Hadron observables from lattice QCD at realistic quark masses

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– QCDSF Collaboration –

With

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Opportunities

- Lattice QCD calculations are now entering the chiral domain
  - Unambiguous test of (HB)ChPT
  - Precise determination of LECs

- With particles becoming unstable, interest is shifting from chiral extrapolation to volume dependence
  - Evaluation of resonances and their properties
  - Derivation of scattering phases and lengths
  - May expect more direct evidence for chiral logs

Outline

Lattice Simulation

Nucleon

Rho

Delta

Nucleon Structure

Conclusions & Outlook

Lattice Simulation

Action

$$N_f = 2$$

$$S = S_G + S_F$$

$$S_G = \beta \sum_{x,\mu<\nu} \left(1 - \frac{1}{3}\mathrm{Re}\,\mathrm{Tr}\, U_{\mu\nu}(x)\right)$$

$$S_F = \sum_x \Big\{ \bar\psi(x)\psi(x) - \kappa\,\bar\psi(x)U_\mu^\dagger(x-\hat\mu)[1+\gamma_\mu]\psi(x-\hat\mu) - \kappa\,\bar\psi(x)U_\mu(x)[1-\gamma_\mu]\psi(x+\hat\mu) - \frac{1}{2}\kappa\, c_{SW}\, g\,\bar\psi(x)\sigma_{\mu\nu}F_{\mu\nu}(x)\psi(x)\Big\}$$

$$\Updownarrow$$

$$\partial_\mu A_\mu^{\mathrm{imp}} = 2m_q P$$

Clover Fermions

Advantages

- Local
- Transfer matrix
- $O(a)$ improved
- Flavor symmetry

  Prerequisite to making contact with $SU(2)$ ChPT

  – Finite size corrections

  – Chiral extrapolation

  – Determination of low-energy constants
- Fast to simulate

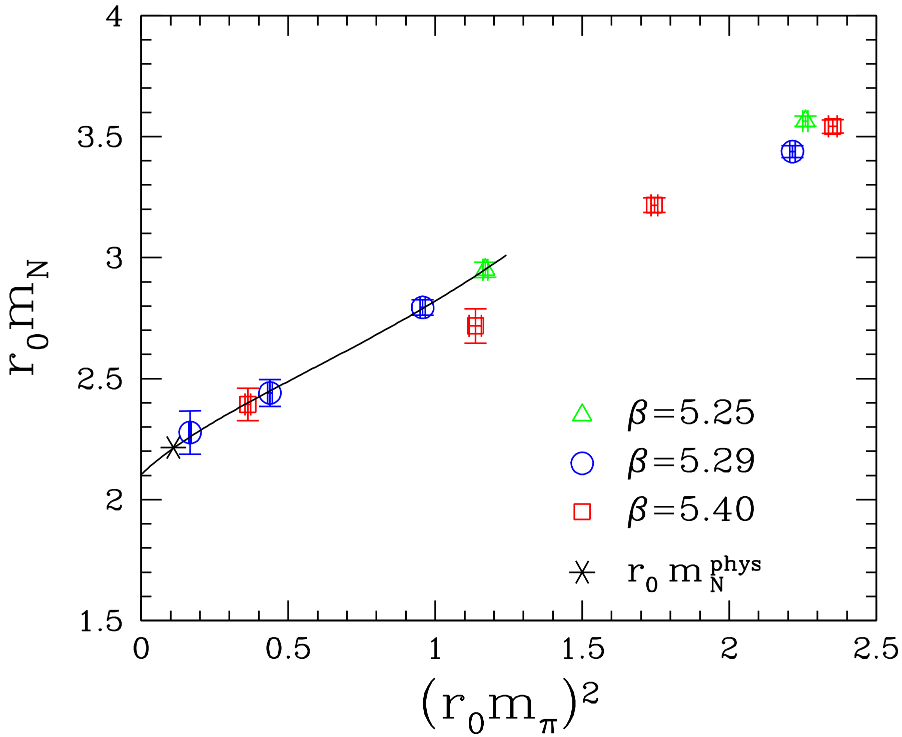
	β	$\kappa_{\rm sea}$	Volume	a [fm]	m_{PS} [MeV]
	5.20	0.13420	$16^3 \times 32$	0.090	1010
	5.20	0.13500	$16^3 \times 32$	0.090	830
	5.20	0.13550	$16^3 \times 32$	0.090	620
	5.25	0.13460	$16^3 \times 32$	0.084	990
	5.25	0.13520	$16^3 \times 32$	0.084	830
	5.25	0.13575	$24^3 \times 48$	0.084	600
	5.25	0.13600	$24^3 \times 48$	0.084	430
	5.26	0.13450	$16^3 \times 32$	0.083	1010
	5.29	0.13400	$16^3 \times 32$	0.080	1170
	5.29	0.13500	$16^3 \times 32$	0.080	930
	5.29	0.13550	$24^3 \times 48$ $16^3 \times 32$ $12^3 \times 32$	0.080	810
	5.29	0.13590	$24^3 \times 48$ $16^3 \times 32$ $12^3 \times 32$	0.080	590
	5.29	0.13605	$24^3 \times 48$	0.080	480
	5.29	0.13620	$24^3 \times 48$	0.080	390
	5.29	0.13632	$40^3 \times 64$ $32^3 \times 64$ $24^3 \times 48$	0.080	250
In progress $\longrightarrow$	5.29	0.13640	$64^3 \times 96$ $40^3 \times 64$	0.080	140
	5.40	0.13500	$24^3 \times 48$	0.072	810
	5.40	0.13560	$24^3 \times 48$	0.072	770
	5.40	0.13610	$24^3 \times 48$	0.072	610
	5.40	0.13625	$24^3 \times 48$	0.072	530
	5.40	0.13640	$32^3 \times 64$	0.072	420
	5.40	0.13660	$32^3 \times 64$	0.072	240

Nucleon

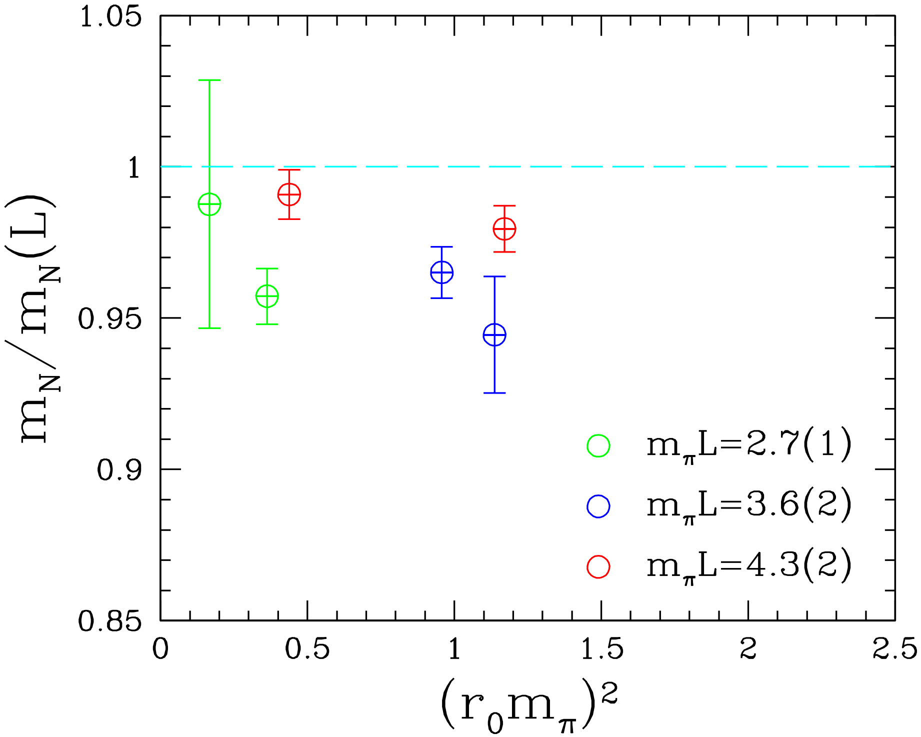
Scale

Nucleon Mass

FS corrected



Corrections



$$r_0 = 0.467\,\text{fm}$$

$$m_N = m_0 - 4c_1 m_{PS}^2 - \frac{3 g_A^{0\,2}}{32\pi f_0^2} m_{PS}^3 + \Bigg[e_1(\mu) - \frac{3}{64\pi^2 f_0^2}\Big(\frac{g_A^{0\,2}}{m_0} - \frac{c_2}{2}\Big) - \frac{3 g_A^{0\,2}}{32\pi^2 f_0^2}\Big(\frac{g_A^{0\,2}}{m_0} - 8c_1 + c_2 + 4c_3\Big) \ln\frac{m_{PS}}{\mu}\Bigg] m_{PS}^4 + \frac{3 g_A^{0\,2}}{256\pi f_0^2 m_0^2} m_{PS}^5 + O(m_{PS}^6)$$

