SU(3)-breaking corrections to the hyperon vector coupling $f_1(0)$ in covariant baryon chiral perturbation theory

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V_{us} ,CKM unitarity, and the $f_1(0)$

Cabibbo-Kobayashi-Maskawa (**CKM**) matrix plays a very important role in our study and understanding of flavor physics

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Particularly, an accurate value of V_{us} is crucial in determinations of the other parameters and in tests of CKM unitarity, of which the most important is the 1st row unitarity:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1.$$

- V_{ub} **:** small, can be neglected at the present precision
- V_{ud} **:** superallowed nuclear beta-decays, neutron decays, pion decays
- V_{us} **:** kaon decays, tau decays, **hyperon decays** ⟷ $f_1(0)$

For exp. Info on V_{us}, see Dr. Achim Denig's talk on Tuesday.

V_{us} from hyperon decays and the Ademollo-Gatto theorem

- To determine V_{us} from hyperon decays, one must know the hyperon vector coupling $f_1(0)$ since experimentally only $|V_{us} f_1(0)|$ is accessible.

- Theoretically, $f_1(0)$ is known up to SU(3) breaking effects due to the hypothesis of Conservation of Vector Current (CVC). To obtain an accurate $f_1(0)$, one then needs to know the size of SU(3) breaking, which could be (naively) ~30%.

- On the other hand, the **Ademollo-Gatto** theorem tells that

$$f_1(0) = g_V + \mathcal{O}((m_s - m)^2)$$

M. Ademollo and R. Gatto, PRL 13, 264 (1964).

which 1) implies that SU(3) breaking corrections are of ~10%;

2) has an important consequence for a ChPT study

Theoretical determination of SU(3) breaking corrections to $f_1(0)$

Theoretical methods used to calculate $f_1(0)$:

- **Quark models:** *J. F. Donoghue et al., 1987; F. Schlumpf,1995;*
  A. Faessler et al., 2008, etc.
- **Large Nc** : *R. Flores-Mendieta, 2004.*
- **Lattice QCD** : *D. Guadagnoli et al., 2007; S. Sasaki et al., 2008.*
- **ChPT** : *A. Krause, 1990; J. Anderson et al., 1993; N. Kaiser, 2001;*
  G. Villadoro, 2006; A. Lacour et al., 2007.

Purpose of the present study:

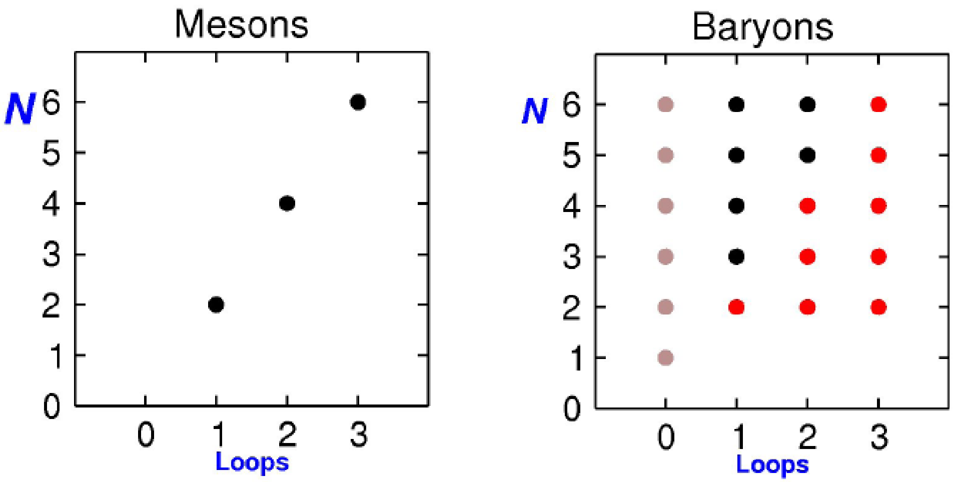
to calculate SU(3) breaking corrections to $f_1(0)$ using covariant ChPT

