Isospin breaking in the pion–nucleon scattering lengths

Martin Hoferichter

Helmholtz-Institut für Strahlen- und Kernphysik (Theorie) and

Bethe Center for Theoretical Physics, Universität Bonn, Germany

Chiral Dynamics 2009

Bern, 6/7/2009

Outline

Introduction

- Isospin symmetry, πN scattering lengths

Formalism

- Baryon chiral perturbation theory
- Virtual photons

Analytic results

Numerical results

Conclusion and outlook

M. Hoferichter, B. Kubis, U.-G. Meißner, PLB 678 (2009) 65

Isospin symmetry and πN scattering lengths

- Isospin is an approximate symmetry of the strong interaction
- Isospin violation (IV) due to EM interactions and $m_{\rm d} - m_{\rm u}$
- πN scattering lengths are a good testing ground for IV

$$a_{\pi^0 p} - a_{\pi^0 n} = \frac{m_{\rm p} c_5 B\,(m_{\rm d} - m_{\rm u})}{\pi\,(m_{\rm p} + M_\pi)\,F_\pi^2} = (-2.3 \pm 0.4) \cdot 10^{-3} M_\pi^{-1}$$

$\Rightarrow$ large effect

Weinberg 1977

Isospin symmetry and πN scattering lengths

- Isospin is an approximate symmetry of the strong interaction
- Isospin violation (IV) due to EM interactions and $m_{\rm d} - m_{\rm u}$
- πN scattering lengths are a good testing ground for IV

$$a_{\pi^0 p} - a_{\pi^0 n} = \frac{m_{\rm p} c_5 B\,(m_{\rm d} - m_{\rm u})}{\pi\,(m_{\rm p} + M_\pi)\,F_\pi^2} = (-2.3 \pm 0.4) \cdot 10^{-3} M_\pi^{-1}$$

$\Rightarrow$ large effect

Weinberg 1977

- $a^{\rm cex}_{\pi^+ n}$ and $a_{\pi^0 p}$ accessible in pion photoproduction

Bernstein et al., arXiv:0902.3650

- Isospin symmetric scattering lengths: $a^- \sim 90$, $a^+ \sim 0$ (leading order in ChPT) $\Rightarrow a^+$ badly constrained and sensitive to IV
- Corrections for IV are essential to extract a^+ and a^- from hadronic atoms

Formalism

- Calculation at third chiral order $\mathcal{O}(p^3)$ in manifestly covariant baryon ChPT $\Rightarrow$ Infrared Regularization

  Becher, Leutwyler, EPJC 9 (1999) 643

- Effective Lagrangian for nucleons, pions, and virtual photons, as constructed in Gasser, Ivanov, Lipartia, Mojžiš, Rusetsky, EPJC 26 (2002) 13
  $\Rightarrow$ extend their work to *all* physical channels

- Calculation at first order in isospin breaking $\delta = \mathcal{O}(e^2, m_\mathrm{d} - m_\mathrm{u})$

Formalism

- Calculation at third chiral order $\mathcal{O}(p^3)$ in manifestly covariant baryon ChPT $\Rightarrow$ Infrared Regularization