$$m_N - m_N(L) = -\frac{3 g_A^{0\,2} m_0 m_{PS}^2}{16\pi^2 f_0^2} \sum_{|\vec{n}|\neq 0} \int_0^\infty dz K_0\left(\sqrt{m_0^2 z^2/m_{PS}^2 + (1-z)}\,\lambda\right) - \frac{3 m_{PS}^4}{4\pi^2 f_0^2} \sum_{|\vec{n}|\neq 0} \left[(2c_1 - c_3)\frac{K_1(\lambda)}{\lambda} + c_2 \frac{K_2(\lambda)}{\lambda^2}\right] + O(m_{PS}^5)$$

$\mu = 1\,\mathrm{GeV}\,, \quad c_2 = 3.2\,\mathrm{GeV}^{-1}\,, \quad c_3 = -3.4\,\mathrm{GeV}^{-1}$ $\qquad \lambda = m_{PS}|\vec{n}|L$

QCDSF

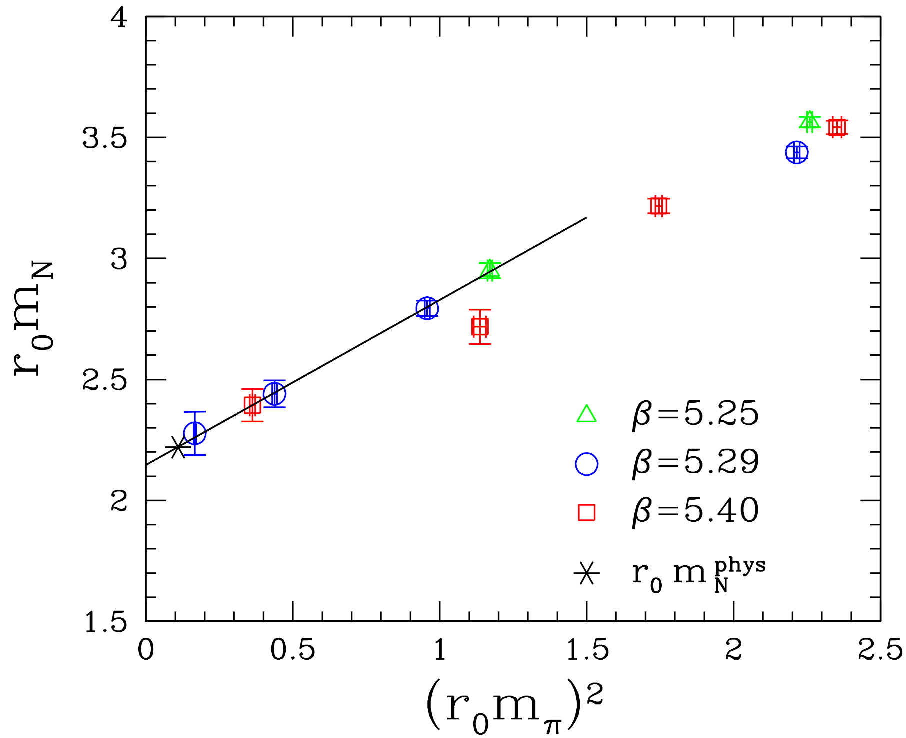
Nucleon Sigma Term

$$\sigma_N = m_\ell \frac{d\, m_N(m_\ell)}{d\, m_\ell} \stackrel{!}{=} m_\pi^2 \frac{d\, m_N(m_\pi)}{d\, m_\pi^2} = -\,4\, c_1 m_\pi^2 - \frac{9\, g_A^{0\;2}}{64\, \pi\, f_0^2} m_\pi^3 + O(m_\pi^4)$$

$$\boxed{\sigma_N = 40.1 \pm 2.8\,\text{MeV}}$$

$$m_\pi \lesssim 400\,\text{MeV}$$

Fitting to larger pion masses gives considerably higher values of σ_N cf. ETM



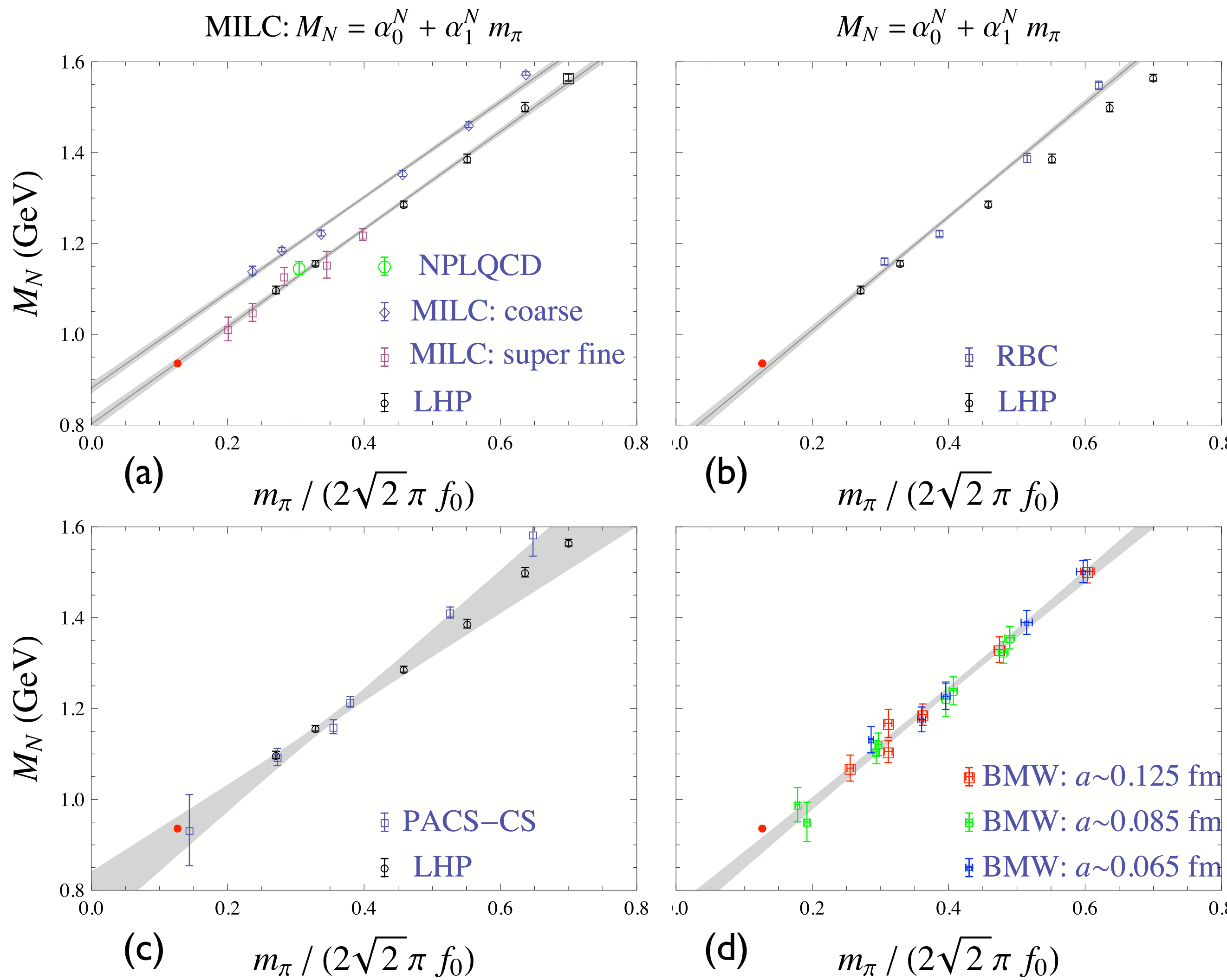
Approximately quadratic in pion mass

$$m_N = m_0 - 4\, c_1\, m_{PS}^2$$

in contrast to BMW, ETM, LHP and MILC, which favor a linear nonanalytic dependence

