Few-nucleon effective field theory with *perturbative* pions

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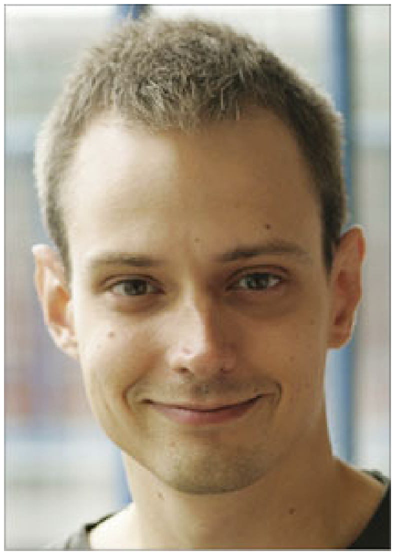
Collaborators



Aleksi Vuorinen (CERN)

David Kaplan (INT)

Outline

- Motivation
- Flavors of pionful EFT
  - Weinberg and its problems
  - KSW and its problems
- A new formulation of $\mathrm{EFT}(\pi)$
- Conclusion

NN phase shifts

↓

↓

NN potentials

Till recently, study of Nuclear Forces has relied on modeling that is disconnected from the Standard Model of particle interactions

Advantages of traditional method

- Explains certain data with high precision
- Until recently, only approach available
- Ease of implementation (*i.e.* modeling inertia)

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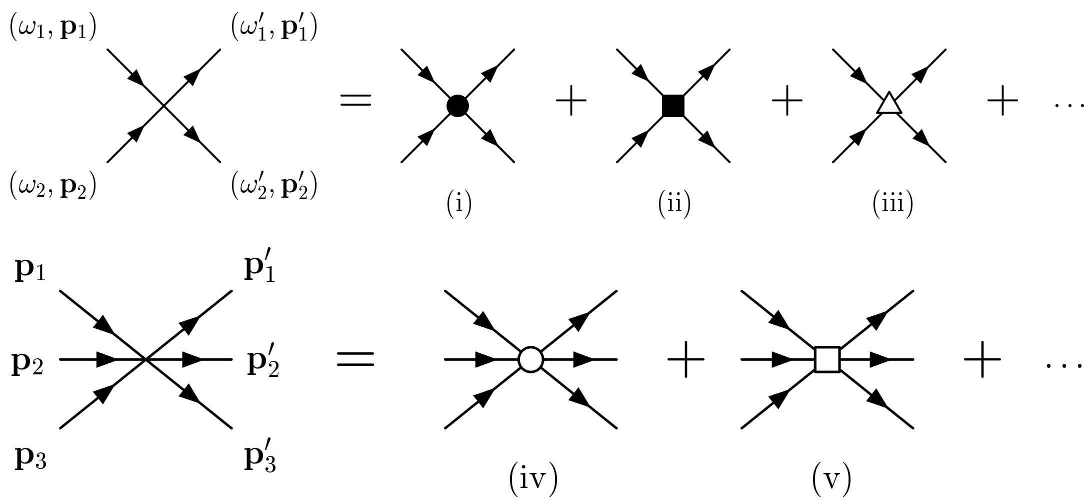
Disadvantages of traditional method

- No systematic expansion (*i.e.* no controlled error estimates)
- "Off-shell" ambiguities
- Difficult to incorporate symmetries, relativity, inelasticities, *etc.*
- Includes vast range of scales ($\equiv$ numerically intensive)

EFT and nuclear physics

Short range interactions represented by contact interactions:



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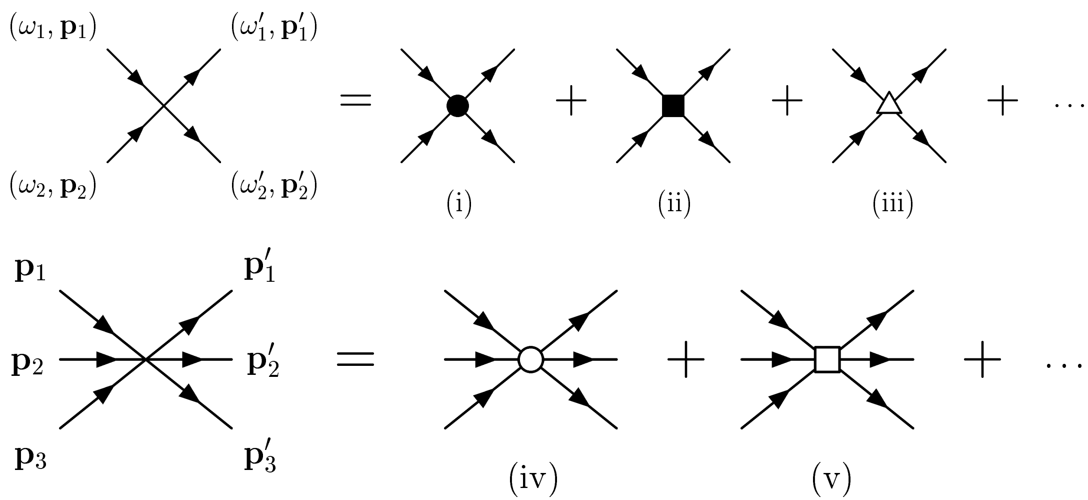


What is "short range"?

- $p \ll m_\pi$ ... everything!
- $m_\pi \leq p < 2m_\pi$ ... everything except one-pion exchange, *etc.*

IF a systematic expansion can be found:

- NO "Off-shell" ambiguities
- Easy to incorporate symmetries, relativity, inelasticities, *etc.*
- Short range interactions are *separable* – greatly simplifies calculations

Pionful EFT

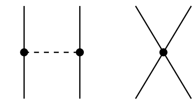
Pionful EFT

Initial proposal due to Weinberg:

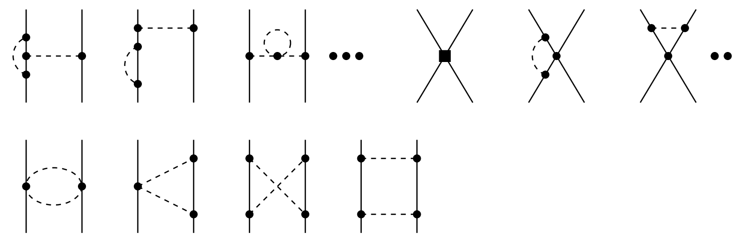
- Two-nucleon irreducible diagrams $\longrightarrow$ the potential
- Plug potential into Lipmann-Schwinger equation $\longrightarrow \delta$
- Regularization and renormalization not considered: operators organized by *dimensional analysis*

$V =$

Leading order



Next–to–leading order



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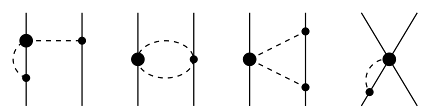
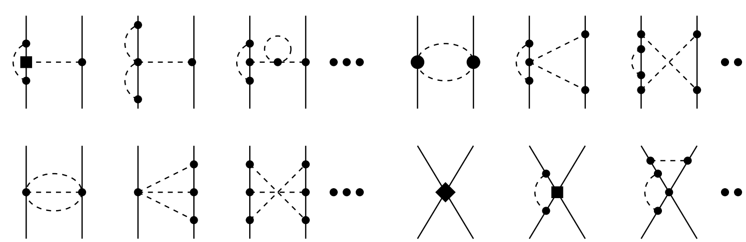
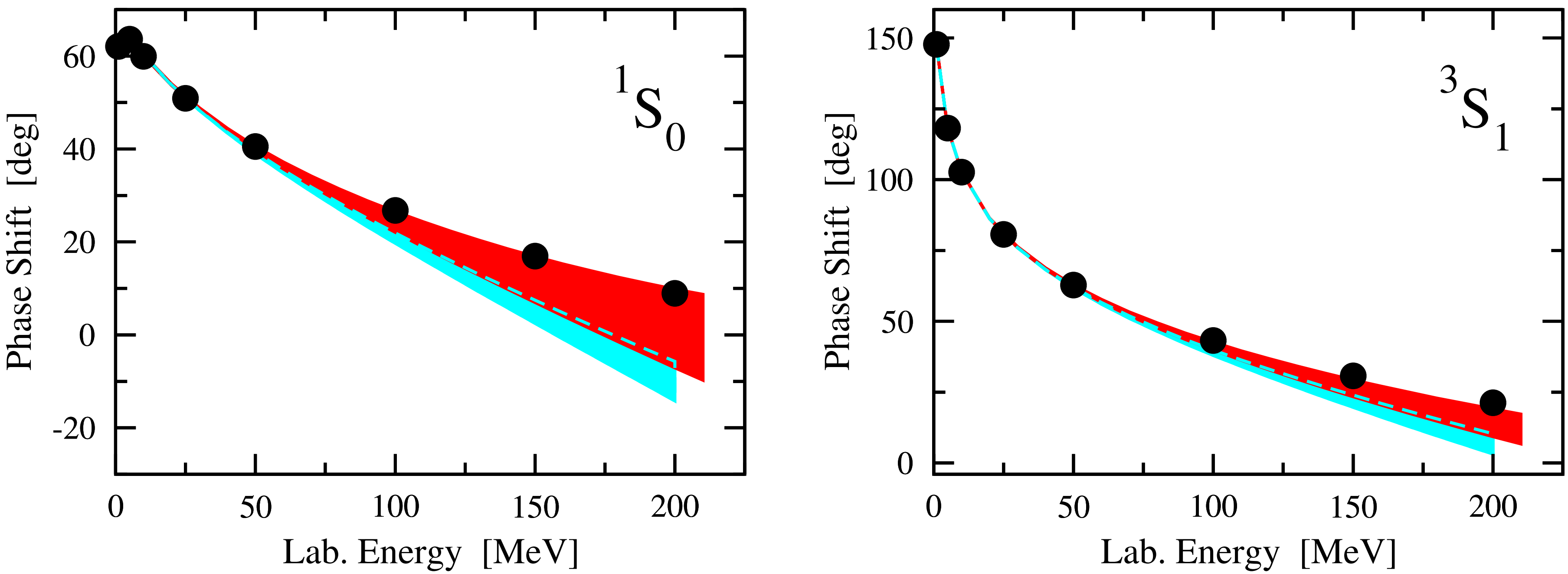


Next–to–next–to–next–to–leading order

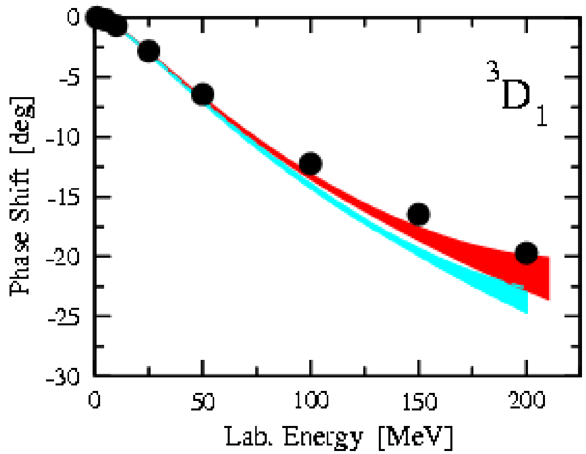


Phase shifts at NNLO (Epelbaum,Glockle,Meissner '04)

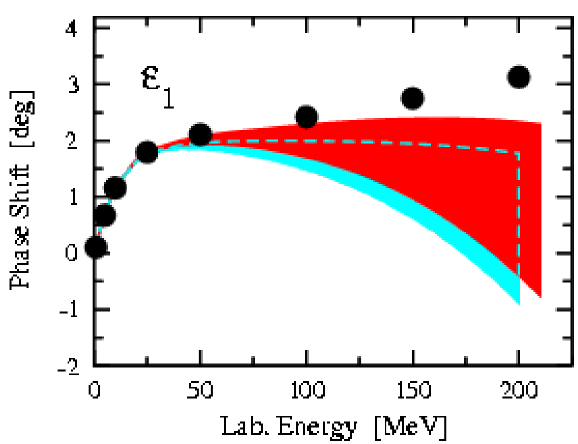


$\Lambda = 450 - 600$ MeV ; $\tilde{\Lambda} = 500 - 700$ MeV

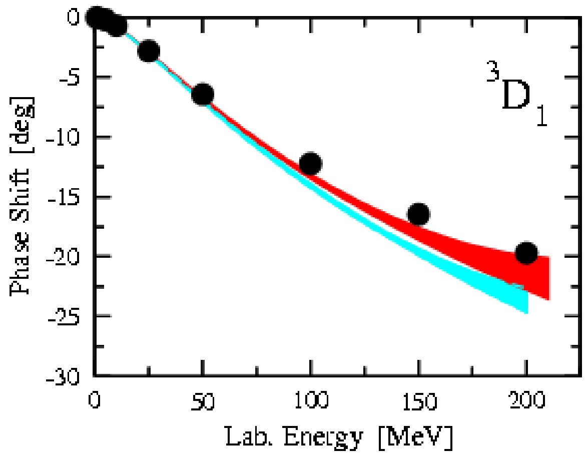
Momentum cutoff in V and in L-S equation!

