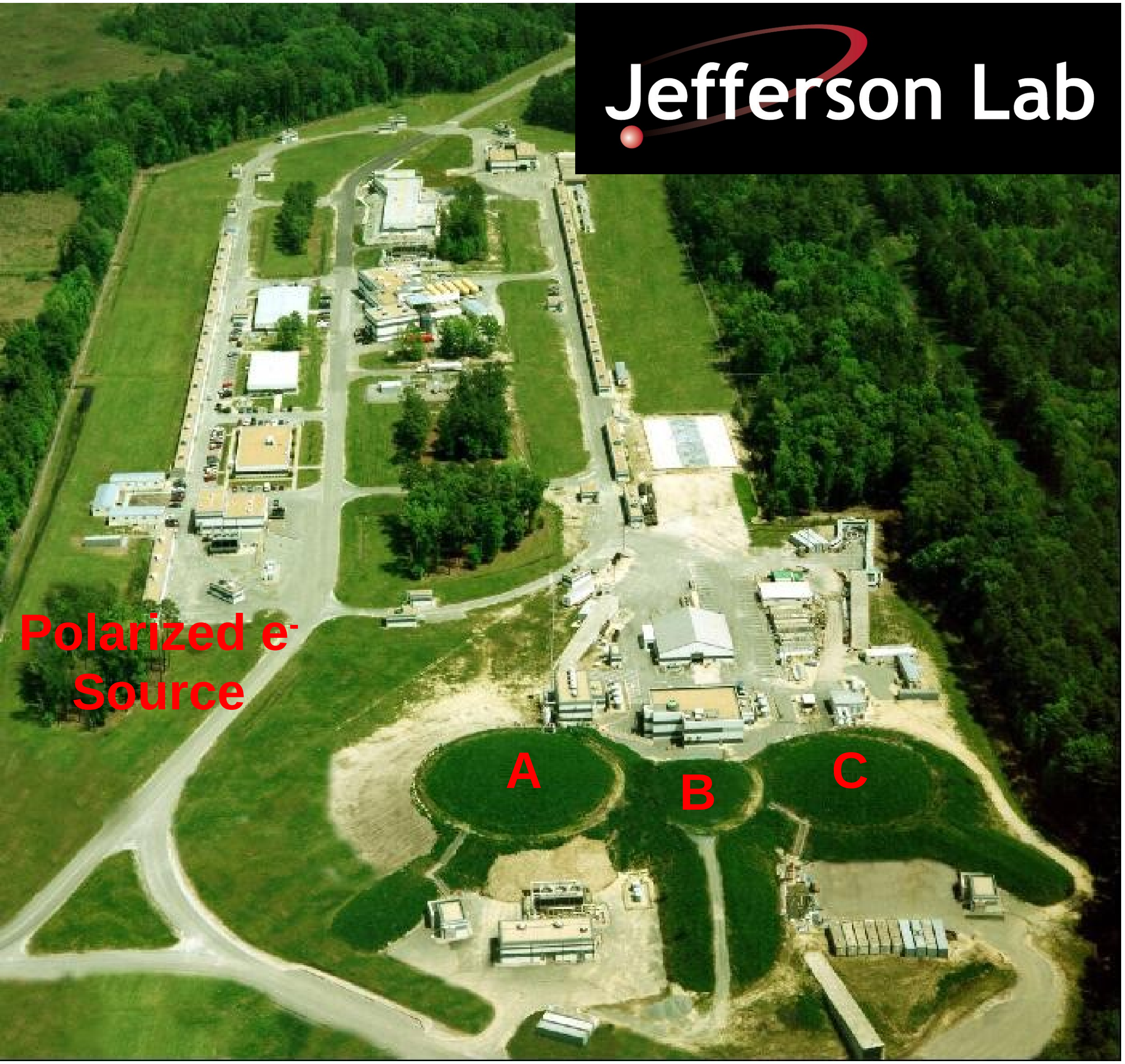
The spin structure functions g_1 and g_2 are related by

$$g_1(x,Q^2) = \frac{MQ^2}{4\alpha^2}\frac{y}{(1-y)(2-y)}\left[\Delta\sigma_{\parallel} + \tan\frac{\theta}{2}\Delta\sigma_{\perp}\right]$$

$$g_2(x,Q^2) = \frac{MQ^2}{4\alpha^2}\frac{y^2}{\text{٢}(1-y)(2-y)}\left[-\Delta\sigma_{\parallel} + \frac{\text{١}+(1-y)\cos\theta}{(1-y)\sin\theta}\Delta\sigma_{\perp}\right]$$

$$y = \frac{E-E'}{E}$$

Spin Structure at Jefferson Lab



Electron beams up to 5.7 GeV with > 80% longitudinal polarization.

Data have been taken in all three experimental halls on spin structure functions

- Data cover from 0.015 to 5 GeV^2
- on proton, deuteron, and ^{3}He targets

Spin Structure with CLAS in Hall B

Cebaf Large Acceptance Spectrometer

- Six-coil toroidal magnetic field
- Six individually instrumented sectors
- Large acceptance

Spin structure measurements in the resonance region:

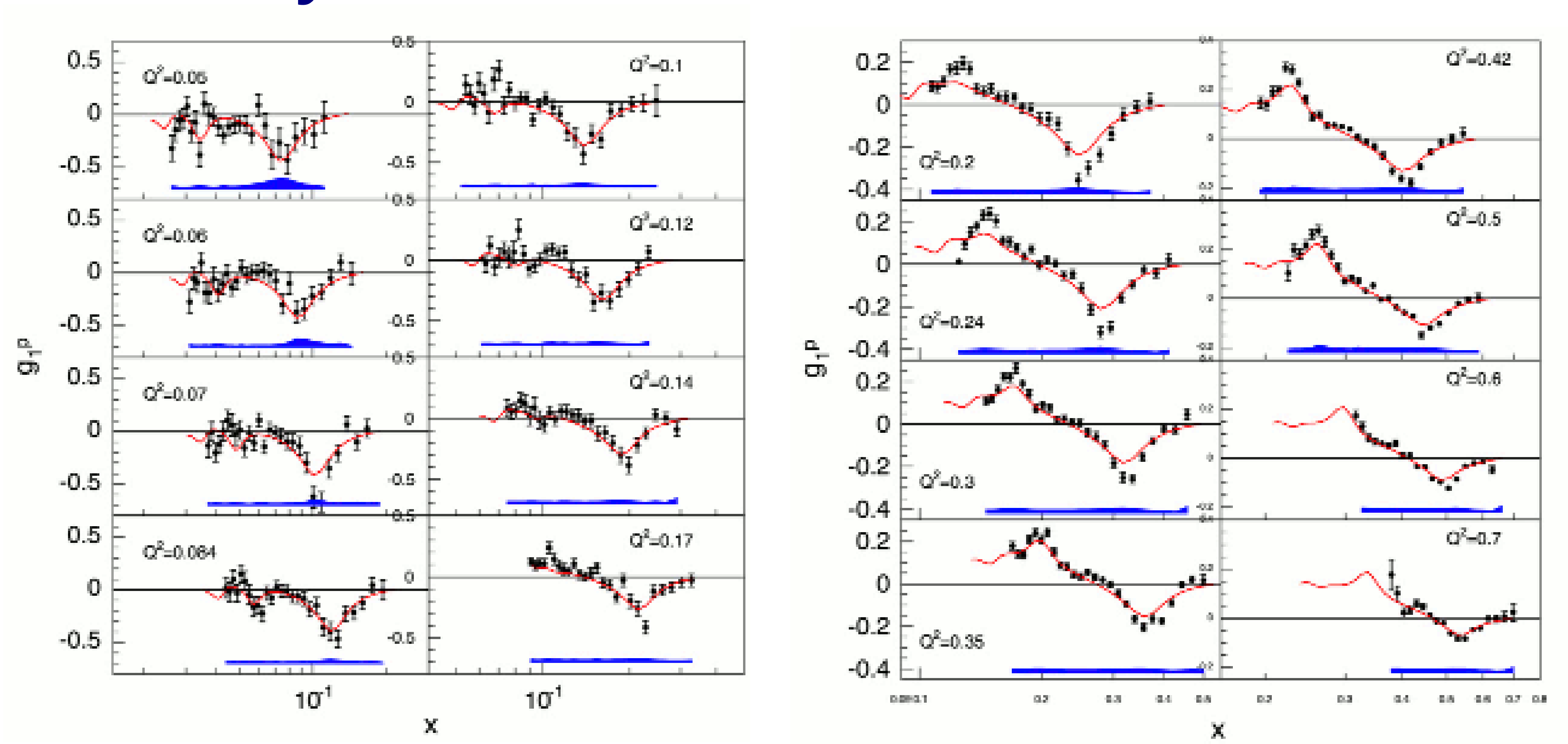
EG1

- $Q^2 = 0.05$ to 5 GeV^2
- Large kinematic coverage

EG4

- Focused on lower Q^2 from 0.015 – 0.5 GeV^2 to test chiral perturbation theory predictions of the GDH sum rule.