  Becher, Leutwyler, EPJC 9 (1999) 643

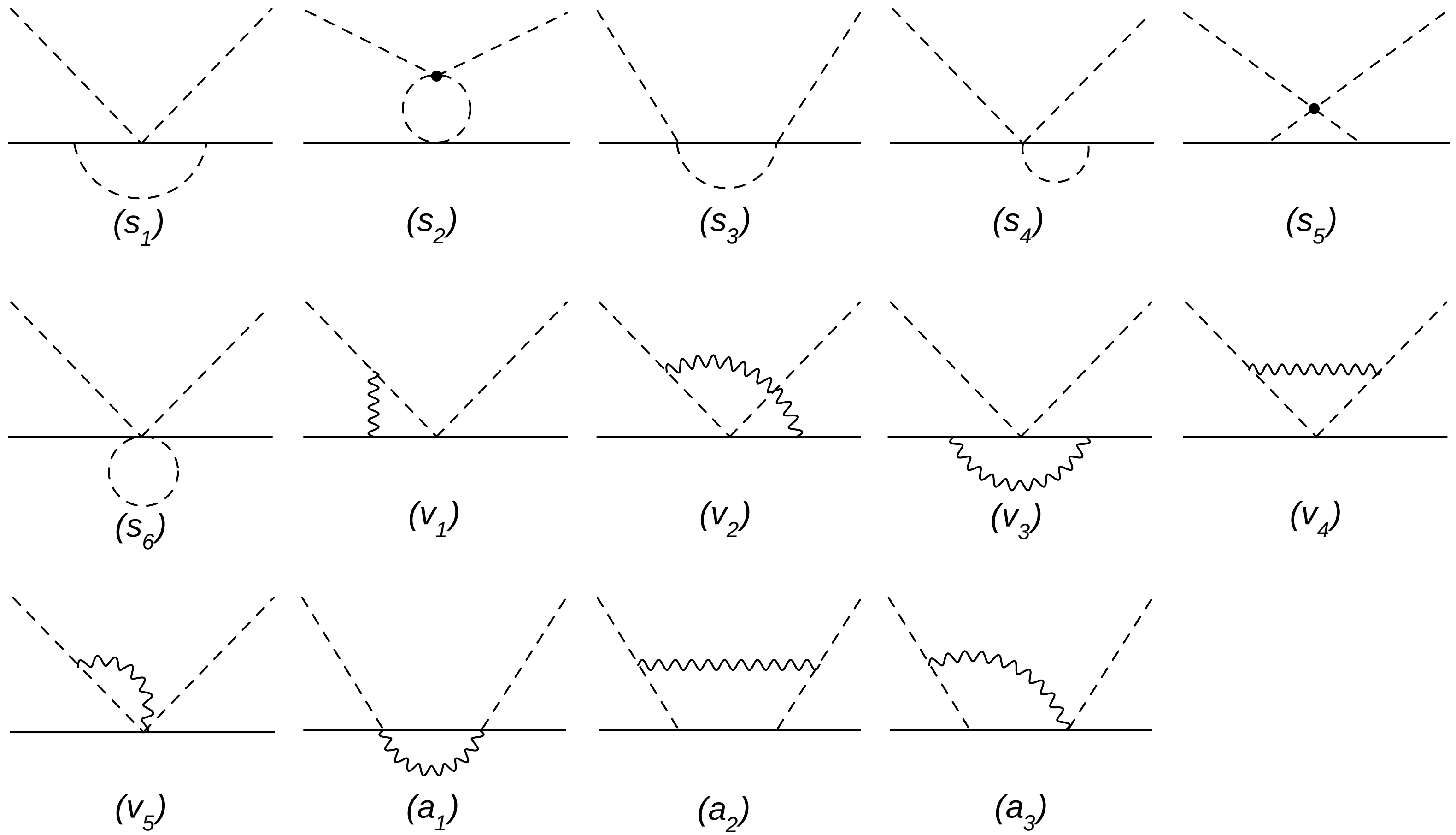
- Effective Lagrangian for nucleons, pions, and virtual photons, as constructed in Gasser, Ivanov, Lipartia, Mojžiš, Rusetsky, EPJC 26 (2002) 13 $\Rightarrow$ extend their work to *all* physical channels
- Calculation at first order in isospin breaking $\delta = \mathcal{O}(e^2, m_\mathrm{d} - m_\mathrm{u})$
- Virtual photons generate Coulomb divergences $\Rightarrow$ subtract one-photon-reducible diagrams $\Rightarrow$ resulting amplitude $\tilde{T}$ has threshold behavior

$$e^{iQ\alpha\theta_\mathrm{C}(|\mathbf{p}|)}\tilde{T}\Big|_{|\mathbf{p}|\to 0} = \frac{\beta_1}{|\mathbf{p}|} + \beta_2 \log\frac{|\mathbf{p}|}{\mu_\mathrm{c}} + T_\mathrm{thr} + \mathcal{O}(|\mathbf{p}|)$$

- Scattering lengths given as $a = \frac{T_\mathrm{thr}}{8\pi\sqrt{s}}$

Feynman diagrams

- One-loop topologies at threshold



- $(v_1) \Rightarrow$ Coulomb pole, $(s_3) \Rightarrow$ Cusp effect, (s_5) triangle graph