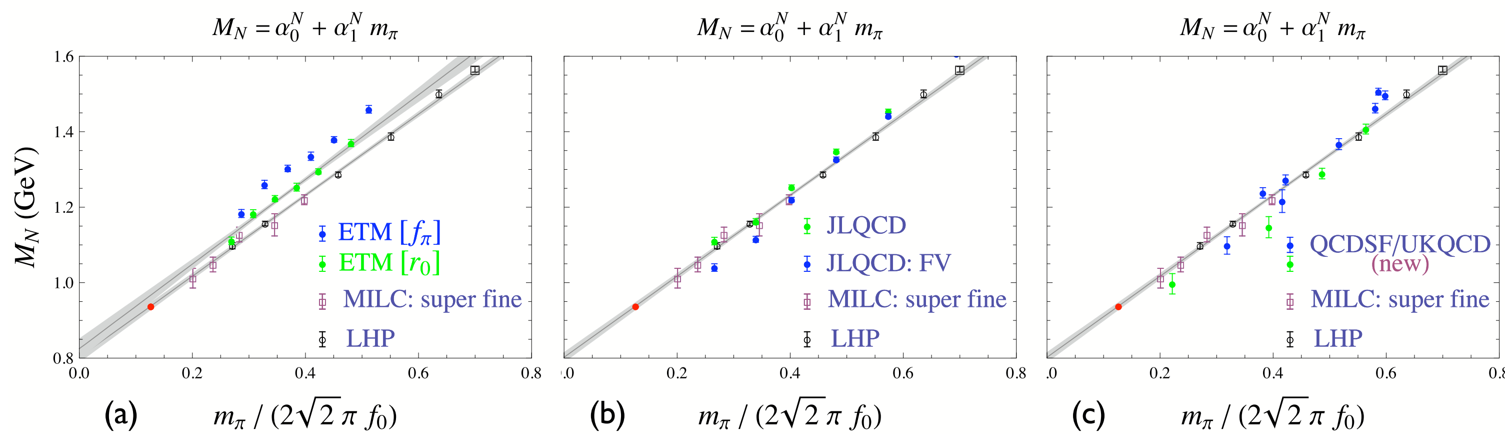
Walker-Loud

$N_f = 2+1$



MILC: $M_N = \alpha_0^N + \alpha_1^N m_\pi$
$M_N = \alpha_0^N + \alpha_1^N m_\pi$
M_N (GeV)
NPLQCD
MILC: coarse
MILC: super fine
LHP
RBC
PACS−CS
BMW: a~0.125 fm
BMW: a~0.085 fm
BMW: a~0.065 fm
(a)
(b)
(c)
(d)
$m_\pi / (2\sqrt{2}\pi f_0)$

$N_f = 2$



$M_N = \alpha_0^N + \alpha_1^N m_\pi$
M_N (GeV)
ETM [f_π]
ETM [r_0]
MILC: super fine
LHP
JLQCD
JLQCD: FV
QCDSF/UKQCD
(new)
(a)
(b)
(c)
$m_\pi / (2\sqrt{2}\pi f_0)$

Rho

The ρ meson is practically a two-pion resonance. It has isospin 1, and the two pions form a p-wave state

We denote the pion momentum in the center-of-mass frame by $k = |\vec{k}|$. Phenomenologically, the scattering phase shift $\delta_{11}(k)$ is very well described by the effective range formula

$$\frac{k^3}{E} \cot \delta_{11}(k) = \frac{24\pi}{g^2_{\rho\pi\pi}} \left(k_\rho^2 - k^2\right)$$

where $E = 2\sqrt{k^2 + m_\pi^2}$ and $k_\rho = \frac{1}{2}\sqrt{m_\rho^2 - 4m_\pi^2}$. The width of the ρ is given by

$$\Gamma_\rho = \frac{g^2_{\rho\pi\pi}}{6\pi} \frac{k_\rho^3}{m_\rho^2}$$

Experimentally, $\Gamma_\rho = 146\,\text{MeV}$, which translates into

$$g_{\rho\pi\pi} = 5.9$$

The physical ρ mass (at any given m_π) is obtained from the momentum k, at which the phase shift $\delta_{11}(k)$ passes through $\pi/2$

In the case of noninteracting pions, the possible energy levels in a periodic box of length L are given by

$$E = 2\sqrt{k^2 + m_\pi^2} \qquad\qquad k = \frac{2\pi|\vec{n}|}{L}\,, \quad \vec{n} \in \mathbb{N}^3$$

In the interacting case, k is the solution of a nonlinear equation involving the phase shift

$$\delta_{11}(k) = \arctan\left\{\frac{\pi^{3/2}q}{\mathcal{Z}_{00}(1, q^2)}\right\} \bmod \pi\,, \quad q = \frac{kL}{2\pi}$$

Task

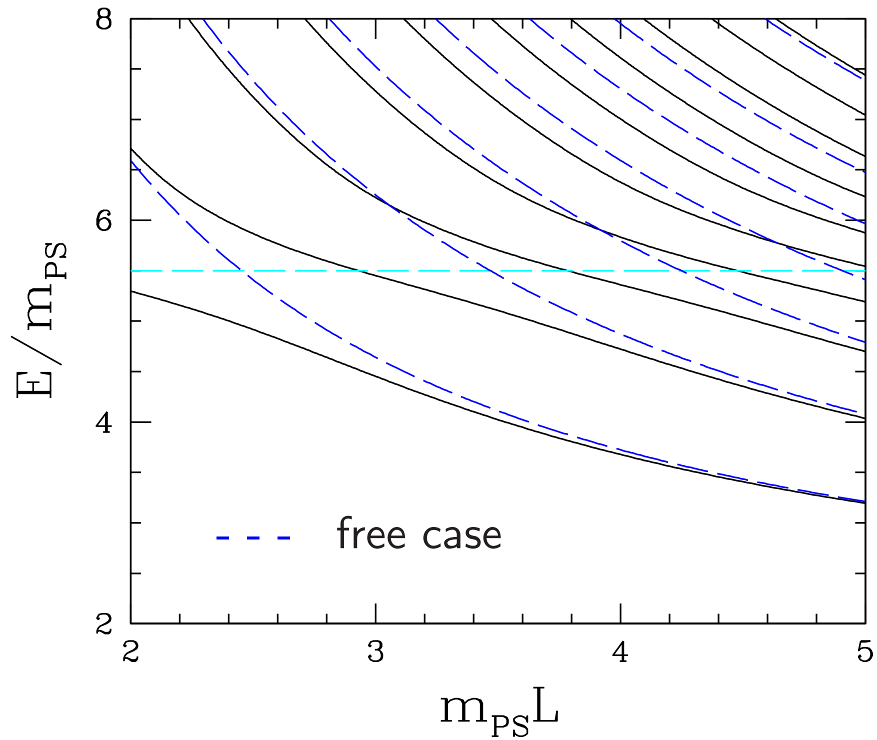
$$E\,|_{m_\pi, L} \longrightarrow k \longrightarrow \delta_{11}(k) \longrightarrow m_\rho\,,\ \Gamma_\rho$$