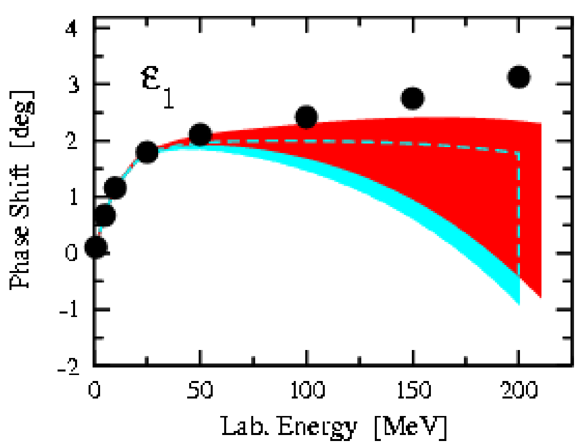


$^3D_1$
Phase Shift [deg]
Lab. Energy [MeV]



ε_1
Phase Shift [deg]
Lab. Energy [MeV]





Weinberg calculations now at NNNLO:

lines through the data!

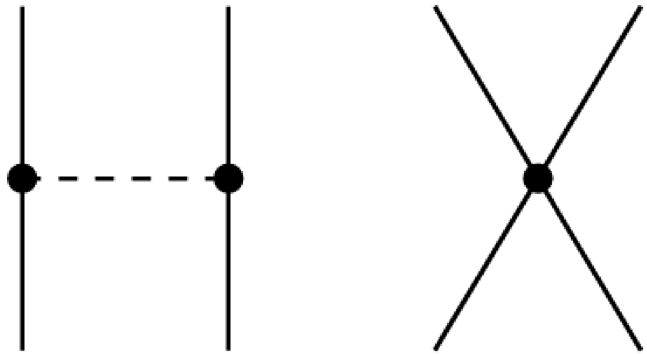
Problems with Weinberg's counting

- Mismatch of quark mass insertions
- Cutoff dependence in higher partial waves

Mismatch of quark mass insertions

(Kaplan,Savage,Wise '96)

$V_{LO}^{^1S_0} =$



$\longrightarrow$ *contains* $\frac{m_\pi^2}{\epsilon}$ *divergence!*

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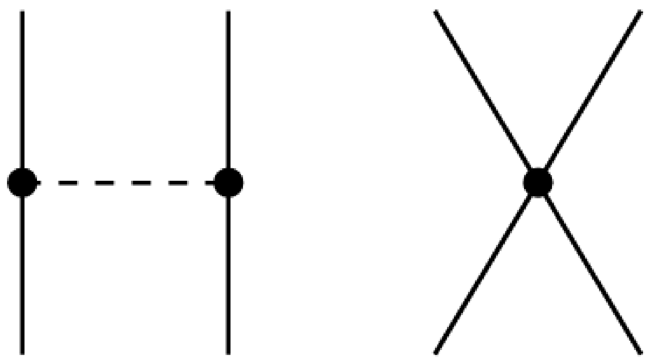
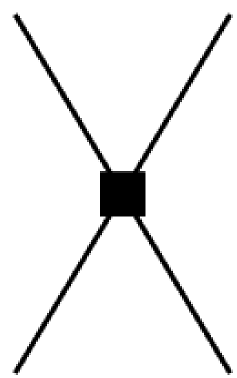


$\longrightarrow$ *contains* $\frac{m_\pi^2}{\epsilon}$ *divergence!*

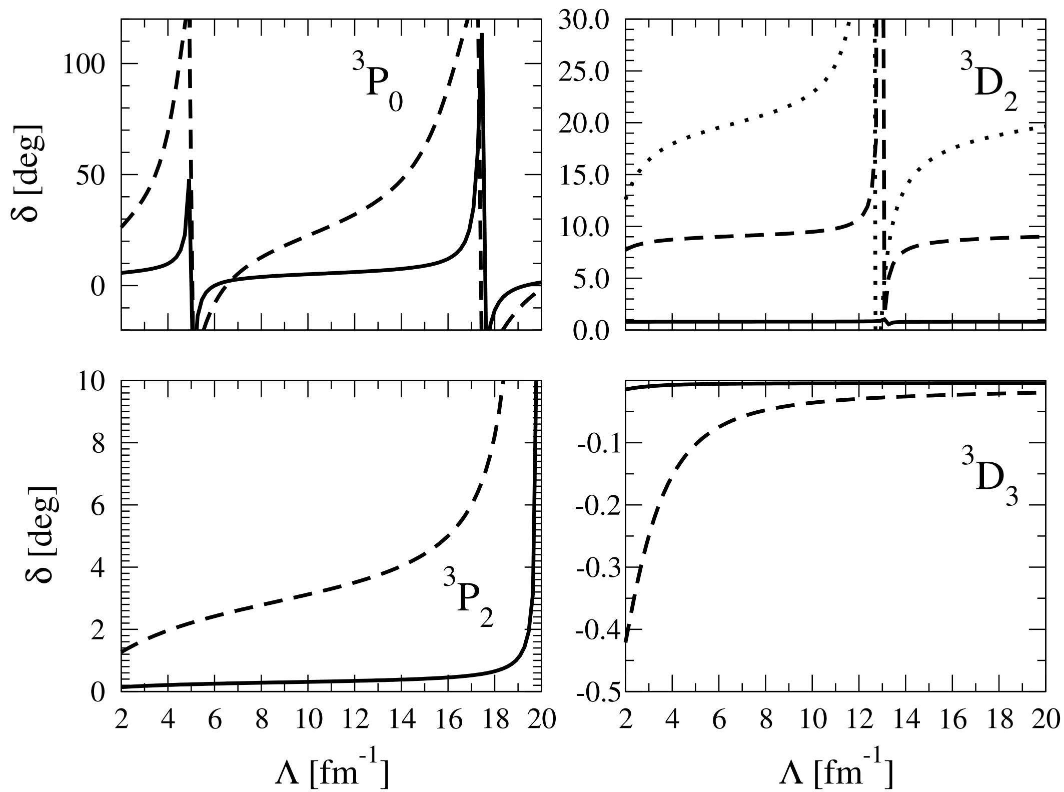
REQUIRES NLO counterterm:



$\sim \quad D_2 \; \mathrm{Tr}\,(M_q \Sigma) \left(N^\dagger N\right)^2$

Cutoff dependence in higher partial waves

(Nogga,Timmermans,van Kolck '05)



Attractive triplet channels at LO

solid: 10 MeV; dashed: 50 MeV; dotted: 100 MeV

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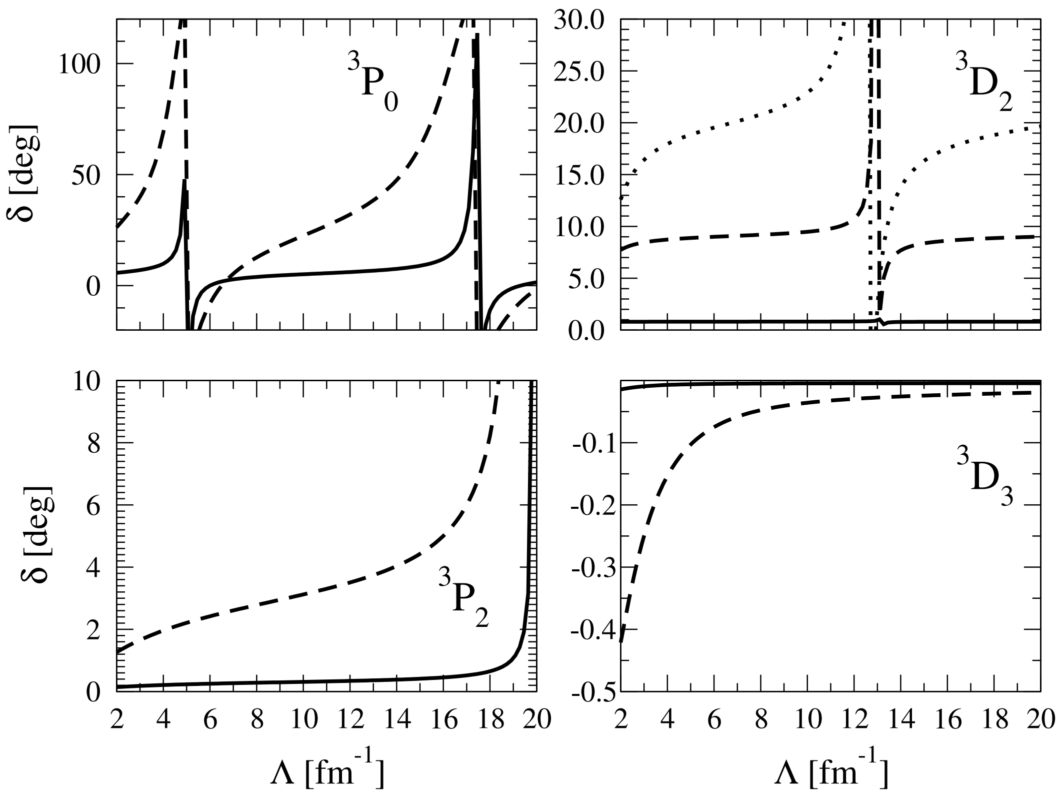

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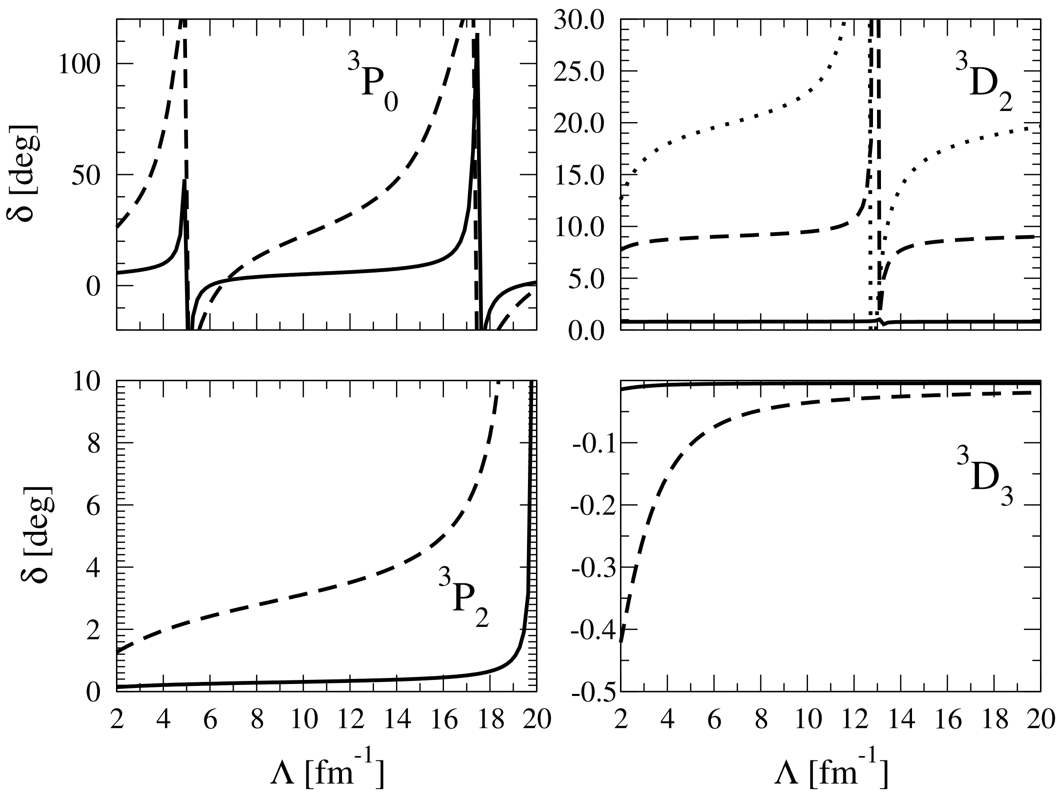

Attractive triplet channels at LO

solid: 10 MeV; dashed: 50 MeV; dotted: 100 MeV

Missing counterterms!