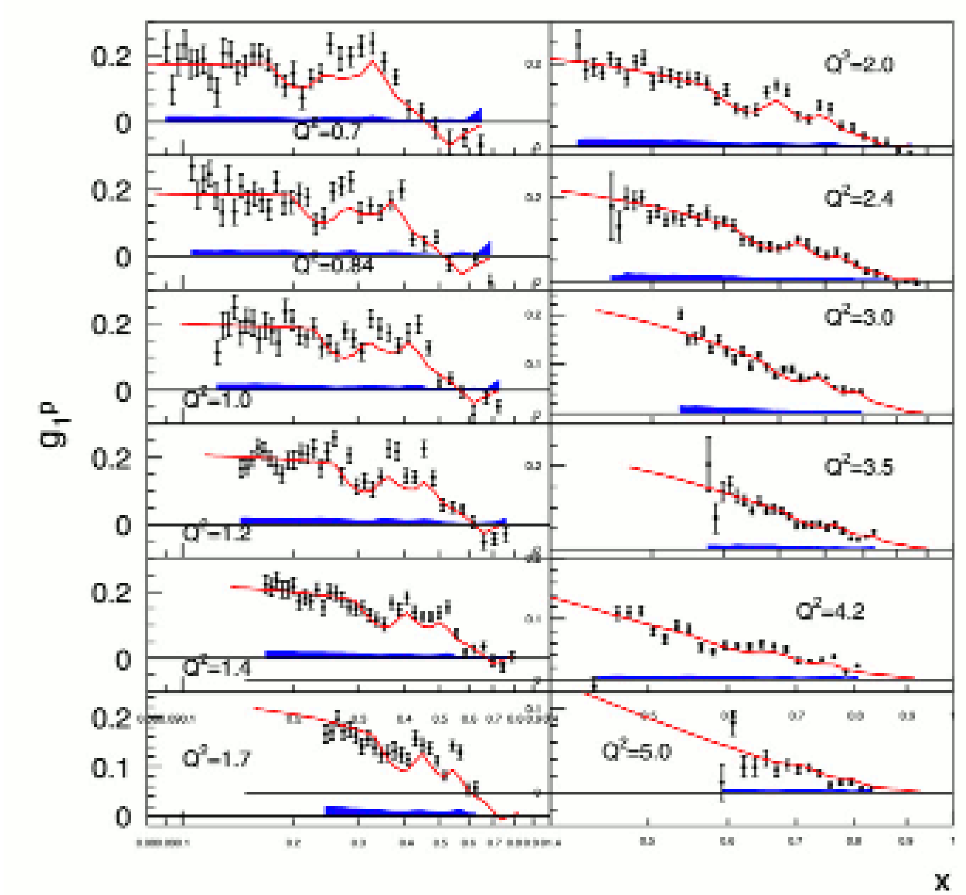
Asymmetries from CLAS EG1



- CLAS EG1 data for g_1^p
- At low Q^2, the Δ(1232) resonance drives the asymmetry (and thus g_1) negative.
- Red curve is the EG1 model used for radiative corrections

Kuhn, Chen, and Leader.
Prog.Part.Nucl.Phys.63:1-50,2009

Asymmetries from CLAS EG1



- CLAS EG1 data for g_1^p
- As Q^2 increases, g_1 becomes positive everywhere.

Kuhn, Chen, and Leader.
Prog.Part.Nucl.Phys.63:1-50,2009

The EG4 Experiment

The CLAS EG4 experiment is focused on the measurement of the generalized GDH sum rule for the proton and neutron (deuteron) at very low Q^2 (0.015 – 0.5 GeV^2)

- Measured polarized electrons scattered off polarized targets down to 6° scattering angles
- Will extract g_1 from the helicity dependent inclusive cross sections

Spokespeople

NH_{3}: M. Battaglieri, A. Deur, R. De Vita, M. Ripani (Contact)

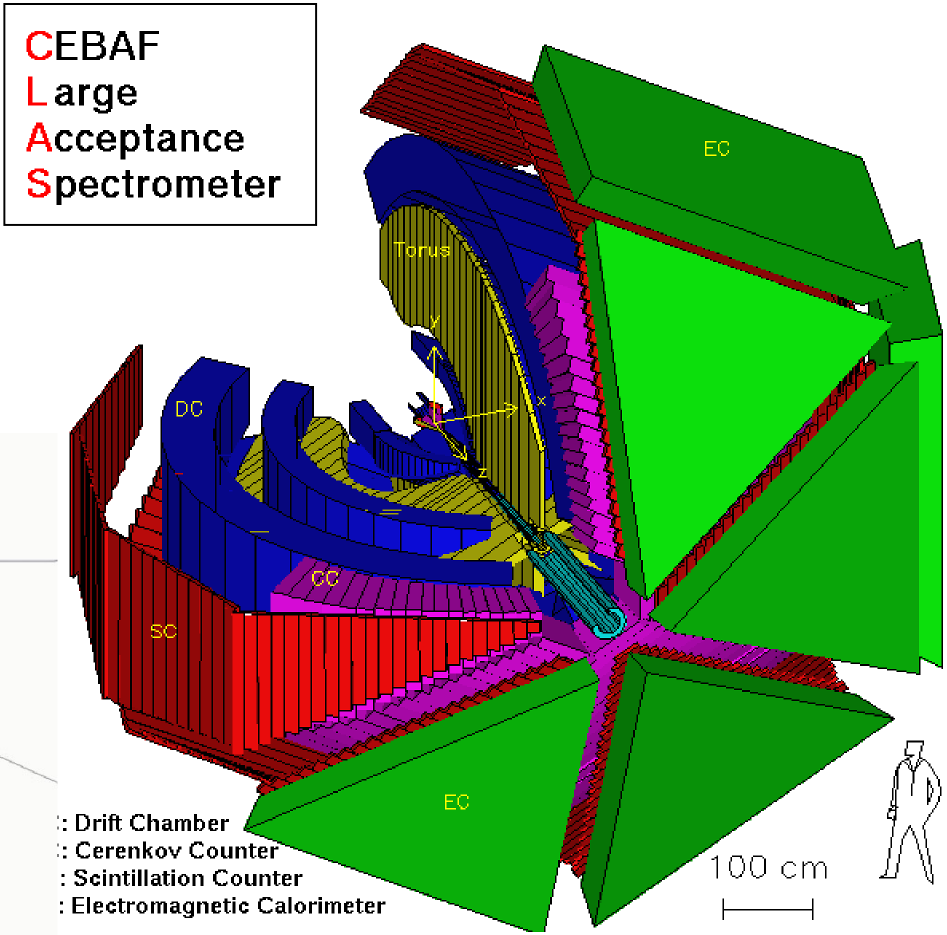
ND_{3}: A. Deur (Contact), G. Dodge, K. Slifer