Two improvements compared to earlier ChPT studies:

- Removal of power-counting-restoration (PCR) dependence
- First covariant, order 4, taking into account dynamical decuplet contributions

Baryon ChPT—power-counting-restoration (PCR) dependence

Baryon chiral perturbation theory has long been complicated by the so-called power-counting-breaking (**PCB**) problem, i.e., the appearance of lower-order analytical terms in nominal higher-order loop calculations.



red dots denote possible PCB terms

J. Gasser et al., NPB 307, 779(1988)

Many power-counting-restoration (PCR) approaches have been proposed:

- Heavy Baryon ChPT : *Jenkins et al., 1993*
- Infrared baryon ChPT: *T. Becher and H. Leutwyler, 1999)*
- EOMS baryon ChPT : *J. Gegelia et al., 1999; T. Fuchs et al.,2003)*

Unfortunately, this necessarily introduces "**PCR dependence**", as mentioned in Dr. J. Martin-Camalich's talk, see also *LSG et al., PRL 101, 222002 (2008)* .

AG theorem's implication and the 1st novelty of the present work

- **First, we have performed a ChPT calculation without introducing PCR dependence** (*equivalent to the EOMS baryon ChPT*)

$$f_1(0) = g_V + \mathcal{O}((m_s - m)^2)$$

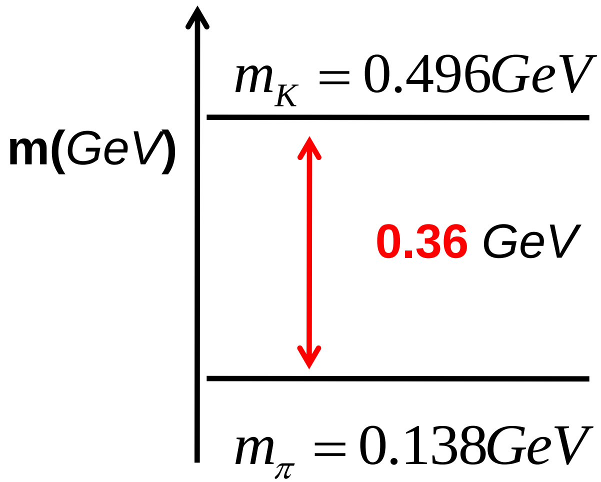
⇓

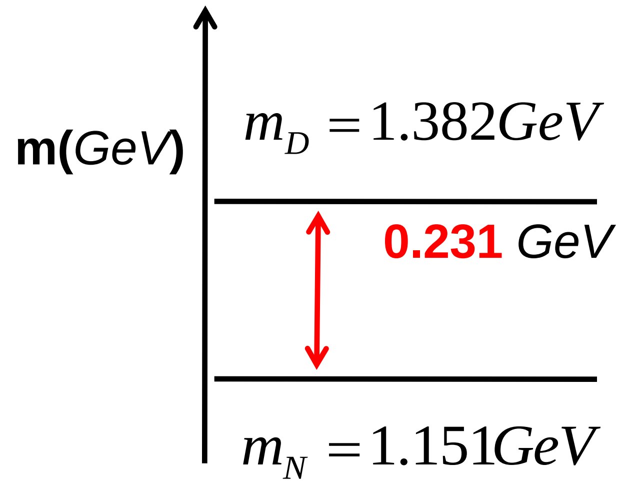
No analytical terms breaking SU(3) symmetry with chiral order less than or equal to 4 in ChPT, both at tree- and loop- levels

⇓

No need to apply any power-counting restoration (PCR) procedures, which thus removes the PCR dependence.