Isospin-breaking corrections

- Example: isospin-breaking shifts to $a_{\pi^\pm p}$

$$\Delta a_{\pi^-p} = a_{\pi^-p} - (a^+ + a^-) = \Delta a^+ + \Delta a^- + i\,\mathrm{Im}\,a_{\pi^-p}$$

$$\Delta a_{\pi^+p} = a_{\pi^+p} - (a^+ - a^-) = \Delta a^+ - \Delta a^-$$

$$\Delta a^+ = \frac{m_\mathrm{p}}{4\pi(m_\mathrm{p}+M_\pi)}\left\{\frac{4\Delta_\pi}{F_\pi^2}c_1 - 2e^2 f_1 - \frac{e^2 f_2}{2} - \frac{33 g_\mathrm{A}^2 M_\pi \Delta_\pi}{128\pi F_\pi^4} - \frac{e^2 g_\mathrm{A}^2 M_\pi}{32\pi F_\pi^2}\right\}$$

$$\Delta a^- = -\frac{m_\mathrm{p} M_\pi}{4\pi(m_\mathrm{p}+M_\pi)}\left\{\frac{\Delta_\pi}{32\pi^2 F_\pi^4}\left(3 + \log\frac{M_\pi^2}{\mu^2}\right) + \frac{8\Delta_\pi}{F_\pi^2}d_5^\mathrm{r}\right.$$
$$\left. + \frac{e^2 g_\mathrm{A}^2}{16\pi^2 F_\pi^2}\left(1 + 4\log 2 + 3\log\frac{M_\pi^2}{\mu^2}\right) - 2e^2\left(g_6^\mathrm{r} + g_8^\mathrm{r} - \frac{5}{9F_\pi^2}\left(k_1^\mathrm{r} + k_2^\mathrm{r}\right)\right)\right\}$$

$$\mathrm{Im}\,a_{\pi^-p} = \frac{m_\mathrm{p}}{4\pi(m_\mathrm{p}+M_\pi)}\left\{\frac{M_\pi^2}{8\pi F_\pi^4}\sqrt{\Delta_\pi - 2M_\pi\Delta_\mathrm{N}} + \frac{e^2 g_\mathrm{A}^2 M_\pi}{4\pi F_\pi^2}\right\}$$

$$\Delta_\pi = M_\pi^2 - M_{\pi^0}^2 \qquad \Delta_\mathrm{N} = m_\mathrm{n} - m_\mathrm{p}$$

- Large contributions from the triangle graph and a cusp effect
- Cusp enhanced by $\sqrt{\delta}$
- Accuracy limited by badly constrained low-energy constants

Numerical results (1)

- Use $f_1 = -2.1^{+3.2}_{-2.2}\,\mathrm{GeV}^{-1}$ (Meißner, Raha, Rusetsky, 2006) and β-functions to estimate unknown low-energy constants
- Isospin-breaking shifts in units of $10^{-3} M_\pi^{-1}$

isospin limit	channel	shift	channel	shift
$a^+ + a^-$	$\pi^- p \to \pi^- p$	$-3.4^{+4.3}_{-6.5} + 5.0i$	$\pi^+ n \to \pi^+ n$	$-4.3^{+4.3}_{-6.5} + 6.0i$
$a^+ - a^-$	$\pi^+ p \to \pi^+ p$	$-5.3^{+4.3}_{-6.5}$	$\pi^- n \to \pi^- n$	$-6.2^{+4.3}_{-6.5}$
$-\sqrt{2}\,a^-$	$\pi^- p \to \pi^0 n$	0.4 ± 0.9	$\pi^+ n \to \pi^0 p$	2.3 ± 0.9
a^+	$\pi^0 p \to \pi^0 p$	-5.2 ± 0.2	$\pi^0 n \to \pi^0 n$	-1.8 ± 0.2

Numerical results (2)