Solution: promote counterterms to lower order!

Weinberg pros

- Long-range pion physics correctly incorporated
- In principle, systematically improvable
- At NNNLO accuracy approaches that of potential models

Weinberg counting $=$  ?

KSW: perturbative pions

Why not perturb around non-trivial fixed point? (Kaplan,Savage,Wise '98)

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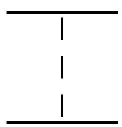
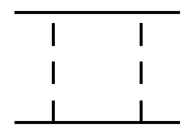


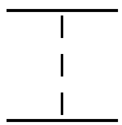
$$= \quad \frac{g_A^2}{2f^2}\, f_1\!\left(\frac{p}{m_\pi}\right)$$

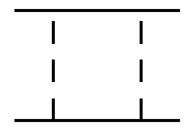


$$= \left(\frac{g_A^2}{2f^2}\right)^2 \frac{M m_\pi}{4\pi} f_2\!\left(\frac{p}{m_\pi}\right)$$

KSW: perturbative pions

Why not perturb around non-trivial fixed point? (Kaplan,Savage,Wise '98)

 $= \frac{g_A^2}{2f^2}\, f_1(\frac{p}{m_\pi})$

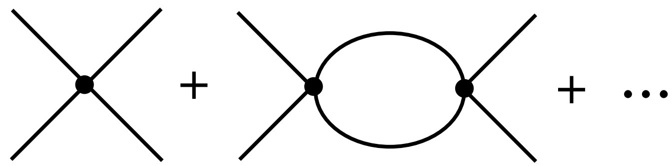
 $= \left(\frac{g_A^2}{2f^2}\right)^2 \frac{M m_\pi}{4\pi} f_2(\frac{p}{m_\pi})$

Suggests expansion parameter:

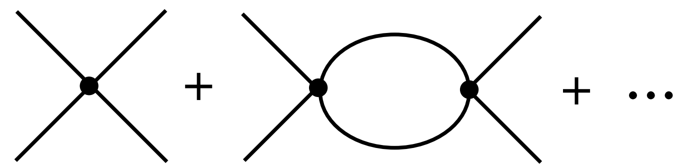
$$\frac{g_A^2 m_\pi M}{8\pi f^2} \equiv \frac{m_\pi}{\Lambda_{NN}} \sim 0.5$$

$\Longrightarrow$ KSW power counting Taken to NNLO!

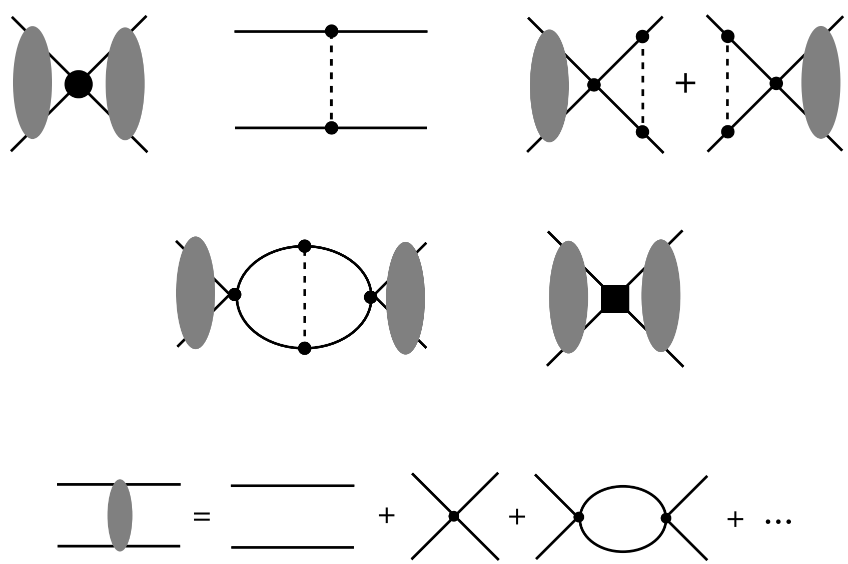
LO :



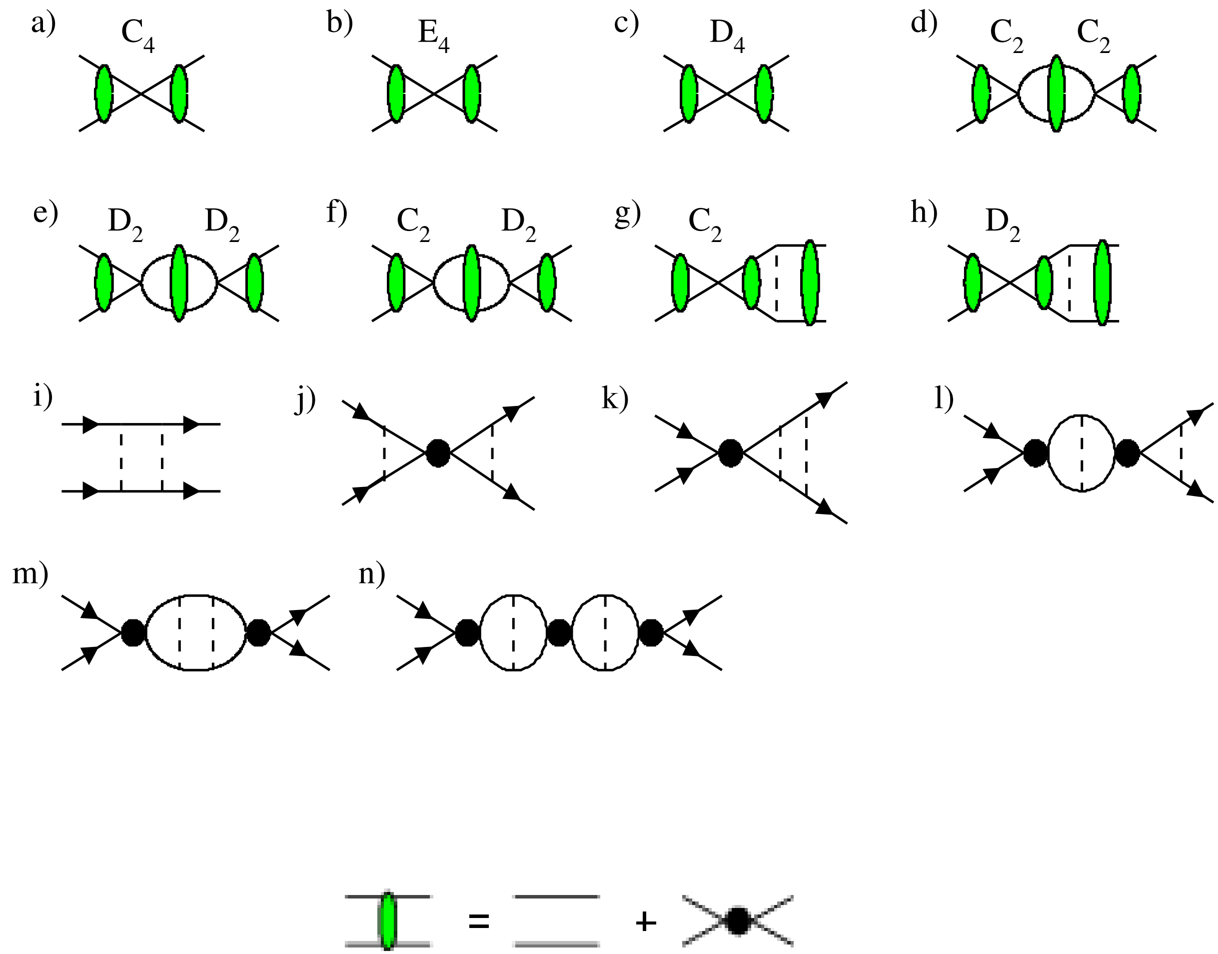
LO :



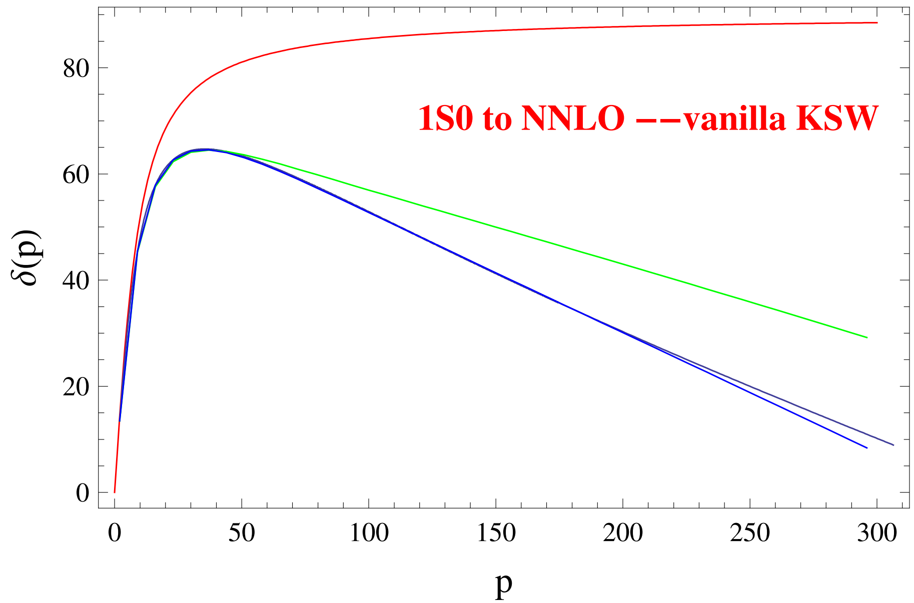
NLO :