2^{nd} improvement: contributions of dynamical decuplet baryons





- **Second, we have taken into account the contributions of virtual decuplet baryons**.
  - ◆They are important because m_D-m_B~**0.231** GeV is similar to pion mass and smaller than kaon (eta) mass. Therefore, in SU(3) ChPT, the exclusion of decuplet baryons **is not well justified**.
  - ◆As I will show, the decuplet baryons **do provide sizable contributions** that **completely change** the results obtained with only dynamical octet baryons

Definition of $f_1(0)$ and notations used in this work

Baryon vector form factors as probed by the charged $\Delta S = 1$ weak current

$$\langle B'|V^\mu|B\rangle = V_{us}\bar{u}(p')\left[\gamma^\mu f_1(q^2) + \frac{2i\sigma^{\mu\nu}q_\nu}{M_{B'}+M_B} f_2(q^2) + \frac{2q^\mu}{M_{B'}+M_B} f_3(q^2)\right] u(p),$$

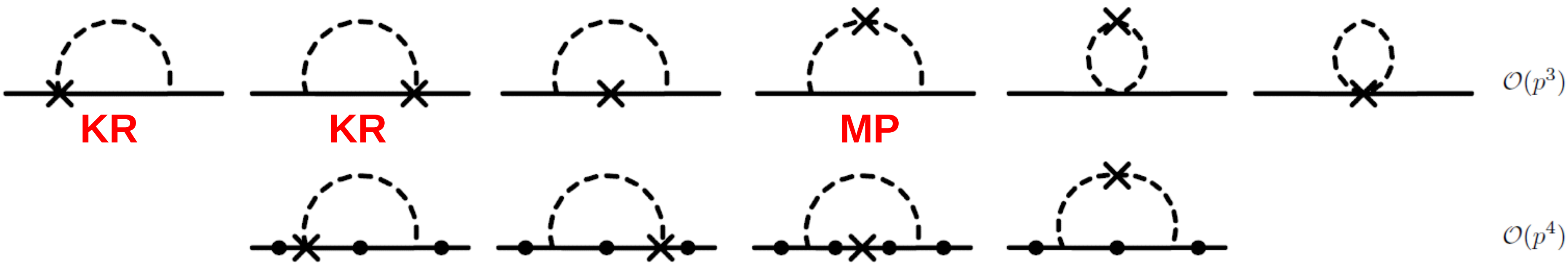
We will parameterize the SU(3)-breaking corrections order-by-order in the covariant chiral expansion as follows:

$$f_1(0) = g_V\left(1 + \delta^{(2)} + \delta^{(3)} + \cdots\right)$$

where $\delta^{(2)}$ and $\delta^{(3)}$ are the LO and NLO SU(3)-breaking corrections induced by loops, corresponding to $\mathcal{O}(p^3)$ and $\mathcal{O}(p^4)$ chiral calculations.

Note :two order schemes, SU(3) breaking and chiral

Dynamical octet contributions



WF normalization diagrams not shown

- Full 4th chiral order analytical results are complicated (not shown here).
- In the case of 3rd chiral order, results are given below as a sum of individual diagrams

$$\begin{aligned}\delta_B^{(2)}(i \to j) \;=\; & \sum_{M=\pi,\eta,K} \beta_M^{\mathrm{BP}} H_{\mathrm{BP}}(m_M) + \sum_{M=\pi,\eta} \beta_M^{\mathrm{MP}} H_{\mathrm{MP}}(m_M, m_K) + \sum_{M=\pi,\eta,K} \beta_M^{\mathrm{KR}} H_{\mathrm{KR}}(m_M) \\ & -\frac{3}{8} \sum_{M=\pi,\eta} H_{\mathrm{TD1}}(m_M, m_K) + \frac{3}{8} \sum_{M=\pi,\eta} H_{\mathrm{TD2}}(m_M) + \frac{3}{4} H_{\mathrm{TD2}}(m_K) \\ & +\frac{1}{2} \sum_{M=\pi,\eta,K} \left(\beta_M^{\mathrm{WF}}(i) + \beta_M^{\mathrm{WF}}(j)\right) H_{\mathrm{WF}}(m_M),\end{aligned}$$