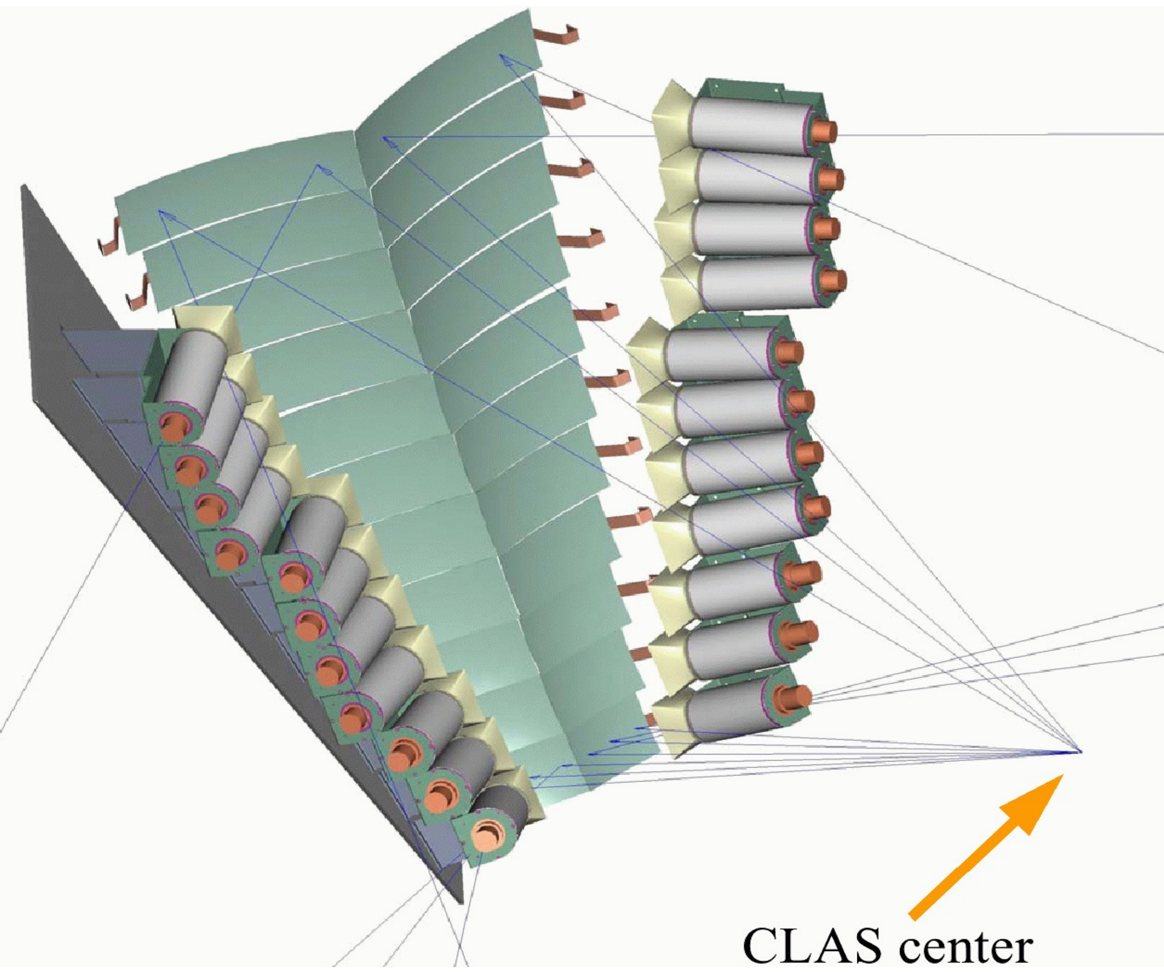
Ph.D. Students

K. Adhikari, H. Kang, K. Kovacs

EG4 Experimental Set-Up

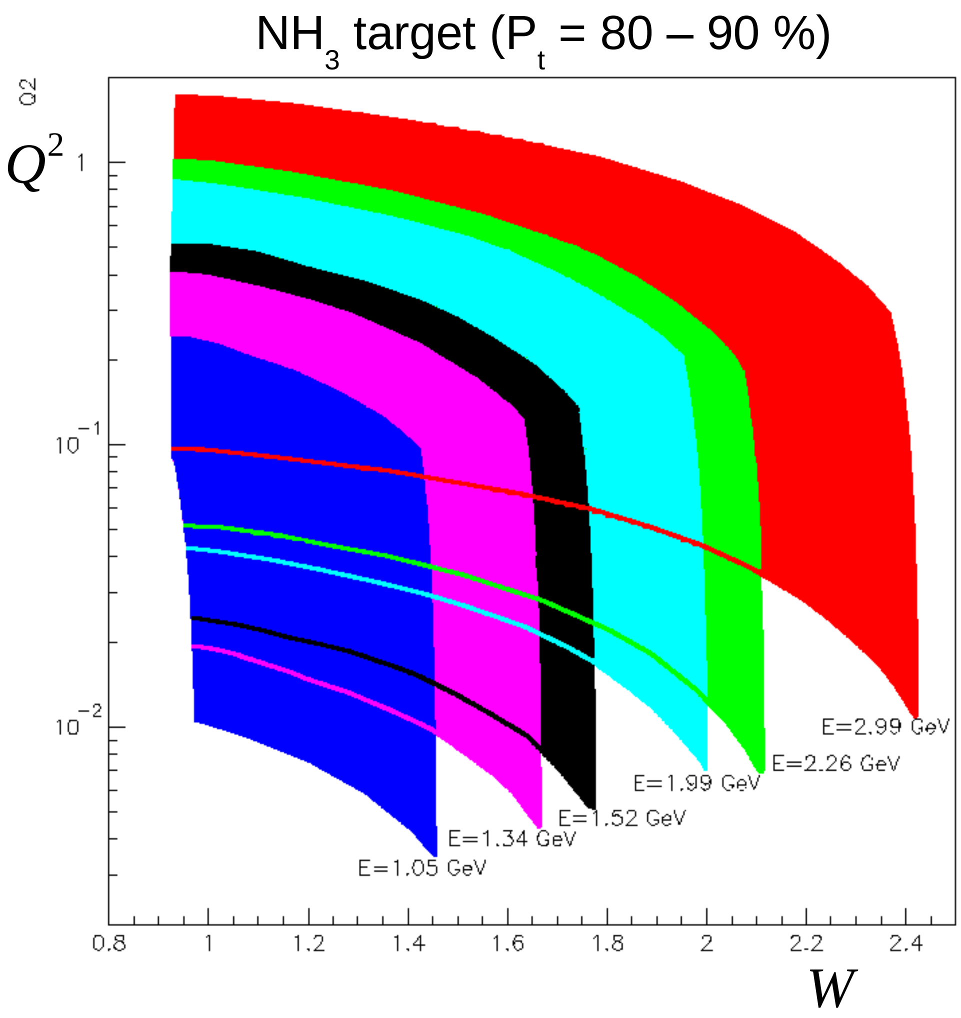
- EG4 ran from February to May 2006 in Hall B using CLAS.
- Longitudinally polarized CLAS NH_3 and ND_3 targets at -1m w.r.t. CLAS center.
- Longitudinally polarized electron beam (P_b ~ 80%) at low energies (1-3 GeV); outbending torus field.

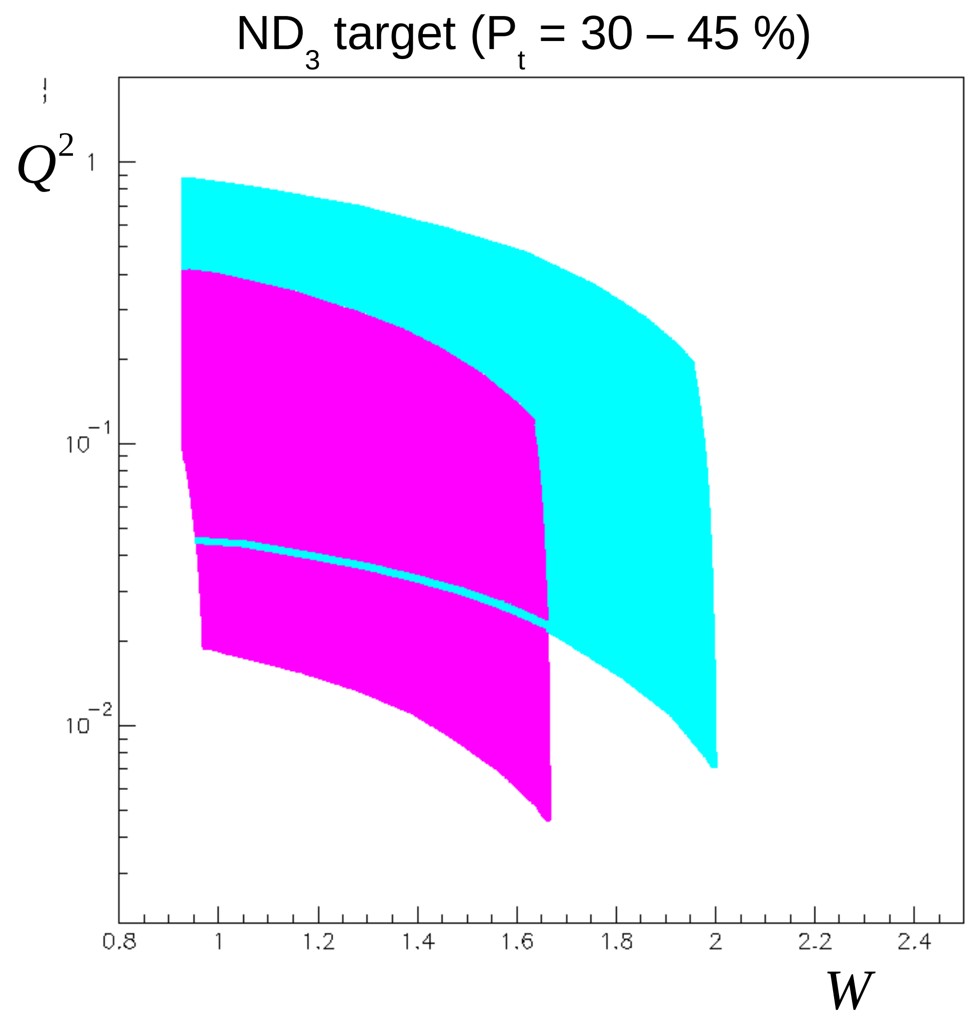




- Cross section measurement requires uniform detection efficiency at low Q^2.
- New Cherenkov detector (INFN – Genova) installed in sector-6 for detecting small angle scatterings down to 6º with uniform and high efficiencies.