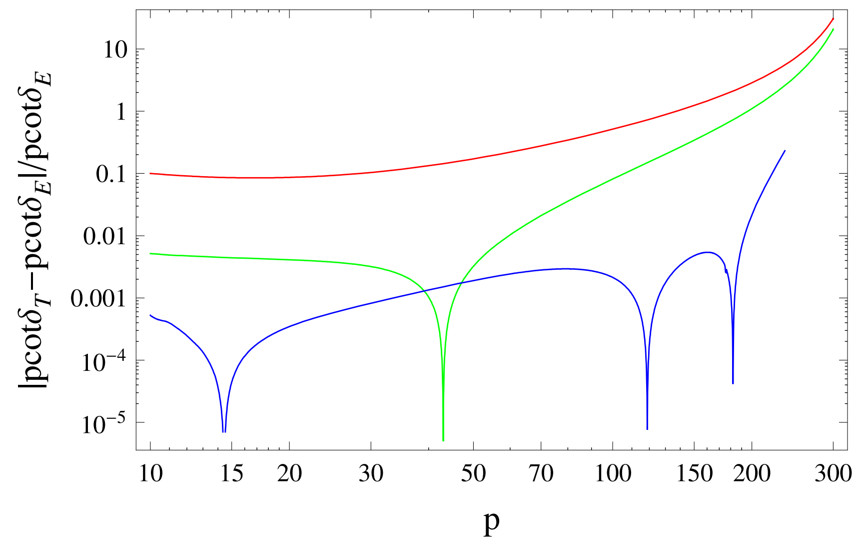


NNLO :



NNLO Results (Fleming,Mehen,Stewart '99)





- ${}^1S_0$ looks good!
- RG invariant at each order
- Two fit parameters at NNLO

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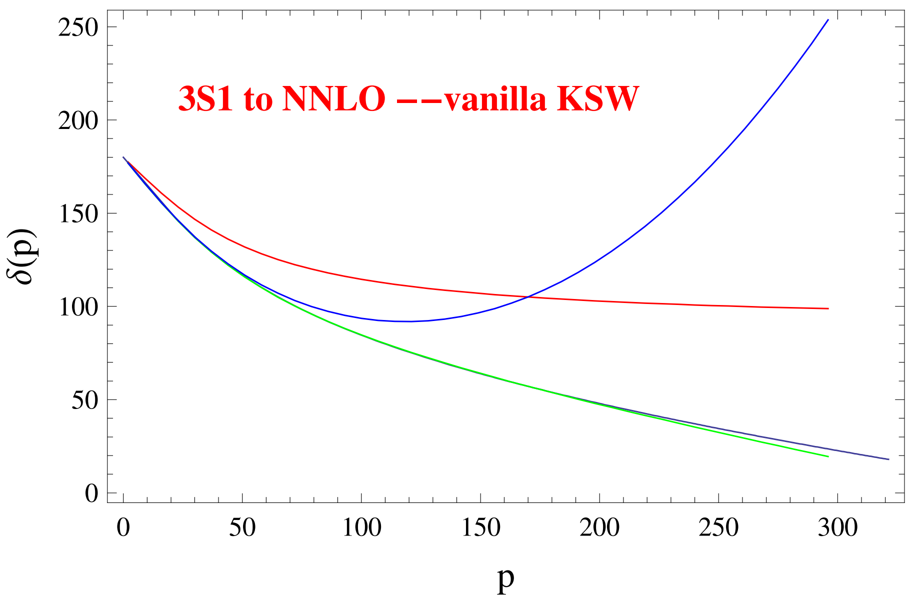

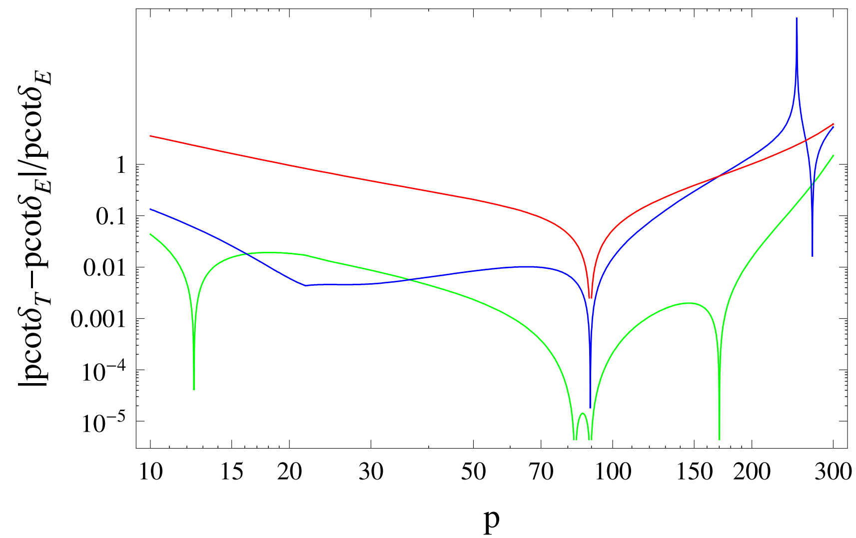

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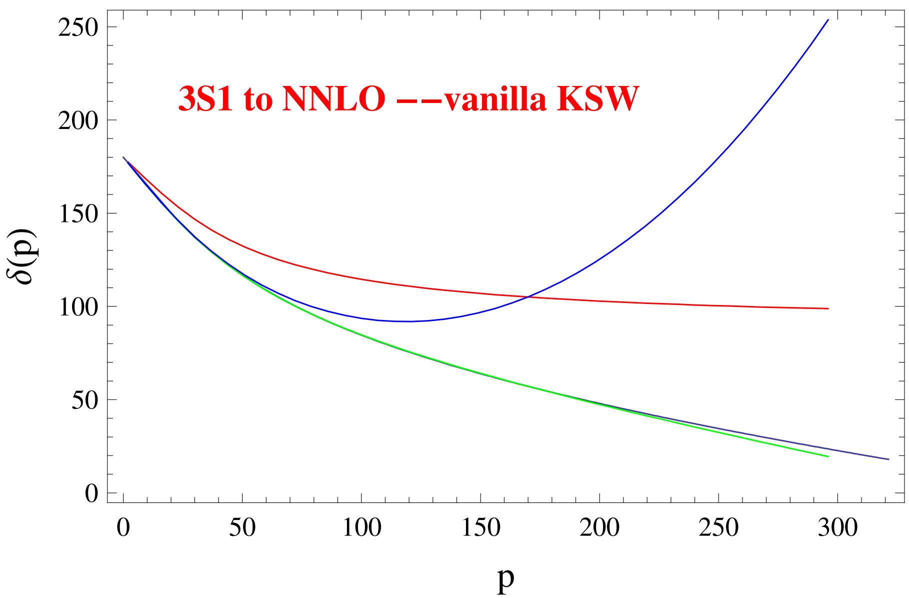

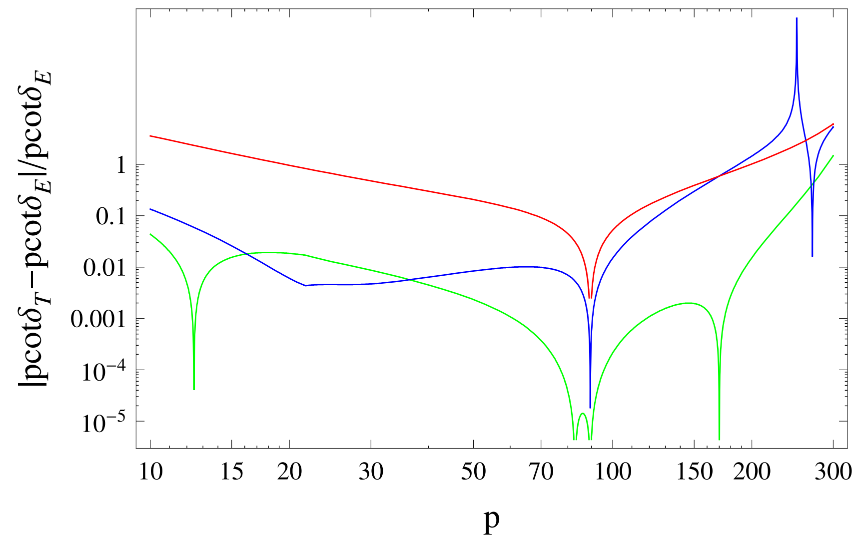

OUCH!! $^3S_1$ does not converge!!

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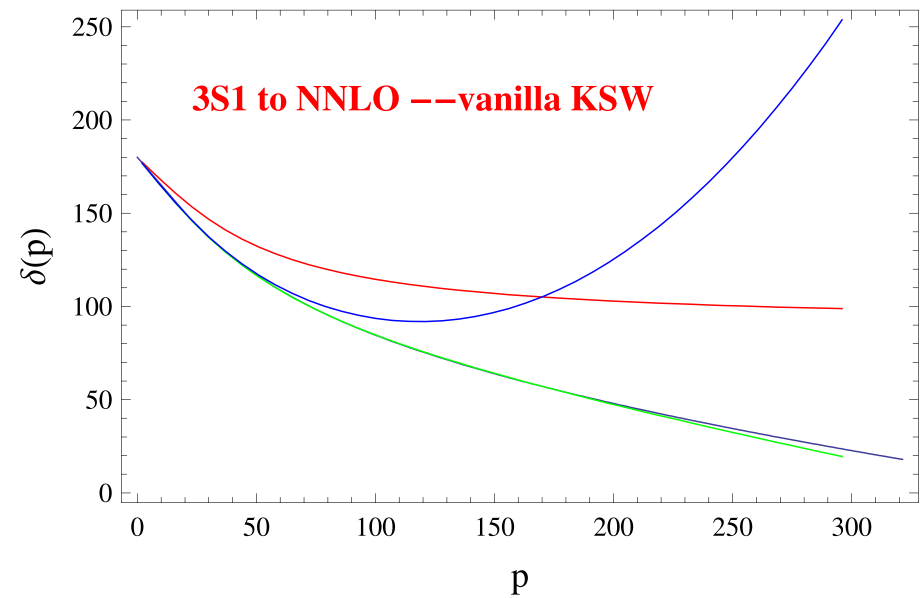
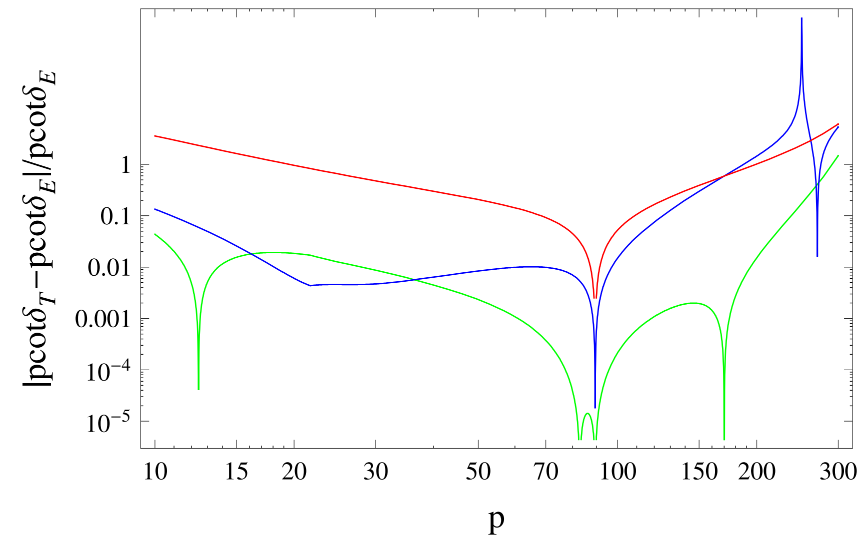