Manifestation of the Ademollo-Gatto (AG) theorem

$$
\begin{aligned}
H_{\rm MP} \;=\; & \frac{1}{(4\pi F_0)^2}\frac{1}{4\,(m_1^2-m_2^2)}\times \\
& \Bigg[\frac{8\left(\frac{m_1^2}{M_B^2}-4\right)\cos^{-1}\left(\frac{m_1}{2M_B}\right)m_1^6}{\sqrt{4m_1^2M_B^2-m_1^4}}-\frac{4\log\left(\frac{m_1^2}{M_B^2}\right)m_1^6}{M_B^2}+\left(6\log\left(\frac{m_1^2}{M_B^2}\right)+2\log\left(\frac{\mu^2}{M_B^2}\right)+11\right)m_1^4 \\
& -\frac{8\left(\frac{m_2^2}{M_B^2}-4\right)\cos^{-1}\left(\frac{m_2}{2M_B}\right)m_2^6}{\sqrt{4m_2^2M_B^2-m_2^4}}+\frac{4\log\left(\frac{m_2^2}{M_B^2}\right)m_2^6}{M_B^2}-\left(6\log\left(\frac{m_2^2}{M_B^2}\right)+2\log\left(\frac{\mu^2}{M_B^2}\right)+11\right)m_2^4 \\
& +8M_B^2(1+\log\left(\frac{\mu^2}{M_B^2}\right))(m_1^2-m_2^2)\Bigg],
\end{aligned}
$$

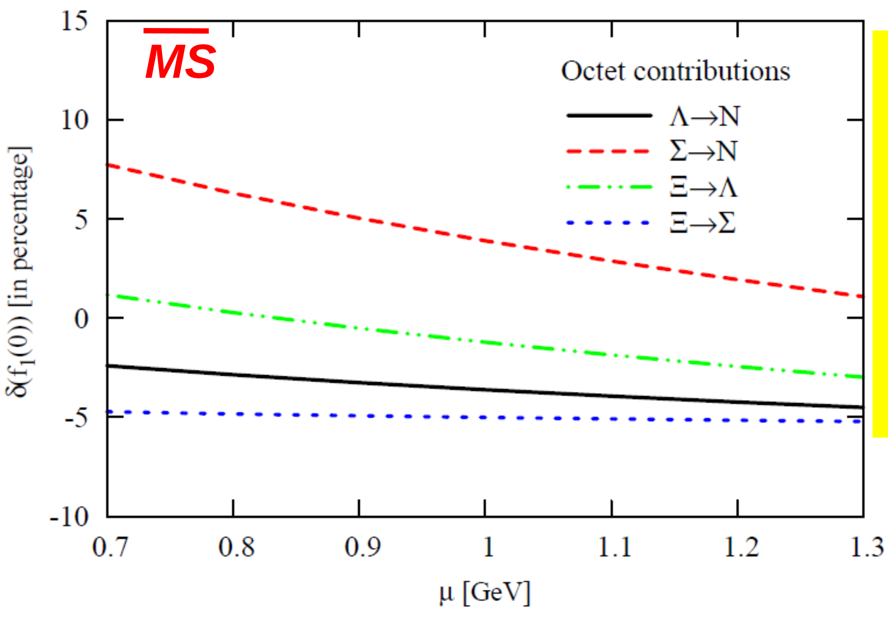
$$
H_{\rm KR}=\frac{1}{(4\pi F_0)^2}\left[\frac{\log\left(\frac{M_B}{m}\right)m^4}{M_B^2}-\frac{\sqrt{4M_B^2-m^2}\cos^{-1}\left(\frac{m}{2M_B}\right)m^3}{M_B^2}+m^2(\log\left(\frac{\mu^2}{M_B^2}\right)+2)+M_B^2(1+\log\left(\frac{\mu^2}{M_B^2}\right))\right]
$$

- Separately, all the loop functions are divergent, but the sum is finite
- H_{MP} and H_{KR} also contain power-counting-breaking pieces, but these pieces cancel in the sum–a manifestation of the AG theorem.

