A New Dispersive Analysis of $\eta \to 3\pi$

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Outline

1. Motivation
2. Method
3. Preliminary Results

Outline

Why $\eta \to 3\pi$?

- $\eta \to 3\pi$ decays allow for a precise determination of the quark mass ratio $Q^2 = \dfrac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}$, because of $\Gamma_{\eta\to 3\pi} \sim |A|^2 \sim Q^{-4}$
- From Kaon mass splitting:

$$Q^2 = \frac{m_K^2}{m_\pi^2} \frac{m_K^2 - m_\pi^2}{(m_{K^0}^2 - m_{K^+}^2)_{QCD}}$$

[Gasser & Leutwyler '84]

Why dispersion relations ?

- Chiral perturbation theory series converges rather slowly:
  - tree level: $\Gamma = 66$ eV [Cronin '67, Osborn & Wallace '70]
  - one-loop: $\Gamma = 160$ eV [Gasser & Leutwyler '84]
  - experiment: $\Gamma = 295$ eV [PDG '08]
- Mainly due to final state rescattering $\Rightarrow$ can be analysed by dispersion relations

Why do it again ?

- This has been done before

[Kambor, Wiesendanger & Wyler '96, Anisovich & Leutwyler '96]

- Better $\pi\pi$ phase shifts

[Ananthanarayan et al. '01, Colangelo et al. '01, Descotes-Genon et al. '01,García-Martín et al. '09]

- New measurements by KLOE, MAMI and WASA
- New two-loop analysis in χPT [Bijnens & Ghorbani '07]

Outline

Amplitude

- $\langle \pi^0\pi^+\pi^-, \mathrm{out}|\eta\rangle = i(2\pi)^4\delta^4(p_\eta - p_{\pi^+} - p_{\pi^-} - p_{\pi^0})A(s,t,u)$
- $A(s,t,u) = -\frac{1}{Q^2}\,\frac{m_K^2(m_K^2 - m_\pi^2)}{2\sqrt{3}m_\pi^2 F_\pi^2}\,M(s,t,u)$
- I only talk about charged channel
- Neutral channel: $\bar{A}(s,t,u) = A(s,t,u) + A(t,u,s) + A(u,s,t)$

Dispersion relations

- Early work [e.g. Khuri & Treiman '60]
- We use $M(s,t,u) =$
  $$M_0(s) + (s-u)M_1(t) + (s-t)M_1(u) + M_2(t) + M_2(u) - \tfrac{2}{3}M_2(s)$$
  [Fuchs, Sazdijan & Stern '93, Anisovich & Leutwyler '96]
- Exact, if Im $t_\ell(s) = 0$, for $\ell \geq 2$
- Chiral counting: Im $t_\ell(s) = \mathcal{O}(p^8), \ell \geq 2$
- Functions of only one variable with only right hand cut

Equations for the M_I

- From unitarity:

$$\text{disc}\, M_I(s) = \theta(s - 4m_\pi^2)\left\{M_I(s) + \hat{M}_I(s)\right\} \sin\delta_I(s) e^{-i\delta_I(s)}$$

- $\hat{M}_I$ are angular averages of the other M_I
- Cauchy representation for $M_I(s)/\Omega_I(s)$ leads to integral equations
- Omnès function:

$$\Omega_I(s) = \left(M_I \text{ with } \hat{M}_I = 0\right) = \exp\left\{\frac{s}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{\delta_I(s')}{s'(s'-s)}\right\}$$

Equations for the M_I

$$M_0(s) = \Omega_0(s) \left\{ \alpha_0 + \beta_0 s + \gamma_0 s^2 + \frac{s^2}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^2} \frac{\sin\delta_0(s')\hat{M}_0(s')}{|\Omega_0(s')|(s' - s - i\epsilon)} \right\}$$

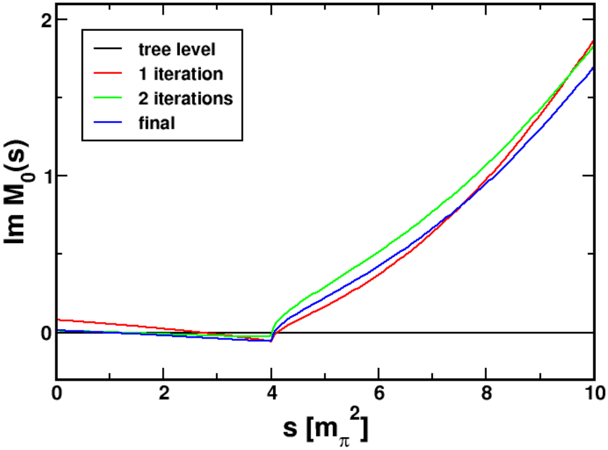
[Anisovich & Leutwyler '96]

Similarly for M_1 and M_2.