by fitting $\delta_{11}(k)$
to effective range
formula

Lüscher, Wiese

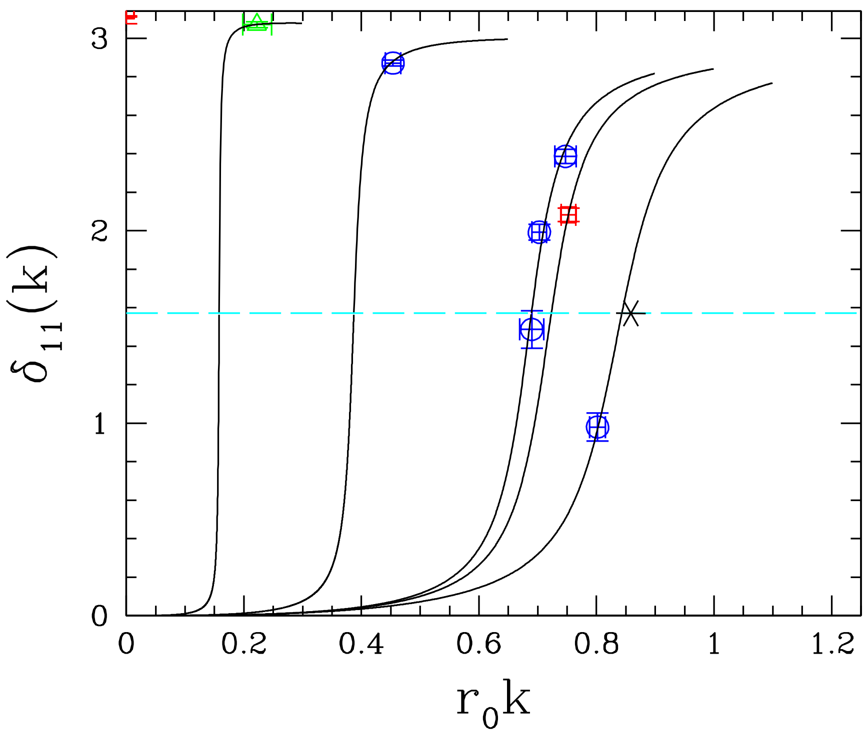
Energy Levels

Physical m_π, m_ρ and Γ_ρ



Useful region

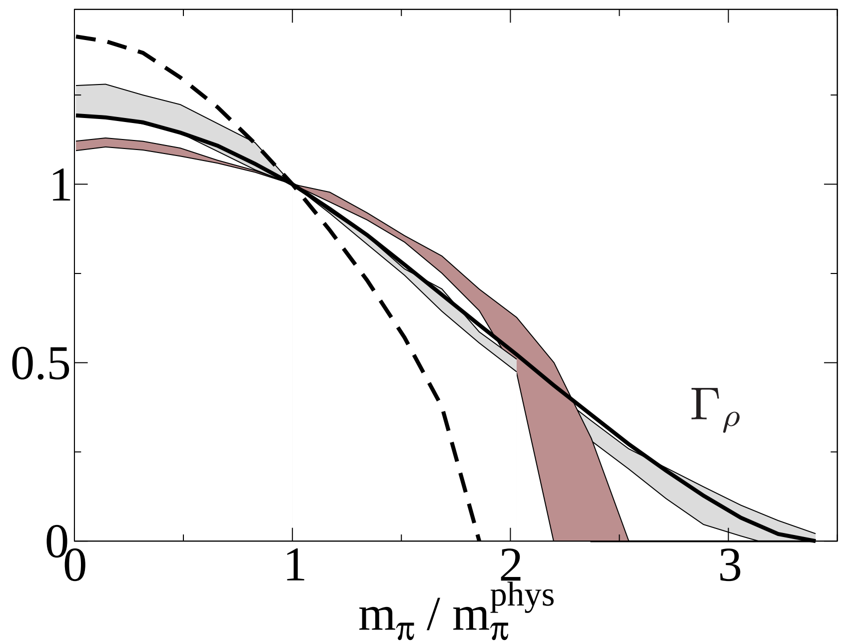
Phase Shift



$m_\pi =$ 430 390 250 150 MeV
240

↑
Fit

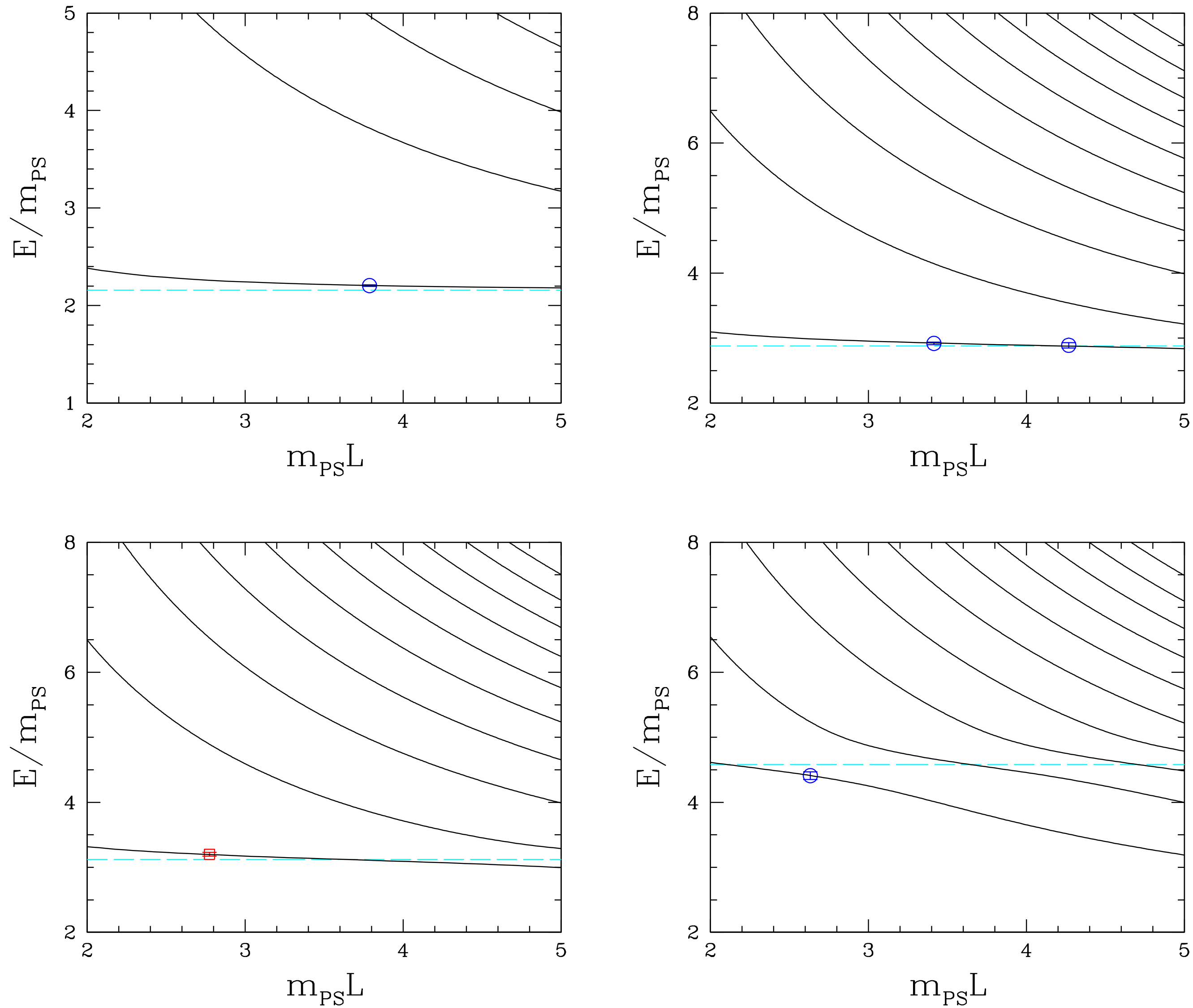
$g_{\rho\pi\pi} = 5.1 \pm 0.4$



Ríos Márquez et al.