Octet contributions to the SU(3) breaking corrections

	present work			HBChPT [17]	IRChPT [18]
	$\delta^{(2)}$	$\delta^{(3)}$	$\delta^{(2)}+\delta^{(3)}$	$\delta^{(2)}+\delta^{(3)}$	$\delta^{(2)}+\delta^{(3)}$
$\Lambda \to N$	-3.8	$0.2^{+1.2}_{-0.9}$	$-3.6^{+1.2}_{-0.9}$	★ 2.7	-5.7 ± 2.1
$\Sigma \to N$	-0.8	$4.7^{+3.8}_{-2.8}$	$3.9^{+3.8}_{-2.8}$	4.1	2.8 ± 0.2
$\Xi \to \Lambda$	-2.9	$1.7^{+2.4}_{-1.8}$	$-1.2^{+2.4}_{-1.8}$	★ 4.3	-1.1 ± 1.7
$\Xi \to \Sigma$	-3.7	$-1.3^{+0.3}_{-0.2}$	$-5.0^{+0.3}_{-0.2}$	★ 0.9	-5.6 ± 1.6

Uncertainties only reflect scale dependence:μ=0.7-1.3 GeV
With central value calculated at μ=1 GeV

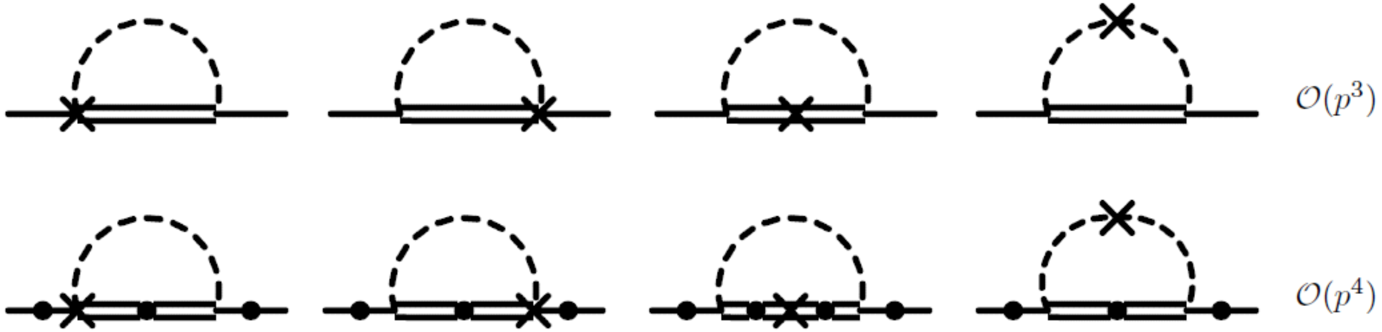


- Our results are similar to those of **IR**, but very different from those of **HB**
- Convergence is **slow**, particularly, in the Sigma-N channel.

[17] G. Villadoro, PRD 74, 014018 (2006).
[18] A. Lacour et al., JHEP 0710, 083 (2007).