EG4 Kinematics





$E_b = 1.1,\ 1.3,\ 1.5,\ 2.0,\ 2.3,\ 3.0\ GeV$

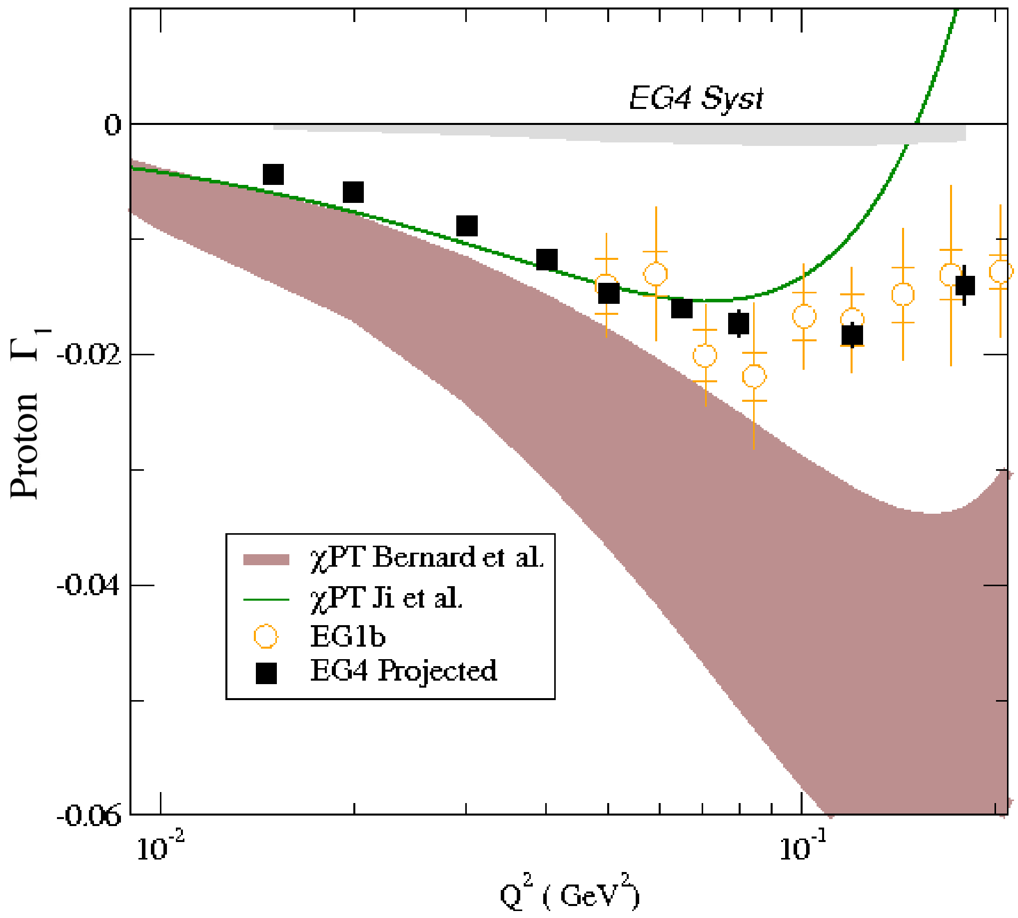
$E_b = 1.3,\ 2.0\ GeV$

$$0.015 < Q^2 < 0.5\ GeV^2$$

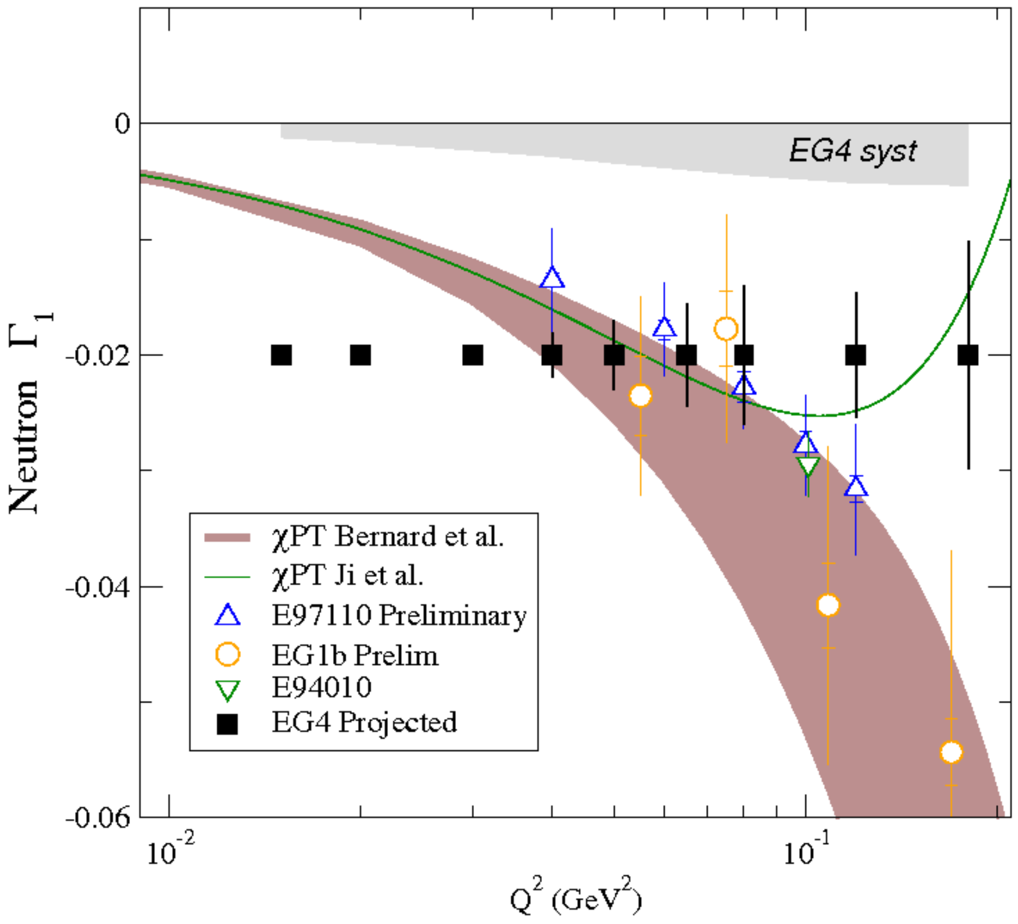
Good coverage of the resonance region

Expected Results on the Generalized GDH Sum Rule

Proton

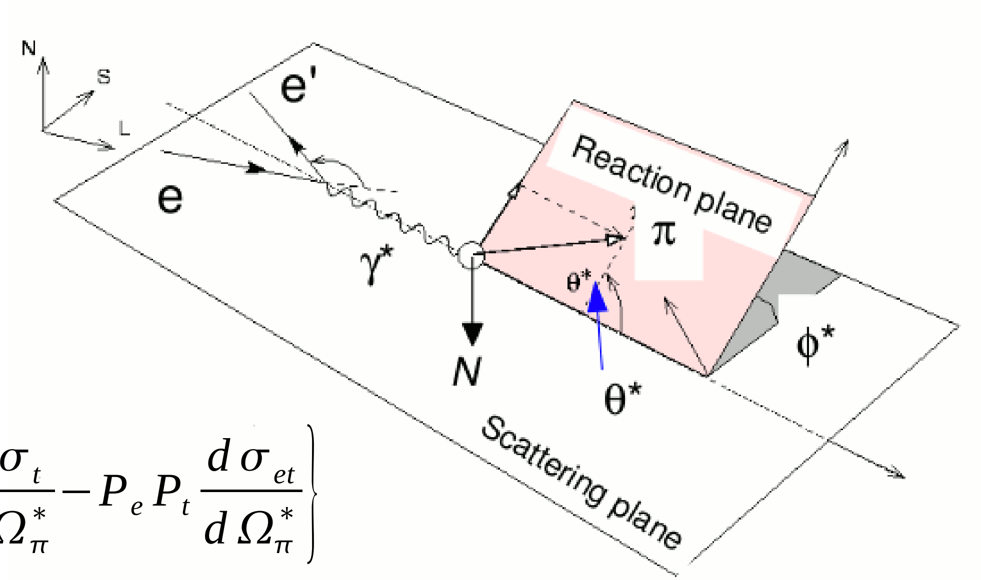


Neutron



Exclusive Channel Analysis

In addition to the inclusive analysis, an exclusive analysis is underway to extract the pion electroproduction asymmetries in the nucleon resonance region.