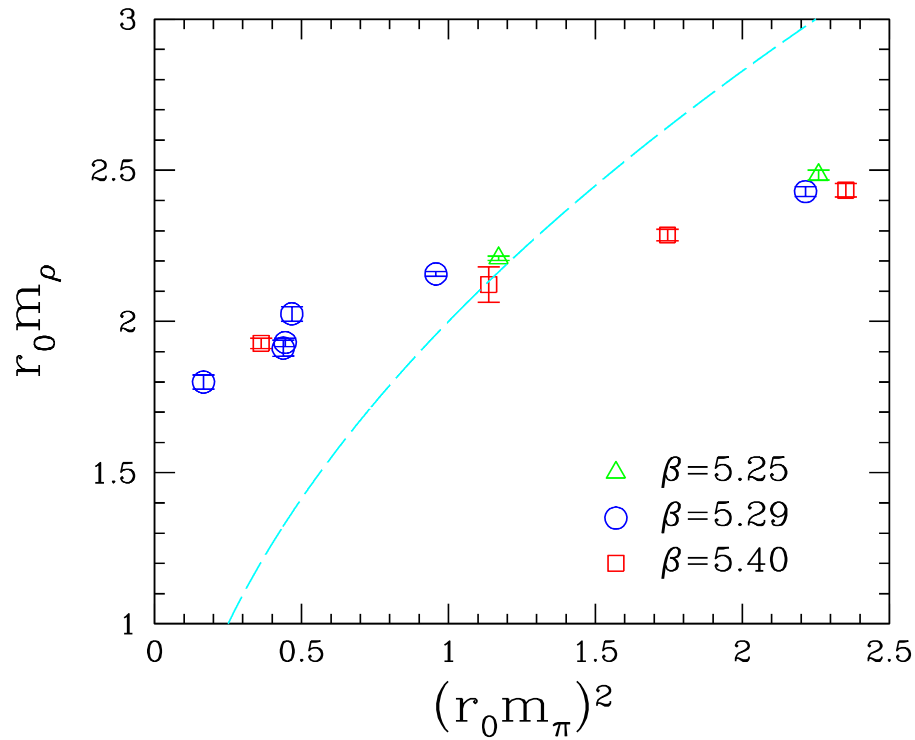
$$g_{\rho\pi\pi}\,|_{m_\pi=140\,\mathrm{MeV}} = 5.9 \pm 0.5$$

Actual levels

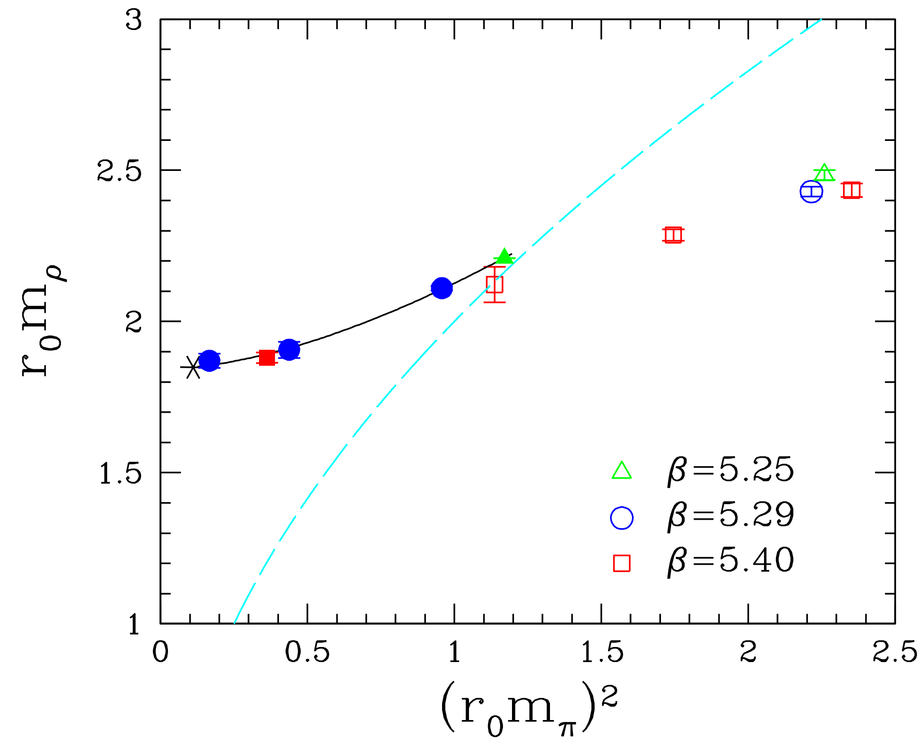


Rho Mass

Lowest energy levels



True ρ mass



Chiral fit: $m_\rho = m_\rho^0 + c_1 m_\pi^2 + c_2 m_\pi^3 + c_3 m_\pi^4 \ln(m_\pi^2)$

Kink ?

Bruns & Meißner

Delta

The $\Delta(1232)$ baryon is an elastic p-wave pion-nucleon resonance with isospin 3/2. Its scattering phase shift $\delta_{3/2\,1}(k)$ is very well described by the effective range formula

$$\frac{k^3}{E}\cot\,\delta_{3/2\,1}(k) = \frac{24\pi}{g^2_{\Delta N\pi}}\left(m^2_{\Delta} - E^2\right)$$

Here

$$E = \sqrt{k^2+m^2_\pi} + \sqrt{k^2+m^2_N}\,, \quad m_\Delta = \sqrt{k^2_\Delta+m^2_\pi} + \sqrt{k^2_\Delta+m^2_N}$$

$$\Gamma_\Delta = \frac{g^2_{\Delta N\pi}}{6\pi}\frac{k^3_\Delta}{m^2_\Delta}$$

Experimentally: $\Gamma_\Delta = 118\,\text{MeV} \implies \frac{g^2_{\Delta N\pi}}{4\pi} = 14.6$

Free case

$$k = \frac{2\pi|\vec{n}|}{L}\,, \quad \vec{n}\in\mathbb{N}^3$$

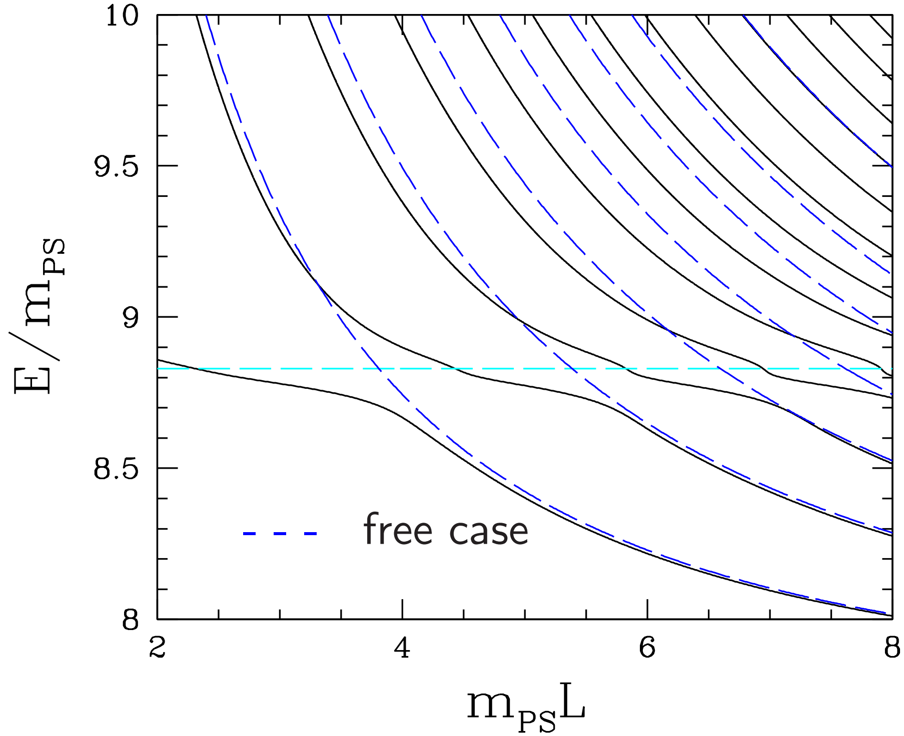
Interacting case

$$\delta_{11}(k) = \arctan\left\{\frac{\pi^{3/2}q}{\mathcal{Z}_{00}(1,q^2)}\right\} \bmod \pi\,, \quad q = \frac{kL}{2\pi}$$

