More effective theory for nuclear forces

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Problem with building an EFT for nuclear forces

Chiral perturbation theory

- expansion in powers of ratios of low-energy scales Q (momenta, m_π, ...) to scales of underlying physics Λ_0 (m_ρ, M_N, $4\pi F_\pi$, ...)
- terms organised by naive dimensional analysis aka "Weinberg power counting" (simply counts powers of low-energy scales)
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- terms organised by naive dimensional analysis aka "Weinberg power counting" (simply counts powers of low-energy scales)
- perturbative: works for weakly interacting systems (eg pions, photons and ≤ 1 nucleon)
- but nucleons interact strongly at low-energies
- bound states exist (nuclei!)

$\longrightarrow$ need to treat some interactions nonperturbatively

Basic nonrelativistic loop diagram

$$\frac{M}{(2\pi)^3}\int\frac{\mathrm{d}^3q}{p^2-q^2+\mathrm{i}\varepsilon}=-\mathrm{i}\,\frac{Mp}{4\pi}+\text{analytic}$$

- of order Q [Weinberg (1991)]
- but potential starts at order Q^0
  (OPE and simplest contact interaction)
- each iteration suppressed by power of Q/Λ_0

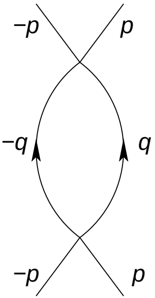
$\longrightarrow$ perturbative provided $Q<\Lambda_0$

- integral linearly divergent

$\longrightarrow$ cut off (or subtract) at $q=\Lambda$

- contributions multiplied by powers of Λ/Λ_0

$\longrightarrow$ again perturbative provided $\Lambda<\Lambda_0$



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- expand potential to some order in Q
- then iterate to all orders in favourite dynamical equation (Schrödinger, Lippmann-Schwinger, . . .)
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- then iterate to all orders in favourite dynamical equation (Schrödinger, Lippmann-Schwinger, . . .)
- widely applied and even more widely invoked
- but no clear power counting for observables
- resums subset of terms to all orders in Q (and some of these depend on regulator)
- not necessarily a problem if these terms are small
- but what if we rely on them to generate bound states?

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- Liberal
  "Let the renormalisation group decide!"

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and the orthdox party seems to be winning the election, so far...

Renormalisation group

General tool for analysing scale-dependence

- first, identify all low-energy scales Q
- including ones to promote leading-order terms to order Q^{-1}
  (cancels Q from loop $\longrightarrow$ iterations not suppressed)
- can, and must, then be iterated to all orders

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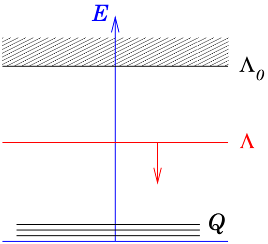
Examples of new scales

- S-wave scattering lengths $1/a \lesssim 40$ MeV [van Kolck; KSW (1998)]
- "unnatural" strength of OPE set by scale

$$\lambda_{NN} = \frac{16\pi F_\pi^2}{g_A^2 M_N} \simeq 290 \text{ MeV}$$

built out of high-energy scales $(4\pi F_\pi, M_N)$ but $\sim 2m_\pi$

Then



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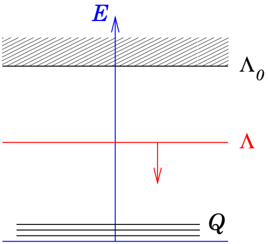

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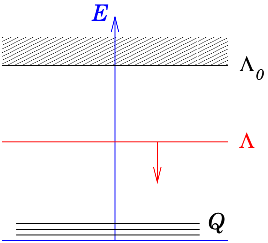

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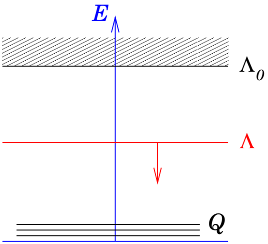

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- look for fixed points (describe scale-free systems)
- expand around these using perturbations that scale like Λ^{ν}

$\longrightarrow$ correspond to terms in EFT of order Q^d where $d = \nu - 1$
(Λ: largest acceptable low-energy scale)

Fixed points of short-range forces

Trivial: $V_0 = 0 \longrightarrow$ weak scattering, Weinberg counting

Nontrivial: $V_0(p,\Lambda) = -\frac{2\pi^2}{M\Lambda}\left[1 - \frac{p}{2\Lambda}\ln\frac{\Lambda+p}{\Lambda-p}\right]^{-1}$ (sharp cutoff)

- order Q^{-1} (so must be iterated)
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- describes "unitary limit": scattering length $a \to \infty$
- expansion around this point

$$V(p,\Lambda) = V_0(p,\Lambda) + V_0(p,\Lambda)^2\,\frac{M}{4\pi}\left(-\frac{1}{a} + \frac{1}{2}\,r_e\,p^2 + \cdots\right)$$

- factor $V_0^2 \propto \Lambda^{-2}$ promotes terms by two orders compared to naive expectation [van Kolck; Kaplan, Savage and Wise (1998)]
- effective-range expansion, "KSW" counting

