Pion photoproduction in a nonrelativistic theory

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Outline

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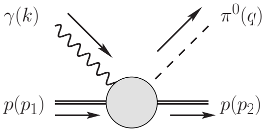
Introduction

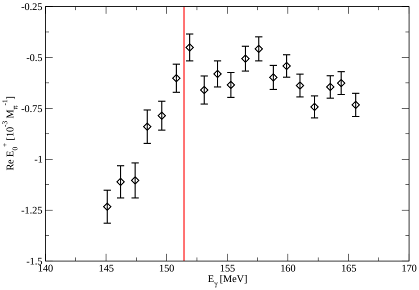
Nonrelativistic framework

Results

Summary and conclusion

Motivation





Schmidt et al. 2001

$$\mathcal{M} = 8\pi\sqrt{s}\,\xi_{t'}^\dagger\,\mathcal{F}\,\xi_t$$

$$\mathcal{F} = i\boldsymbol{\tau}\cdot\boldsymbol{\epsilon}\,\mathcal{F}_1 + \boldsymbol{\tau}\cdot\hat{\mathbf{q}}\,\boldsymbol{\tau}\cdot(\hat{\mathbf{k}}\times\boldsymbol{\epsilon})\,\mathcal{F}_2 + i\boldsymbol{\tau}\cdot\hat{\mathbf{k}}\,\hat{\mathbf{q}}\cdot\boldsymbol{\epsilon}\,\mathcal{F}_3 + i\boldsymbol{\tau}\cdot\hat{\mathbf{q}}\,\hat{\mathbf{q}}\cdot\boldsymbol{\epsilon}\,\mathcal{F}_4$$

$$E_{0+} = \frac{1}{2}\int_{-1}^{1} dz\left[\mathcal{F}_1 - z\mathcal{F}_2 + \frac{1}{3}(1-P_2(z))\mathcal{F}_4\right]$$

Chew, Goldberger, Low, Nambu, 1957

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$\longrightarrow$ Physical values for the masses

$\longrightarrow$ Correct reproduction of singularity structure for $|\mathbf{q}| \ll M_{\pi}$

Lagrangian

- Kinetic part: Decompose relativistic propagator

$$\frac{1}{M^2 - p^2} = \frac{1}{2\omega(\mathbf{p})}\frac{1}{\omega(\mathbf{p}) - p^0} + \frac{1}{2\omega(\mathbf{p})}\frac{1}{\omega(\mathbf{p}) + p^0}$$

$$\omega(\mathbf{p}) = \sqrt{M^2 + \mathbf{p}^2}$$

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Lyubovitskij, Rusetsky 2000, Zemp 2004

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- Matching:

$$C_x = \sqrt{2}\, 8\pi (m_p + M_\pi) a_{0+}^- + \delta_x$$

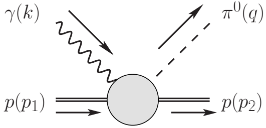
Lyubovitskij, Rusetsky 2000

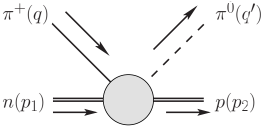
Power counting

Masses : $O(1)$

Mass differences : $M_\pi^2 - M_{\pi^0}^2 = O(\epsilon^2)$

$m_n^2 - m_p^2 = O(\epsilon^2)$





Incoming momenta : $O(1)$

Outgoing momenta : $O(\epsilon)$

All momenta : $O(\epsilon)$

πN vertex : $O(a)$

Is this consistent?

$$\mathcal{L}_\gamma = G_0 \psi^\dagger \boldsymbol{\tau} \psi \, \mathbf{A} \, \pi_0^\dagger + X \psi^\dagger \boldsymbol{\tau} \psi \, \triangle \mathbf{A} \, \pi_0^\dagger + \cdots$$

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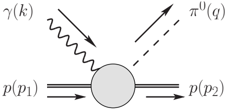

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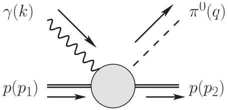

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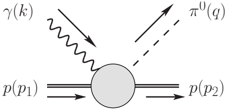

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Comments:

- Derivatives on $\mathbf{A}$ $\longrightarrow$ dependence on $\cos\Theta$

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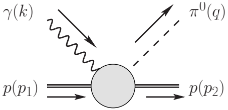

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Comments:

- Derivatives on $\mathbf{A} \longrightarrow$ dependence on $\cos\Theta$
- No predictions about dependence on $\mathbf{k}$

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The phase of E_{0+} below the $\pi^+ n$ threshold agrees with the phase of the $\pi^0 p \to \pi^0 p$ S-wave at leading order in e.

