(Manifestly Lorentz-Invariant) Baryon Chiral Perturbation Theory

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1. Introduction

Effective field theory

> ... if one writes down the most general possible Lagrangian, including all terms consistent with assumed symmetry principles, and then calculates matrix elements with this Lagrangian to any given order of perturbation theory, the result will simply be the most general possible S−matrix consistent with analyticity, perturbative unitarity, cluster decomposition and the assumed symmetry principles. ... [1]

> ... if we include in the Lagrangian all of the infinite number of interactions allowed by symmetries, then there will be a counterterm available to cancel every ultraviolet divergence. ... [2]

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[1] S. Weinberg, Physica A 96, 327 (1979)

[2] S. Weinberg, The Quantum Theory of Fields, Vol. I, 1995, Chap. 12

Perturbative calculations in effective field theory require two main ingredients

1. Knowledge of the most general effective Lagrangian

   (a) Mesonic ChPT [SU(3)×SU(3)] [3] (π, K, η)

$$\underbrace{2}_{\mathcal{O}(q^2)} \quad + \quad \underbrace{10+2}_{\mathcal{O}(q^4)} \quad + \quad \underbrace{90+4+23}_{\mathcal{O}(q^6)} \quad + \quad \cdots$$

   - q: Small quantity such as a pion mass
   - Even powers
   - Two-loop level

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[3] J. Gasser and H. Leutwyler, Nucl. Phys. B250, 465 (1985);
H. W. Fearing and S. S. , Phys. Rev. D 53, 315 (1996);
J. Bijnens, G. Colangelo, G. Ecker, JHEP 02, 020 (1999);
T. Ebertshäuser, H. W. Fearing, S. S., Phys. Rev. D 65, 054033 (2002);
J. Bijnens, L. Girlanda, P. Talavera, Eur. Phys. J. C 23, 539 (2002)

(b) Baryonic ChPT [SU(2)×SU(2)×U(1)] [4] (π, N)

$$\underbrace{2}_{\mathcal{O}(q)} + \underbrace{7}_{\mathcal{O}(q^2)} + \underbrace{23}_{\mathcal{O}(q^3)} + \underbrace{118}_{\mathcal{O}(q^4)} + \quad \cdots$$

– Odd and even powers (spin)

– One-loop level

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[4] J. Gasser, M. E. Sainio, A. Švarc, Nucl. Phys. B307, 779 (1988);
V. Bernard, N. Kaiser, U.-G. Meißner, Int. J. Mod. Phys. E 4, 193 (1995);
G. Ecker and M. Mojžiš, Phys. Lett. B 365, 312 (1996);
N. Fettes, U.-G. Meißner, M. Mojžiš, S. Steininger, Ann. Phys. (N.Y.) 283, 273 (2000)

2. Consistent expansion scheme for observables

(a) Tree-level diagrams, loop diagrams $\rightsquigarrow$ ultraviolet divergences, regularization (of infinities)

(b) Renormalization condition

(c) Power counting scheme for renormalized diagrams

(d) Remove regularization

Commonly used methods

(a) Expansion in powers of coupling constants (e. g., QED)

(b) Loop expansion (expansion in $\hbar$)

(c) ChPT: Momentum and quark mass expansion at fixed ratio m_{quark}/q^2 [5]

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[5] J. Gasser and H. Leutwyler, Annals Phys. 158, 142 (1984)

2. Renormalization and power counting

- **Most general Lagrangian**

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi} + \mathcal{L}_{\pi N} = \mathcal{L}_{\pi}^{(2)} + \mathcal{L}_{\pi}^{(4)} + \cdots + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \cdots$$

Basic Lagrangian

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}\left(i\gamma_{\mu}\partial^{\mu} - \boxed{m}\right)\Psi - \frac{1}{2}\frac{\boxed{g_A}}{F}\bar{\Psi}\gamma_{\mu}\gamma_5\tau^a\partial^{\mu}\pi^a\Psi + \cdots$$

m, g_A, and F denote the chiral limit of the physical nucleon mass, the axial-vector coupling constant, and the pion-decay constant, respectively

- **Power counting:** Associate chiral order D with a diagram
  - Square of the lowest-order pion mass:
    $M^2 = B(m_u + m_d) \sim \mathcal{O}(q^2)$
  - Nucleon mass in the chiral limit $m \sim \mathcal{O}(q^0)$
  - Loop integration in n dimensions $\sim \mathcal{O}(q^n)$
  - Vertex from $\mathcal{L}_\pi^{(2k)} \sim \mathcal{O}(q^{2k})$
  - Vertex from $\mathcal{L}_{\pi N}^{(k)} \sim \mathcal{O}(q^k)$
  - Nucleon propagator $\sim \mathcal{O}(q^{-1})$
  - Pion propagator $\sim \mathcal{O}(q^{-2})$

- Renormalization
  - Regularize (typically dimensional regularization)

$$
\begin{aligned}
I(M^2,\mu^2,n) &= \mu^{4-n}\int\frac{d^nk}{(2\pi)^n}\frac{i}{k^2-M^2+i0^+}\\
&= \frac{M^2}{16\pi^2}\left[R+\ln\left(\frac{M^2}{\mu^2}\right)\right]+O(n-4),
\end{aligned}
$$

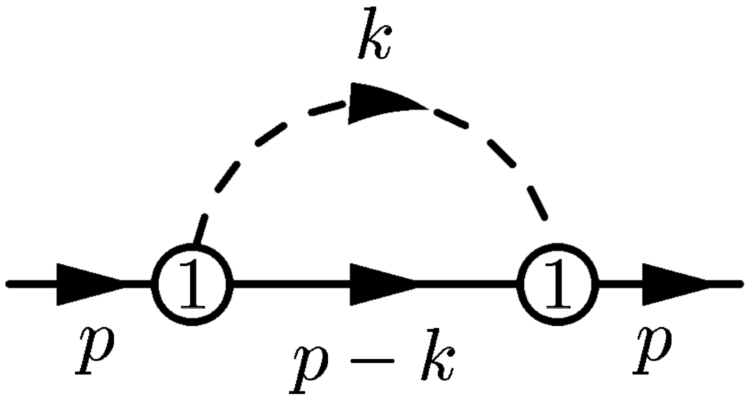
where

$$
\boxed{\mathrm{R}} = \frac{2}{n-4} - [\ln(4\pi)+\Gamma'(1)] - 1 \rightarrow \boxed{\infty}
$$

Scale μ: 't Hooft parameter (integral has the same dimension for arbitrary n)

  - Adjust counterterms such that they absorb all the divergences occurring in the calculation of loop diagrams
  - Renormalization prescription: Adjust finite pieces such that renormalized diagrams satisfy a given power counting

- **Example: Contribution to nucleon mass**



Goal: $\boldsymbol{D = n \cdot 1 - 2 \cdot 1 - 1 \cdot 1 + 2 \cdot 1 = n - 1}$

$$\Sigma = -\frac{3g_{A0}^2}{4F_0^2}\left[(\not{p}+m)I_N + M^2(\not{p}+m)I_{N\pi}(-p,0)+\cdots\right]$$

Apply $\widetilde{\mathrm{MS}}$ renormalization scheme

$$\begin{aligned}\Sigma_r &= -\frac{3g_{Ar}^2}{4F_r^2}[M^2(\not{p}+m)\underbrace{I_{N\pi}^r(-p,0)}_{-\frac{1}{16\pi^2}+\cdots}+\cdots] \\ &= \mathcal{O}(q^2)\end{aligned}$$

GSS [6]: It turns out that loops have a much more complicated low-energy structure if baryons are included. Because the nucleon mass m_N does not vanish in the chiral limit, the mass scale m (nucleon mass in the chiral limit) occurs in the effective Lagrangian $\mathcal{L}_{\pi N}^{(1)}$ ... . This complicates life a lot.

[6] J. Gasser, M. E. Sainio, A. Švarc, Nucl. Phys. B307, 779 (1988)

Solutions

- Heavy-baryon chiral perturbation theory [7]

- Infrared regularization (IR) [8]

  Special treatment of (the Feynman parameterization of) one-loop integrals

$$\frac{1}{ab} = \int_0^1 \frac{dx}{[ax + b(1-x)]^2}$$

$$a = (k-p)^2 - m^2 + i0^+, \quad b = k^2 - M^2 + i0^+$$

$$H = \int_0^1 dx \cdots = \int_0^\infty dx \cdots - \int_1^\infty dx \cdots \equiv I + R$$

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[7] E. Jenkins and A. V. Manohar, Phys. Lett. B 255, 558 (1991); V. Bernard, N. Kaiser, J. Kambor, U.-G. Meißner, Nucl. Phys. B388, 315 (1992)

[8] T. Becher and H. Leutwyler, Eur. Phys. J. C 9, 643 (1999)

- I: power counting o.k.
- R: violates power counting; regular, i.e., can be absorbed in counterterms

- Extended on-mass-shell (EOMS) scheme [9]

  Main idea: Perform additional subtractions such that renormalized diagrams satisfy the power counting

  Motivation for this approach [10]

  Terms violating the power counting are analytic in small quantities (and can thus be absorbed in a renormalization of counterterms)

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[9] T. Fuchs, J. Gegelia, G. Japaridze, S. S., Phys. Rev. D 68, 056005 (2003)

[10] J. Gegelia and G. Japaridze, Phys. Rev. D 60, 114038 (1999)

– Example (chiral limit)

$$H(p^2, m^2; n) = \int \frac{d^n k}{(2\pi)^n} \frac{i}{[(k-p)^2 - m^2 + i0^+][k^2 + i0^+]}$$

Small quantity

$$\Delta = \frac{p^2 - m^2}{m^2} = \mathcal{O}(q)$$

We want the renormalized integral to be of order

$$D = n - 1 - 2 = n - 3$$

Result of integration [11]

$$H \sim F(n, \Delta) + \Delta^{n-3} G(n, \Delta)$$

F and G are hypergeometric functions; analytic in Δ for arbitrary n

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[11] J. Gegelia, G. Japaridze, K. S. Turashvili, Theor. Math. Phys. 101, 1313 (1994)

F corresponds to first expanding the integrand in small quantities and then performing the integration

$\Rightarrow$ Algorithm: Expand integrand in small quantities and subtract those (integrated) terms whose order is smaller than suggested by the power counting

Here:

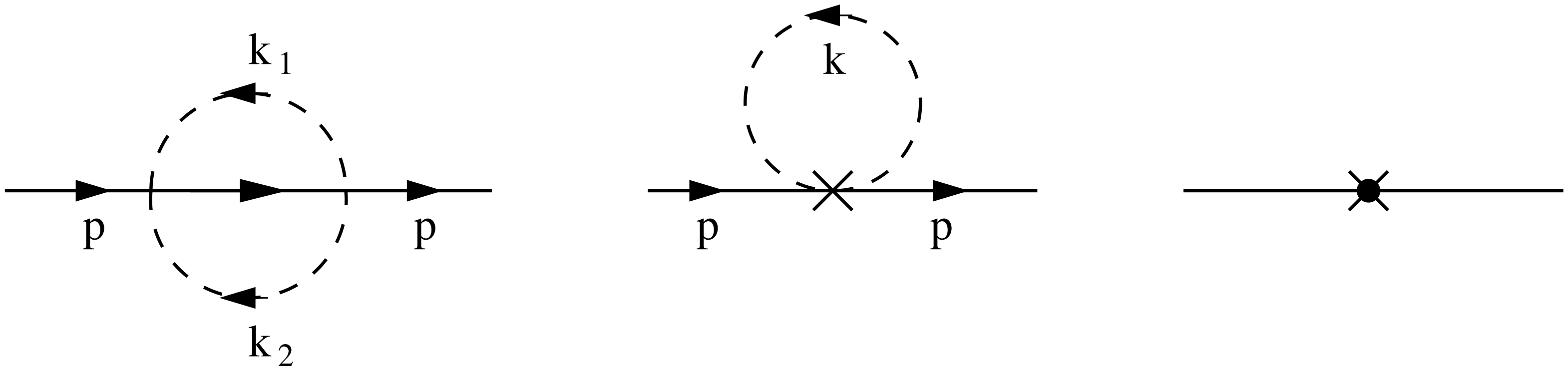
$$
\begin{aligned}
H^{\text{subtr}} &= \int \frac{d^n k}{(2\pi)^n} \frac{i}{(k^2 - 2k\cdot p + i0^+)(k^2 + i0^+)}\bigg|_{p^2=m^2} \\
&= -2\bar{\lambda} + \frac{1}{16\pi^2} + O(n-4)
\end{aligned}
$$

where

$$
\bar{\lambda} = \frac{m^{n-4}}{(4\pi)^2}\left\{\frac{1}{n-4} - \frac{1}{2}\left[\ln(4\pi) + \Gamma'(1) + 1\right]\right\}
$$

$$
H^R = H - H^{\text{subtr}} = \mathcal{O}(q^{n-3})
$$

- Reformulation of IR in terms of EOMS
  - Formal equivalence shown at one-loop level [12]
  - Higher-order loops [13]



(a) (b) (c)

  - heavy degrees of freedom [14]

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[12] M. R. Schindler, J. Gegelia, S. S., Phys. Lett. B 586, 258 (2004)
[13] M. R. Schindler, J. Gegelia, S. S., Nucl. Phys. B 682, 367 (2004)
[14] T. Fuchs, M. R. Schindler, J. Gegelia, S. S., Phys. Lett. B 575, 11 (2003)

3. Applications I

Mass of the nucleon at $\mathcal{O}(q^3)$

- **GSS ($\widetilde{\mathrm{MS}}$)** [15]

$$m_N = m - 4c_{1r}M^2 + \boxed{\frac{3g_{Ar}^2 M^2}{32\pi^2 F_r^2}m} - \frac{3g_{Ar}^2 M^3}{32\pi^2 F_r^2}$$

Solution to power counting problem

Term violating the power counting is analytic in small quantities and can thus be absorbed in counterterms

Rewrite

$$c_{1r} = c_1 + \delta c_1, \quad \delta c_1 = \frac{3mg_A^2}{128\pi^2 F^2} + \cdots$$

Final result for the nucleon mass at order $\mathcal{O}(q^3)$

$$m_N = m - 4c_1 M^2 - \frac{3g_A^2 M^3}{32\pi^2 F^2} + \mathcal{O}(M^4)$$

[15] J. Gasser, M. E. Sainio, A. Švarc, Nucl. Phys. B307, 779 (1988)

Mass of the nucleon at $\mathcal{O}(q^4)$ [16]

$$m_N = m + k_1 M^2 + k_2 M^3 + k_3 M^4 \ln\left(\frac{M}{m}\right) + k_4 M^4 + O(M^5)$$

$$k_3 = \frac{3}{32\pi^2 F^2}\left(8c_1 - c_2 - 4c_3 - \frac{g_A^2}{m}\right),$$

$$k_4 = \frac{3g_A^2}{32\pi^2 F^2 m}(1 + 4c_1 m) + \frac{3}{128\pi^2 F^2}c_2 - 16e_{38} - 2e_{115} - 2e_{116}.$$

$$m = [938.3 - 74.8 + 15.3 + 4.7 + 1.6 - 2.3]\,\text{MeV} = 882.8\,\text{MeV}$$

$$\boxed{\Delta m = 55.5\,\text{MeV}}$$

Remark: $m = m_N(m_u = 0, m_d = 0, m_s)$

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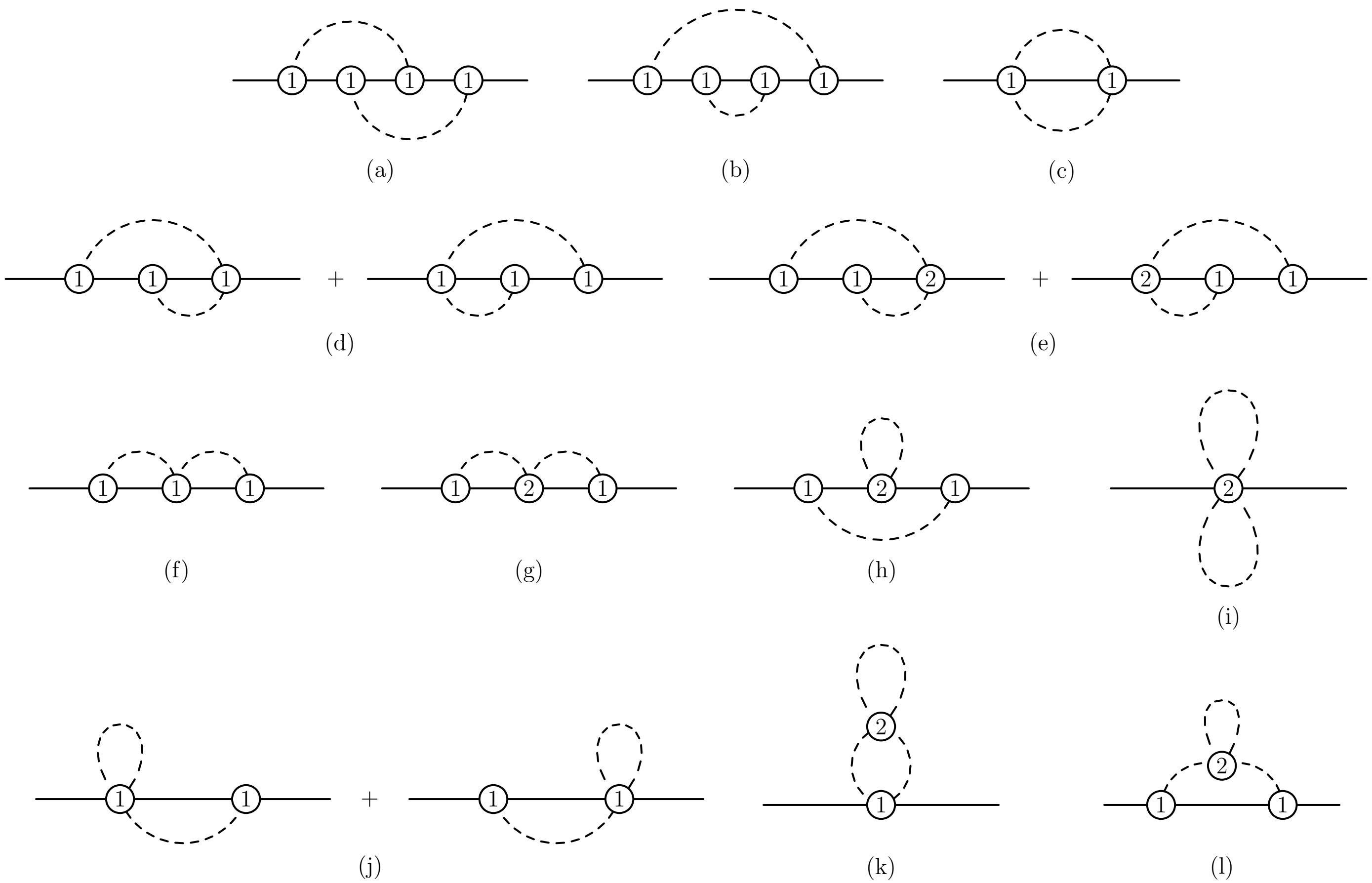
[16] T. Becher and H. Leutwyler, Eur. Phys. J. C 9, 643 (1999); T. Fuchs, J. Gegelia, S. S., Eur. Phys. J. A 19, 35 (2004)

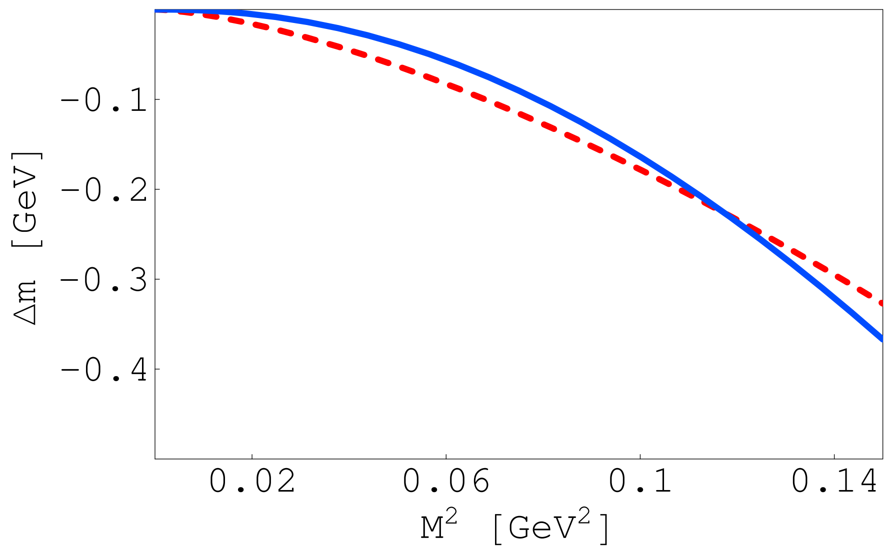
Mass of the nucleon at $\mathcal{O}(q^6)$ [17]

Two-loop contributions (M. R. Schindler, PhD thesis, 2007)



[17] M. R. Schindler, D. Djukanovic, J. Gegelia, S. S., Phys. Lett. B 649, 390 (2007); Nucl. Phys. A 803, 68 (2008)

$$m_N = m + k_1 M^2 + k_2\, M^3 + k_3 M^4 \ln\frac{M}{\mu} + k_4 M^4$$
$$\underbrace{+k_5 M^5 \ln\frac{M}{\mu} + k_6 M^5 + k_7 M^6 \ln^2\frac{M}{\mu} + k_8 M^6 \ln\frac{M}{\mu} + k_9 M^6}_{\text{two loop}}$$



$M_0 \approx 360\,$**MeV** (convergence)

At physical pion mass: $-\,4.8\,$**MeV** $=$ 31% of $k_2\, M^3$

Remarks

- Expressions of the coefficients in the chiral expansion of a physical quantity differ in various renormalization schemes

- However, the leading nonanalytic terms have to agree in all renormalization schemes

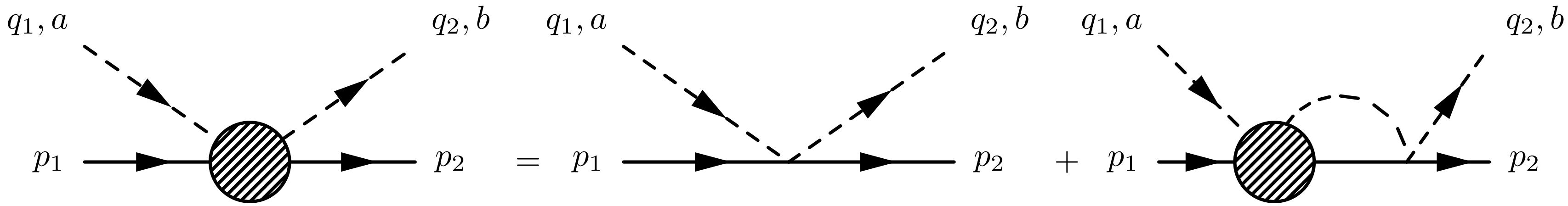
- Comparison with HBChPT [18]: Agreement for k_2, k_3, and k_5 (consistent!)