Enhancement follows from form of wave functions as $r \to 0$

Two particles in unitary limit

- irregular solutions: $\psi(r) \propto r^{-1}$ (S wave)
- cutoff smears contact interaction over range $R \sim \Lambda^{-1}$

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3 bosons or 3 distinct fermions in unitary limit (triton)

- naive dimensional analysis $\longrightarrow$ leading contact term of order Q^3
- as hyperradius $R \to 0$ wave functions behave like $\psi(R) \propto R^{-2 \pm i s_0}$ with $s_0 \simeq 1.006$ [Efimov (1971)]

$\longrightarrow$ leading three-body force promoted to order Q^{-1} (limit cycle of RG) [Bedaque, Hammer and van Kolck (1999)]

Effects of iterated one-pion exchange forces

Central OPE (spin-singlet waves)

- $1/r$ singularity – not enough to alter power-law forms of wave functions at small r
- $L \geq 1$ waves: weak scattering $\rightarrow$ Weinberg power counting
- $^1S_0$: similar to expansion around unitary fixed point

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Tensor OPE (spin-triplet waves)

- $1/r^3$ singularity
- wave functions $\psi(r) \propto r^{-1/4}$ multiplied by either sine or exponential function of $1/\sqrt{\lambda_{NN} r}$

$\longrightarrow$ new counting needed [Nogga, Timmermans and van Kolck (2005)]

- leading contact interaction of order $Q^{-1/2}$ in waves with $L \geq 1$
- very slowly converging expansion $\longrightarrow$ better to iterate

Importance of tensor OPE does depend on cutoff Λ

- higher partial waves protected by centrifugal barrier
- only waves above critical momentum resolve singularity
  $\longrightarrow$ OPE not perturbative
- $L \geq 3$: $p_c \gtrsim 2$ GeV $\longrightarrow$ Weinberg counting OK for $\Lambda \lesssim 600$ MeV
- $L \leq 2$: $p_c \lesssim 3m_\pi$ $\longrightarrow$ NTvK counting needed

Three-body forces

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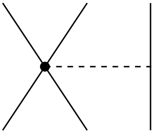
One-pion exchange ("c_D")

- contains two-body contact vertices like $(N^\dagger N)(N^\dagger \sigma N) \cdot \nabla \pi$
- promoted in same way as contact interactions for $L \leq 2$



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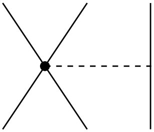

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Contact interaction (“c_E”)

- counting still not known: need to solve 3-body problem with $1/r^3$ potentials [L Platter]
- expect to be promoted $\rightarrow$ order Q^d, $-1 < d < 3$?

A new road map for nuclear EFT

To order Q^3 (N2LO in Weinberg's counting)

Order	NN	NNN
Q^{-1}	${}^1S_0$, ${}^3S_1$ C_0's, LO OPE	
$Q^{-1/2}$	${}^3P_J$, ${}^3D_J$ C_0's	
Q^0	${}^1S_0$ C_2	
$Q^{1/2}$	${}^3S_1$ C_2	
Q^1		${}^1S_0$ C_{D0} OPE
$Q^{3/2}$	${}^3P_J$, ${}^3D_J$ C_2's	${}^3S_1$ C_{D0} OPE
Q^2	${}^1S_0$ C_4, ${}^1P_1$ C_0, NLO OPE, LO TPE	
$Q^{5/2}$	${}^3S_1$ C_4	${}^3P_J$, ${}^3D_J$ C_{D0}'s OPE
Q^3	NLO TPE	${}^1S_0$ C_{D2} OPE, LO 3N TPE
$Q^?$		C_E

- orange terms absent from "N2LO chiral potential"
- red terms absent from "N3LO"
- order Q^{-1}: have to iterate; order $Q^{-1/2}$: probably better to

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- resumming subset of higher-order terms
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- but problems don't arise, provided higher-order terms are small
- general way to ensure this: keep cutoff small, $\Lambda < \Lambda_0$
- introduces artefacts $\propto (Q/\Lambda)^n \longrightarrow$ radius of convergence Λ not Λ_0
- want to keep Λ as large as possible

$\longrightarrow$ leaves only a narrow window: Λ just below Λ_0

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- renormalise all potentially divergent integrals
- iterate all fixed-point or marginal terms, order Q^{-1}
- do not iterate irrelevant terms, order Q^d with $d \geq 0$
- otherwise . . .

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- otherwise . . .

$\longrightarrow$ if very lucky, might discover a new power counting
eg tensor OPE in low partial waves [NTvK]

$\longrightarrow$ more generally, lose any consistent counting
eg effective-range term in short-range potential
[Phillips, Beane and Cohen (1997); and many others]

Effective potential and scattering observables

Contact interactions directly related to "observables" (phase shifts)

- distorted-wave K matrix $\widetilde{K}(p) = -\dfrac{4\pi}{Mp}\tan\big(\delta_{\rm PWA}(p) - \delta_{\rm OPE}(p)\big)$

$\longrightarrow$ either DWBA: expand $\widetilde{K}(p)$ in powers of energy (peripheral w's)

- or DW effective-range expansion: expand $1/\widetilde{K}(p)$ (S waves)
- need to work with finite radial cutoff
  since OPE and centrifugal barrier both singular as $r \longrightarrow 0$
  (but can take this to be very small, provided we keep to our power counting)