

Chiral Dynamics 2009

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Thanks to organizers for
an outstanding conference!

Warning: This will not be a conference
summary.

Why not?

i) Organizers said I didn't have to

ii) Should conference be summarized?

Should I do it?

Theorem: Answer is NO to any paper with question mark!

iii) Nathan Isgur method: personal reflection

Chiral Dynamics: 1994-2009

Anti-global warming: $37^\circ \rightarrow 20^\circ$

Kolcker room \rightarrow Lecture hall 099

Invariants: Gasser, Leutwyler, Bernstein, Weinberg,

.....

Recent retirements: J. Gasser, G. Ecker, BRH

Looking backward:

Selected Topics:

i) Weak nonleptonic decays: $K \rightarrow 3\pi$

ii) Electromagnetic polarizabilities

iii) NN chiral potential

Nonleptonic weak Decay: $K \rightarrow 3\pi$

Thesis (1969) used current algebra/PCAC methods to predict $K \rightarrow 3\pi$ amplitudes in terms of $K \rightarrow 2\pi$ data

Idea is to write general low energy expansion of $K \rightarrow 3\pi$ amplitude, *e.g.*

$$\langle \pi^+ \pi^- \pi^0 | \mathcal{H}_w | K^0 \rangle = (a + bs_{+-} + c(s_{+0} + s_{-0}) + \dots)$$

and then require

$$\lim_{p_0 \rightarrow 0} \langle \pi^+ \pi^- \pi^0 | \mathcal{H}_w | K^0 \rangle = -\frac{i}{F_\pi} \langle \pi^+ \pi^- | [F_5^3, \mathcal{H}_w] | K^0 \rangle$$

Since $J_\mu = V_\mu + A_\mu$ have

$$[F_5^3, \mathcal{H}_w] = [I_3, \mathcal{H}_w] = \frac{1}{2} \mathcal{H}_w$$

$$\lim_{p_0 \rightarrow 0} \langle \pi^+ \pi^- \pi^0 | \mathcal{H}_w | K^0 \rangle = \frac{-i}{2F_\pi} \langle \pi^+ \pi^- | \mathcal{H}_w | K^0 \rangle$$

and similarly for $p_+, p_- \rightarrow 0$. Yields good predictions for amplitudes and slopes of all $K \rightarrow 3\pi$ amplitudes.

Now fastforward to 1990. Joachim Kambor thesis looks at $K \rightarrow 2\pi$ and $K \rightarrow 3\pi$ using χ pt. Agrees with current algebra/PCAC constraints— a, b are counterterms but also have unitarity terms and nonanalytic pieces.

Why not done in 1970? Pagels and others calculate chiral loops and generate nonanalytic terms. Gasiorowicz and Geffen generate effective chiral Lagrangian.

Why didn't happen? Focused at the time on renormalizable theories. Weinberg insight came in 1979 that allowed dealing with nonrenormalizable theories in terms of energy-momentum expansion. Program carried out by Gasser and Leutwyler in 1984.

2009: Bijmens, Prades, . . .

Lowest order $\mathcal{O}(p^2)$ Lagrangian \mathcal{L}_2^{nl} with three terms— $G_8 G'_8 G_{27}$ —must be determined experimentally from $K \rightarrow 2\pi$.

$$\mathcal{L}_{W2} = C F_0^4 \left[G_8 \langle \Delta_{32} u_\mu u^\mu \rangle + G'_8 \langle \Delta_{32} \chi_+ \rangle \right. \\ \left. + G_{27} t^{ij,kl} \langle \Delta_{ij} u_\mu \rangle \langle \Delta_{kl} u^\mu \rangle \right] + \text{h.c.}$$

Also higher order terms K_i $i=1,2,\dots,48$. Too many to determine experimentally—must make assumptions. Fits to precision $K \rightarrow 3\pi$ data very good:

Decay	Width [GeV]	ChPT [GeV]	Fact. [GeV]
$K^+ \rightarrow \pi^+\pi^0$	$(1.1231 \pm 0.0078) \cdot 10^{-17}$	$1.123 \cdot 10^{-17}$	$1.127 \cdot 10^{-17}$
$K_S \rightarrow \pi^0\pi^0$	$(2.2828 \pm 0.0104) \cdot 10^{-15}$	$2.282 \cdot 10^{-15}$	$2.283 \cdot 10^{-15}$
$K_S \rightarrow \pi^+\pi^-$	$(5.0691 \pm 0.0108) \cdot 10^{-15}$	$5.069 \cdot 10^{-15}$	$5.069 \cdot 10^{-15}$
$K_L \rightarrow \pi^0\pi^0\pi^0$	$(2.6748 \pm 0.0358) \cdot 10^{-18}$	$2.618 \cdot 10^{-18}$	$2.698 \cdot 10^{-18}$
$K_L \rightarrow \pi^+\pi^-\pi^0$	$(1.5998 \pm 0.0271) \cdot 10^{-18}$	$1.658 \cdot 10^{-18}$	$1.711 \cdot 10^{-18}$
$K^+ \rightarrow \pi^0\pi^0\pi^+$	$(9.195 \pm 0.0213) \cdot 10^{-19}$	$8.934 \cdot 10^{-19}$	$8.816 \cdot 10^{-19}$
$K^+ \rightarrow \pi^+\pi^+\pi^-$	$(2.9737 \pm 0.0174) \cdot 10^{-18}$	$2.971 \cdot 10^{-18}$	$2.933 \cdot 10^{-18}$

Decay	Quantity	Experiment	ChPT	Fact.
$K_L \rightarrow \pi^0\pi^0\pi^0$	h	-0.0050 ± 0.0014	-0.0062	-0.0025
$K_L \rightarrow \pi^+\pi^-\pi^0$	g	0.678 ± 0.008	0.678	0.654
	h	0.076 ± 0.006	0.088	0.083
	k	0.0099 ± 0.0015	0.0057	0.0068
$K_S \rightarrow \pi^+\pi^-\pi^0$	γ_S	$(3.3 \pm 0.5) \cdot 10^{-8}$	$3.0 \cdot 10^{-8}$	$2.9 \cdot 10^{-8}$
$K^\pm \rightarrow \pi^0\pi^0\pi^\pm$	g	0.638 ± 0.020	0.636	0.648
	h	0.051 ± 0.013	0.077	0.080
	k	0.004 ± 0.007	0.0047	0.0069
$K^+ \rightarrow \pi^+\pi^+\pi^-$	g	-0.2154 ± 0.0035	-0.215	-0.226
	h	0.012 ± 0.008	0.012	0.019
	k	-0.0101 ± 0.0034	-0.0034	-0.0033
$K^- \rightarrow \pi^-\pi^-\pi^+$	g	-0.217 ± 0.007		
	h	0.010 ± 0.006		
	k	-0.0084 ± 0.0019		

$\pi\pi$ Scattering

Isospin invariance:

$$T_{\alpha\beta;\gamma\delta}(s, t, u) = A(s, t, u)\delta_{\alpha\beta}\delta_{\gamma\delta} + A(t, s, u)\delta_{\alpha\gamma}\delta_{\beta\delta} + A(u, t, s)\delta_{\alpha\delta}\delta_{\beta\gamma}$$