[18] J. A. McGovern and M. C. Birse, Phys. Lett. B 446, 300 (1999)

Probing the convergence of perturbative series [19]

Sum up sets of an infinite number of diagrams by solving equations exactly and compare the solutions with the perturbative contributions

Example: Pion-nucleon scattering



$$\Gamma^I = V^I + V^I\, G\, \Gamma^I, I = \frac{3}{2} \text{ or } \frac{1}{2},$$

$$V^{ba}(p_2, q_2; p_1, q_1) = -\frac{\epsilon^{bac}\tau^c}{4\,F^2}\,(\not q_1 + \not q_2) - \frac{i\, g_A^2\, \tau^b\tau^a}{4\,F^2}\,\frac{\not q_2\,(\not p - m)\,\not q_1}{p^2 - m^2}.$$

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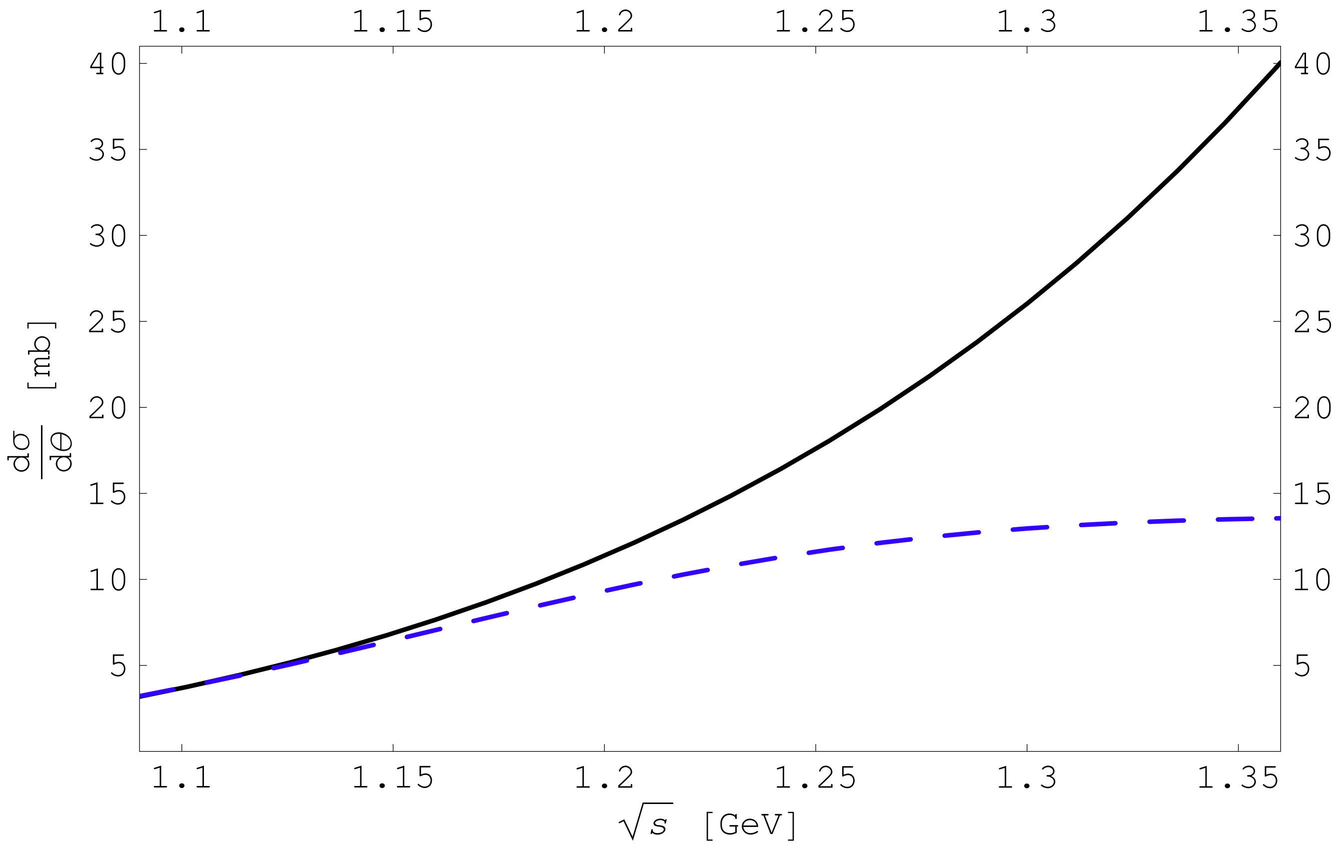
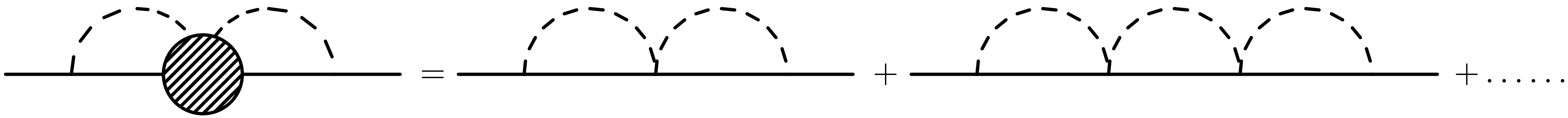
[19] D. Djukanovic, J. Gegelia, S. S., Eur. Phys. J. A 29, 337 (2006)

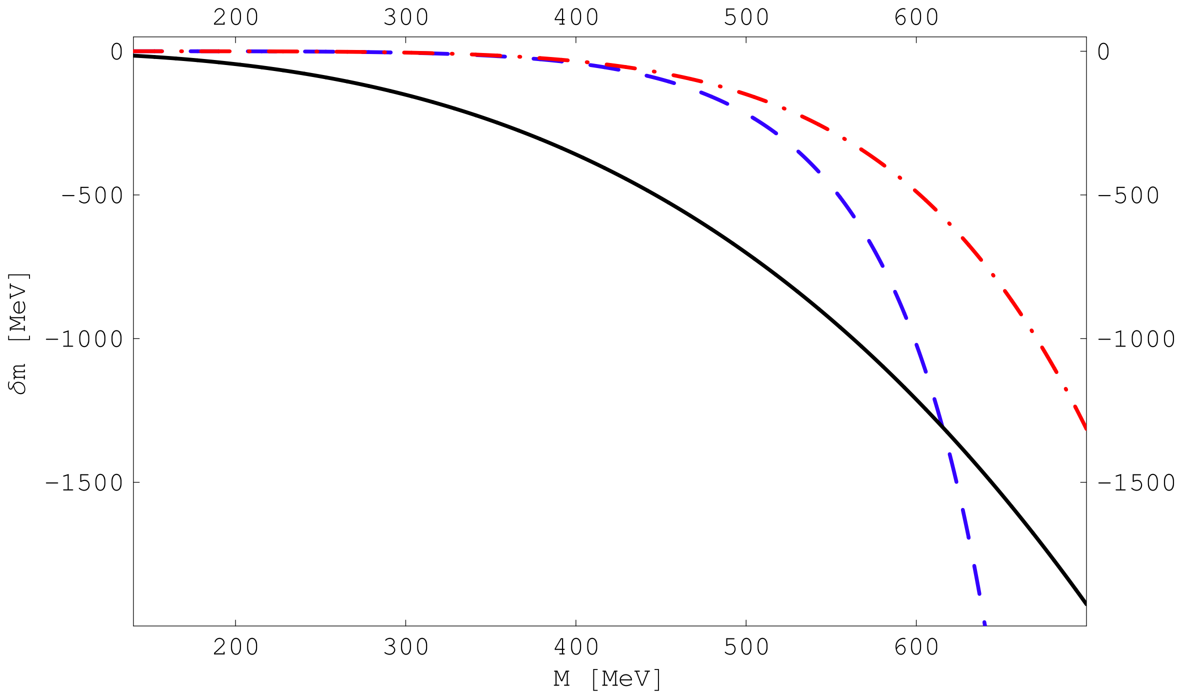


Sum of differential cross sections for $\pi^- p \to \pi^- p$ and $\pi^- p \to \pi^0 n$ in forward direction.
Solid line: non-perturbative result;
dashed line: perturbative (tree plus one-loop order) result

Example: Nucleon self-energy



$$\delta m = -0.00233530\,\mathrm{MeV} = (-0.00230219 - 0.00003305 - 0.00000007 + \cdots)\ \mathrm{MeV}\,.$$

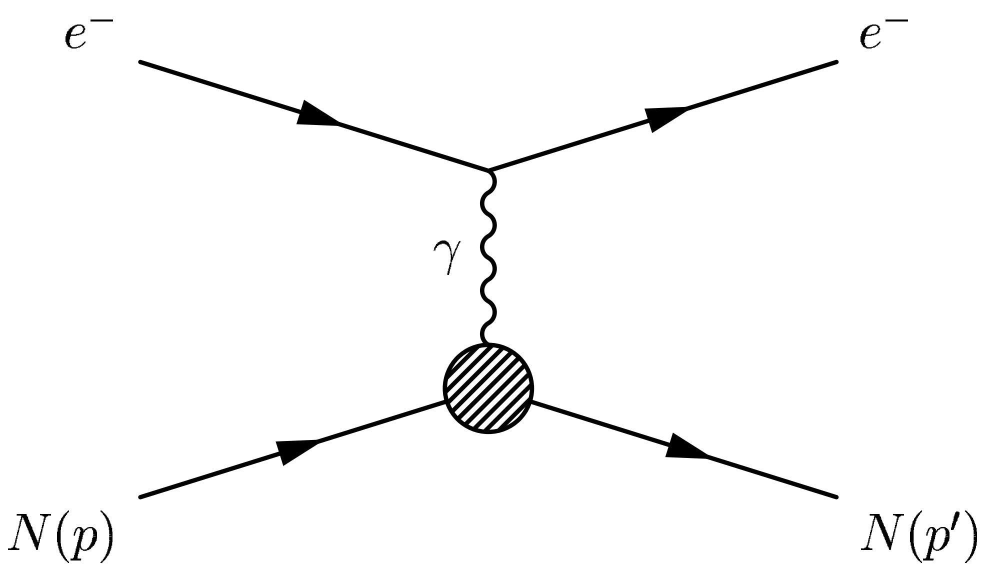


Contributions to the nucleon mass as functions of M.