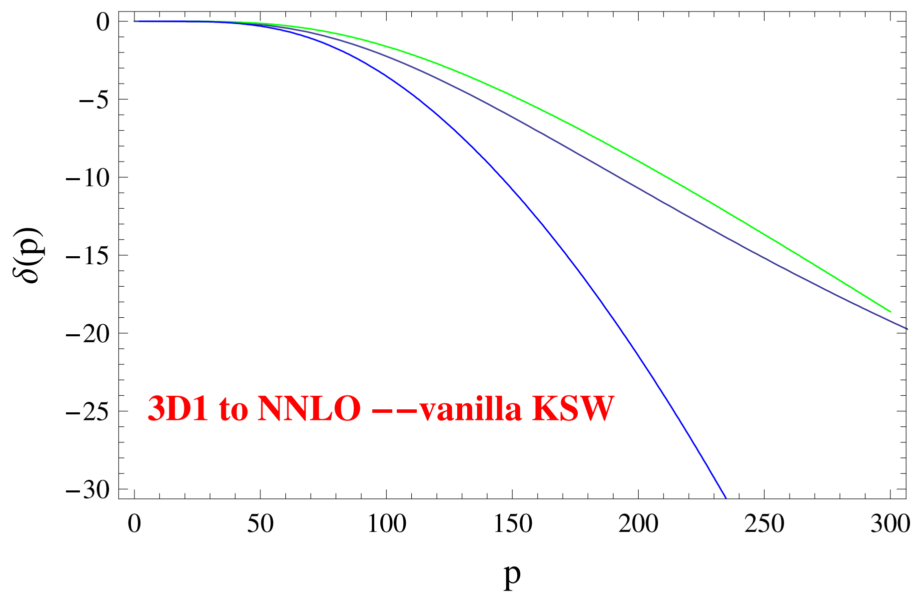


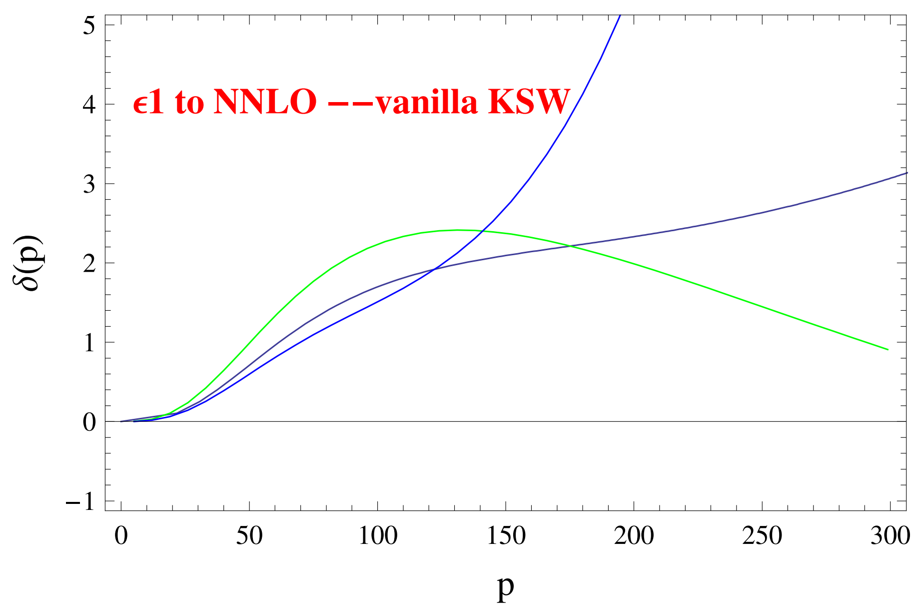
OUCH!! $^3S_1$ does not converge!!

$$\mathcal{A} \sim 6 \left(\frac{4\pi}{M}\frac{1}{\gamma + ip}\right)^2 \frac{M}{4\pi}\left(\frac{g_A^2 M}{8\pi f^2}\right)^2 p^3 \tan^{-1}\left(\frac{p}{m_\pi}\right) \underset{p\to\infty}{\longrightarrow} p$$

Survives in the chiral limit!

NNLO Results





Configuration space viewpoint

$$V_C(r;m_\pi) \;=\; -\frac{\alpha_\pi}{r}\,m_\pi^2\,e^{-m_\pi r} \;=\; -\frac{\alpha_\pi}{r}\,m_\pi^2 \;+\; \vartheta(r^0)$$

$$V_T(r;m_\pi) \;=\; -\frac{\alpha_\pi}{r}\,m_\pi^2\,e^{-m_\pi r}\left(1+\frac{3}{m_\pi r}+\frac{3}{m_\pi^2 r^2}\right) \;=\; -\frac{3\alpha_\pi}{r^3} \;+\; \frac{\alpha_\pi}{2r}\,m_\pi^2 \;+\; \vartheta(r)$$

$$\alpha_\pi = \frac{g_A^2}{16\pi F_\pi^2}$$

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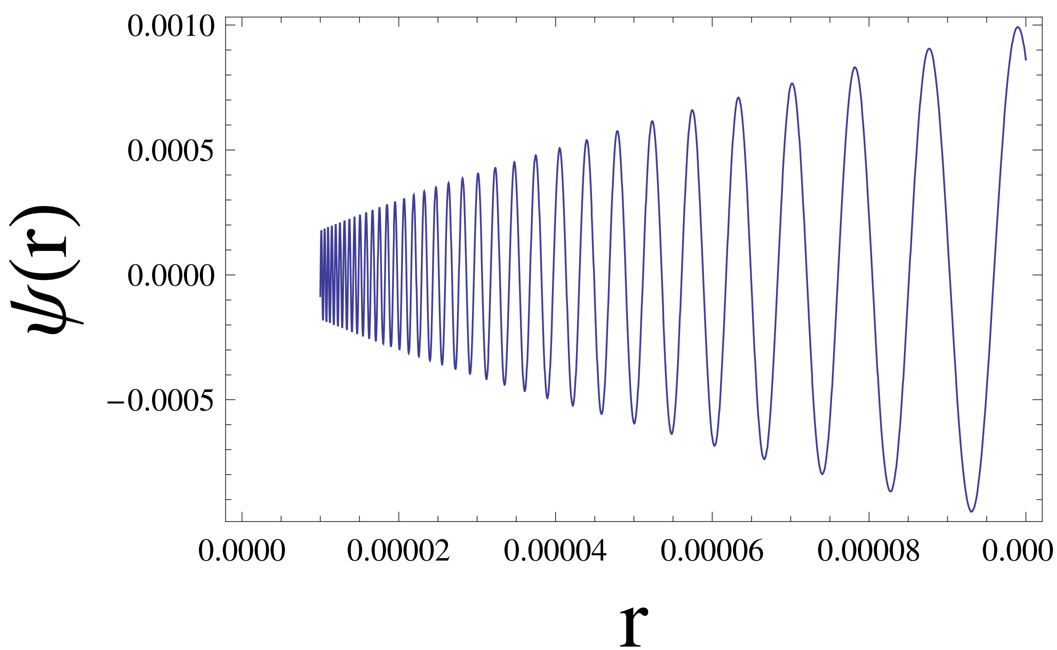

Breakdown of perturbation theory is due to singular tensor interaction

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Expand about the chiral limit? (Bedaque,Savage,van Kolck,SB '02)

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Expand about the chiral limit? (Bedaque,Savage,van Kolck,SB '02)

Pauli-Villars Regularization (Kaplan,Vuorinen,SB '08)

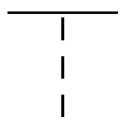
$$V_C^{PV}(r; m_\pi, \lambda) = V_C(r; m_\pi) - V_C(r; \lambda) = \frac{\alpha_\pi}{r}\left(\lambda^2 - m_\pi^2\right) + \vartheta(r^0)$$

$$V_T^{PV}(r; m_\pi, \lambda) = V_T(r; m_\pi) - V_T(r; \lambda) = -\frac{\alpha_\pi}{2r}\left(\lambda^2 - m_\pi^2\right) + \vartheta(r)$$

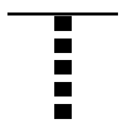
Absorb effect of singular interaction into contact operators

We would like to leave ${}^1S_0$ unaffected

Modification of the pion propagator:



$$= \quad G_\pi(p,m) = i\,\frac{g_A^2}{4f_\pi^2}\,\frac{(\mathbf{q}\cdot\boldsymbol{\sigma}_1)(\mathbf{q}\cdot\boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)}{\mathbf{q}^2+m^2}$$



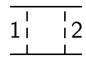
$$= \quad G_{(1,0)}(p,\lambda) = i\,\frac{g_A^2}{4f_\pi^2}\,\frac{\lambda^2}{\mathbf{q}^2+\lambda^2}\,(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)$$

$$G_\pi(p,m_\pi) \to \widetilde{G}_\pi(p,m_\pi,\lambda) \quad = \quad G_\pi(p,m_\pi) - G_\pi(p,\lambda) + G_{(1,0)}(p,\lambda)$$

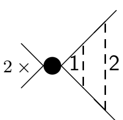
Modification of NNLO KSW (FMS) results?

Modification of NNLO KSW (FMS) results?

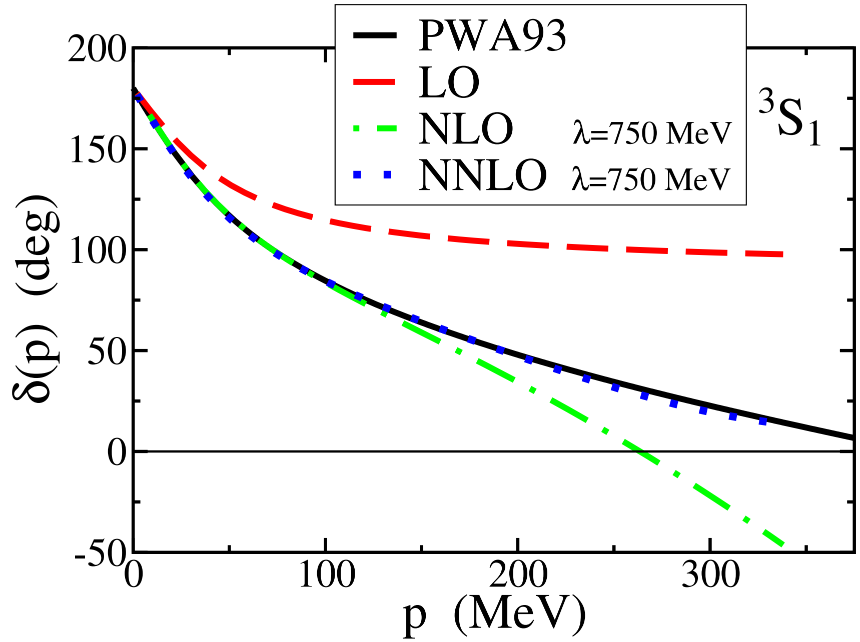
- Count $\lambda \sim m_\pi \sim Q$ (with $\lambda \geq 2\Lambda_{NN}$!)
- Positive powers of λ absorbed into C.T.s
- $\lambda \to \infty \to$ KSW