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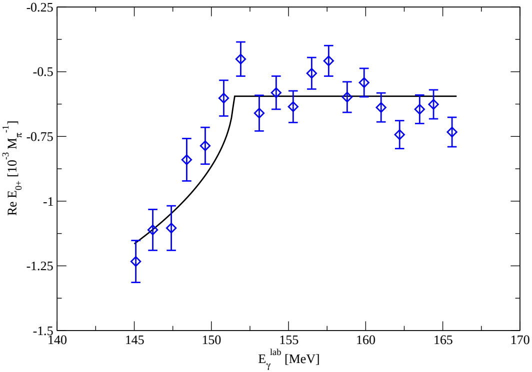
$\longrightarrow$ Up to and including $O(a^2\epsilon^4)$: $\delta_{E_{0+}} = \delta_{\pi^0 p}$

Comment:

- $\delta_{\pi^0 p}$ differs from the pertinent phase in isospin limit

Determination of πN scattering length



- $E_{0+}(s) = G_0 + G_0 C_0 J_{p0} + C_x H_0 J_{n+}$
- Fix $C_0 = 8\pi(m_p + M_\pi)a^+_{0+} + \delta_0$

  Hoferichter, Kubis, Meißner 2009
- H_0 from experiment $\longrightarrow a^-_{0+} = 51 \cdot 10^{-3}\, M_\pi^{-1}$

  Data from Fuchs et al. 1996 and Schmidt et al. 2001

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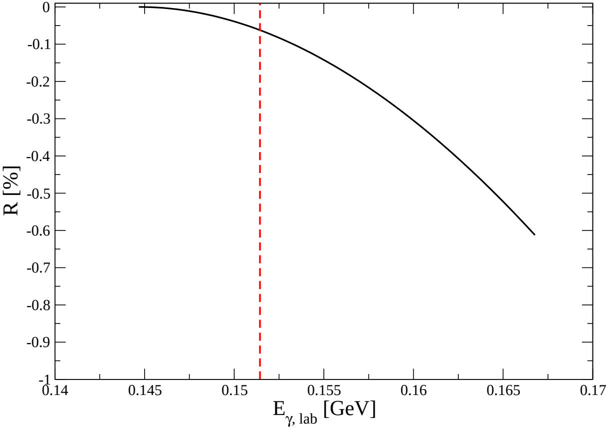
Summary and Conclusion

- Construction of nonrelativistic framework for photoproduction of pions with consistent power counting
- Analytical results for S- and P-wave multipoles up to and including $O(\epsilon^3, a\epsilon^4, a^2\epsilon^5)$
- Determination of scattering length not (yet?) very promising

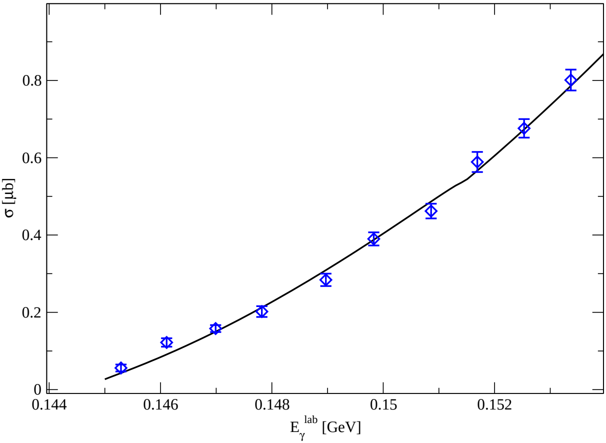
SPARES

Nucleon pole

Expand the chiral tree level result in $\mathbf{q}$ up to and including $O(\mathbf{q}^2)$.



$$R = \frac{E_{0+}^{\text{exp}} - E_{0+}}{E_{0+}}$$


0.8
0.6
0.4
0.2
0
σ [μb]
0.144
0.146
0.148
0.15
0.152
E_γ^lab [GeV]