Solid line: $\mathcal{O}(q^3)$; dashed line: δm; dashed-dotted line: two-loop diagram.

4. Applications II

Electromagnetic form factors



$\mathcal{J}^\mu$

Electromagnetic current operator

$$\mathcal{J}^\mu(x) = \frac{2}{3}\,\bar{u}(x)\gamma^\mu u(x) - \frac{1}{3}\,\bar{d}(x)\gamma^\mu d(x) + \cdots = \bar{q}(x)\,Q\,q(x) + \cdots$$

Definition of Dirac and Pauli form factors

$$\langle N(p')|\mathcal{J}^{\mu}(0)|N(p)\rangle = \bar{u}(p')\left[\boxed{F_1^N(Q^2)}\gamma^{\mu} + i\frac{\sigma^{\mu\nu}q_{\nu}}{2m_p}\boxed{F_2^N(Q^2)}\right]u(p)$$

$$N = p, n, \quad q^{\mu} = p'^{\mu} - p^{\mu}, \quad Q^2 = -q^2$$

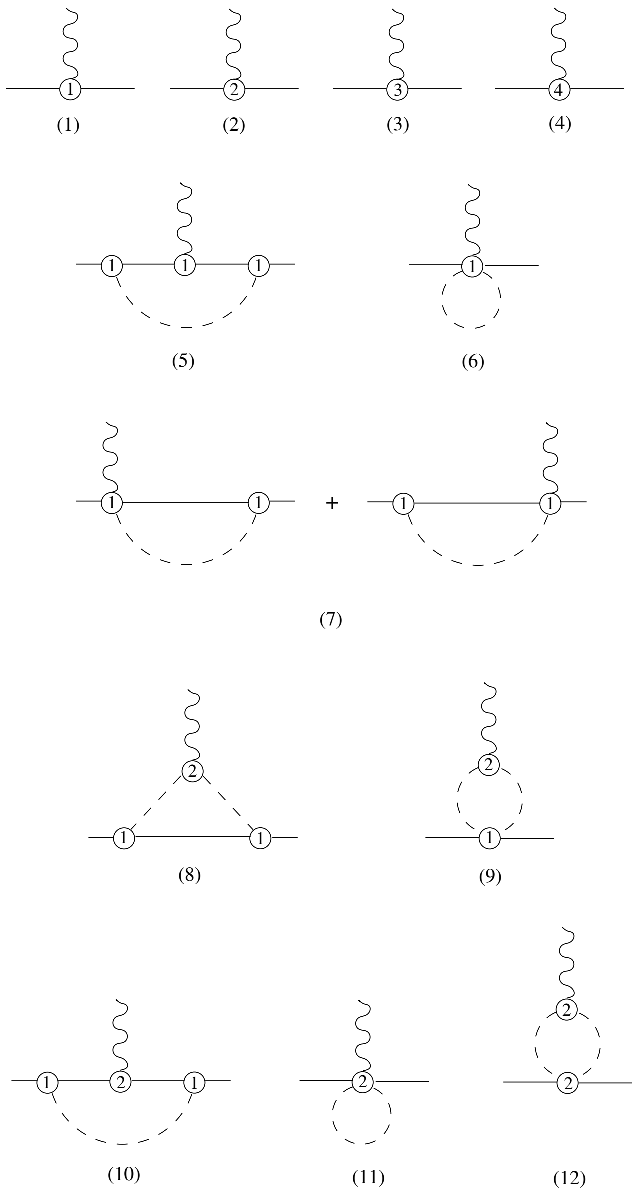
$$F_1^p(0) = 1,\ F_1^n(0) = 0,\ F_2^p(0) = 1.793,\ F_2^n(0) = -1.913.$$

Sachs form factors

$$G_E^N(Q^2) = F_1^N(Q^2) - \frac{Q^2}{4m_N^2}F_2^N(Q^2)$$

$$G_M^N(Q^2) = F_1^N(Q^2) + F_2^N(Q^2)$$

Diagrams at $\mathcal{O}(q^4)$



Diagrams potentially violating power counting: (5), (8), and (10).

EOMS subtractions

- Dirac form factor

$$\Delta F_1^{10} = \frac{g_A^2 m}{64\pi^2 F^2}\left(3c_7 - 2c_6\tau_3\right) t,$$

- Pauli form factor

$$\begin{aligned}
\Delta F_2^{5} &= -\frac{g_A^2 m_N (m - 4c_1 M^2)}{32\pi^2 F^2}\left(3 - \tau_3\right), \\
\Delta F_2^{8} &= \frac{g_A^2 m_N (m - 4c_1 M^2)}{8\pi^2 F^2}\tau_3, \\
\Delta F_2^{10} &= -\frac{g_A^2 m_N (m^2 - 8c_1 M^2 m)}{16\pi^2 F^2}\left(3c_7 - 2c_6\tau_3\right).
\end{aligned}$$

Parameters

	c_2	c_4	$\tilde{c}_6$	$\tilde{c}_7$	d_6	d_7	e_{54}	e_{74}
EOMS	2.66	2.45	1.26	-0.13	-0.57	-0.44	0.27	1.71
IR	2.66	2.45	0.47	-1.87	0.32	-0.89	0.33	1.65

The LECs c_i are given in units of GeV^{-1}, the d_i in units of GeV^{-2}, and the e_i in units of GeV^{-3}.

c_2 and c_4 from πN scattering;

$\tilde{c}_6$ and $\tilde{c}_7$ from anomalous magnetic moments;

d_6, d_7, e_{54}, and e_{74} from charge and magnetic radii: [20]

$r_E^p = 0.848\,\text{fm}$,

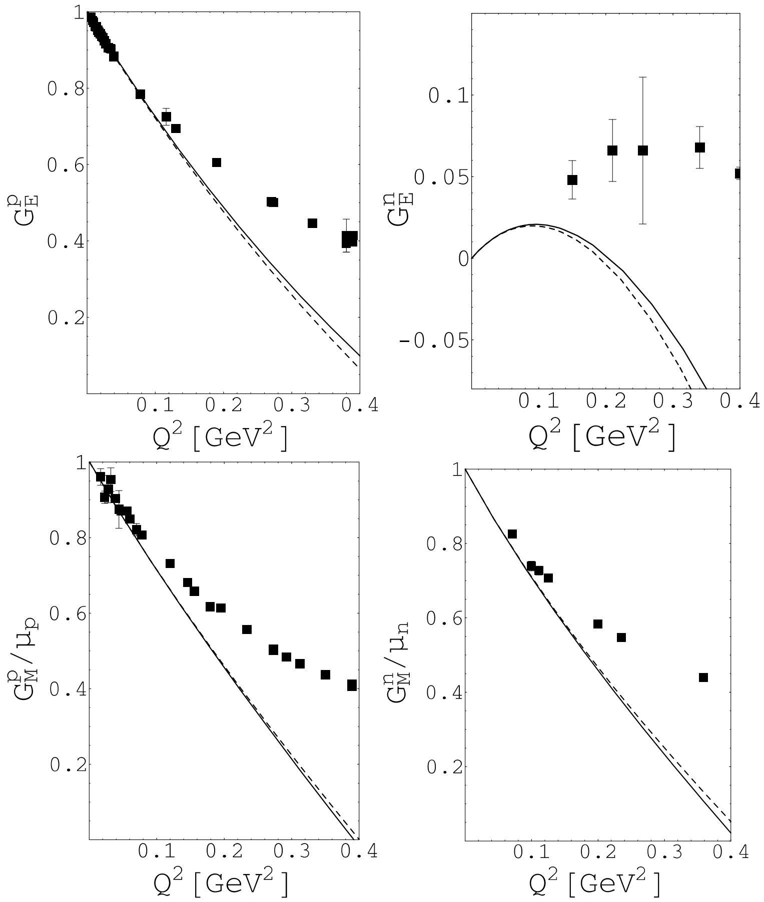
$r_M^p = 0.857\,\text{fm}$,

$r_E^n = 0.113\,\text{fm}$,

$r_M^n = 0.879\,\text{fm}$.

[20] H. W. Hammer and U.-G. Meißner, Eur. Phys. J. A 20, 469 (2004)

Sachs form factors [21] (T. Fuchs, PhD thesis, 2003)



[21] B. Kubis and U.-G. Meißner, Nucl. Phys. A679, 698 (2001); T. Fuchs, J. Gegelia, S. S., J. Phys. G 30, 1407 (2004); M. R. Schindler, J. Gegelia, S. S., Eur. Phys. J. A 26, 1 (2005); data taken from J. Friedrich and Th. Walcher, Eur. Phys. J. A 17, 607 (2003)

Vector meson dominance model $\rightarrow$ Important contributions to the electromagnetic form factors [22]

In standard ChPT: Vector meson contributions in low-energy constants

$$\frac{1}{q^2 - M_V^2} = -\frac{1}{M_V^2}\left[1 + \frac{q^2}{M_V^2} + \left(\frac{q^2}{M_V^2}\right)^2 + \mathcal{O}(q^6)\right]$$

Inclusion of vector mesons $\Rightarrow$ re-summation of higher-order contributions

Reformulated IR regularization and EOMS scheme allow for consistent inclusion of vector mesons

[22] B. Kubis and U.-G. Meißner, Nucl. Phys. A679, 698 (2001)

Inclusion of ρ, ω, and ϕ mesons [23]

Vector representation [24]

$$\mathcal{L}_{\pi V}^{(3)} = -f_\rho \mathrm{Tr}(\rho^{\mu\nu} f_{\mu\nu}^{+}) - f_\omega \omega^{\mu\nu} f_{\mu\nu}^{(s)} - f_\phi \phi^{\mu\nu} f_{\mu\nu}^{(s)} + \ldots,$$

$$\mathcal{L}_{NV}^{(0)} = \frac{1}{2} \sum_{V=\rho,\omega,\phi} g_V \bar{\Psi}\gamma^\mu V_\mu \Psi,$$

$$\mathcal{L}_{NV}^{(1)} = \frac{1}{4} \sum_{V=\rho,\omega,\phi} G_V \bar{\Psi}\sigma^{\mu\nu} V_{\mu\nu}\Psi.$$

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[23] M. R. Schindler, J. Gegelia, S. S., Eur. Phys. J. A 26, 1 (2005)