Then

$$T^0(s, t, u) = 3A(s, t, u) + A(t, s, u) + A(u, t, s)$$

$$T^1(s, t, u) = A(t, s, u) - A(u, t, s),$$

$$T^2(s, t, u) = A(t, s, u) + A(u, t, s).$$

1966 Weinberg

$$A(s, t, u) = \frac{s - m_\pi^2}{F_\pi^2}$$

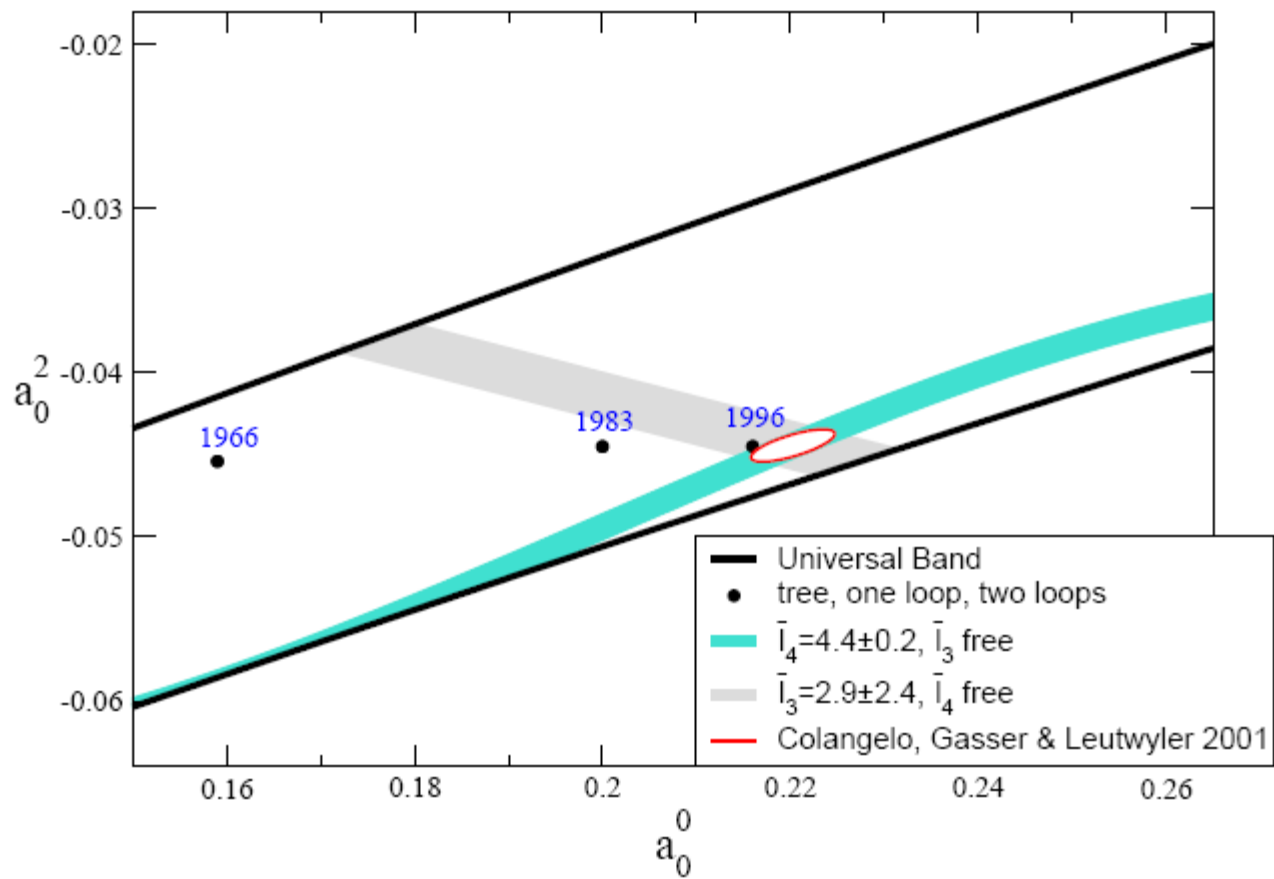
Weinberg scattering lengths:

$$a_0^0 = \frac{7m_\pi^2}{32\pi F_\pi^2}, \quad a_0^2 = -\frac{m_\pi^2}{16\pi F_\pi^2}, \quad a_1^1 = -\frac{m_\pi^2}{24\pi F_\pi^2}$$
$$b_0^0 = \frac{m_\pi^2}{4\pi F_\pi^2}, \quad b_0^2 = \frac{m_\pi^2}{8\pi F_\pi^2},$$

1984 Gasser-Leutwyler one loop:

2009 Gasser-Leutwyler-Colangelo:
Roy Equations

Chiral predictions for a_0^0 and a_0^2



$\pi\pi$ case: Weinberg generated lowest order chiral predictions in 1966

$$a_0 = 0.16m_\pi^{-1} \quad \text{and} \quad a_2 = -0.05m_\pi^{-1}$$

Gasser-Leutwyler show one loop correction gives $a_0 = 0.20m_\pi^{-1}$. In higher order, result depends on chiral condensate

$$\text{Std. Chipt} : \frac{\langle |\bar{q}q| \rangle}{F_\pi^2} \sim 1\text{GeV} \quad a_0 = 0.22$$

$$\text{Gen. Chipt} : \frac{\langle |\bar{q}q| \rangle}{F_\pi^2} \ll 1\text{GeV} \quad a_0 = 0.26$$

Chipt-Roy Equation result is

$$a_0 = (0.220 \pm 0.005)m_\pi^{-1}$$

$$a_2 = (-0.044 \pm 0.001)m_\pi^{-1}$$

Now use $K \rightarrow 3\pi$ to yield $\pi\pi$ scattering lengths

Analysis of cusp structure in $K \rightarrow 3\pi$ and/or $\eta \rightarrow \pi^+\pi^-\pi^0$

Basic idea (Cabibbo) exploits mass difference between π^+ and π^0

Well known example: π^0 photoproduction. Interference between basic $\gamma p \rightarrow \pi^0 p$ and $\gamma p \rightarrow \pi^+ n \rightarrow \pi^0 p$ amplitudes—near threshold

$$E_{0+}(\gamma p \rightarrow \pi^0 p; s) = e^{i\delta_0(s)} [A(s) + i\beta q_+]$$