Observables in pion electroproduction

$$\frac{d\sigma}{d\Omega_\pi^*} = \frac{|\vec{q}|}{q_\gamma^{CM}} \left\{ \frac{d\sigma_0}{d\Omega_\pi^*} + P_e \frac{d\sigma_e}{d\Omega_\pi^*} + P_t \frac{d\sigma_t}{d\Omega_\pi^*} - P_e P_t \frac{d\sigma_{et}}{d\Omega_\pi^*} \right\}$$

Single-beam $$A_e = \frac{d\sigma_e}{d\sigma_{unp}} = \frac{\sigma(+h_e) - \sigma(-h_e)}{\sigma(+h_e) + \sigma(-h_e)}$$

Single-target $$A_t = \frac{d\sigma_t}{d\sigma_{unp}} = \frac{\sigma(+h_N) - \sigma(-h_N)}{\sigma(+h_N) + \sigma(-h_N)}$$

Double beam-target

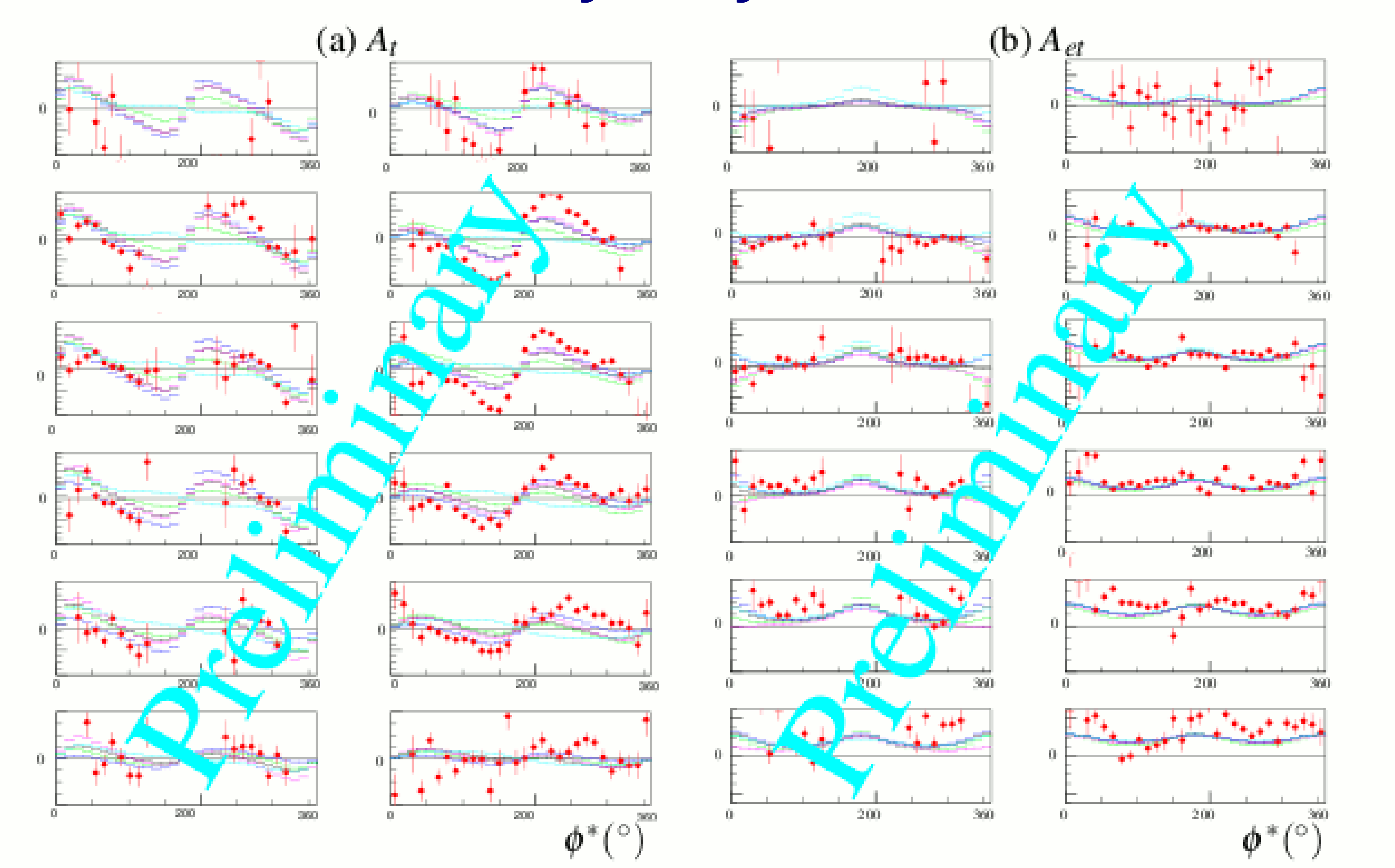
$$A_{et} = \frac{d\sigma_{et}}{d\sigma_{unp}} = \frac{\sigma(+h_e,+h_N) + \sigma(-h_e,-h_N) - \sigma(+h_e,-h_N) - \sigma(-h_e,+h_N)}{\sigma(+h_e,+h_N) + \sigma(-h_e,-h_N) + \sigma(+h_e,-h_N) + \sigma(-h_e,+h_N)}$$

EG4 Exclusive Channel Analysis

- This analysis will extract A_t and A_{et} from EG4 data for
  - NH3 target: $\vec{e}\,\vec{p} \rightarrow e'\pi^{+} n$ and $\vec{e}\,\vec{p} \rightarrow e'\pi^{\cdot} p$
  - ND3 target: $\vec{e}\,\vec{n} \rightarrow e'\pi^{-} p$ and $\vec{e}\,\vec{p} \rightarrow e'\pi^{+} n$

- These results will help to constrain models and chiral perturbation theory predictions at low Q^2

Preliminary Asymmetries

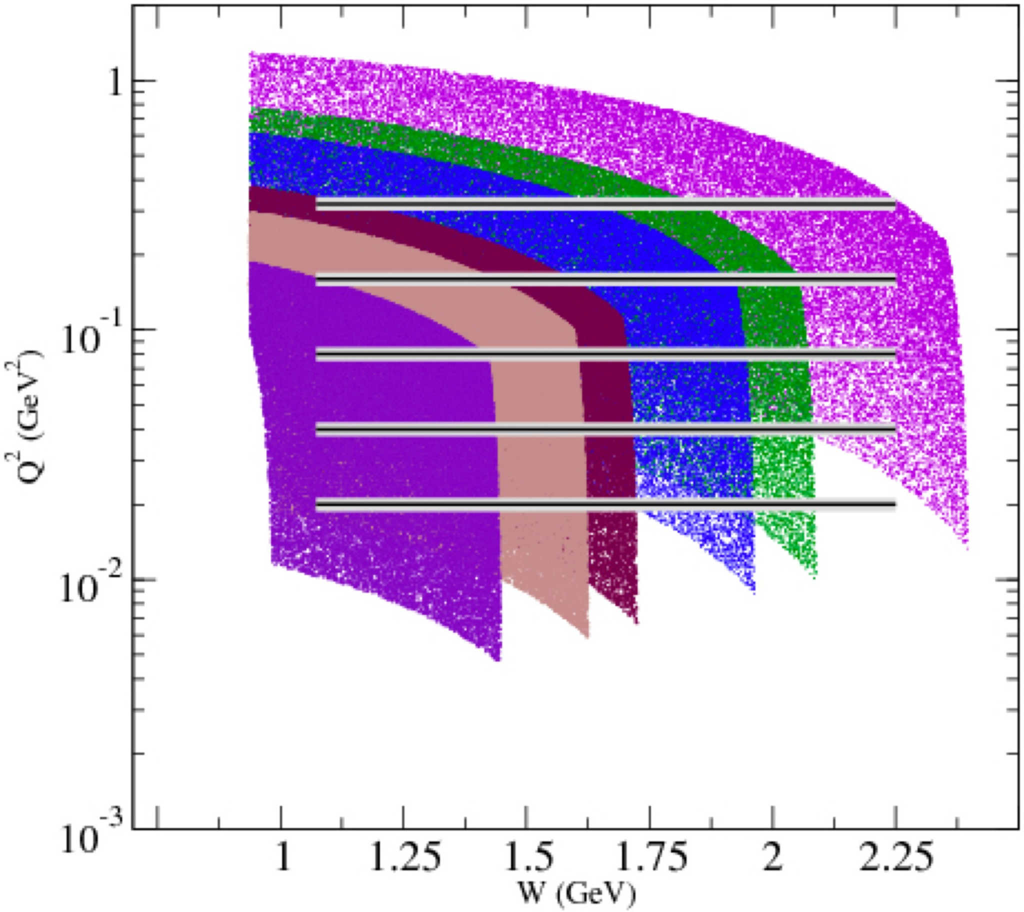


- Asymmetries not corrected for contribution from unpolarized nucleons in target
- Data indicates about 20% of events are from polarized protons in the NH3 target
- Models are scaled by 0.2 to compare with data