- Significant modification of Weinberg's prediction due to a cusp effect

$$
\begin{aligned}
a_{\pi^0 p} - a_{\pi^0 n} = \frac{m_\mathrm{p}}{4\pi(m_\mathrm{p}+M_\pi)} \Bigg\{ \frac{4c_5 B(m_\mathrm{d}-m_\mathrm{u})}{F_\pi^2} \\
- \frac{M_\pi^2}{8\pi F_\pi^4}\left(\sqrt{\Delta_\pi + 2M_\pi\Delta_\mathrm{N}} - \sqrt{\Delta_\pi - 2M_\pi\Delta_\mathrm{N}}\right)\Bigg\} \\
= (-3.4 \pm 0.4)\cdot 10^{-3} M_\pi^{-1}
\end{aligned}
$$

Numerical results (2)

- Significant modification of Weinberg's prediction due to a cusp effect

$$a_{\pi^0 p} - a_{\pi^0 n} = \frac{m_{\mathrm{p}}}{4\pi(m_{\mathrm{p}} + M_\pi)} \left\{ \frac{4c_5 B(m_{\mathrm{d}} - m_{\mathrm{u}})}{F_\pi^2} - \frac{M_\pi^2}{8\pi F_\pi^4} \left(\sqrt{\Delta_\pi + 2M_\pi \Delta_{\mathrm{N}}} - \sqrt{\Delta_\pi - 2M_\pi \Delta_{\mathrm{N}}} \right) \right\} = (-3.4 \pm 0.4) \cdot 10^{-3} M_\pi^{-1}$$

- Quantify IV in terms of measurable quantities by means of the triangle relation

$$R = 2\frac{a_{\pi^+ p} - a_{\pi^- p} - \sqrt{2}\, a^{\mathrm{cex}}_{\pi^- p}}{a_{\pi^+ p} - a_{\pi^- p} + \sqrt{2}\, a^{\mathrm{cex}}_{\pi^- p}} = (1.5 \pm 1.1)\,\%$$

Conclusion and outlook

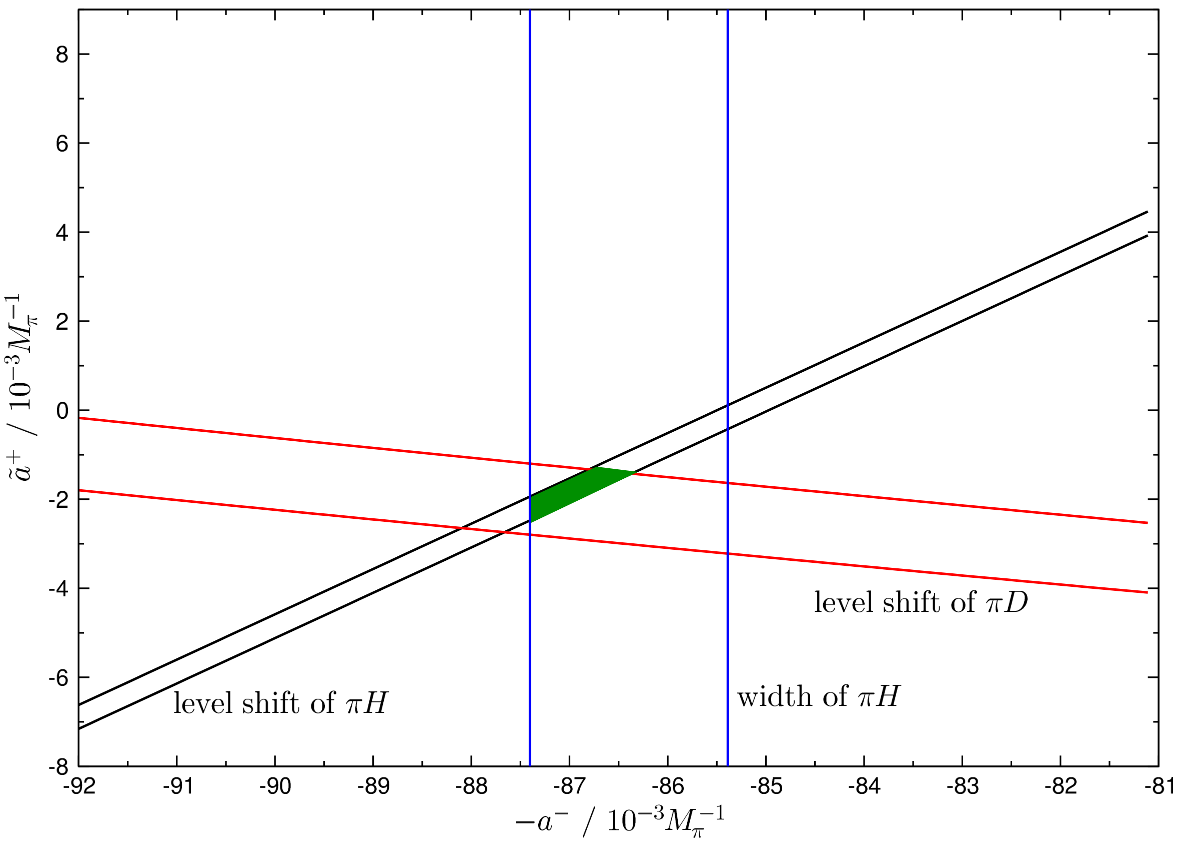
- Small corrections to the charge exchange reactions with rather well-controlled uncertainties
- Sizeable shifts in the charged-pion elastic channels, but large uncertainties due to f_1
- Triangle relation violated by about 1.5%
- Substantial modification of Weinberg's prediction for $a_{\pi^0 p} - a_{\pi^0 n}$