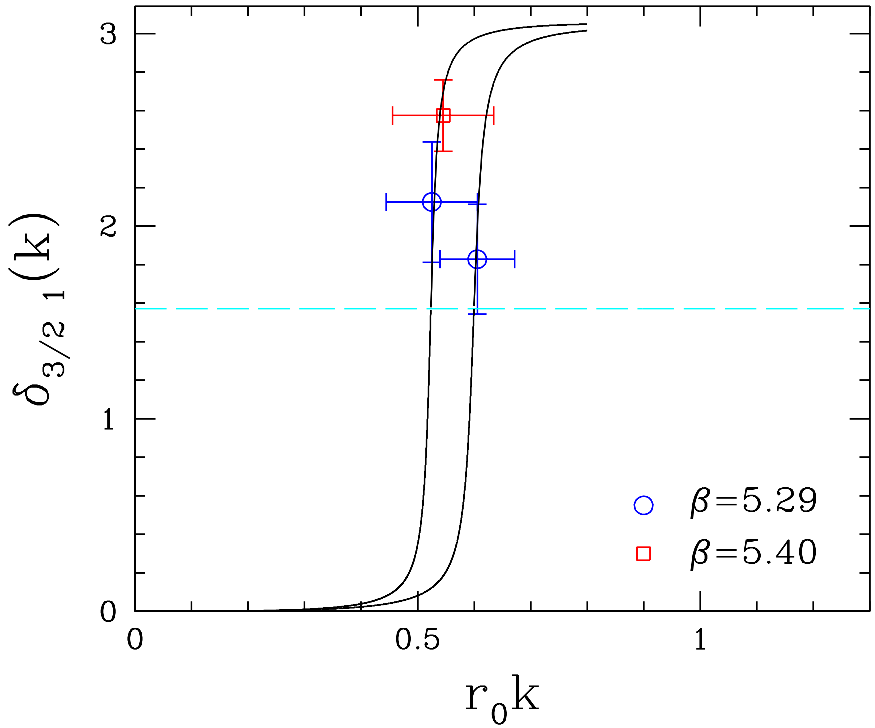
Bernard et al.