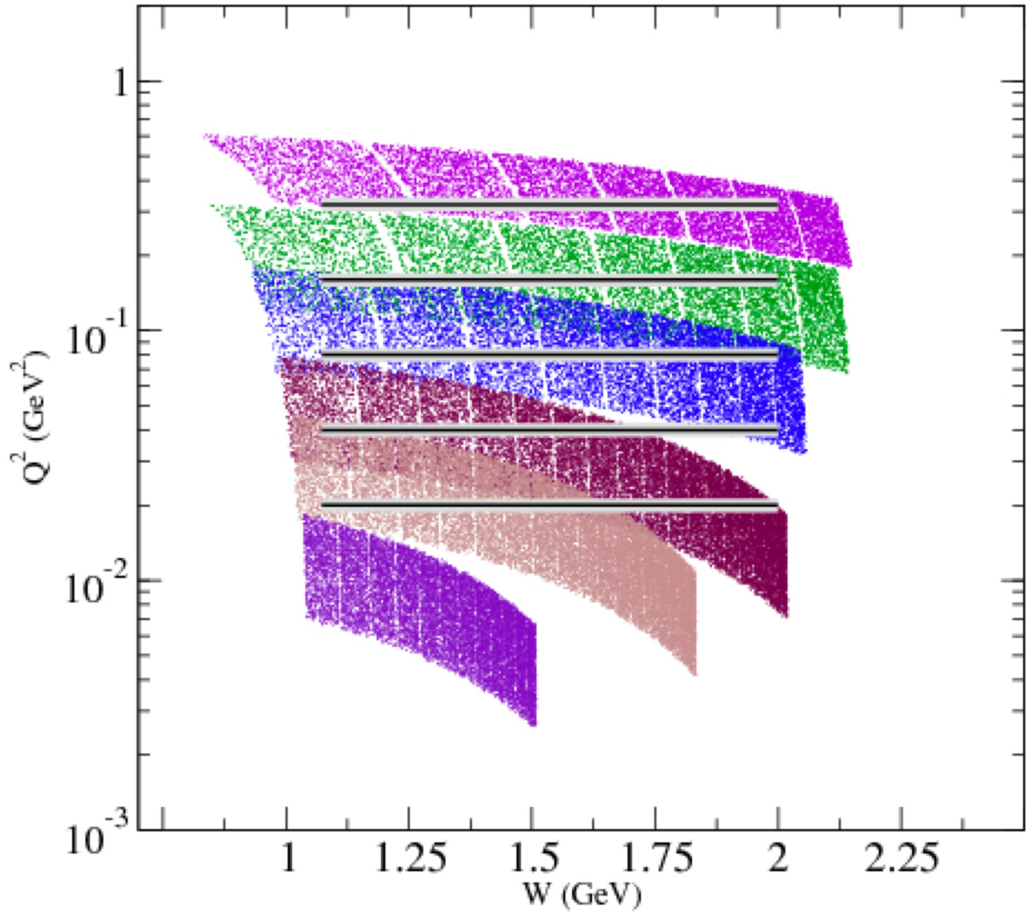
(X. Zheng)

More Measurements to Come...

- EG4 measured g_1^p and g_1^d at low Q^2 (0.015 – 0.5 GeV^2)
- The g_2^p structure function will be determined by E08-027 in JLab Hall A in the resonance region for $0.02 < Q^2 < 0.4$ GeV^2.
  - Can evaluate the BC sum and the longitudinal-transverse polarizability δ_{LT} from these data.