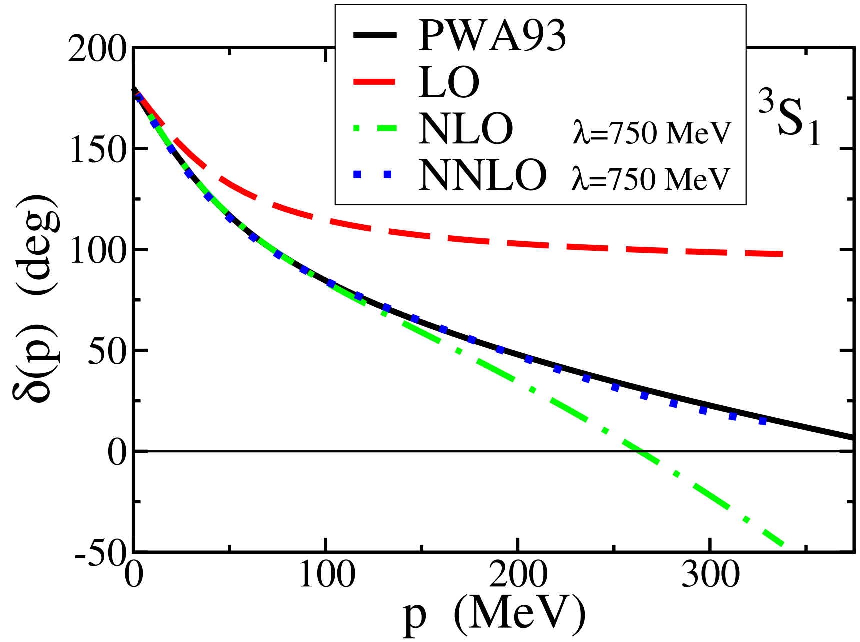


$$
\begin{aligned}
&= 3\frac{iM}{4\pi}\left(\frac{g_A^2}{2f^2}\right)^2\Bigg\{\frac{3im_2^2m_1^2}{8p^3}-\frac{m_1m_2(m_1+m_2)}{4p^2}-\frac{i(m_1^2+m_2^2)}{4p}-(m_1+m_2)+\frac{ip}{2}+\frac{4\mu}{3}\\
&+\left(\frac{3m_1^2m_2^2(m_1^2+m_2^2)}{16p^5}-\frac{m_1^4+m_2^4-4m_1^2m_2^2}{8p^3}-\frac{m_1^2+m_2^2}{p}-2p\right)\tan^{-1}\left(\frac{2p}{m_1+m_2}\right)\\
&-\frac{i}{4}\left(\frac{3m_1^4m_2^2}{4p^5}-\frac{m_1^4}{2p^3}+\frac{m_1^2m_2^2}{p^3}\right)\log\left(1-\frac{2ip}{m_1}\right)\\
&-\frac{i}{4}\left(\frac{3m_1^2m_2^4}{4p^5}-\frac{m_2^4}{2p^3}+\frac{m_1^2m_2^2}{p^3}\right)\log\left(1-\frac{2ip}{m_2}\right)\\
&+\frac{1}{8}\left(\frac{3m_1^3m_2^3(m_1+m_2)}{4p^6}+\frac{m_1^3m_2^2}{p^4}+\frac{m_1^2m_2^3}{p^4}\right)\log\left(1+\frac{4p^2}{(m_1+m_2)^2}\right)\\
&-\frac{1}{4}\left(\frac{3m_1^4m_2^4}{8p^7}+\frac{m_1^2m_2^2(m_1^2+m_2^2)}{2p^5}+\frac{m_2^2m_1^2}{p^3}\right)\times\\
&\Bigg[\operatorname{Im}\mathsf{Li}_2\left(-\frac{m_1-2ip}{m_2}\right)+\operatorname{Im}\mathsf{Li}_2\left(-\frac{m_2-2ip}{m_1}\right)+\operatorname{Im}\mathsf{Li}_2\left(-\frac{m_1+2ip}{m_2-2ip}\right)\\
&-\frac{1}{2}\tan^{-1}\left(\frac{2p}{m_2}\right)\log\left(\frac{m_2^2+4p^2}{m_1^2}\right)-\frac{i}{2}\log\left(1-\frac{2ip}{m_1}\right)\log\left(1+\frac{4p^2}{m_2^2}\right)\Bigg]\Bigg\}\,;\\
&\equiv 3\frac{iM}{4\pi}\left(\frac{g_A^2}{2f^2}\right)^2\mathcal{K}_i\,.
\end{aligned}
$$

2 × 

$$
\begin{aligned}
&= 3i\,\mathcal{A}_{-1}\left(\frac{Mg_A^2}{8\pi f^2}\right)^2\Bigg\{\frac{3im_1^3m_2^2}{4p^3}-\frac{m_1^3m_2}{2p^2}-\frac{m_1^2m_2^2}{p^2}-\frac{im_1(m_1^2+m_1m_2+m_2^2)}{2p}\\
&+\frac{11m_1^2}{6}-m_1m_2+\frac{4m_2^2}{3}-2i(m_1+m_2)p+\frac{8i\mu p}{3}+\frac{4\mu^2}{3}\\
&-2(2p^2+m_1^2+m_2^2)\ln\frac{2\mu}{m_1+m_2-2ip}\\
&+\left(\frac{3m_1^4m_2^2}{4p^4}-\frac{m_1^4}{2p^2}+\frac{m_1^2m_2^2}{p^2}\right)\log\left(\frac{m_1-2ip}{m_1+m_2-2ip}\right)\\
&-\frac{1}{2}\left(\frac{3im_1^3m_2^4}{4p^5}-\frac{3m_1^2m_2^4}{4p^4}+\frac{im_1^3m_2^2}{p^3}-\frac{m_1^2m_2^2}{p^2}+\frac{m_2^4}{2p^2}\right)\log\left(\frac{m_1+m_2}{m_2}\,\frac{m_2-2ip}{m_1+m_2-2ip}\right)\\
&+\frac{1}{2}\left(\frac{3im_1^4m_2^3}{4p^5}+\frac{3m_1^4m_2^2}{4p^4}+\frac{im_1^2m_2^3}{p^3}-\frac{m_1^4}{2p^2}+\frac{m_1^2m_2^2}{p^2}\right)\log\left(1-\frac{2ip}{m_1+m_2}\right)\\
&-\frac{1}{2}\left(\frac{3m_1^4m_2^4}{8p^6}+\frac{m_1^4m_2^2}{2p^4}+\frac{m_1^2m_2^4}{2p^4}+\frac{m_1^2m_2^2}{p^2}\right)\times\\
&\Bigg[\mathsf{Li}_2\left(-\frac{m_1-2ip}{m_2}\right)+\mathsf{Li}_2\left(-\frac{m_2-2ip}{m_1}\right)+\mathsf{Li}_2\left(-\frac{m_2+2ip}{m_1-2ip}\right)-\mathsf{Li}_2\left(-\frac{m_2}{m_1}\right)\\
&+\log\left(1-\frac{2ip}{m_1}\right)\log\left(1-\frac{2ip}{m_2}\right)+\frac{1}{2}\log^2\left(\frac{m_1-2ip}{m_2}\right)+\frac{\pi^2}{6}\Bigg]\Bigg\}\,;\\
&\equiv 3i\,\mathcal{A}_{-1}\left(\frac{Mg_A^2}{8\pi f^2}\right)^2\mathcal{K}_k\,.
\end{aligned}
$$


PWA93
LO
NLO λ=750 MeV
NNLO λ=750 MeV
$^3S_1$
δ(p) (deg)
p (MeV)
200
150
100
50
0
-50
0
100
200
300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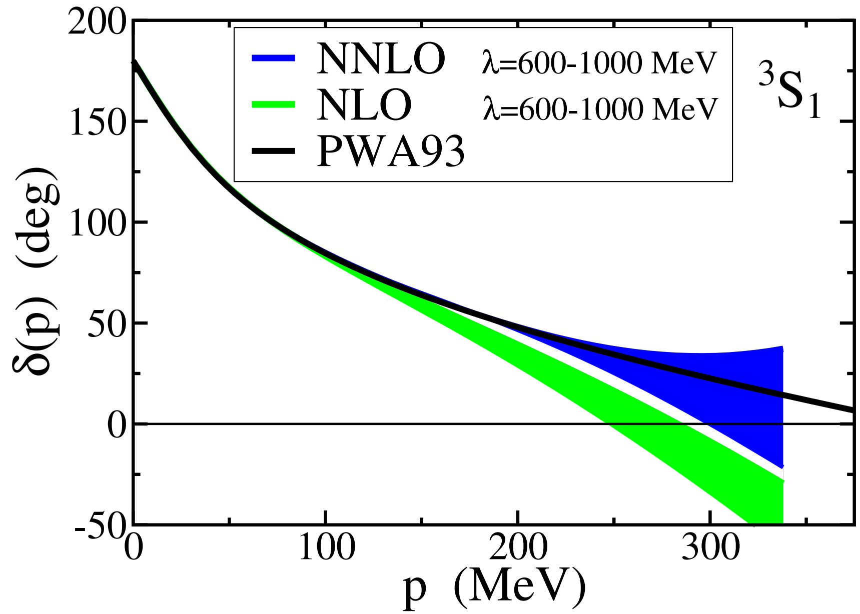
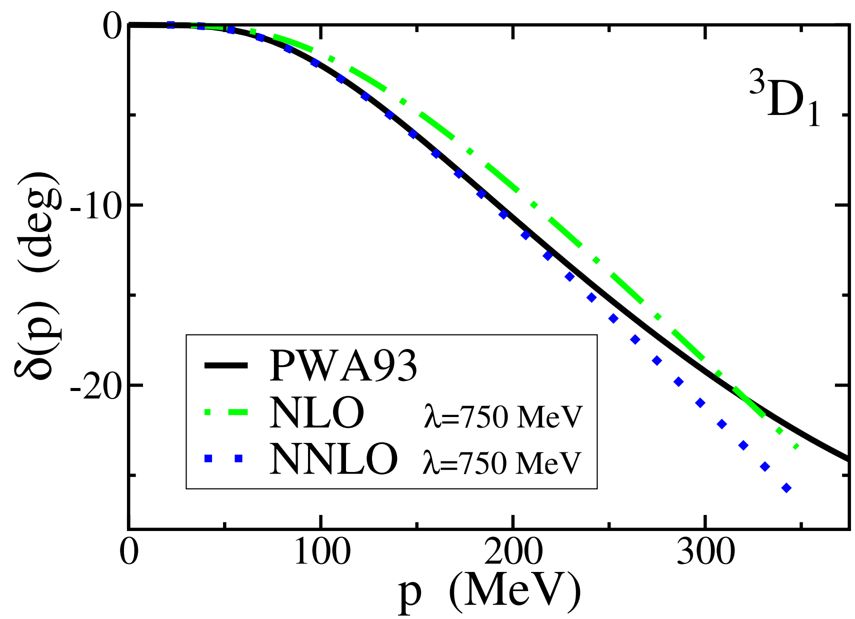
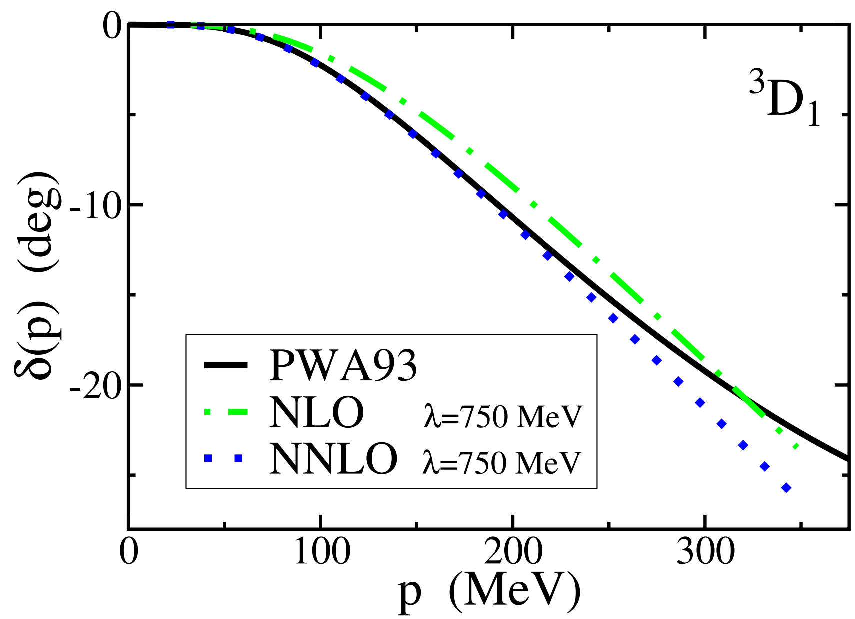

NNLO λ=600-1000 MeV
NLO λ=600-1000 MeV
PWA93
$^3S_1$
δ(p) (deg)
p (MeV)