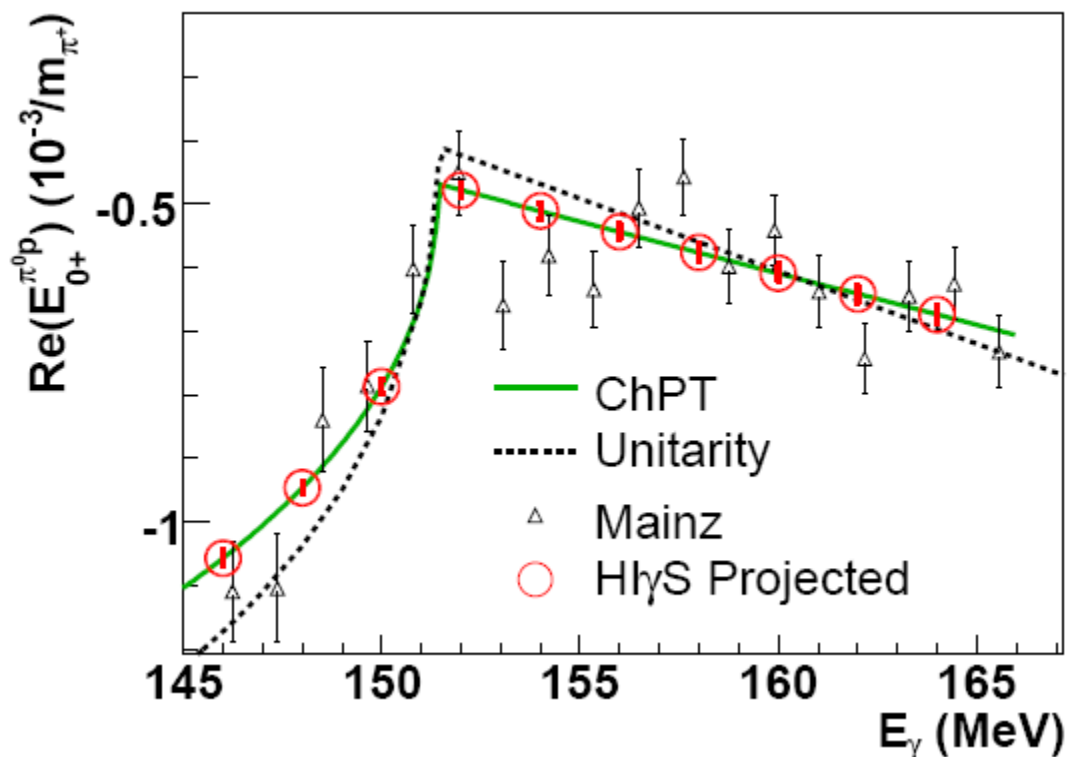
with $q_+ = \frac{1}{2}\sqrt{s - 4m_{\pi^+}^2}$ charged pion momentum and

$$\beta \sim E_{0+}(\gamma p \rightarrow \pi^+ n; s) a_{\pi^+ n \rightarrow \pi^0 p}$$

Above threshold q_+ real and $|E_{0+}(\gamma p \rightarrow \pi^0 p; s)|^2 \propto |A(s)|^2 - \beta^2 q_+^2$ and smooth function of s .

In region $4m_0^2 < s < 4m_+^2$ $q^+ = i|q_+|$ so $|E_{0+}(\gamma p \rightarrow \pi^0 p; s)|^2 \propto |A(s)|^2 + \beta^2 |q_+|^2 - 2A(s)\beta|q_+|$ —slope discontinuity.

This is unitarity cusp—confirmed experimentally at MAMI



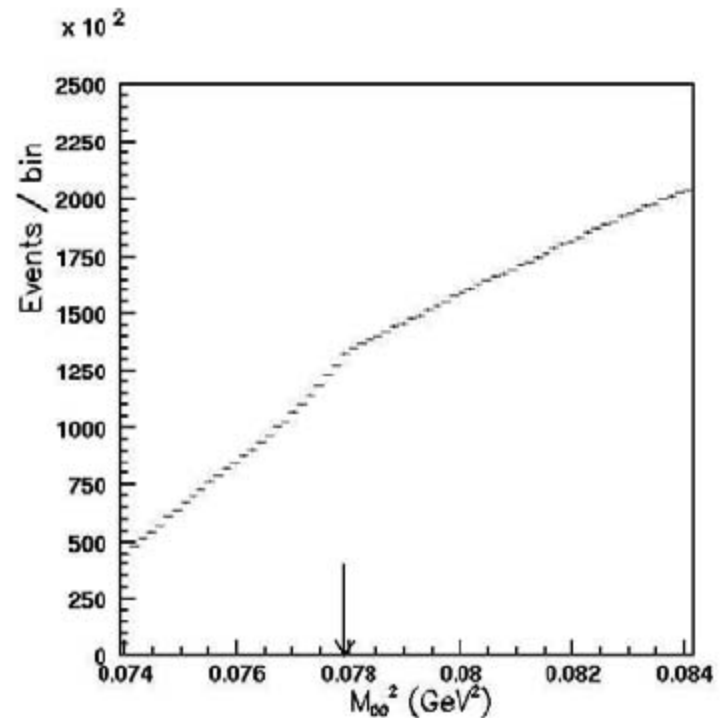
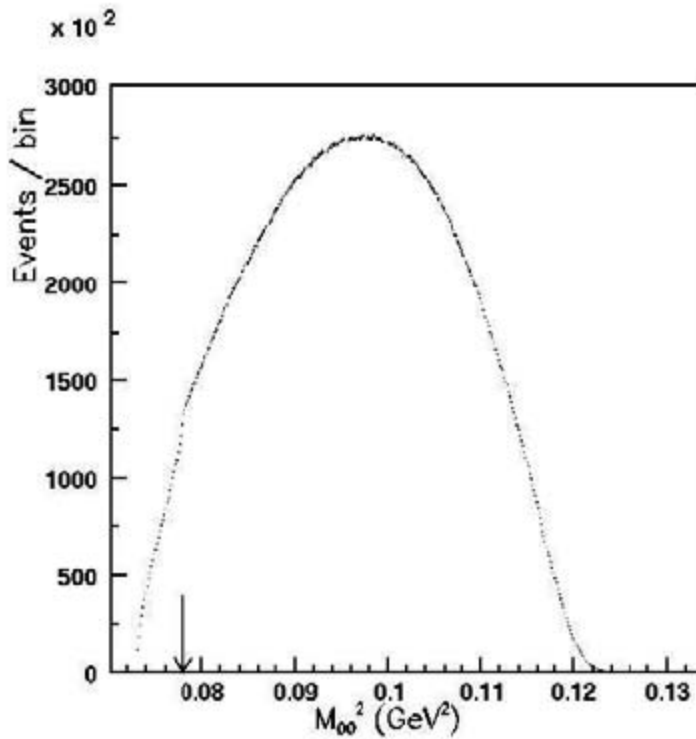
Same idea in kaon decay—interfere

$$K^+ \rightarrow \pi^+\pi^0\pi^0 \quad \text{and} \quad K^+ \rightarrow \pi^+\pi^+\pi^- \rightarrow \pi^+\pi^0\pi^0$$

Examine region wherein $4m_{\pi^0}^2 < s_{00} < 4m_{\pi^+}^2$

Have $\text{Amp}(K^+ \rightarrow \pi^+\pi^0\pi^0; s_{00}) \propto A(s_{00}) + i\beta q_+$
where $A(s_{00})$ smooth function of s_{00} and $\beta \sim 2A(s_{00})a(\pi^+\pi^- \rightarrow \pi^0\pi^0)$

Here $a(\pi^+\pi^- \rightarrow \pi^0\pi^0) = a_0 - a_2$ so sensitive to this quantity. Calculate decay amplitudes carefully and fit to $|\text{Amp}(K^+ \rightarrow \pi^+\pi^0\pi^0; s_{00})|^2$ —expect cusp as in photoproduction case



Results from NA48/2 (preliminary)

$$a_0 - a_2 = (0.261 \pm 0.006 \pm 0.003 \pm 0.013)m_{\pi^+}^{-1}$$

$$a_2 = (-0.037 \pm 0.009 \pm 0.013)m_{\pi^+}^{-1}$$

Polarizabilities

What is a polarizability?