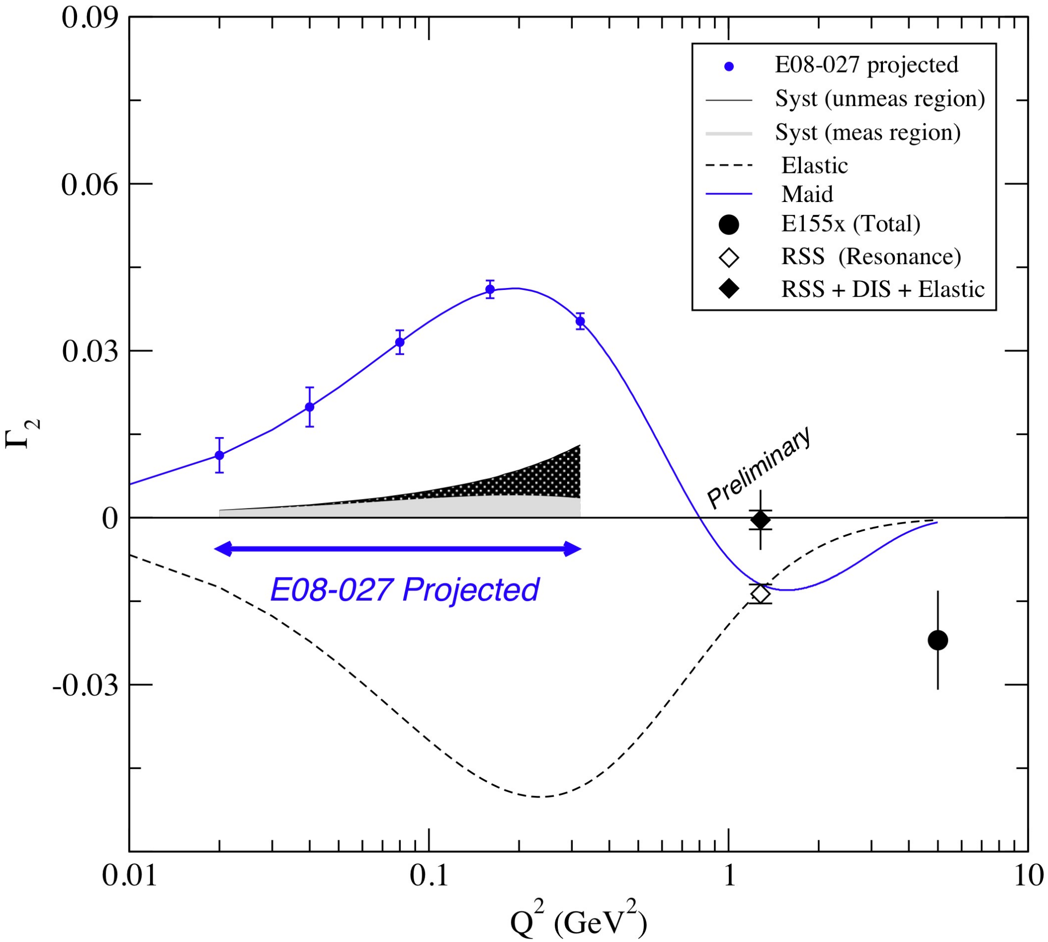
EG4: g1p



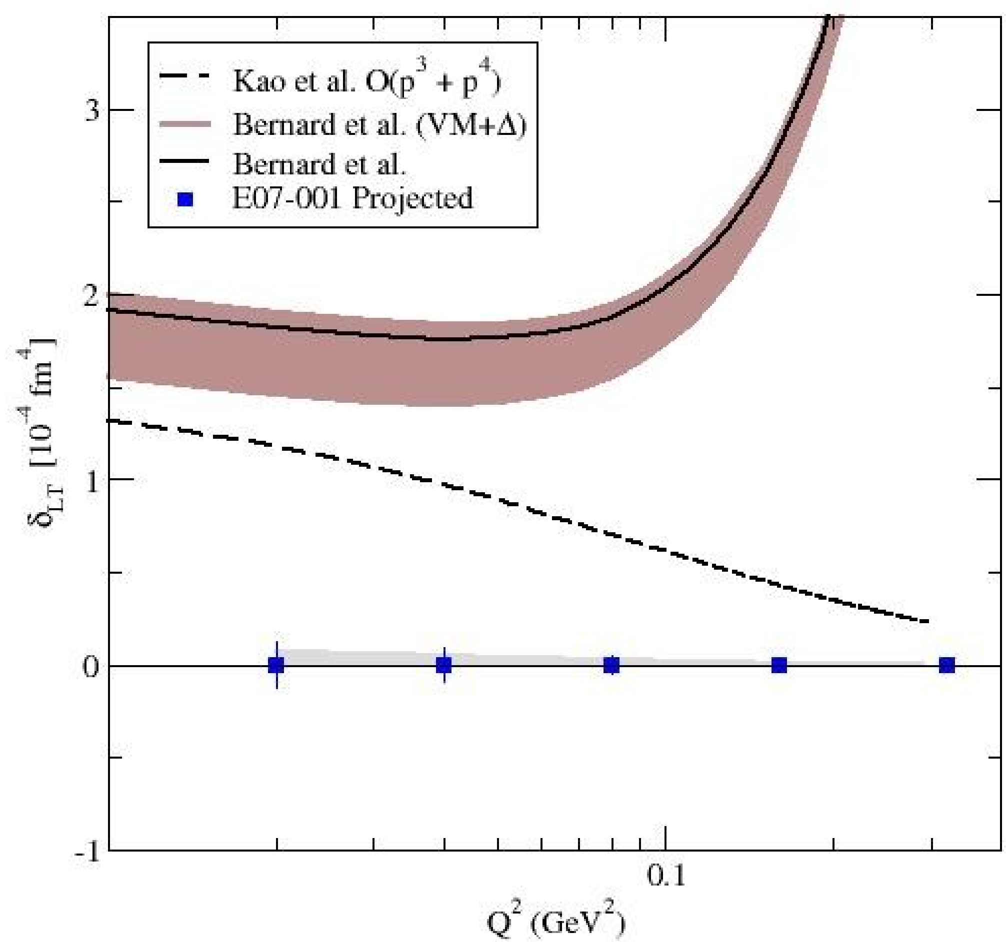
E08-027 : g_2^p
Will run in 2011

The Hall A g_2^p Experiment (E08-027)

- Inclusive measurement at forward angle of the proton spin-dependent cross sections to determine g_2^p in the resonance region for $0.02 < Q^2 < 0.4$ GeV^2.
  - Can evaluate the BC sum and the longitudinal-transverse polarizability δ_{LT} from these data.



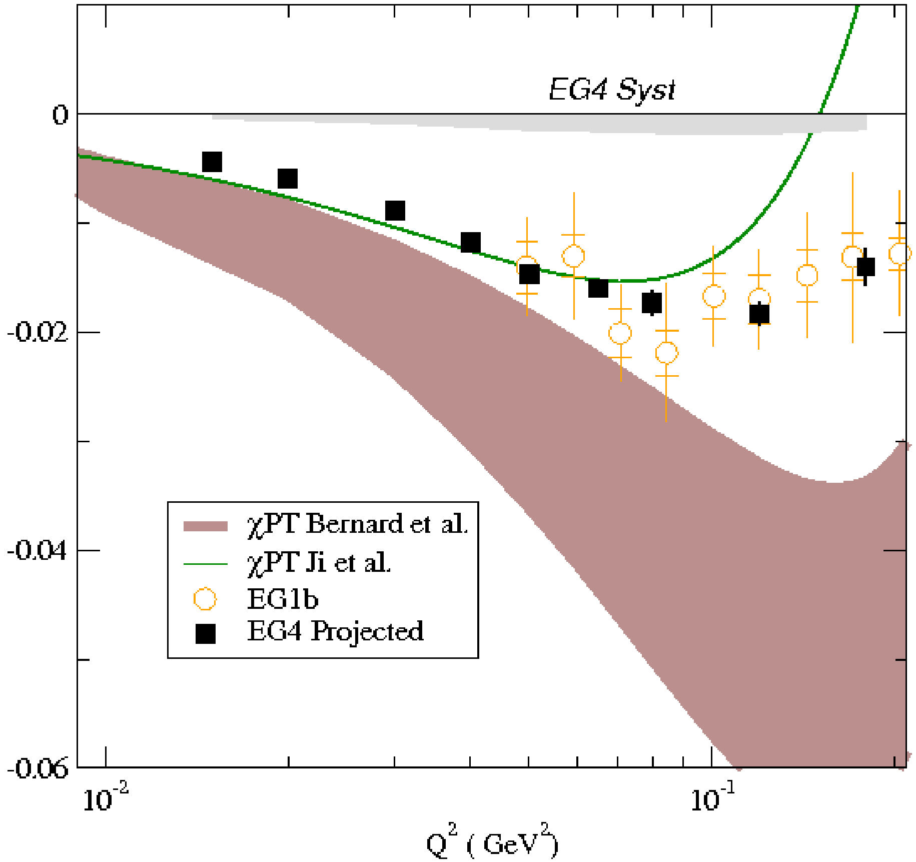
$$\int g_2(x,Q^2)\,dx$$



$$\delta_{LT}(Q^2) = \frac{16\,\alpha\,M^2}{Q^6}\int_0^{x_0} x^2\left[g_1(x,Q^2)+g_2(x,Q^2)\right]dx$$

Summary

- JLab and CLAS has (and will take more) structure function data in the resonance region.
- Analysis on the EG4 data is well underway! EG4 will
  - Determine the behavior of $g_1(x,Q^2)$ at very low Q^2
  - Extract the proton and the neutron GDH sums at very low Q^2;
  - Extract pion electroproduction asymmetries A_t and A_{et};
  - Compare to Chiral Perturbation Theory calculations.



- Previous data from EG1b show large contributions from resonance; EG4 results should be interesting!
- Stay tuned for our new results, and data yet to come!

Uncertainties

Uncertainties on Γ^d_1

Q^2 (GeV^2)	δ_{DIS}	δ_{trans}	$\delta\sigma_{born}$	δ_{syst}	δ_{stat}
0.015	1.9	0.5	8.9	9.1	2
0.02	2.2	0.7	8.9	9.2	3
0.05	1.5	1.1	8.9	9.1	8
0.10	1.1	1.7	8.9	9.1	13
0.15	0.2	2.2	8.9	9.2	22
0.20	1.1	2.7	8.9	9.4	30

- δ_{DIS}: the uncertainty due to the unmeasured contribution to the integral from W = Wmax to W = ∞.
- δ_{trans}: due to lack of transverse target spin data
- $\delta\sigma_{born}$: uncertainty on the polarized cross section difference after radiative corrections
- δ_{syst}: total systematic uncertainty, added in quadrature
- δ_{stat}: the statistical uncertainty

Systematic Errors

Errors on the generalized GDH sum for the proton:

Electron Efficiency	< 5 %
Beam and Target Polarization	1-2 %
^{15}N Background	1-2 %
Beam Charge Asymmetry	---
Luminosity and Filling Factor	3%
Modeling of g_2	1 – 10 % (depending on Q^2)
Extrapolation	1 – 10 % (depending on Q^2)
Radiative Corrections	5%