Conclusion and outlook

- Small corrections to the charge exchange reactions with rather well-controlled uncertainties
- Sizeable shifts in the charged-pion elastic channels, but large uncertainties due to f_1
- Triangle relation violated by about 1.5%
- Substantial modification of Weinberg's prediction for $a_{\pi^0 p} - a_{\pi^0 n}$

- Extend the analysis beyond threshold
- Application to hadronic atoms $\Rightarrow$ extract a^+ and a^-

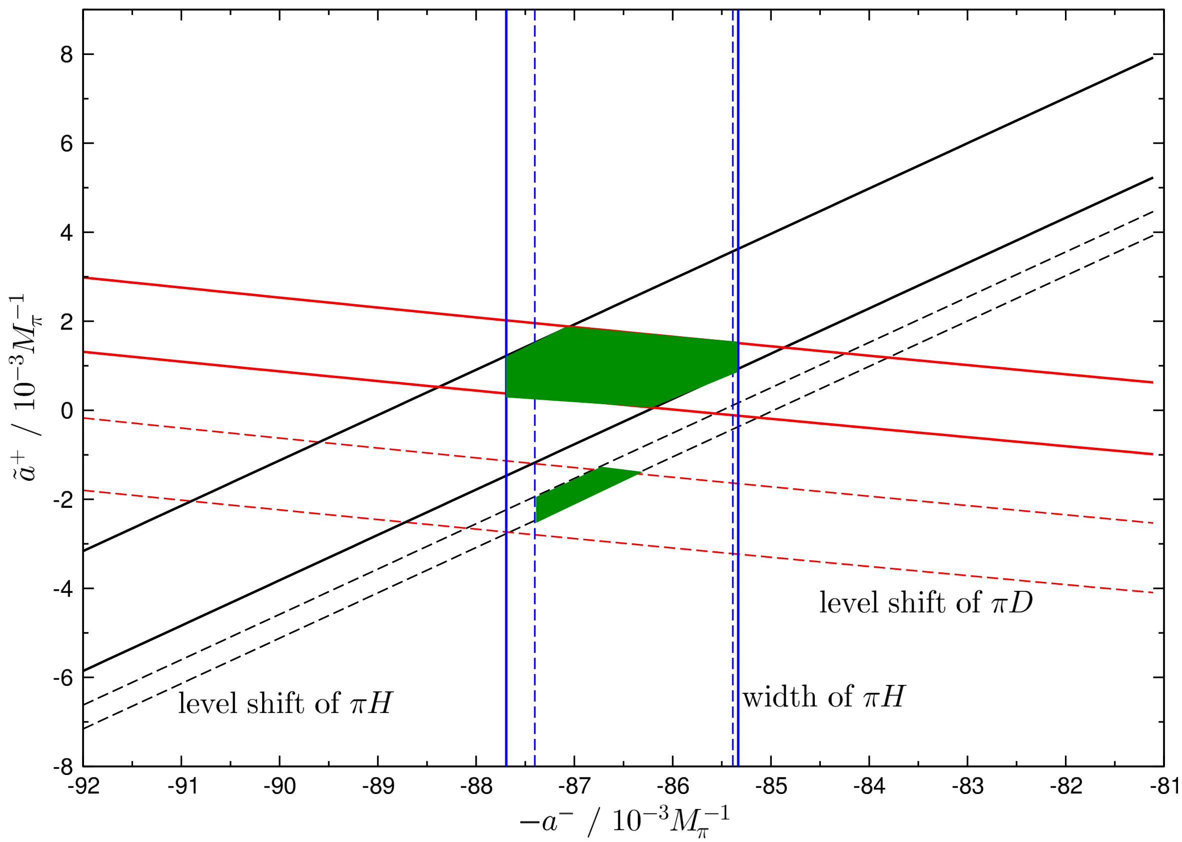
Outlook: extraction of a^+ and a^- from πH and πD