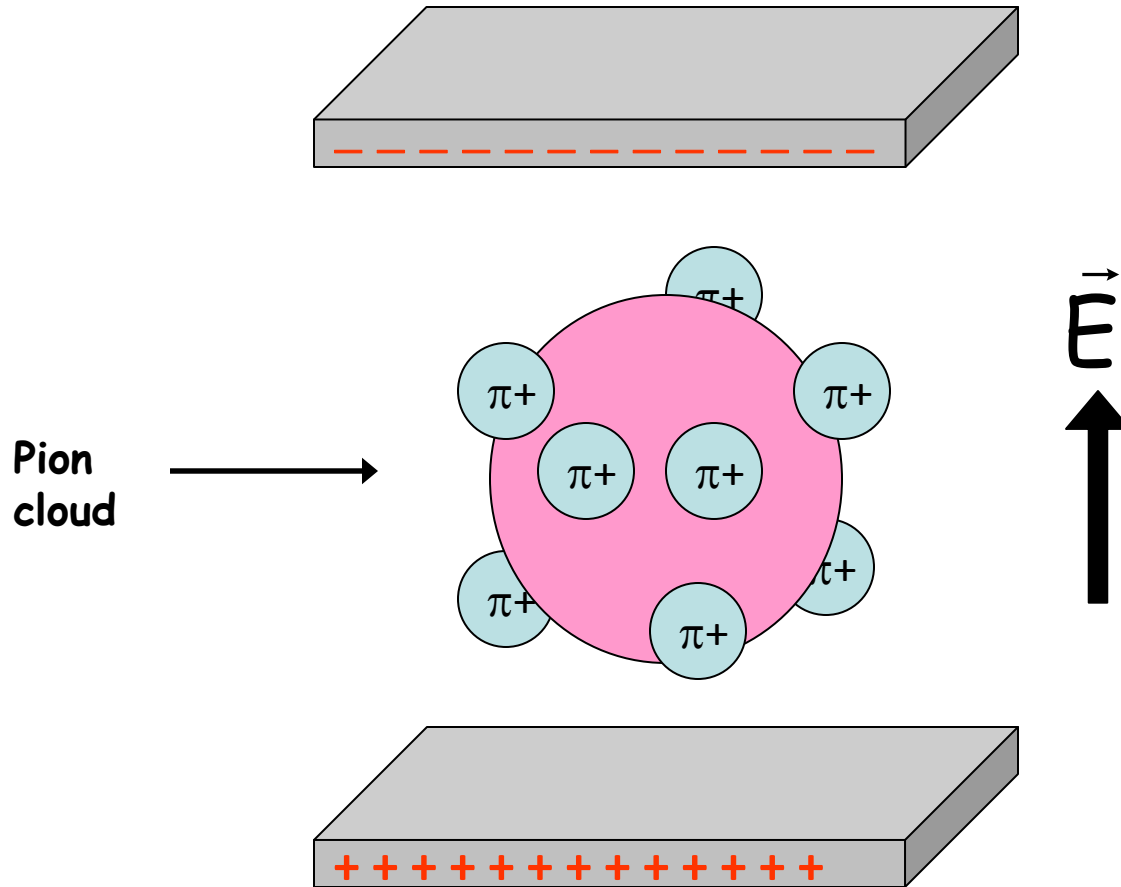
Answer: A measure of response of system to quasi-static electric and/or magnetic field. Simplest example: electric polarizability α_E —applied electric field \vec{E} induces EDM \vec{p}

$$\vec{p} = 4\pi\alpha_E\vec{E}$$

Equivalently energy density is

$$u = -\frac{1}{2}4\pi\alpha_E\vec{E}^2$$

Proton electric polarizability



Electric polarizability: proton between charged parallel plates

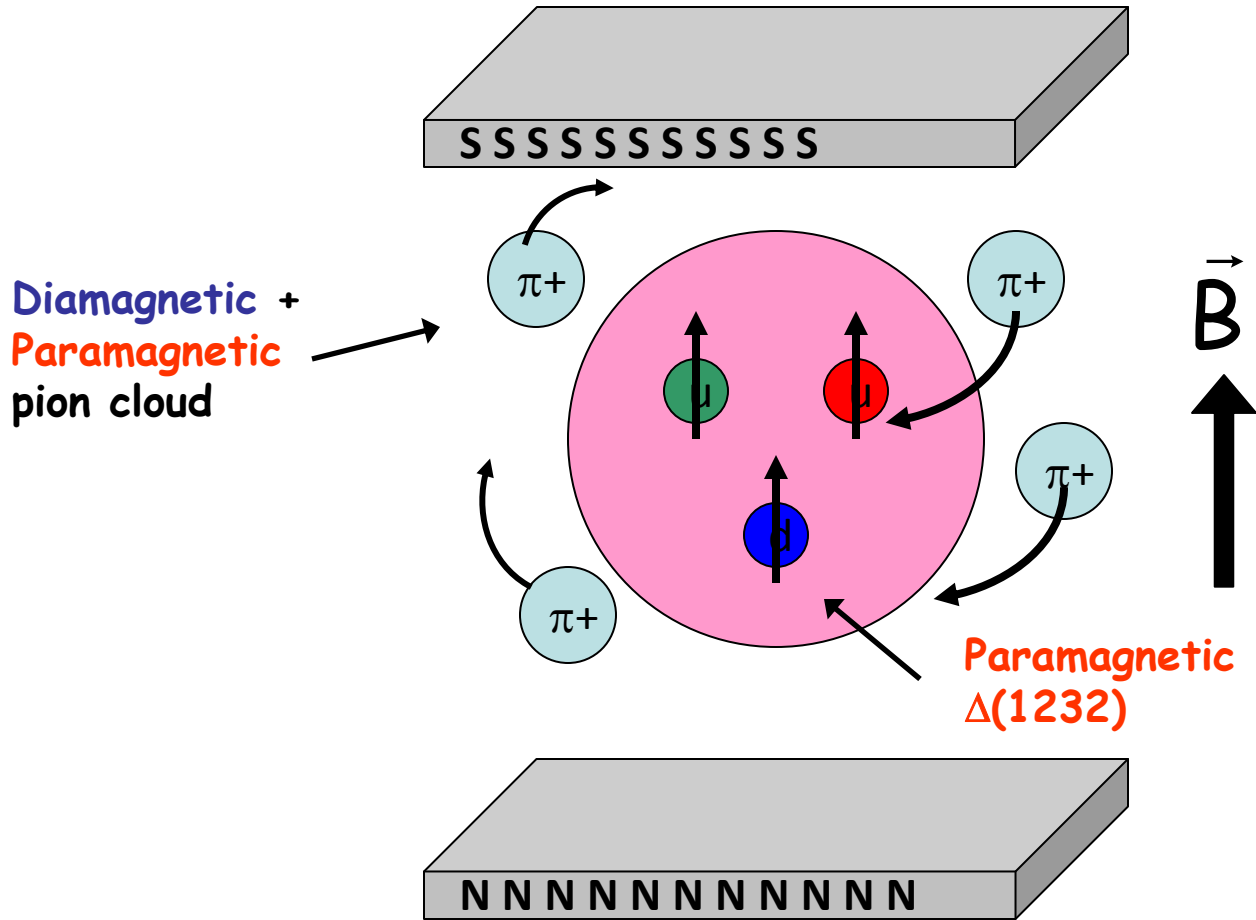
Similarly magnetic polarizability β_M —applied
magnetic field \vec{H} induces MDM \vec{m}