Dynamical decuplet contributions: Diagrams

WF normalization diagrams not shown

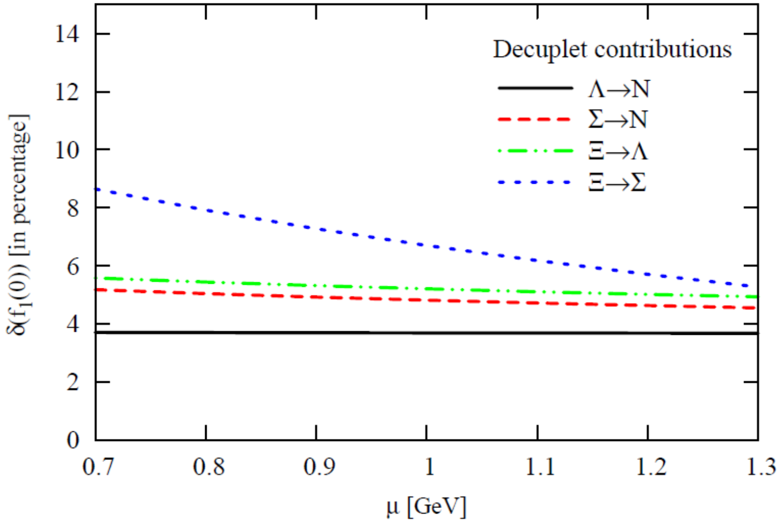


- Description of decuplet baryons and their interactions is **subtle**, as you may have heard from other talks in this workshop. For further details, see e.g.

V. Pascalutsa, Phys. Lett. B 503, 85 (2001)
V. Pascalutsa, et al., Phys. Rep. 437, 125 (2007)
C. Hacker et al., PRC 72 (2005) 055203.

- In the work, we have adopted the **“consistent coupling scheme”** advocated by V. Pascalutsa and collaborators. Details can be found in *PRD 79, 094022 (2009).*

Dynamical decuplet contributions: Results



scale dependence

	Present work			HBChPT		
	$\delta^{(2)}$	$\delta^{(3)}$	$\delta^{(2)}+\delta^{(3)}$	$\delta^{(2)}$	$\delta^{(3)}$	$\delta^{(2)}+\delta^{(3)}$
$\Lambda \to N$	0.7	$3.0^{+0.1}_{-0.1}$	$3.7^{+0.1}_{-0.1}$	1.8	1.3	3.1
$\Sigma \to N$	-1.4	$6.2^{+0.4}_{-0.3}$	$4.8^{+0.4}_{-0.3}$	-3.6	8.8	5.2
$\Xi \to \Lambda$	-0.02	$5.2^{+0.4}_{-0.3}$	$5.2^{+0.4}_{-0.3}$	-0.05	4.2	4.1
$\Xi \to \Sigma$	0.7	$6.0^{+1.9}_{-1.4}$	$6.7^{+1.9}_{-1.4}$	1.9	-0.2	1.7

- NLO corrections are **larger** than LO corrections
- Overall, virtual decuplet baryons provide **positive** SU(3) breaking corrections
- They are **sizable** compared to the octet results

Full results including both octet and decuplet contributions:

	$\delta^{(2)}$	$\delta^{(3)}$	$\delta^{(2)} + \delta^{(3)}$
$\Lambda \to N$	-3.1	$3.2^{+1.3}_{-1.0}$	$0.1^{+1.3}_{-1.0}$
$\Sigma \to N$	-2.2	$10.9^{+4.2}_{-3.1}$	$8.7^{+4.2}_{-3.1}$
$\Xi \to \Lambda$	-2.9	$6.9^{+2.8}_{-2.1}$	$4.0^{+2.8}_{-2.1}$
$\Xi \to \Sigma$	-3.0	$4.7^{+2.2}_{-1.6}$	$1.7^{+2.2}_{-1.6}$

◆**NLO** breaking corrections are **larger than LO** breakings, which might indicate slow convergence but also that one should not trust LO results

◆**Positive** SU(3) breaking corrections for all the 4 channels

Comparison with the results of other approaches

TABLE V: SU(3)-breaking corrections (in percentage) to $f_1(0)$ obtained in different approaches.