Energy Levels

Physical m_π, m_Δ and Γ_Δ

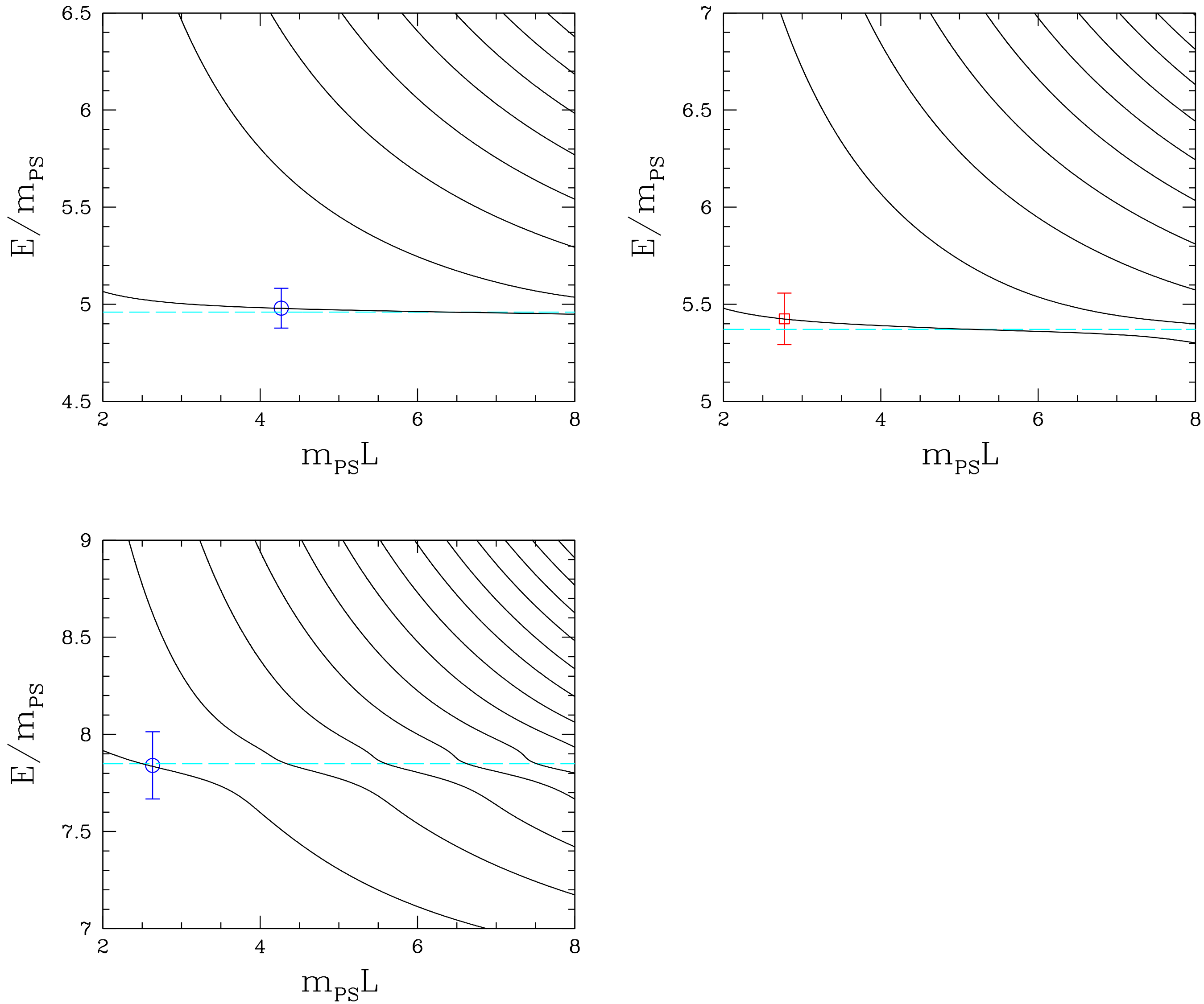


Phase Shift



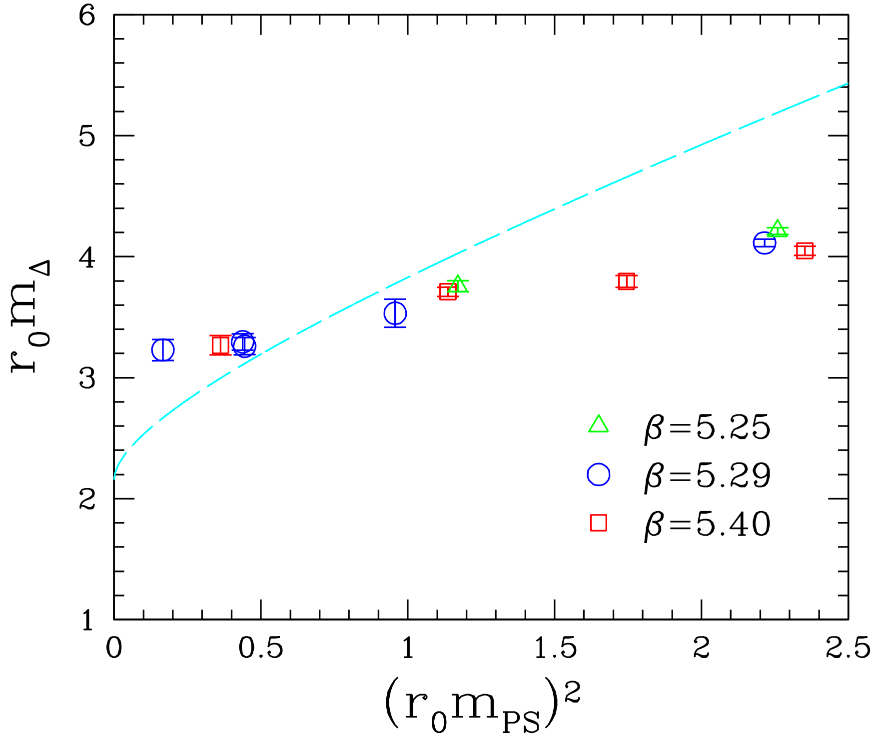
$m_\pi =$ 250 150 MeV

Actual levels

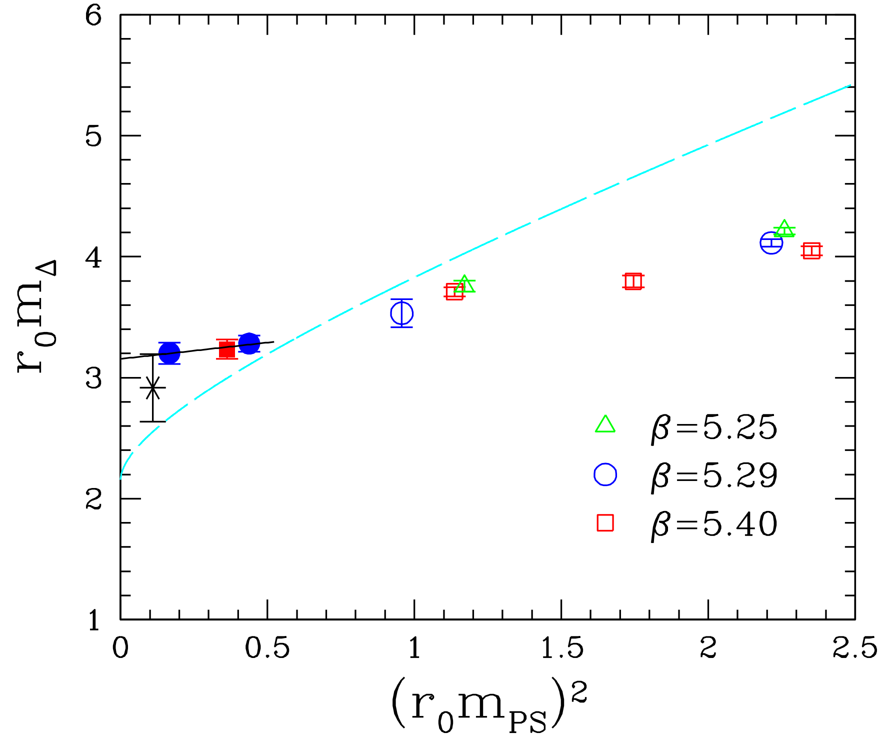


Delta Mass

Lowest energy levels



True Δ mass



Chiral fit: $m_\Delta = m_\Delta^0 - 4c_1 m_\pi^2 + c_2 m_\pi^3$

Bernard et al.

Beyond Delta

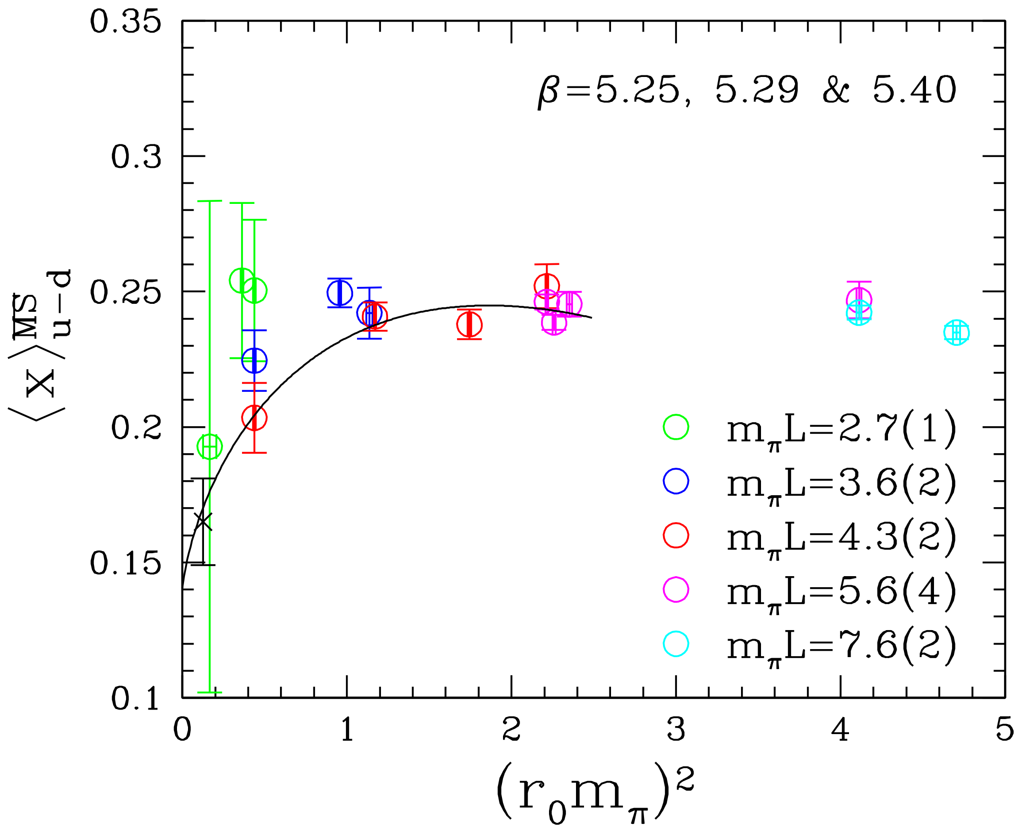
$$\left.\begin{aligned} N(1440) \to\ & N\pi \quad \text{Roper} \\ & \Delta\pi \\ & N\eta \\ N^*(1535) \to\ & N\pi \\ & N\eta \\ \vdots \end{aligned}\right\} \Longrightarrow \text{Multichannel Lüscher formalism}$$

Lage, Meißner & Rusetsky

Nucleon Structure

Moment of Parton Distribution

$$\langle x \rangle = \int dx\, x \left[u(x, Q^2) - d(x, Q^2) \right]$$



Fit to largest $m_\pi L$

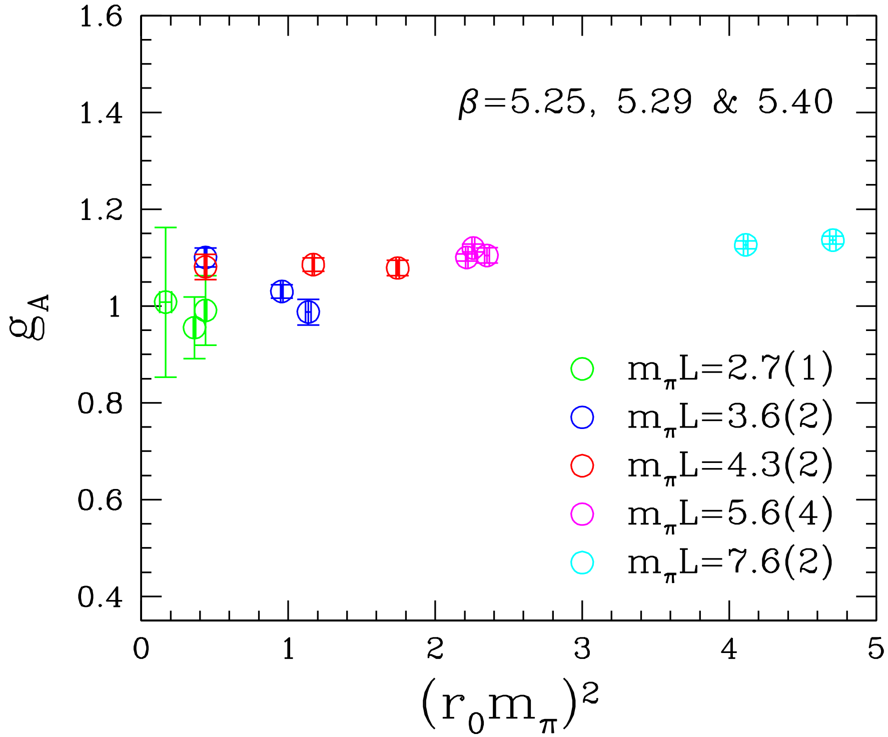
$$
\begin{aligned}
\langle x \rangle_{u-d} &\equiv v_2 \\
&= v_2^0 + \frac{v_2^0 m_{PS}^2}{(4\pi f_0)^2}\Bigg\{ -(3g_A^{0\,2}+1)\ln\frac{m_{PS}^2}{\lambda^2} - 2g_A^{0\,2} + g_A^{0\,2}\frac{m_{PS}^2}{m_0^2}\left(1+3\ln\frac{m_{PS}^2}{m_0^2}\right) \\
&- \frac{1}{2}g_A^{0\,2}\frac{m_{PS}^4}{m_0^4}\ln\frac{m_{PS}^2}{m_0^2} + g_A^{0\,2}\frac{m_{PS}}{\sqrt{4m_0^2-m_{PS}^2}}\left(14-8\frac{m_{PS}^2}{m_0^2}+\frac{m_{PS}^4}{m_0^4}\right) \\
&\times \arccos\left(\frac{m_{PS}}{2m_0}\right)\Bigg\} + \frac{\Delta v_2^0 g_A^0 m_{PS}^2}{3(4\pi f_0)^2}\Bigg\{2\frac{m_{PS}^2}{m_0^2}\left(1+3\ln\frac{m_{PS}^2}{m_0^2}\right) - \frac{m_{PS}^4}{m_0^4}\ln\frac{m_{PS}^2}{m_0^2} \\
&+ \frac{2m_{PS}(4m_0^2-m_{PS}^2)^{\frac{3}{2}}}{m_0^4}\arccos\left(\frac{m_{PS}}{2m_0}\right)\Bigg\} + 4m_{PS}^2\frac{c_8^{(r)}(\lambda)}{M_0^2} + \mathcal{O}(p^3)
\end{aligned}
$$