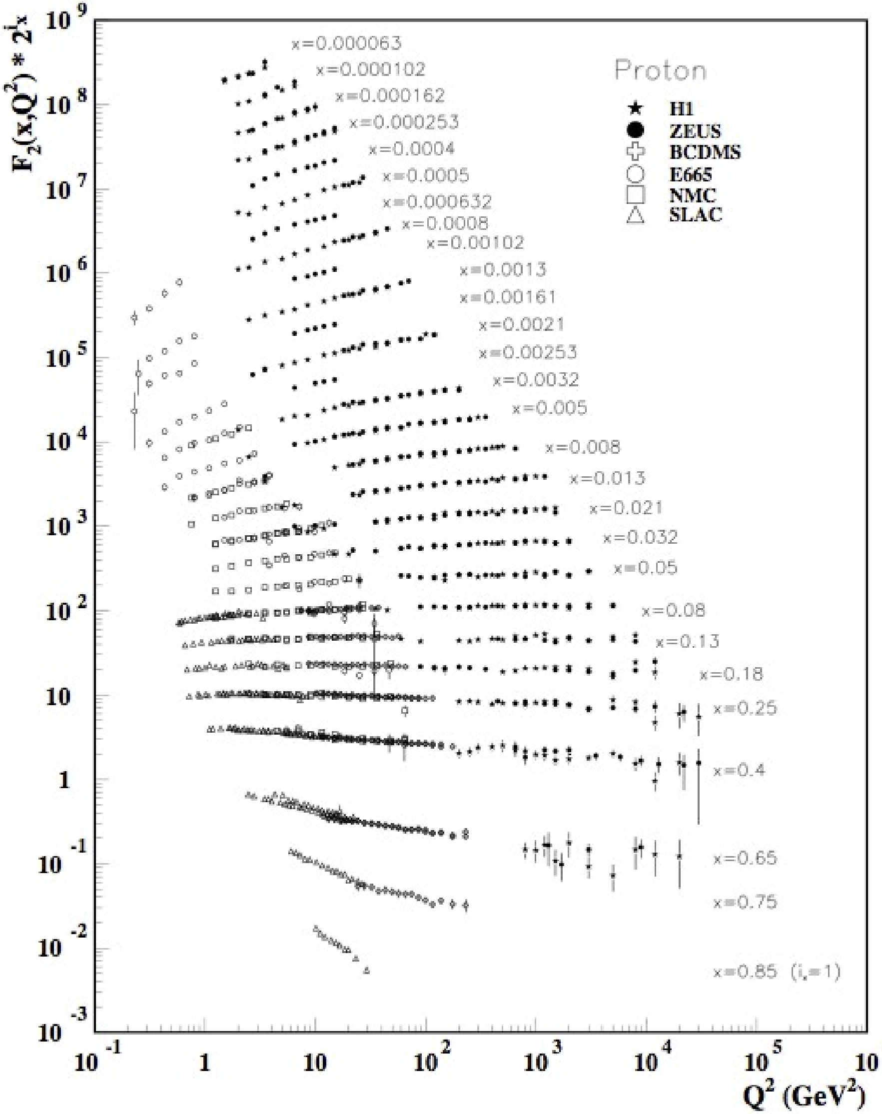
Neutron Extraction

Kahn, Melnitchouk, and Kulagin, PRC 79, 035205 (2009)

Kulagin and Melnitchouk, PRC 77, 015210 (2008)

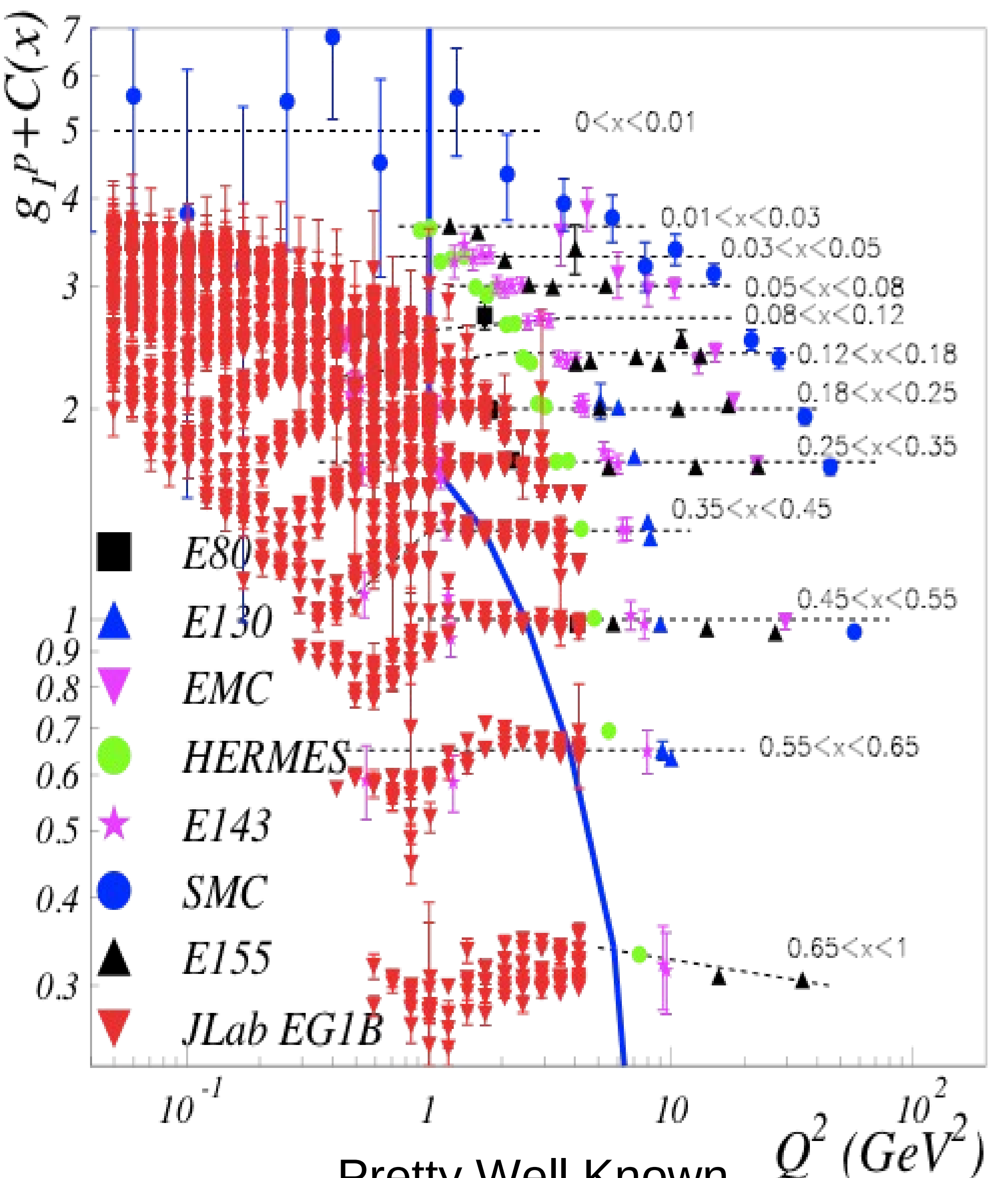
C. Ciofi degli Atti and S. Scopetta, Phys. Lett. B404, 223 (1997)

World Data



Well Known

$$F_2(x) = 2x\, F_1(x)$$



Pretty Well Known

$$g_1^p$$

Hydrogen Hyperfine Structure

The hyperfine splitting of hydrogen has been measured to a relative accuracy of 10^{-13}, but calculations are only accurate to a few ppm.

- Due to lack of knowledge of nucleon structure at low Q^2!

$$\Delta E = 1420.405\ 751\ 766\ 7(9)\ MHz$$

$$= (1+\delta) E_F \qquad \text{Fermi energy}$$

$$\delta = 1 + (\delta_{QED} + \delta_R + \delta_\epsilon) + \Delta_S \qquad \text{Proton structure correction}$$

$$\Delta_S = \Delta_Z + \Delta_{pol} \qquad \Delta_{pol} \approx (\Delta_1 + \Delta_2)$$

$$\Delta_1 = \frac{9}{4} \int_0^\infty \frac{dQ^2}{Q^2} \left\{ F_2^2(Q^2) + \frac{8 m_p^2}{Q^2} B_1(Q^2) \right\} \qquad \Delta_2 = -24 m_p^2 \int_0^\infty \frac{dQ^2}{Q^4} B_2(Q^2)$$

$$B_1(Q^2) = \int_0^{x_{th}} dx\, \beta_1(\tau)\, g_1(x, Q^2) \qquad B_2(Q^2) = \int_0^{x_{th}} dx\, \beta_2(\tau)\, g_2(x, Q^2)$$

- Q^2 weighting of Δ_1 and Δ_2 ensures low momentum transfer region dominates integrals
- Precise measurements of g_1, g_2 at low Q^2 needed!

Nazaryan, Carlson, and Griffioen, Phys.Rev.Lett 96:163001 (2006)

Resonance and Spin Structure

Nucleon resonances can generally be described in terms of three helicity amplitudes:

- $A_{3/2}(Q^2)$ – transverse photons leading to a final state helicity 3/2
- $A_{1/2}(Q^2)$ – transverse photons leading to a final state helicity 1/2
- $S_{1/2}(Q^2)$ – longitudinal photons

These amplitudes are directly related to the photon asymmetries:

$$A_1 = \frac{|A_{1/2}|^2 - |A_{3/2}|^2}{|A_{1/2}|^2 + |A_{3/2}|^2} \qquad A_2 = \sqrt{2}\,\frac{Q}{q*}\,\frac{S^*_{1/2} A_{1/2}}{|A_{1/2}|^2 + |A_{3/2}|^2}$$

By studying the Q^2 dependency, information on the relative strength of resonances and transitions can be determined.