$^3D_1$
δ(p) (deg)
p (MeV)
0
-10
-20
0
100
200
300
PWA93
NLO λ=750 MeV
NNLO λ=750 MeV



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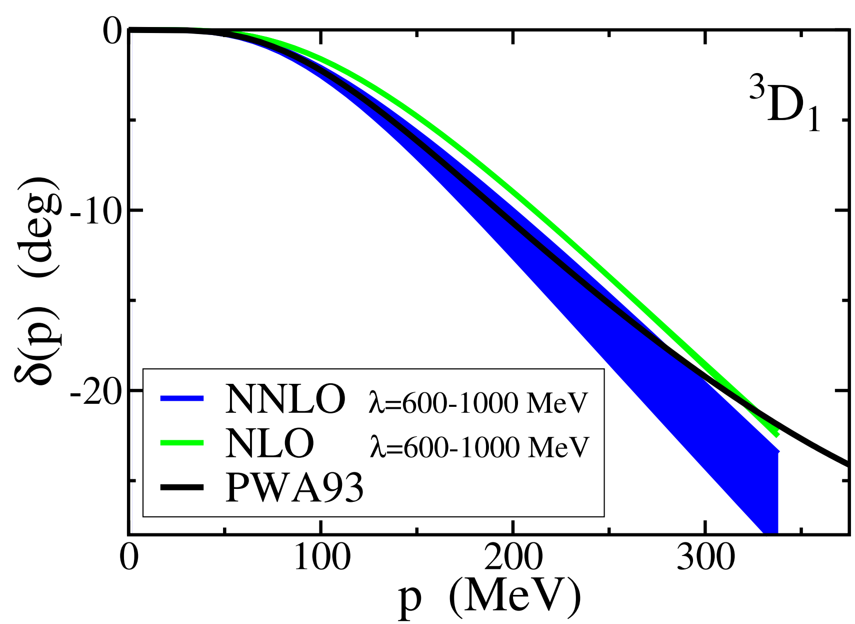

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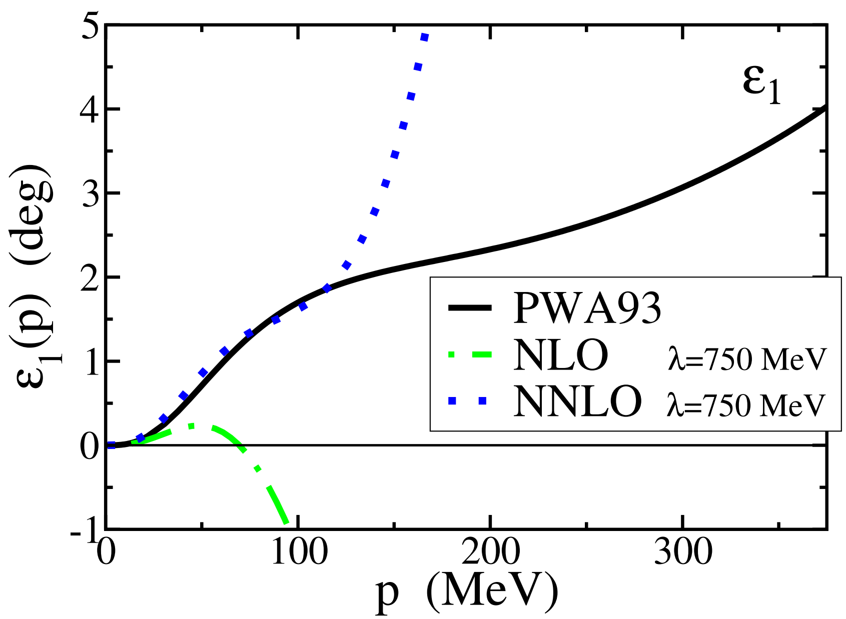
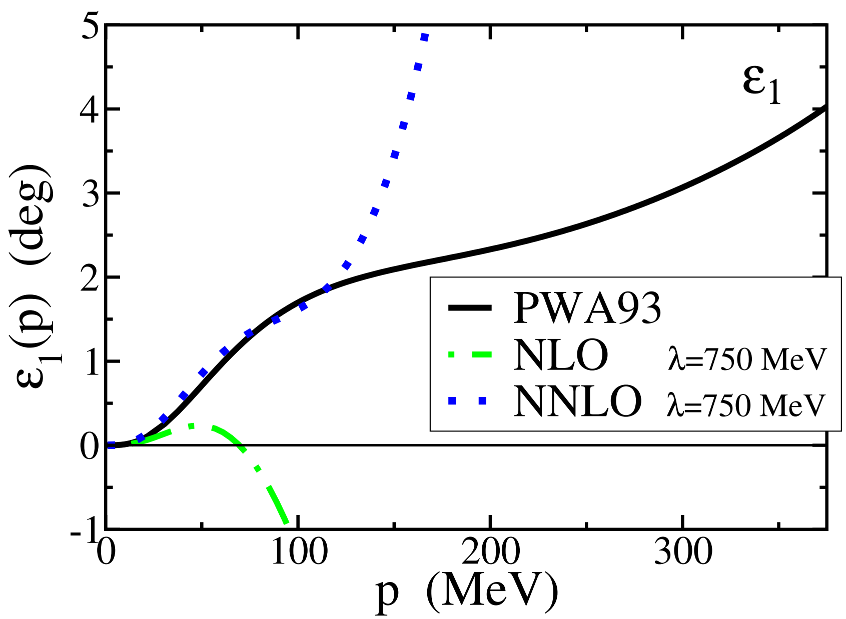


5
4
3
2
1
0
-1
0
100
200
300
p (MeV)
$\varepsilon_1(p)$ (deg)
ε_1
PWA93
NLO λ=750 MeV
NNLO λ=750 MeV


ε1
ε1(p) (deg)
p (MeV)
PWA93
NLO λ=750 MeV
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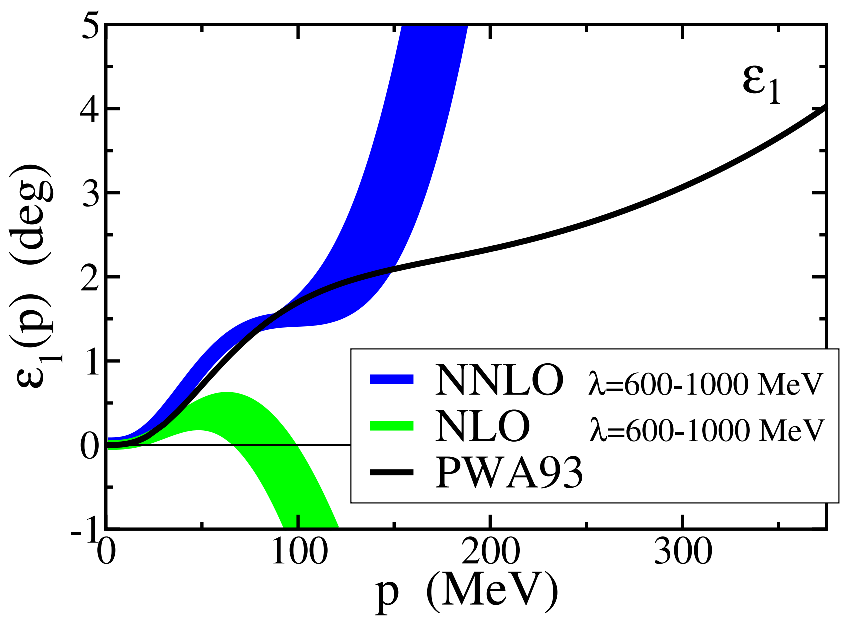
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What is the meaning of λ?

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Like μ in PQCD:

- Unphysical
- Controls resummation of log divergences into the coupling constant
- Controls convergence of perturbative expansion

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λ is:

- Unphysical
- Controls resummation of $1/r^3$ effects into contact interactions
- Controls convergence of perturbative expansion

Conclusion

- New EFT scheme for NN scattering with perturbative pions seems to cure convergence problems of KSW. Dimensionful parameter λ regulates singular tensor interaction. New scheme describes s and d waves well.

- Analytic formulas for amplitudes available, which allows detailed study of renormalization issues. However, as with KSW scheme, amplitudes are slowly converging.

- Many processes to calculate: deuteron form factor, πd scattering, Compton, *etc.*!

- In progress: beta functions, higher partial waves, starting point for many-body theory.

Effective field theory strategy:

- Fit LO, NLO, *etc.* couplings in $\mathcal{L}_{EFF}$ to low-energy NN scattering data.
- Use these couplings to compute:

  electromagnetic form factors of deuteron

  deuteron compton scattering, polarizability

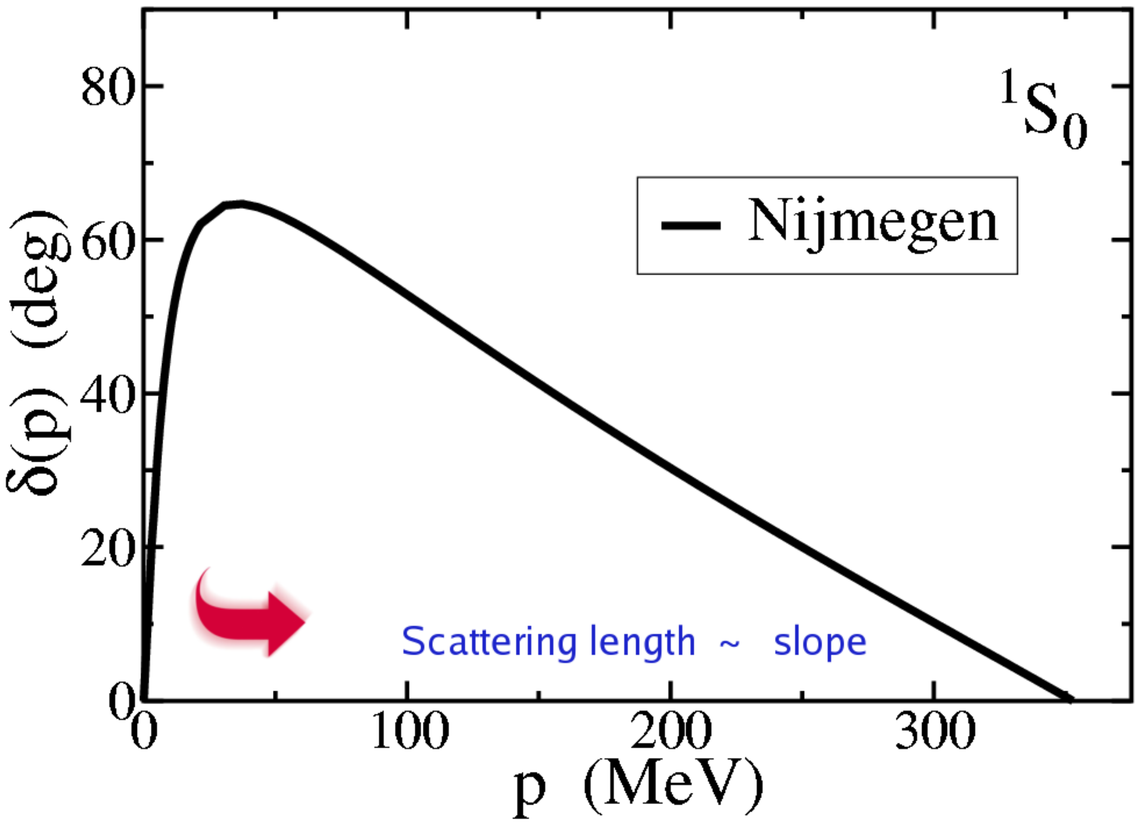
  $np \to d\gamma$

  anapole moment

  muon capture, *etc.*
- Fit three-nucleon forces from Nd scattering data.
- Use these couplings to compute:

  three-body processes!