$$\vec{m} = 4\pi\beta_M\vec{H}$$

with energy density

$$u = -\frac{1}{2}4\pi\beta_M\vec{H}^2$$

Proton magnetic polarizability



Magnetic polarizability: proton between poles of a magnetic

How to measure? Compton scattering—if

$$H = \frac{(\vec{p} - e\vec{A})^2}{2m} - \frac{1}{2}4\pi\alpha_E\vec{E}^2 - \frac{1}{2}4\pi\beta_M\vec{H}^2$$

with

$$\vec{E} = -\frac{\partial}{\partial t}\vec{A}, \quad \vec{B} = \vec{\nabla} \times \vec{A}$$

then

$$T = \hat{\epsilon} \cdot \hat{\epsilon}' \left(-\frac{Q^2}{m} + \omega\omega'4\pi\alpha_E \right) + \hat{\epsilon} \times \vec{k} \cdot \hat{\epsilon}' \times \vec{k}'4\pi\beta_M$$

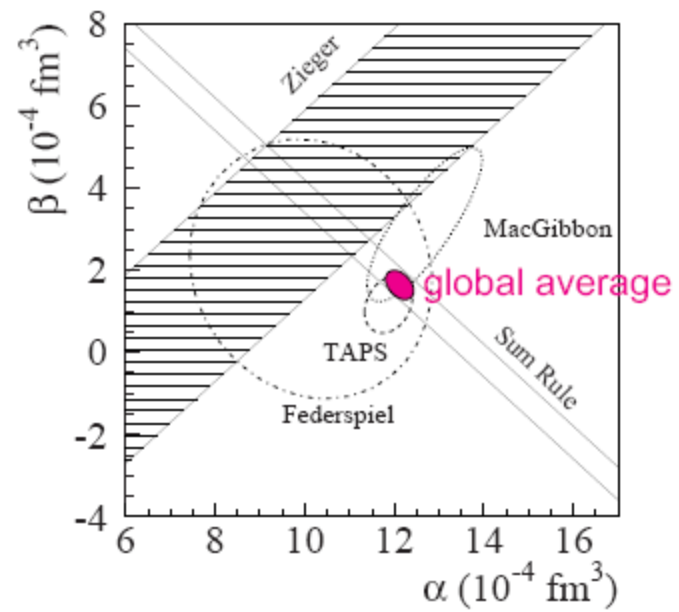
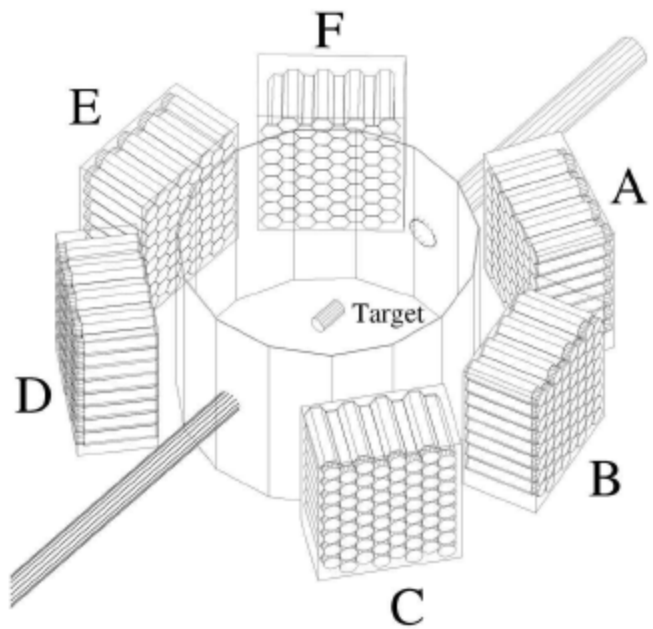
and

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{m^2} \left(\frac{\omega'}{\omega} \right)^2 \left(\frac{1}{2}(1 + \cos^2 \theta) - \frac{m\omega\omega'}{\alpha} \right.$$

$$\left. \cdot \left[\frac{\alpha_E + \beta_M}{2} (1 + \cos \theta)^2 + \frac{\alpha_E - \beta_M}{2} (1 - \cos \theta)^2 \right] + \dots \right)$$

Results from MAMI (TAPS), Illinois, Saskatoon for proton

$$\alpha_E^p = (12.0 \pm 0.6) \times 10^{-4} \text{ fm}^3, \quad \beta_M = (1.9 \mp 0.6) \times 10^{-4} \text{ fm}^3$$



What have we learned?

- i) Size of α_E measures "stiffness" of system. For H atom well known calculation yields

$$\alpha_E^H = \frac{9}{2}a_B^3 = \frac{27}{8\pi}\text{Vol}$$

while for proton

$$\alpha_E^P \simeq 3 \times 10^{-4}\text{Vol}$$

so proton is very "stiff". Handwaving estimate is

$$\frac{\alpha_E^P/\text{Vol.}}{\alpha_E^H/\text{Vol.}} \sim \frac{E_{bind}^H/m}{E_{bind}^P/m} \sim \frac{\alpha_{em}^2}{\alpha_{strong}^2} \sim 10^{-4}$$

ii) Δ pole makes a very strong paramagnetic contribution to $\beta_M \sim +10 \times 10^{-4} \text{ fm}^3$ so proton must have strong diamagnetic component to cancel this.

iii) Presumably comes from meson cloud component—indeed simple quark model picture gives

$$\alpha_E^p = 2\alpha m_p \langle r_p^2 \rangle^2 \gg \alpha_E^{exp}$$

so meson contribution needed.

Estimate via chiral perturbation theory (Bernard, Kaiser, Meissner):

$$\alpha_E^p = \frac{\alpha g_A^2}{48\pi^2 F_\pi^2 M_n} \left[\frac{5\pi}{2\mu} + 18 \log \mu + \frac{33}{2} + \mathcal{O}(\mu) \right] = 7.4$$

$$\beta_M^p = \frac{\alpha g_A^2}{48\pi^2 F_\pi^2 M_n} \left[\frac{\pi}{4\mu} + 18 \log \mu + \frac{63}{2} + \mathcal{O}(\mu) \right] = -2.0$$

with $\mu = m_\pi/m_n$ in units of 10^{-4} fm^3 .