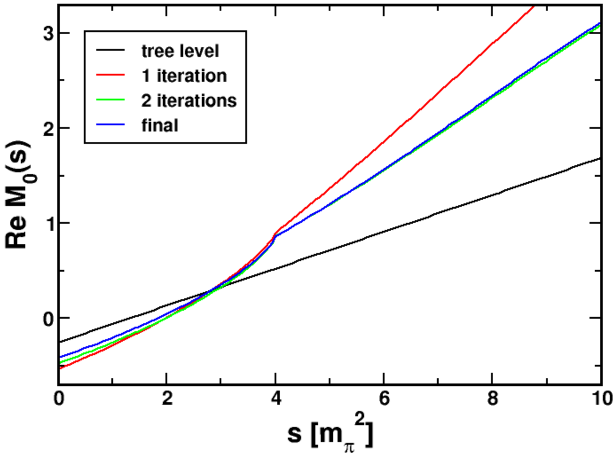
- Solve equations numerically by iterative procedure.
- Determine 4 subtraction constants by matching to one-loop χPT (Adler zero)

Outline

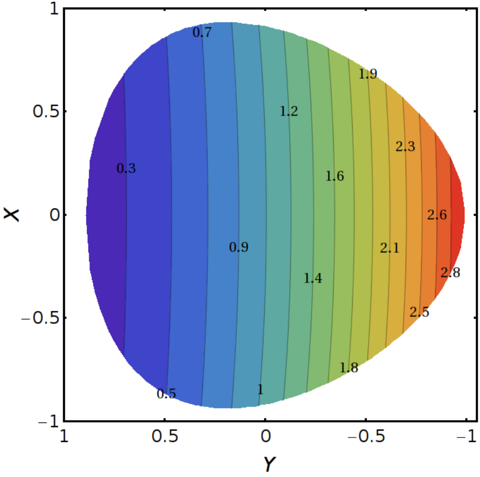
Im $M_0(s)$


tree level
1 iteration
2 iterations
final
Im M0(s)
s [mπ2]
0
1
2
0
2
4
6
8
10

Re $M_0(s)$


tree level
1 iteration
2 iterations
final
Re M0(s)
s [mπ2]
0
1
2
3
0
2
4
6
8
10

Dalitz plot for $\eta \to \pi^0\pi^+\pi^-$



Measured, e.g. [KLOE '07]

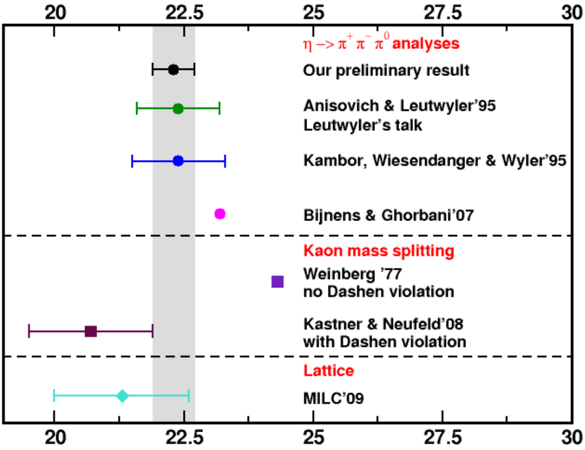
Dalitz plot variables:

$$X = \frac{\sqrt{3}}{2m_\eta Q_\eta}(u - t)$$

$$Y = \frac{3}{2m_\eta Q_\eta} \times ((m_\eta - m_{\pi^0})^2 - s) - 1$$

$$Q_\eta = m_\eta - 2m_{\pi^+} - m_{\pi^0}$$

Q

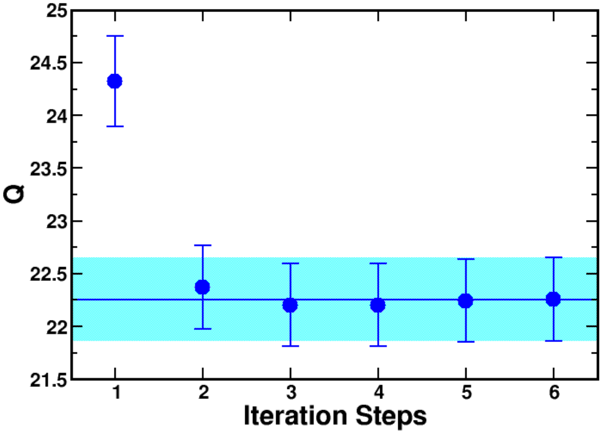


Preliminary result:

$Q = 22.3 \pm 0.4$

Experimental error only!

Iteration Steps



Future improvements

- $m_{\pi^+} - m_{\pi^0}$ effects [Kupść, Rusetsky & Gullström '08, Kubis & Schneider '09]
- Electromagnetic corrections [Ditsche, Kubis, Meißner '08]
- Inelasticity
- Use experimental data to determine subtraction constants [KLOE '07, MAMI '08, WASA '08]
- Imaginary parts of D and higher waves
- Error Analysis

Appendix

Dispersion Integrals for the M_I

$$M_0(s) = \Omega_0(s)\left\{\alpha_0 + \beta_0 s + \gamma_0 s^2 + \frac{s^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'^2}\frac{\sin\delta_0(s')\hat{M}_0(s')}{|\Omega_0(s')|(s'-s-i\epsilon)}\right\}$$

$$M_1(s) = \Omega_1(s)\left\{\beta_1 s + \frac{s}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'}\frac{\sin\delta_1(s')\hat{M}_1(s')}{|\Omega_1(s')|(s'-s-i\epsilon)}\right\}$$

$$M_2(s) = \Omega_2(s)\frac{s^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'^2}\frac{\sin\delta_2(s')\hat{M}_2(s')}{|\Omega_2(s')|(s'-s-i\epsilon)}$$

R

- $R = \dfrac{m_s - \hat{m}}{m_d - m_u} = \dfrac{2}{m_s/\hat{m} + 1}\ Q^2$

- With $m_s/\hat{m} = 24.4$ and $Q = 22.3$, we get $R \approx 39$

[Leutwyler '96]