[24] G. Ecker, J. Gasser, H. Leutwyler, A. Pich, E. de Rafael, Phys. Lett. B 223, 425 (1989)

Values of the vector-meson coupling constants [25]

f_ρ	f_ω	f_ϕ	g_ρ	g_ω	g_ϕ	G_ρ [GeV^{-1}]	G_ω [GeV^{-1}]	G_ϕ [GeV^{-1}]
0.10	0.03	0.05	4.0	42.8	-20.6	13.0	0.96	-3.3

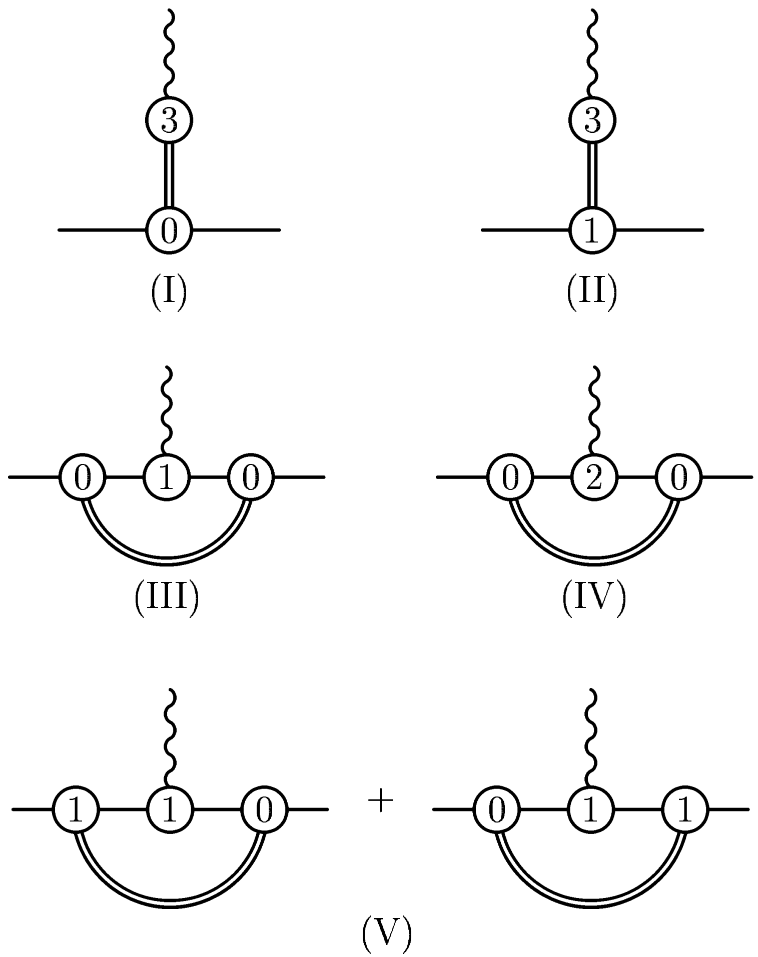
$\Rightarrow$ Modified couplings d_6, d_7, e_5 and e_{74}

	d_6	d_7	e_{54}	e_{74}
EOMS	1.21	1.30	-0.76	1.65
IR	0.98	0.24	-0.26	-0.90

Additional rules:

- Vector meson propagator $\sim \mathcal{O}(q^0)$
- Vertex from $\mathcal{L}_V^{(i)} \sim \mathcal{O}(q^i)$

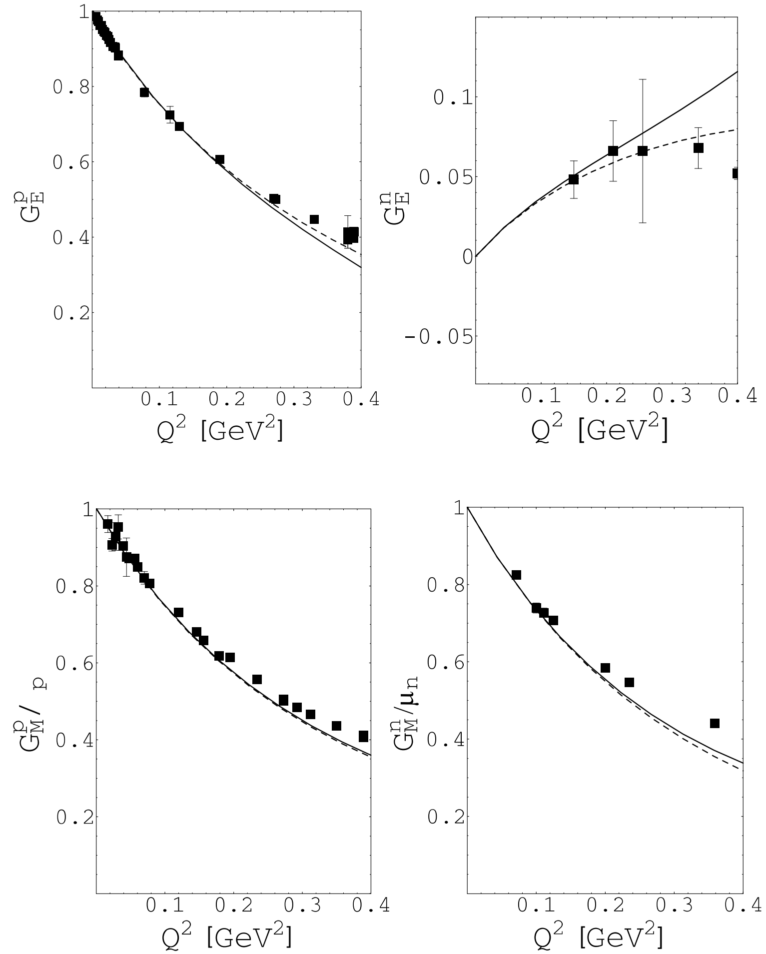
[25] H. W. Hammer and U.-G. Meißner, Eur. Phys. J. A 20, 469 (2004)



Feynman diagrams involving vector mesons contributing to the electromagnetic form factors up to and including $\mathcal{O}(q^4)$

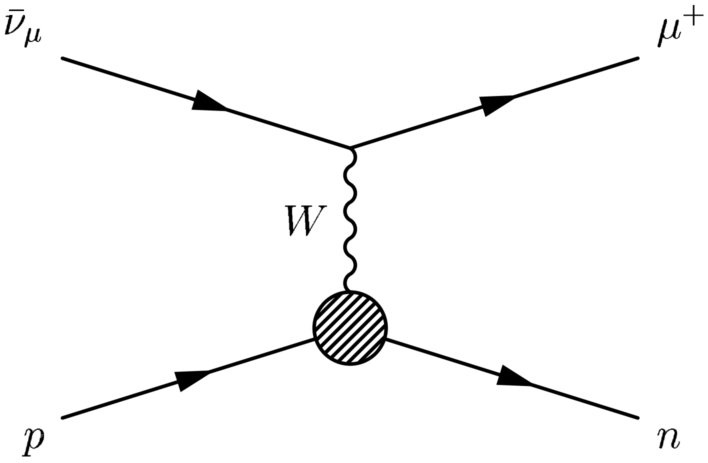
E.m. form factors including vector mesons at $\mathcal{O}(q^4)$ [26]

(M. R. Schindler, thesis, 2004)



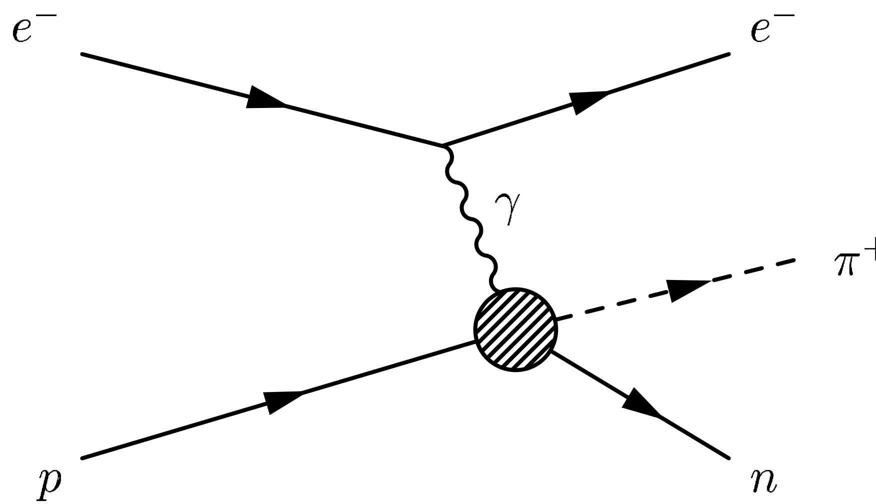
[26] M. R. Schindler, J. Gegelia, and S. S., Eur. Phys. J. A 26, 1 (2005); data taken from J. Friedrich and Th. Walcher, Eur. Phys. J. A 17, 607 (2003)