If take only leading term ($\mathcal{O}(q^3)$ HBChipt), then

$$\alpha_E^p = 10\beta_M^p = \frac{5g_A^2}{96\pi F_\pi^2 m_\pi} = 12.2$$

in perfect agreement with experiment. Clearly
accidental since $\mathcal{O}(q^4)$ calculation gives

$$\alpha_E^p = 10.5 \pm 2.0 \quad \text{and} \quad \beta_M^p = 3.5 \pm 3.6$$

Neutron measurements much more difficult since

i) no neutron target available

ii) no Thomson term to interfere with

Has been done via

All agree that $\alpha_E^n \simeq \alpha_E^p$ and $\beta_M^n \simeq \beta_M^p$. Schumacher recommends values

$$\alpha_E^n = 12.5 \pm 1.8 \quad \text{and} \quad \beta_M = 2.7 \mp 1.8$$

in good agreement with HBchipt prediction that primarily isoscalar.

Charged Pion Polarizabilities

Another interesting case is charged pion. Here lowest order chiral symmetry $\mathcal{O}(p^4)$ gives

$$\alpha_E^{\pi^+} + \beta_M^{\pi^+} = 0 \quad \text{and} \quad \alpha_E^{\pi^+} - \beta_M^{\pi^+} = 5.4 \times 10^{-4} \text{ fm}^3$$

where here size of polarizability is given by axial term in radiative pion decay— $\pi^+ \rightarrow e^+ \nu_e \gamma$. **Two loop also done, generating small corrections**

$$\alpha_E^{\pi^+} = 2.9 \times 10^{-4} \text{ fm}^3 \quad \text{and} \quad \beta_M^{\pi^+} = -2.5 \times 10^{-4} \text{ fm}^3$$

as expected.

Experimental results are varied. Three methods:

i) Primakoff Effect: $\pi^+ + N \rightarrow \pi^+ + N + \gamma$ Antipov et al. (1985) $\alpha_E^{\pi^+} = (6.8 \pm 1.4 \pm 1.2) \times 10^{-4} \text{ fm}^3$

ii) Pion Pole: $\gamma + N \rightarrow \gamma + N + \pi^+$ Aibergenov et al. (1986) $\alpha_E^{\pi^+} = (20 \pm 12) \times 10^{-4} \text{ fm}^3$

Ahrens et al. (MAMI-2005): $\alpha_E^{\pi^+} - \beta_M^{\pi^+} = (11.6 \pm 1.5 \pm 3.0 \pm 0.5) \times 10^{-4} \text{ fm}^3$

iii) Annihilation: $\pi^+ \pi^- \rightarrow \gamma \gamma$

Babusci et al. (1992) $\alpha_E^{\pi^+} = (2.2 \pm 1.6) \times 10^{-4} \text{ fm}^3$

Future: Spin Polarizabilities

Basic Idea: Higher order in photon energy expansion:

$$\alpha_E = \alpha_{E1E1} \quad \beta_M = \beta_{M1M1}$$

If include spin then allow terms

$$\gamma_{E1E1}, \gamma_{M1M1}, \gamma_{E1M2}, \gamma_{M1E2}$$

such that

$$H = -\frac{1}{2}4\pi \left[\gamma_{E1E1} \vec{\sigma} \cdot \vec{E} \times \dot{\vec{E}} + \gamma_{M1M1} \vec{\sigma} \cdot \vec{B} \times \dot{\vec{B}} \right. \\ \left. + 2\gamma_{E1M2} \sigma_i E_j B_{ij} - 2\gamma_{M1E2} \sigma_i B_j E_{ij} \right]$$

with $E_{ij} = \frac{1}{2}(\nabla_i E_j + \nabla_j E_i)$. Note terms are $\mathcal{O}(\omega^3)$ vs. $\mathcal{O}(\omega^2)$ for α_E, β_M

HBChipt predictions at $\mathcal{O}(q^3)$ are

$$\gamma_{E1E1}^P = -5\gamma_{M1M1}^P = 5\gamma_{E1M2}^P = 5\gamma_{M1E2}^P = -\frac{1}{\pi m_\pi} \alpha_E^P$$

At this order (large) contribution from pion pole graph is

$$\gamma_{E1E1}^P = -\gamma_{M1M1}^P = -\gamma_{M1E2}^P = \gamma_{E1M2}^P = \frac{1}{\pi m_\pi g_A} 2.4 \alpha_E^P$$

dominates other terms.

At present no direct measurements of spin polarizabilities. However, analysis of Compton scattering data gives values in backward directions

$$LEGS : \gamma_\pi = (-27.1 \pm 2.2 \pm 2.6) \times 10^{-4} \text{ fm}^4$$

$$MAMI - TAPS : \gamma_\pi = (-35.9 \pm 2.3) \times 10^{-4} \text{ fm}^4$$

$$MAMI - LARA : \gamma_\pi = (-40.9 \pm 0.6 \pm 2.2) \times 10^{-4} \text{ fm}^4$$

Schumacher recommends $\gamma_\pi = (-38.7 \pm 1.8) \times 10^{-4} \text{ fm}^4$ and dispersion relation (higher order GDH) gives value in forward direction

$$\gamma_0 = (-1.0 \pm 0.8 \pm 0.10) \times 10^{-4} \text{ fm}^4$$

Note $|\gamma_\pi^P| \gg |\gamma_0^P|$. Reason is

$$\gamma_0^P = -\gamma_{E1E1}^P - \gamma_{M1M1}^P - \gamma_{E1M2}^P - \gamma_{M1E2}^P$$

so large pion pole contribution cancels, while

$$\gamma_\pi^P = -\gamma_{E1E1}^P + \gamma_{M1M1}^P - \gamma_{E1M2}^P + \gamma_{M1E2}^P$$

wherein pion pole does NOT cancel—

$$pole \gamma_\pi^P = -\frac{9.6\alpha_E^P}{\pi m_\pi g_A} = -42.7 \times 10^{-4} \text{ fm}^4$$

Clearly pole dominates γ_{π}^p . Lowest order chiral prediction for

$$\gamma_0^p = \frac{6}{5\pi m_{\pi}} \alpha_E^p = +6.2 \times 10^{-4} \text{ fm}^4$$

Wrong sign, so higher order terms must be important.

For neutron, have "experimental" results

$$\gamma_{\pi}^n = (58.6 \pm 4.0) \times 10^{-4} \text{ fm}^4$$

$\mathcal{O}(q^3)$ HBChipt predictions:

$$\gamma_{\pi}^n = 42.7(\text{pole}) + 6.1(\text{loop}) \times 10^{-4} \text{ fm}^4$$

$$\gamma_0^n = 6.8 \times 10^{-4} \text{ fm}^4$$

At higher chiral order, evidence for significant cancelations.

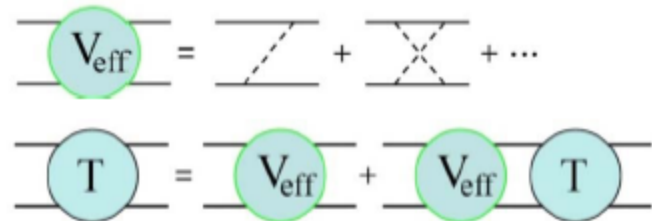
On experimental side, programs at MAMI and Hiγs should allow measurements of individual spin polarizabilities.