Consider neutron-proton scattering in the $^1S_0$ channel



$$a_s^{^1S_0} \simeq -23 \text{ fm} \simeq \frac{1}{8 \text{ MeV}}$$

Phase shift varies over $\Delta p \sim 8$ MeV: NO Taylor expansion in $\frac{p}{m_\pi}$!

Dynamically generated length scale much longer than scale of underlying physics

$$a \gg \Lambda_{QCD}^{-1} \;!!$$

Resembles QFT at a non-trivial fixed point!

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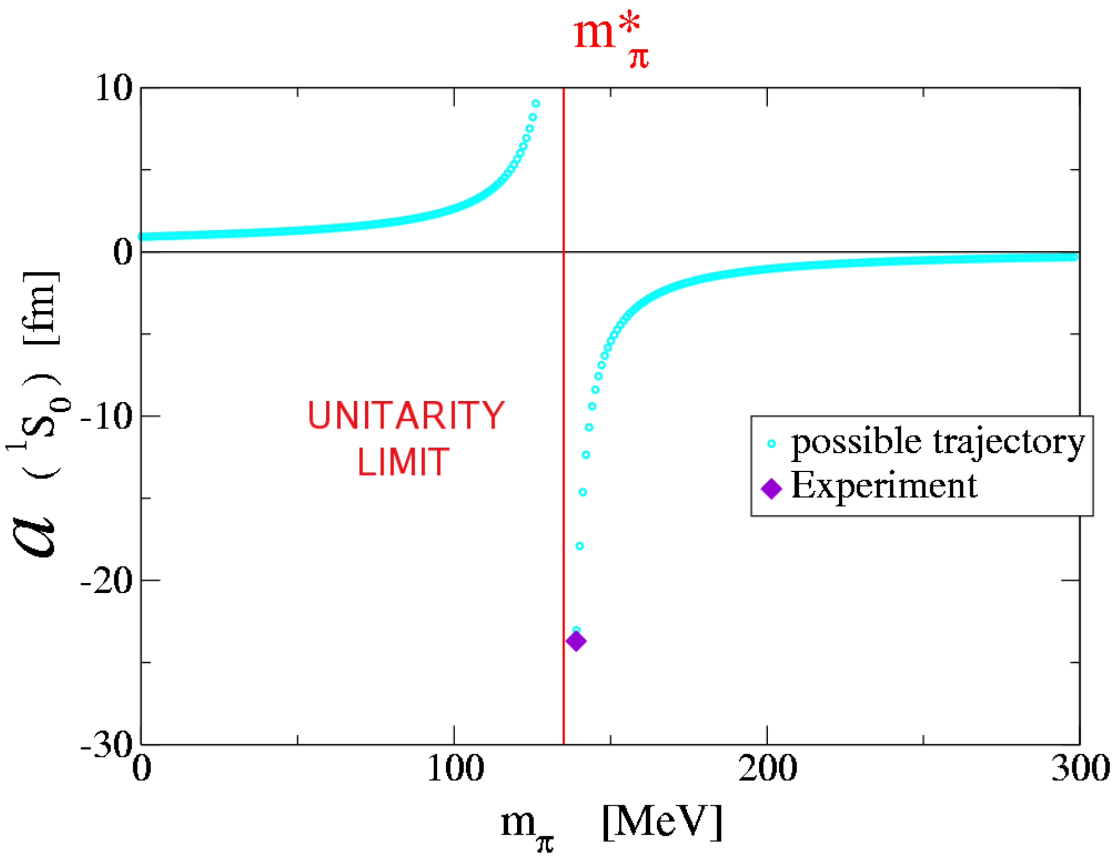
$$a \gg \Lambda^{-1}_{QCD} \text{ !!}$$

Resembles QFT at a non-trivial fixed point!

EFT is nonperturbative

(van Kolck '09)



$$a_s^{-1} \sim \frac{m_\pi - m_\pi^*}{m_\pi} \Lambda_{QCD}$$

Low-Energy S-wave Nucleon-Nucleon Scattering

$$A(p) \;=\; \frac{4\pi}{Mp}\sin\delta(p)e^{i\delta(p)} \;=\; \frac{4\pi}{M}\,\frac{1}{p\cot\delta(p) - ip} \;=\; \frac{4\pi}{M}\,\frac{1}{-\frac{1}{a_s} + \frac{1}{2}r_s\;p^2 + v_2 p^4 + \cdots - ip}$$

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neutron-proton (np) S-wave:

$$a_s^{^1S_0} = -23.714\ \text{fm} \qquad r_s^{^1S_0} = 2.73\ \text{fm}$$

$$a_s^{^3S_1} = 5.425\ \text{fm} \qquad r_s^{^3S_1} = 1.749\ \text{fm}$$

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Expand in p with $a_s p \sim 1$:

$$A(p) = -\frac{4\pi}{M}\frac{1}{(a_s^{-1} + ip)}\left[1 + \frac{r_s}{2(a_s^{-1} + ip)}p^2 + \left(\frac{r_s^2}{4(a_s^{-1} + ip)^2} + \frac{v_2}{(a_s^{-1} + ip)}\right)p^4 + \ldots\right]$$

EFT $\not\pi$

$p \ll m_\pi \quad \Longrightarrow \quad$ Integrate out the pion

Expansion in $\frac{p}{m_\pi}$, $\frac{p}{M}$

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EFT of contact operators:

$$\mathcal{L} = -\ C_0\ (N^\dagger N)^2 -\ C_2\ (N^\dagger \nabla^2 N)(N^\dagger N) + h.c. + \ldots$$

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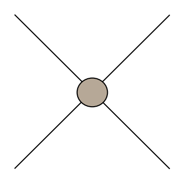
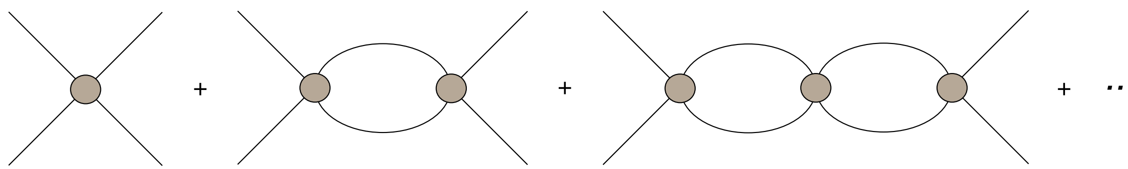
EFT of contact operators:

$$\mathcal{L} = -\, C_0\, (N^\dagger N)^2 - \, C_2\, (N^\dagger \nabla^2 N)(N^\dagger N) + h.c. + \ldots$$

$$V(p) = C_0 \, + \, C_2 \, p^2 \, + \, \ldots \equiv$$

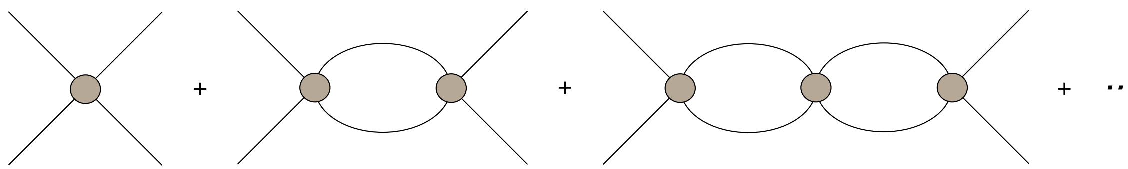


$A(p) =$



$$= \frac{\sum C_{2n}(\mu)\ p^{2n}}{1 - I_0 \sum C_{2n}(\mu)\ p^{2n}}$$

$$A(p) =$$



$$= \frac{\sum C_{2n}(\mu)\ p^{2n}}{1 - I_0 \sum C_{2n}(\mu)\ p^{2n}}$$

$$I_0 = \left(\frac{\mu}{2}\right)^{4-D} \int \frac{d^{D-1}\mathbf{q}}{(2\pi)^{D-1}} \frac{1}{E - \frac{|\mathbf{q}|^2}{M} + i\epsilon}$$

$$\xrightarrow[PDS]{} \quad -\frac{M}{4\pi}\ (\mu + ip)$$

Power counting

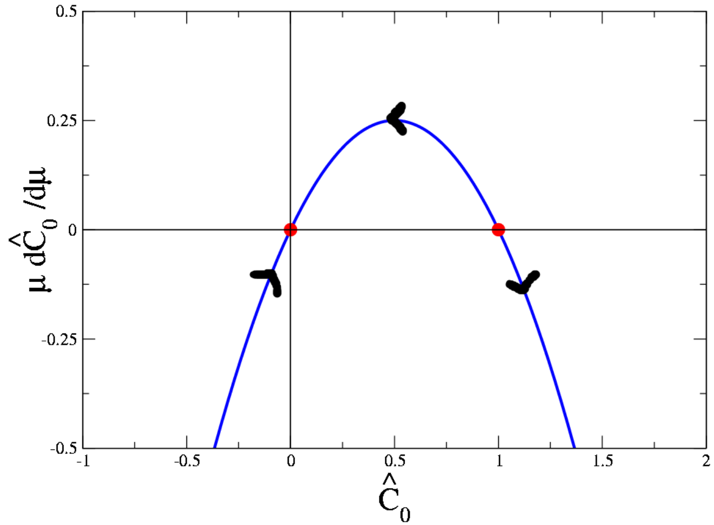
$$C_0(\mu) = -\frac{4\pi}{M}\frac{1}{\mu - 1/a_s}, \qquad C_2(\mu) = \frac{4\pi}{M}\frac{r_s}{(\mu - 1/a_s)^2}, \quad \ldots$$

$$A(p) = -\frac{C_0}{\left(1+\frac{C_0 M}{4\pi}(\mu+ip)\right)}\left[1+\frac{C_2 p^2/C_0}{\left(1+\frac{C_0 M}{4\pi}(\mu+ip)\right)}+\ldots\right]$$

$$Q \sim \mu \sim m_\pi$$

C_0 operator treated to all orders! $\quad$ C_n with $n \geq 2$ perturbative!

Non-Trivial Fixed Point



$$\hat{C}_0(\mu) \equiv -\frac{M\mu}{4\pi} C_0(\mu) = \frac{\mu}{\mu - 1/a_s}$$

$$\mu \frac{d}{d\mu} \hat{C}_0(\mu) = \hat{C}_0(\mu)\left(1 - \hat{C}_0(\mu)\right)$$

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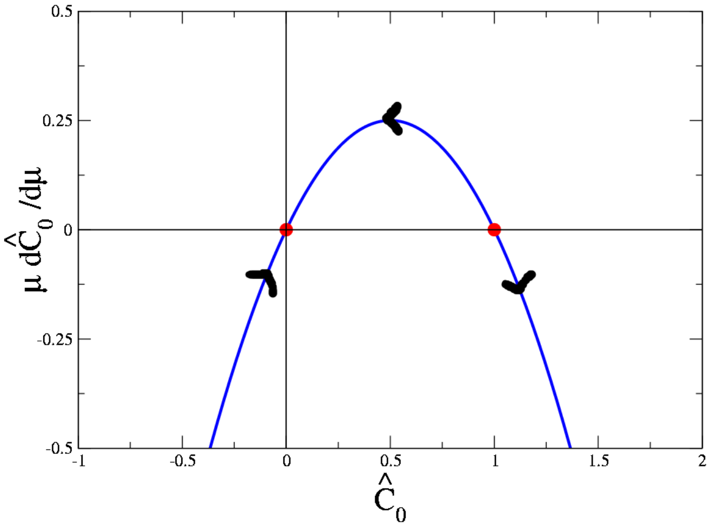

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$$a_s \to \pm\infty \leftrightarrow \hat{C}_0(\mu) = 1$$

$$\Downarrow$$

$\mathrm{EFT}(\not\pi)$ defines conformal field theory!!