Dorati, Gail & Hemmert

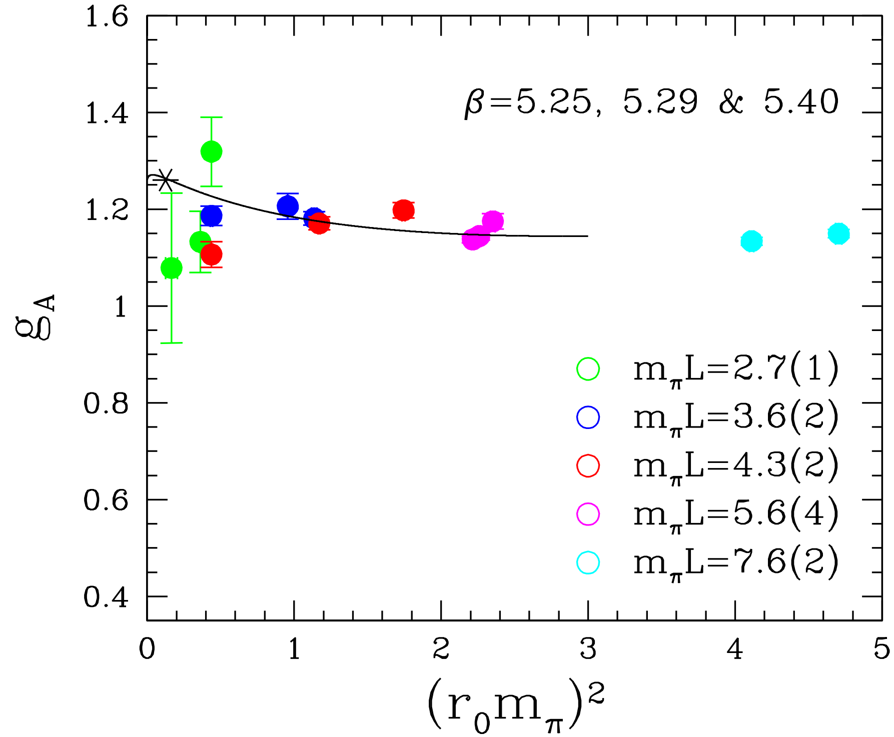
Finite size corrections not known so far

Axial Coupling

Uncorrected



FS corrected



Nonperturbatively renormalized

Combined fit

$$g_A = g_A^0 + \left[4\, B_9^r(\lambda) - 8\, g_A^0 B_{20}^r(\lambda) - \frac{g_A^{0\,3}}{16\pi^2 f_0^2} - \frac{25 c_A^2 g_1}{324\pi^2 f_0^2} + \frac{19 c_A^2 g_A^0}{108\pi^2 f_0^2}\right] m_{PS}^2$$

$$- \frac{m_{PS}^2}{4\pi^2 f_0^2}\left[g_A^{0\,3} + \frac{1}{2}\, g_A^0\right] \ln\frac{m_{PS}}{\lambda} + \frac{4 c_A^2 g_A^0}{27\pi \Delta_0 f_0^2} m_{PS}^3$$

$$+ \left[25 c_A^2 g_1 \Delta_0^2 - 57 c_A^2 g_A^0 \Delta_0^2 - 24 c_A^2 g_A^0 m_{PS}^2\right] \frac{\sqrt{m_{PS}^2 - \Delta_0^2}}{81\pi^2 f_0^2 \Delta_0} \arccos\frac{\Delta_0}{m_{PS}}$$

$$+ \frac{25 c_A^2 g_1 \left(2\Delta_0^2 - m_{PS}^2\right)}{162\pi^2 f_0^2} \ln\frac{2\Delta_0}{m_{PS}} + \frac{c_A^2 g_A^0 \left(3 m_{PS}^2 - 38\Delta_0^2\right)}{54\pi^2 f_0^2} \ln\frac{2\Delta_0}{m_{PS}} + \mathcal{O}(m_{PS}^4)$$

$$\Delta_0 = 0.271\ \text{GeV}$$

$$
\begin{aligned}
g_A - g_A(L) \ &= \frac{g_A^0 m_{PS}^2}{4\pi^2 f_0^2} \sum_{|\vec{n}|\neq 0} \frac{K_1(\lambda)}{\lambda} - \frac{g_A^{0\,3} m_{PS}^2}{6\pi^2 f_0^2} \sum_{|\vec{n}|\neq 0} \left[K_0(\lambda) - \frac{K_1(\lambda)}{\lambda} \right] \\
&- \left(\frac{25 c_A^2 g_1}{81\pi^2 f_0^2} - \frac{c_A^2 g_A^0}{\pi^2 f_0^2} \right) \int_0^\infty dy\, y \sum_{|\vec{n}|\neq 0} \left[K_0(\lambda_f) - \frac{\lambda_f}{3} K_1(\lambda_f) \right] \\
&- \frac{8 c_A^2 g_A^0}{27\pi^2 f_0^2} \int_0^\infty dy \sum_{|\vec{n}|\neq 0} \frac{f(m_{PS}, y)^2}{\Delta_0} \left[K_0(\lambda_f) - \frac{K_1(\lambda_f)}{\lambda_f} \right] \\
&+ \frac{4 c_A^2 g_A^0}{27\pi f_0^2} \frac{m_{PS}^3}{\Delta_0} \sum_{|\vec{n}|\neq 0} \frac{\mathrm{e}^{-\lambda}}{\lambda} + \mathcal{O}(m_{PS}^4)
\end{aligned}
$$

$$
f(m_{PS}, y) = \sqrt{m_{PS}^2 + y^2 + 2y\Delta_0}\,, \ \lambda_f = f(m_{PS}, y)|\vec{n}|L
$$

Conclusions & Outlook

- Simulations at the physical pion mass with Wilson-type fermions progressing
  - Improvement of algorithms
  - Increase of computing power

- Successful (?) test of HBChPT

  Dispersion method of Lüscher can be mapped upon HBChPT

  QCDSF

- Computation of phase shifts and resonance masses of ρ and Δ in chiral regime progressing

  Lowest energy level E sufficient

- To exploit the full potential of lattice calculations, a major investment in finite volume corrections is needed
  - Finite volume corrections of hadron observables
  - Inelastic scattering in finite volume