Nucleon-Nucleon Interaction

ARV18 fits NN phase shifts up to ~ 300 MeV with ~ 40 empirical parameters—phenomenological 3-nucleon potential

Chiral approach: systematic within pionful picture

- irreducible contributions can be calculated using ChPT
- enhanced reducible contributions must be summed up to infinite order

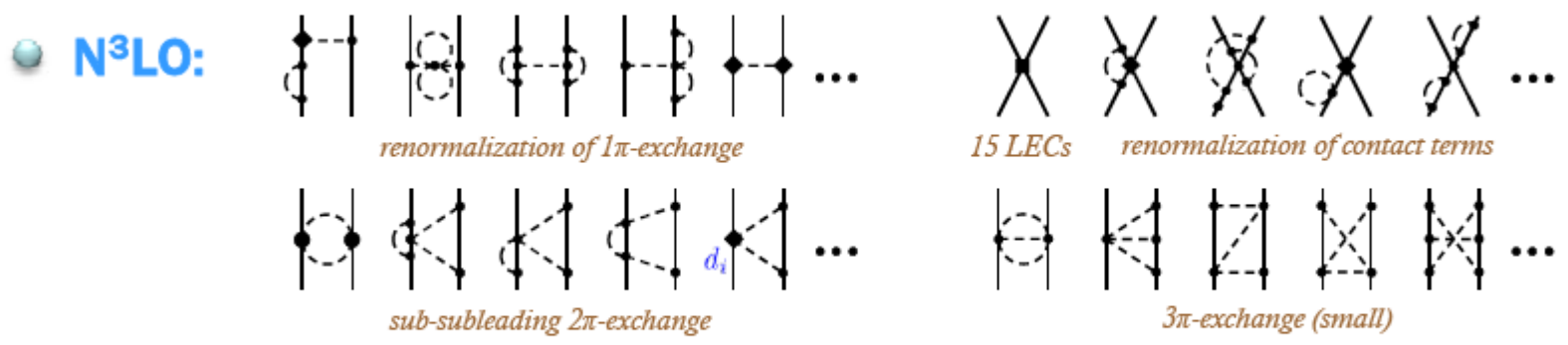
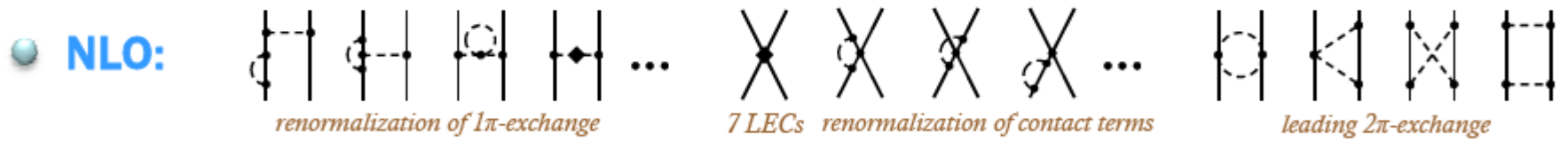


	Two-nucleon force	Three-nucleon force	Four-nucleon force
Q^0		—	—
Q^2		—	—
Q^3			—
Q^4			

Explains the observed hierarchy of nuclear forces:

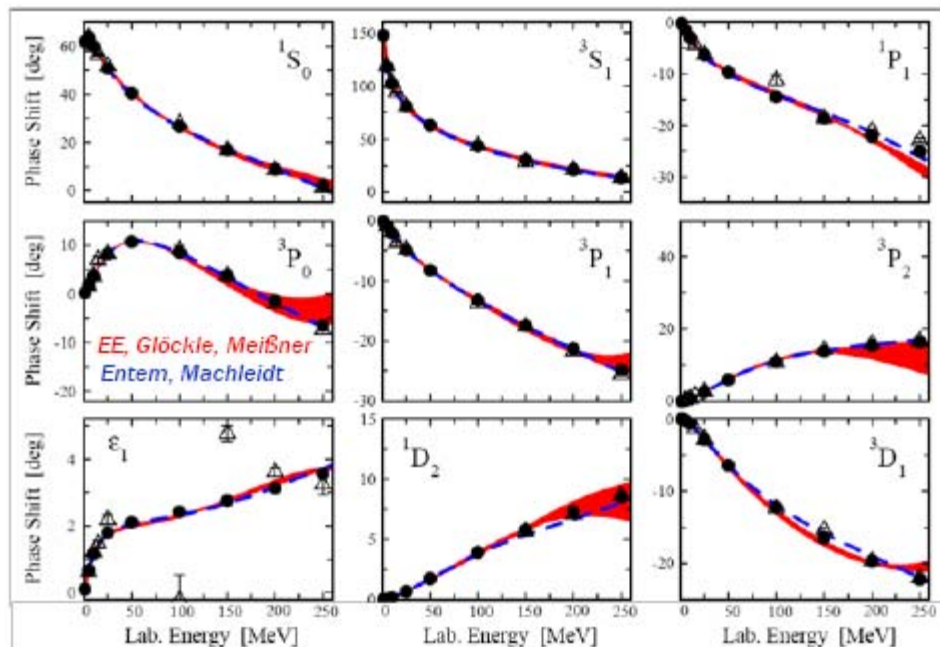
$$\langle V_{2N} \rangle \sim 20 \text{ MeV/pair} \gg \langle V_{3N} \rangle \sim 1 \text{ MeV/triplet} \gg \langle V_{4N} \rangle \sim 0.1 \text{ MeV/quartet}$$

Chiral expansion of the 2N force: $V_{2N} = V_{2N}^{(0)} + V_{2N}^{(2)} + V_{2N}^{(3)} + V_{2N}^{(4)} + \dots$

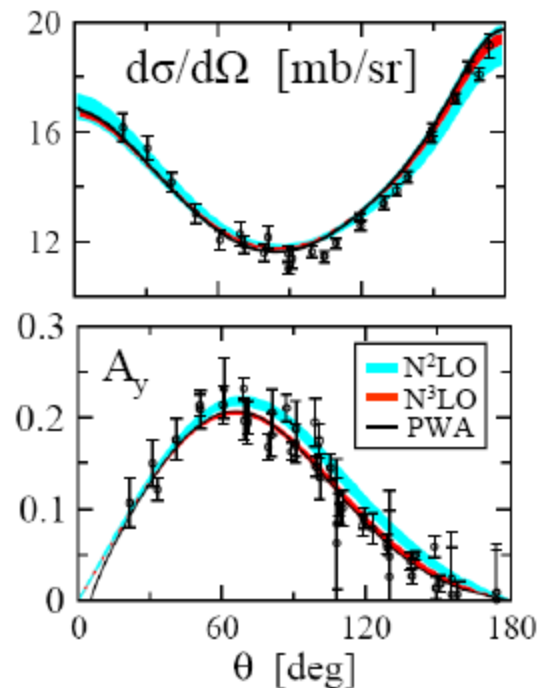


+ isospin-breaking corrections...
 van Kolck et al. '93,'96; Friar et al. '99,'03,'04; Niskanen '02; Kaiser '06; E.E. et al. '04,'05,'07; ...