Axial and induced pseudoscalar form factors G_A and G_P

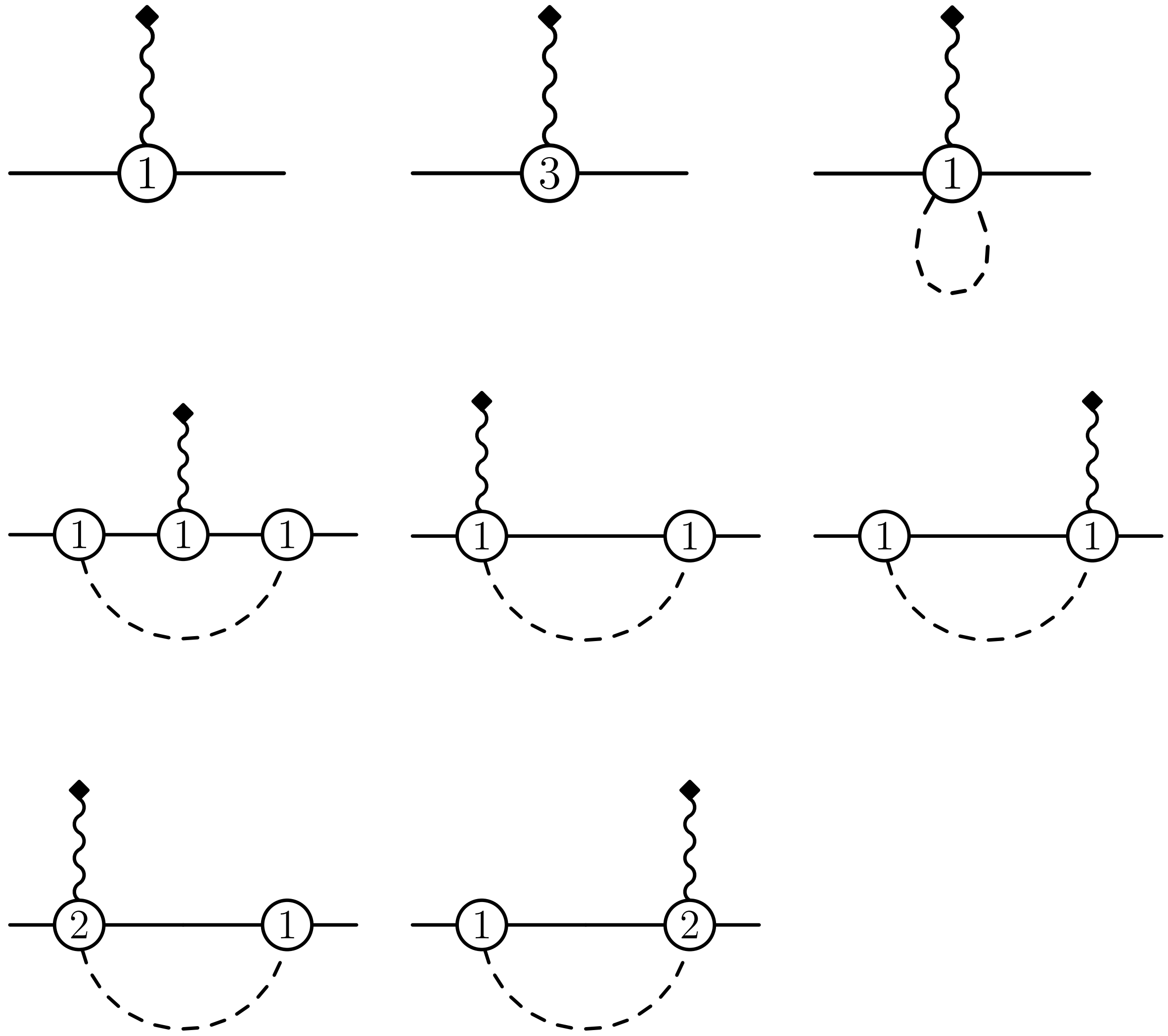


$\mathcal{V}^\mu - \mathcal{A}^\mu$

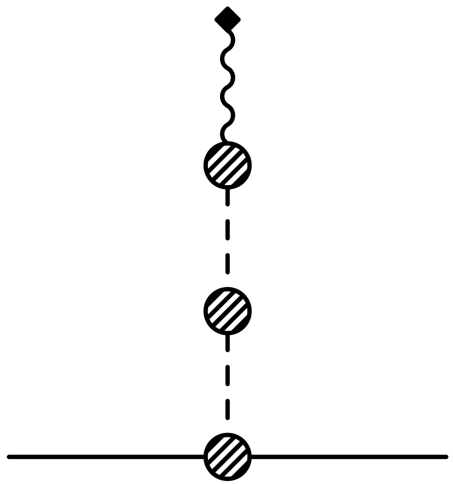
Partially
Conserved
Axial-vector
Current
hypothesis



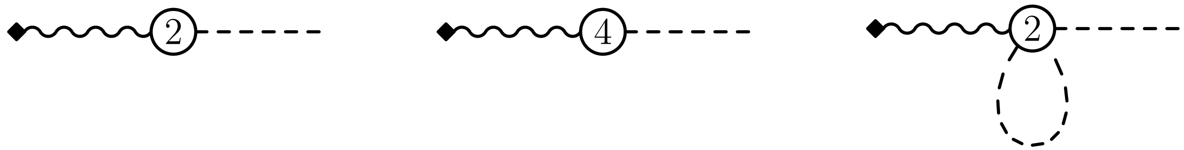
$$\langle n|\mathcal{A}^{\mu,-}(0)|p\rangle = \bar{u}(p')\left[\gamma^\mu\gamma_5 \boxed{G_A(Q^2)} + \frac{q^\mu}{2m_N}\gamma_5 \boxed{G_P(Q^2)}\right] u(p)$$



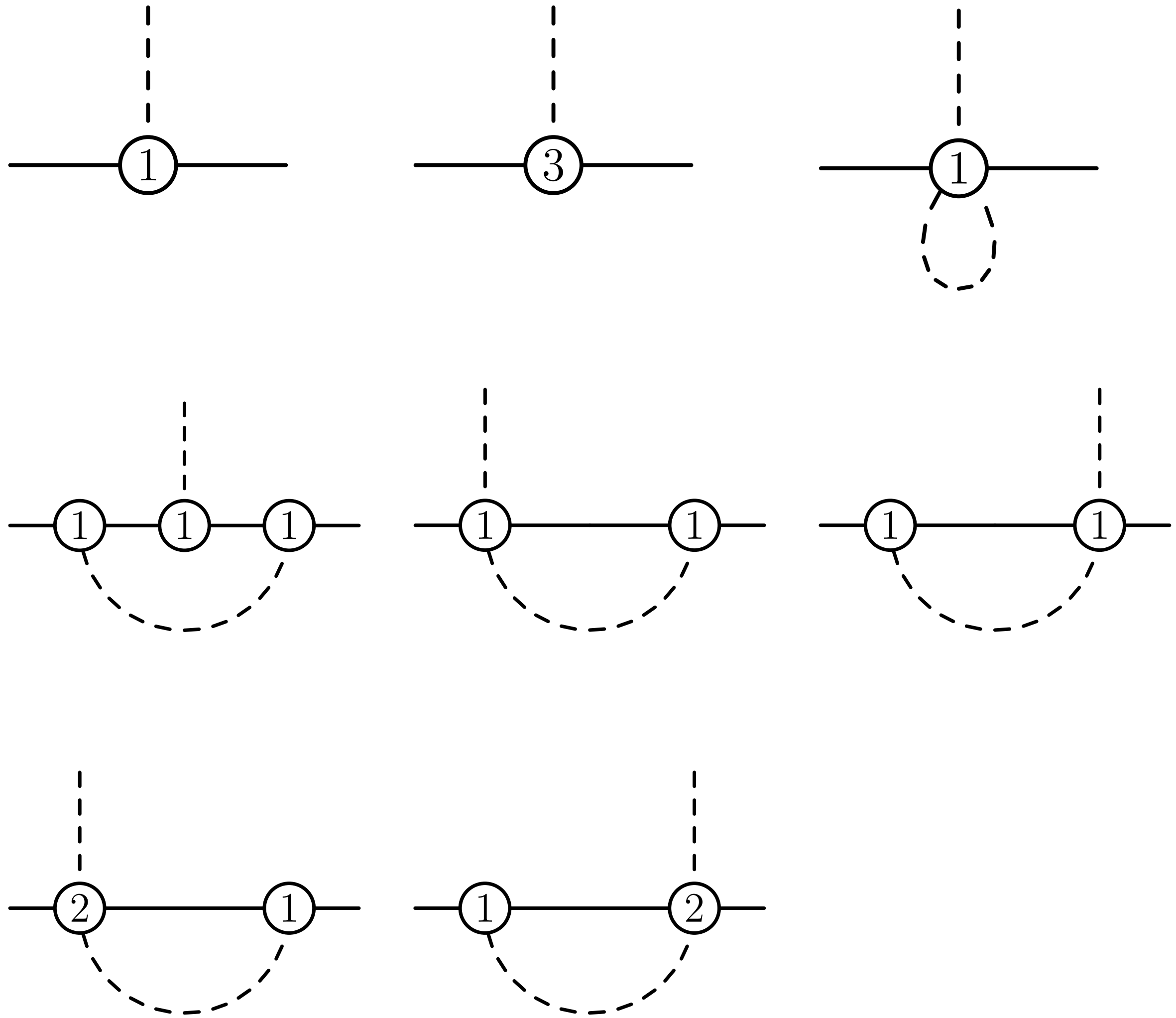
One-particle-irreducible diagrams contributing to the nucleon matrix element of the isovector axial-vector current.



Pion pole graph of the isovector axial-vector current.



Diagrams contributing to the coupling of the isovector axial-vector current to a pion up to $\mathcal{O}(q^4)$.



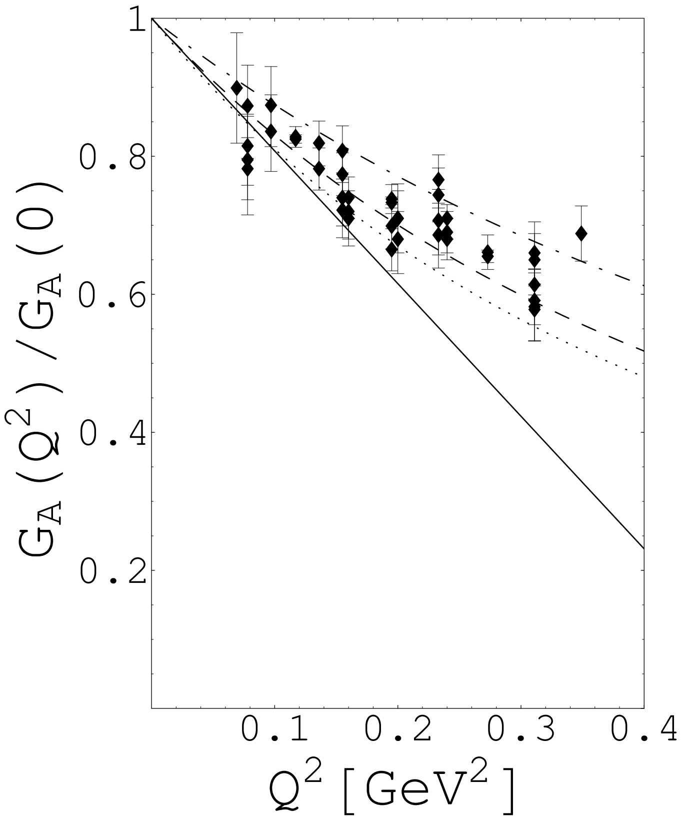
Diagrams contributing to the πN vertex up to $\mathcal{O}(q^4)$.

Result for G_A is of the form

$$G_A(Q^2) = g_A - \frac{1}{6} g_A \langle r_A^2 \rangle Q^2 + \frac{g_A^3}{4F^2} \bar{H}(Q^2).$$

$\langle r_A^2 \rangle$: axial mean-square radius (LEC)
$\bar{H}(Q^2)$: loop contributions
$\bar{H}(0) = \bar{H}'(0) = 0$.