	Present work	Large N_c	Quark model			quenched LQCD
		Ref. [3]	Ref. [11]	Ref. [12]	Ref. [13]	
$\Lambda \to N$	$0.1^{+1.3}_{-1.0}$	2 ± 2	-1.3	-2.4	0.1	
$\Sigma \to N$	$8.7^{+4.2}_{-3.1}$	4 ± 3	-1.3	-2.4	0.9	$-1.2 \pm 2.9 \pm 4.0$ [19]
$\Xi \to \Lambda$	$4.0^{+2.8}_{-2.1}$	4 ± 4	-1.3	-2.4	2.2	
$\Xi \to \Sigma$	$1.7^{+2.2}_{-1.6}$	8 ± 5	-1.3	-2.4	4.2	-1.3 ± 1.9 [20]

- Consistent with the large Nc results and those of chiral quark models
- Agree marginally with the quenched LQCD results

[3] R. Flores-Mendieta, PRD 70, 114036 (2004).
[11] J. F. Donoghue et al., P.RD 35, 934 (1987).
[12] F. Schlumpf, PRD 51, 2262 (1995).
[13] A. Faessler et al., PRD 78, 094005 (2008).
[19] D. Guadagnoli et al., NPB 761, 63 (2007)
[20] S. Sasaki et al., arXiv:0811.1406 [hep-ph].

Implications for the value of V_{us}

◆Full analysis using all data sets is complicated.

◆A simple (naive) calculation

- Using only experimental decay rates and g_1/f_1 ratios
- Using SU(3) symmetric values of $g_2=0$ and f_2
- Using our calculated $f_1(0)$
- **We obtain**

$$V_{us} = 0.2177 \pm 0.0030$$

Dr. Nicola Cabibbo et al. have performed a similar calculation but have used SU(3) **symmetric $f_1(0)$, they obtained**

$$V_{us} = 0.2250(27)$$

PRL 92, 251803 (2004)

◆**Comparing these two numbers, one can easily see the importance of $f_1(0)$.**

◆**Of course, both numbers should be taken with caution.**

Summary and conclusions

- We have calculated the SU(3) breaking corrections to $f_1(0)$ using the covariant baryon chiral pertrubation theory.

- We find that both the **NLO** and the dynamical **decuplet** baryon contributions are sizable, which also implies their inclusions are important.

- We predict positive SU(3) breaking corrections for all the four independent channels, in agreement with large Nc and chiral QM.

- We encourage their uses in future determinations of V_{us} from hyperon decay data.

Hyperon semileptonic decay data

	$\Lambda \to p$	$\Sigma^- \to n$	$\Xi^- \to \Lambda$	$\Xi^- \to \Sigma^0$	$\Xi^0 \to \Sigma^+$
R	3.161 ± 0.058	6.88 ± 0.24	3.44 ± 0.19	0.53 ± 0.10	0.93 ± 0.14
$\alpha_{e\nu}$	-0.019 ± 0.013	0.347 ± 0.024	0.53 ± 0.10		
α_e	0.125 ± 0.066	-0.519 ± 0.104			
α_ν	0.821 ± 0.060	-0.230 ± 0.061			
α_B	-0.508 ± 0.065	0.509 ± 0.102			
A			0.62 ± 0.10		
g_1/f_1	0.718 ± 0.015	-0.340 ± 0.017	0.25 ± 0.05		1.32 ± 0.22

Table 2: Experimental data on $|\Delta S| = 1$ hyperon semileptonic decays [7]. R is given in units of $10^6\,\mathrm{s}^{-1}$.

	$\Lambda \to p$	$\Sigma^- \to n$	$\Xi^- \to \Lambda$	$\Xi^0 \to \Sigma^+$
$\vert\tilde{f}_1 V_{us}\vert$	0.2221 (33)	0.2274 (49)	0.2367 (97)	0.216 (33)

Table 3: Results for $|\tilde{f}_1 V_{us}|$ obtained from the measured rates and $g_1(0)/f_1(0)$ ratios. The quoted errors only reflect the statistical uncertainties.

V. Mateu and A. Pich, JHEP 0510, 041 (2005).