Neutron-proton phase shifts at N³LO



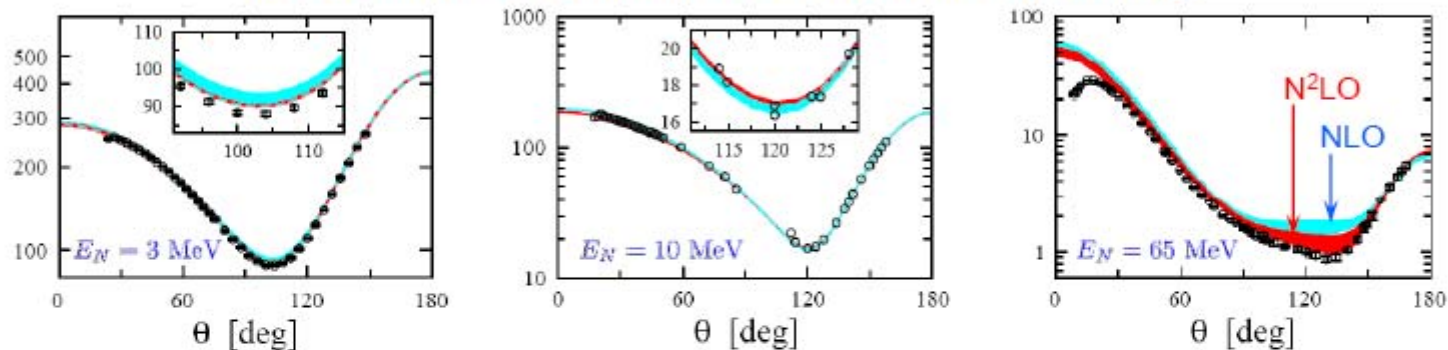
np scattering at 50 MeV



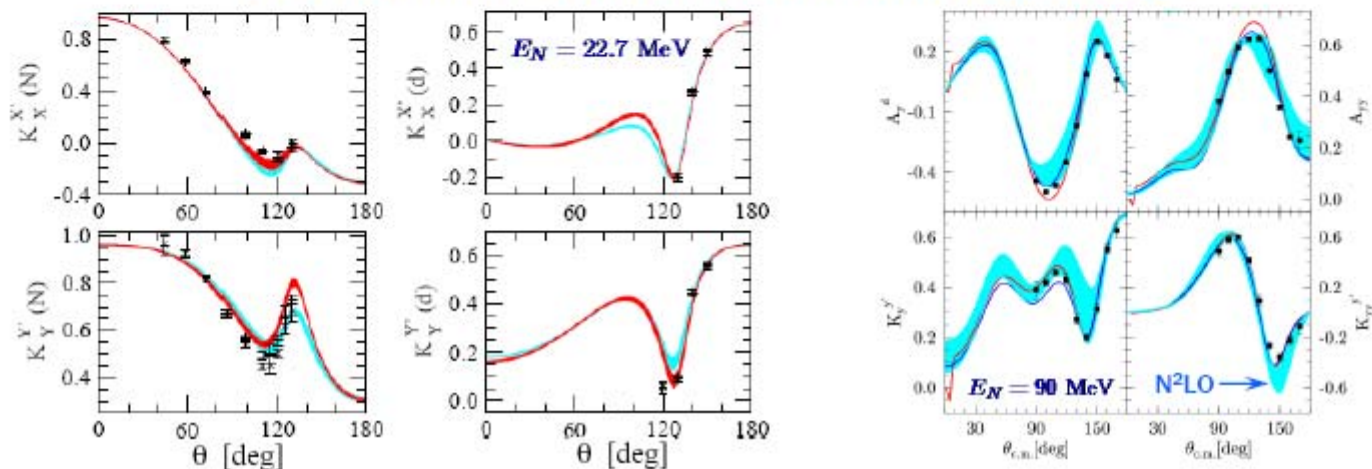
Three nucleons up to N²LO

E.E. et al. '02; Kistryn et al. '05; Witala et al. '06; Ley et al. '06; Stephan et al. '07; ...

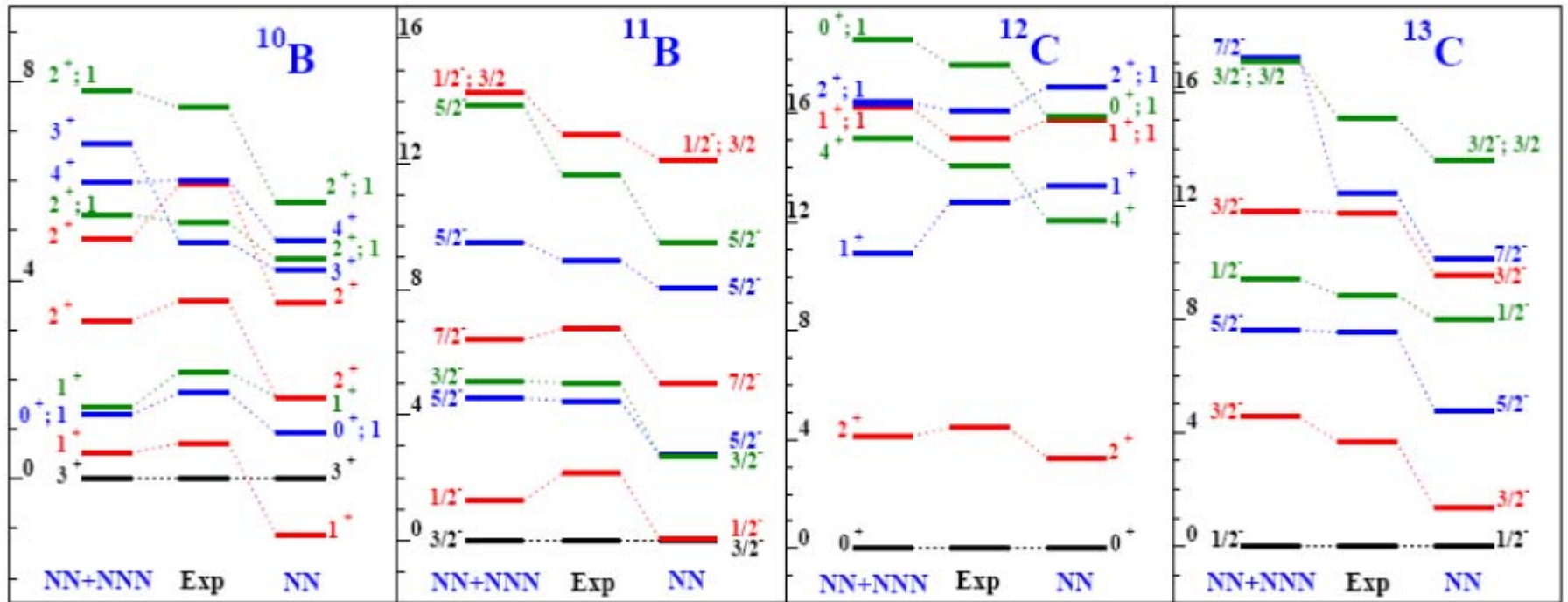
Differential cross section in elastic Nd scattering



Polarization observables in elastic Nd scattering



^{10}B , ^{11}B , ^{12}C and ^{13}C states dominated by p-shell configurations



Navratil et al., PRL 99 (2007) 042501

Summary

Still lots of chiral challenges

See you all in USA in 2012!