Full line: result in infrared renormalization.
Again: No curvature!
Dashed line: Dipole, $M_A = 1.026$ GeV;
Dotted line: Dipole; $M_A = 0.95$ GeV;
Dashed-dotted line: Dipole $M_A = 1.20$ GeV,



Inclusion of $a_1(1260)$ meson [27]

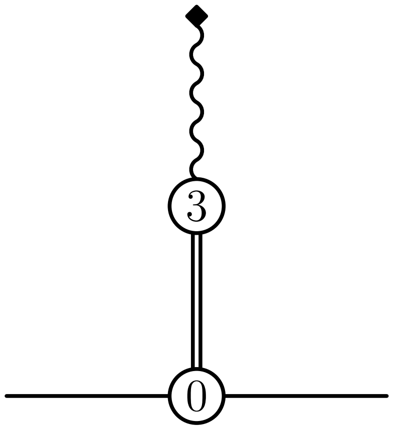
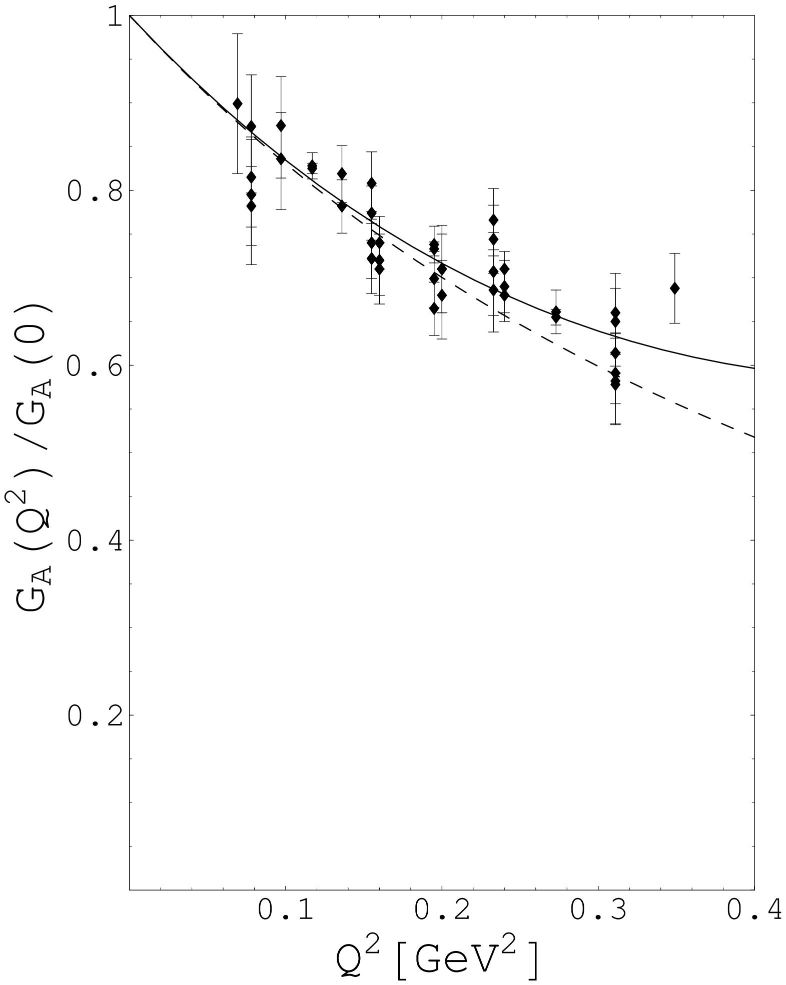


Diagram containing axial-vector meson (double line) contributing to the form factors G_A and G_P.

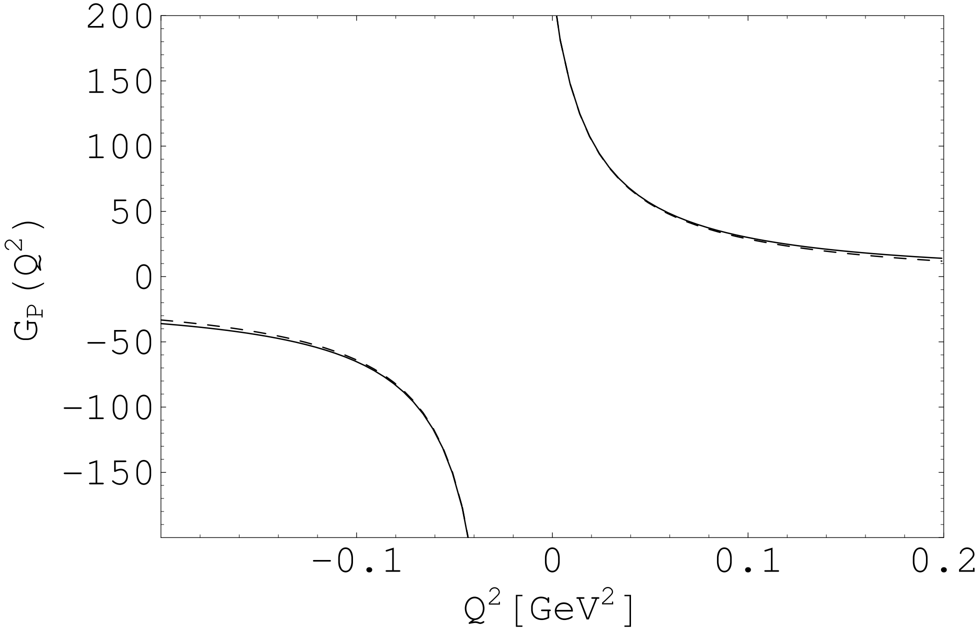
$$G_A^{AVM}(q^2) = -f_A g_{a_1} \frac{q^2}{q^2 - M_{a_1}^2},$$

$$f_A g_{a_1} \approx 8.70.$$

[27]M. R. Schindler, T. Fuchs, J. Gegelia, S. S, Phys. Rev. C 75, 025202 (2007)



G_A including a_1 (M. R. Schindler, PhD thesis, 2007)



G_P at $\mathcal{O}(q^4)$

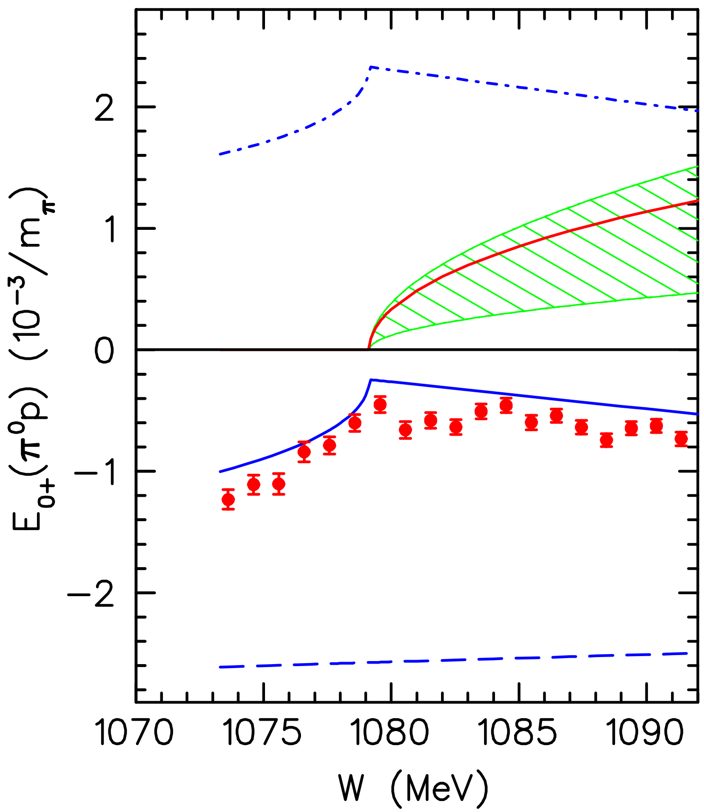
Full line: result with axial-vector meson, dashed line: result without axial-vector meson.

Work in progress

Present: χ MAID (B. C. Lehnhart, PhD thesis 2007)

$\mathcal{O}(q^4)$: 20 tree-level diagrams + 85 loop diagrams

Example: $\gamma + p \to p + \pi^0$ at $\mathcal{O}(q^3)$ [28]



Solid blue line: real part

Solid red line: imaginary part

Cusp from taking m_n and m_{π^+} in loop

Long-dashed blue line: tree-level contribution

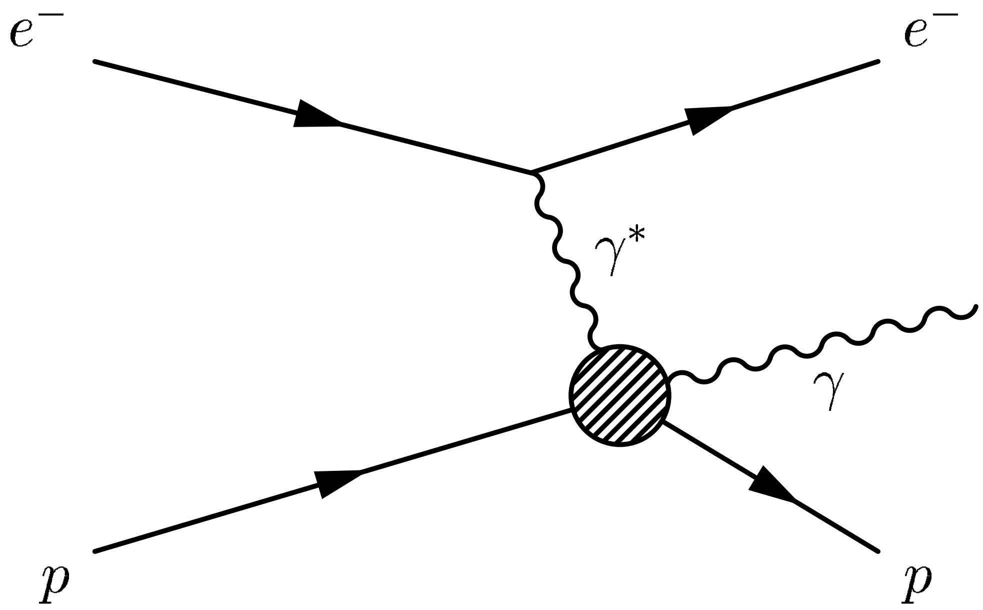
Dashed-dotted blue line: loop contribution

Green band: Imaginary part from ansatz $\mathrm{Im}(E_{0+}) = \beta|\vec{q}|$

[28] Data taken from A. Schmidt et al., Phys. Rev. Lett. 87, 232501 (2001)

(Virtual) Compton scattering off the nucleon

- **Virtual Compton scattering $\gamma^* p \to \gamma p$ through $ep \to ep\gamma$**



+ Bethe-Heitler terms

- **6 generalized polarizabilities (GPs(q^2))**

- **Starting point:**

  Program Compton Scattering Observables
  (B. Pasquini, Pavia)

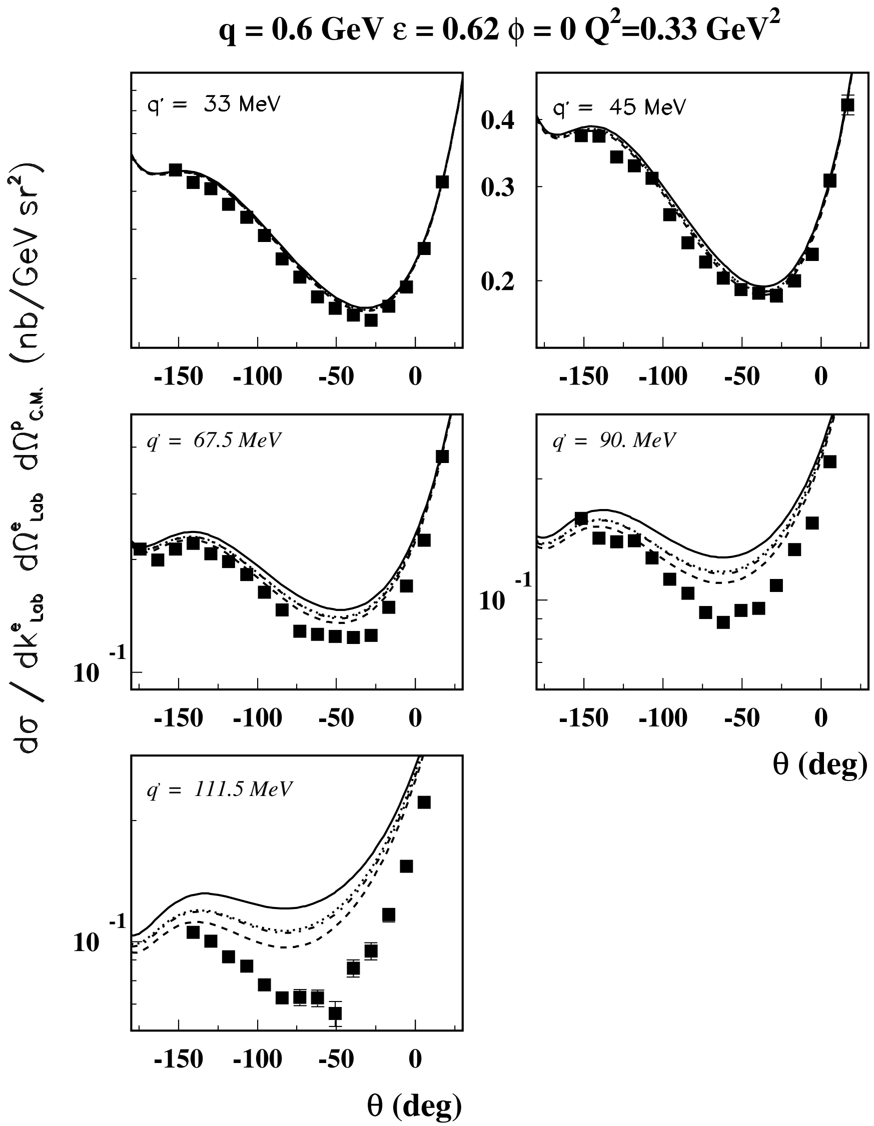
- **Present**

  Development of χ CSO for RCS, VCS, VVCS

  (Manifestly Lorentz-invariant one-loop ChPT to $\mathcal{O}(q^4)$)

- **At $\mathcal{O}(q^4)$ two new parameters related to α and β of RCS**

  (D. Djukanovic, PhD thesis, 2008)



Differential cross section for $ep \to ep\gamma$ as function of the photon scattering angle in the MAMI kinematics specified in the plot. [a]

[a] Data taken from J. Roche et al., Phys. Rev. Lett. 85, 708, (2000)

5. Summary

- Baryonic ChPT: Renormalization condition $\leftrightarrow$ Consistent power counting
- IR and EOMS renormalization (manifestly Lorentz-invariant)
- Inclusion of heavy degrees of freedom/Two-loop calculation
- Applications: Mass of the nucleon and form factors
- Present and future
  - Electromagnetic processes: Real and virtual Compton scattering, pion photo- and electroproduction, etc.
  - Complex mass renormalization (see talk by J. Gegelia)

Thanks to my collaborators

- Dalibor Djukanovic
- Dr. Thomas Fuchs
- Dr. Jambul Gegelia
- Dr. Christian Hacker
- Dr. Björn C. Lehnhart
- Dr. Matthias R. Schindler
- Natalia Wies

Thank You!

Inclusion of the $\Delta(1232)$ into ChPT [29]

$$\boldsymbol{\Delta(1232): \quad I(J^P) = \frac{3}{2}(\frac{3}{2}^+)}$$

Description in terms of a vector-spinor isovector-isospinor

$$\Psi_{\mu,\alpha;i,m}$$

Too many components $\Rightarrow$ Constraints

Dirac's analysis using the Hamiltonian method:

$$L(q,\dot{q}) \quad \to \quad p_i = \frac{\partial L}{\partial \dot{q}_i} \quad \to \quad H(q,p) = p_i \dot{q}_i - L$$

But

$$\Phi_m(q,p) = 0 \quad \text{primary constraints}$$

Introduce constraints in terms of Lagrange multipliers into Hamiltonian

$$H_T = H + u_m \Phi_m$$

[29] C. Hacker, N. Wies, J. Gegelia, S. S., Phys. Rev. C 72, 055203 (2005); N. Wies, J. Gegelia, S. S., Phys. Rev. D 73, 094012 (2006)

Consider time evolution (in terms of Poisson brackets)

$\{H_T, \Phi_m\} = 0 \quad \Rightarrow$ new (secondary) constraints

Iterate until all Lagrange multipliers have been solved

In a consistent theory

initial # of d.o.f $-$ # of constraints $=$ correct # of d.o.f.

$\Rightarrow$ Restrictions on the possible interaction terms

Example [30]

$$\begin{aligned}\mathcal{L}_{\pi\Delta} &= -\bar{\Psi}^{\mu}[\frac{g_1}{2}g_{\mu\nu}\gamma^{\alpha}\gamma_5\partial_{\alpha}\phi \\ &+\frac{g_2}{2}(\gamma_{\mu}\partial_{\nu}\phi + \partial_{\mu}\phi\gamma_{\nu})\gamma_5 \\ &+\frac{g_3}{2}\gamma_{\mu}\gamma^{\alpha}\gamma_5\gamma_{\nu}\partial_{\alpha}\phi]\Psi^{\nu}\end{aligned}$$

[30] T. R. Hemmert, B. R. Holstein, J. Kambor, J. Phys. G 24, 1831 (1998)

Analysis of constraints [31] [32]

$$\begin{aligned} g_2 &= A g_1\,, \\ g_3 &= -\frac{1}{2}(1+2A+3A^2)g_1 \end{aligned}$$

Applications so far

- **Mass of the nucleon**
- **Pole of the Δ**
- **πN scattering** [33]
- **Magnetic moment of the Δ resonance** [34]

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[31](A parameter of the lowest-order Lagrangian)

[32]N. Wies, J. Gegelia, S. S., Phys. Rev. D 73, 094012 (2006)

[33]N. Wies, thesis, Mainz, 2005

[34]C. Hacker, N. Wies, J. Gegelia, S. S., Eur. Phys. J. A 28, 5 (2006)

Infrared regularization reformulated [35]

Basic idea

$$\frac{1}{ab} = \int_0^1 \frac{dx}{[ax + b(1-x)]^2}$$

$$\begin{aligned} a &= (k-p)^2 - m^2 + i0^+ \\ b &= k^2 - M^2 + i0^+ \end{aligned}$$

$$H = \int_0^1 dx \cdots = \int_0^\infty dx \cdots - \int_1^\infty dx \cdots \equiv I + R$$

In R expand the integrand in small momenta and masses and interchange summation and integration [36]

$\Rightarrow$ integrals over x of the type

$$I_i = -\int_1^\infty dx x^{n+i}, \quad i \text{ integer number}$$

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[35] M. R. Schindler, J. Gegelia, and S. Scherer, Phys. Lett. B 586, 258 (2004)

[36] T. Becher and H. Leutwyler, Eur. Phys. J. C 9, 643 (1999)

I_i are calculated by analytic continuation from the domain of n in which they converge, i.e.

$$I_i = - \left. \frac{x^{n+i+1}}{n+i+1} \right|_1^\infty = \frac{1}{n+i+1}$$

EOMS:

- Expand integrand in small momenta and masses

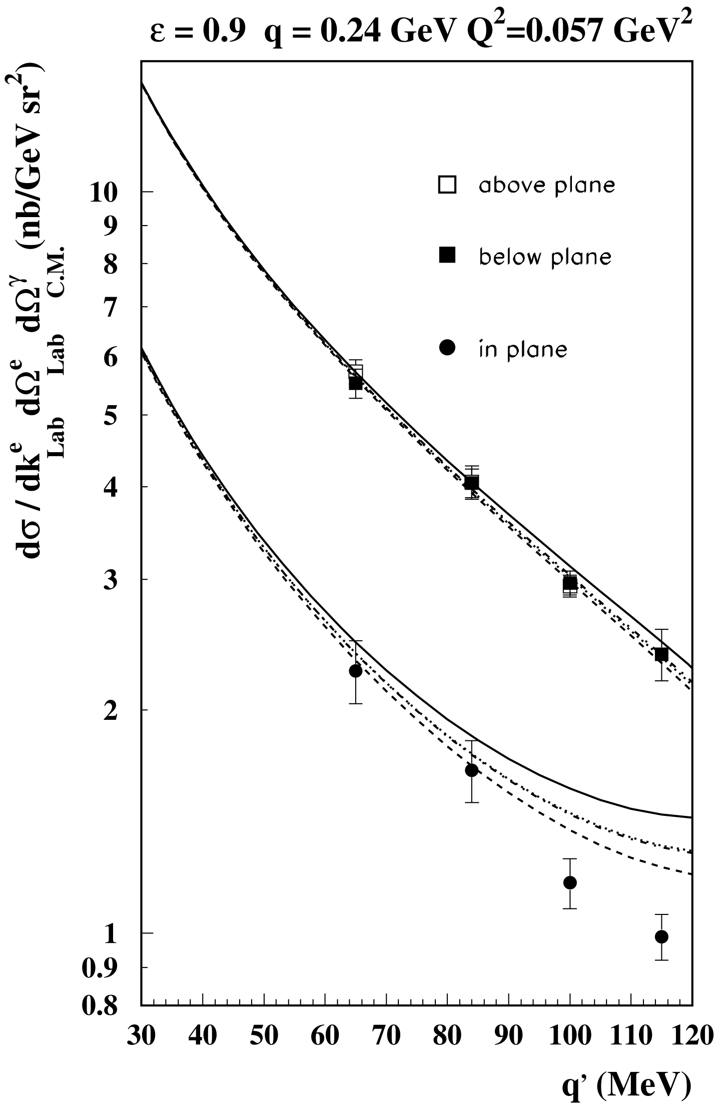
- Interchange summation and integration

$\Rightarrow$ exactly the same expansion as for the IR regular part of the IR regularization with the only difference that instead of the integrals I_i we now have

$$J_i = \int_0^1 dx x^{n+i}$$

Calculating these integrals by analytical continuation from the domain of n in which they converge, we obtain:

$$J_i = \left.\frac{x^{n+i+1}}{n+i+1}\right|_0^1 = \frac{1}{n+i+1}$$



Differential cross section for $ep \rightarrow ep\gamma$ as function of the photon the outgoing photon energy in the MIT-Bates kinematics specified in the plot. [a]

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[a] Data taken from P. Bourgeois et al., Phys. Rev. Lett. 97, 212001 (2006)