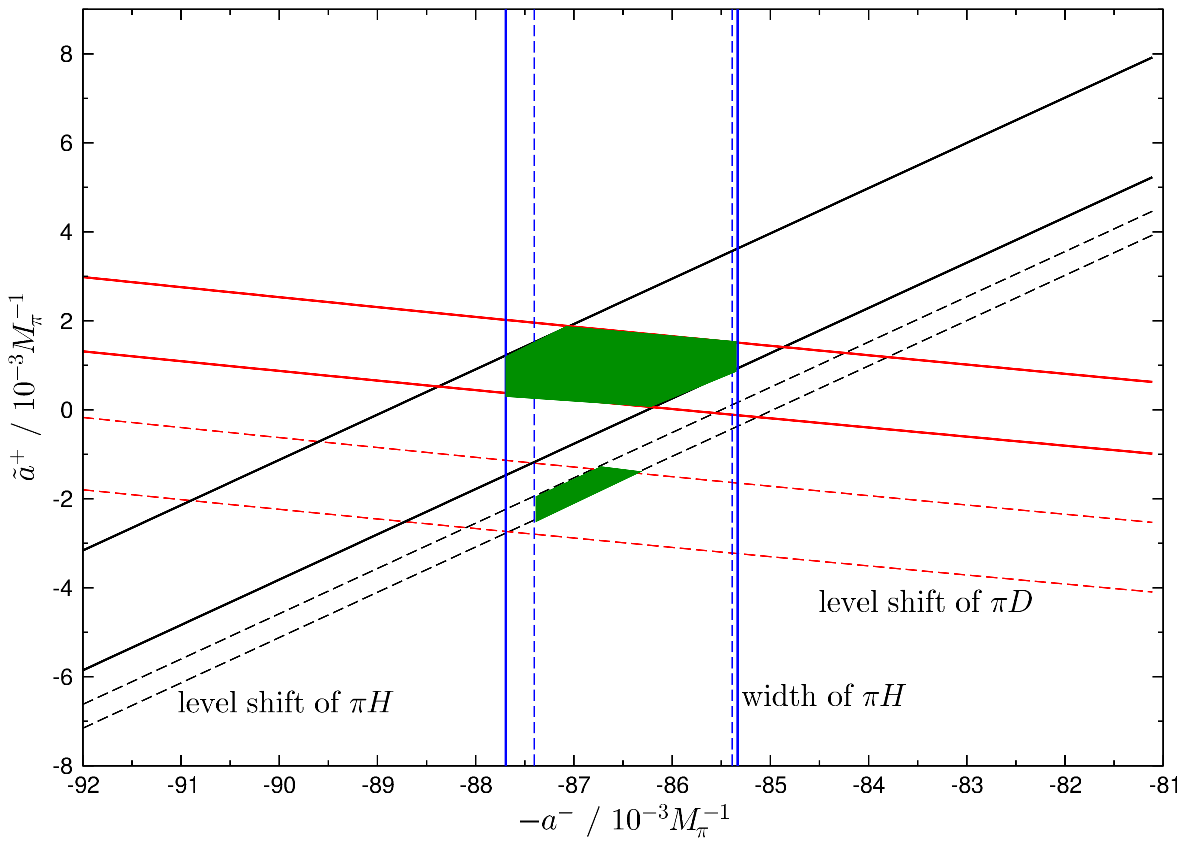
- IV at order $\mathcal{O}(p^2)$ only
- Instead of a^+ we consider

$$\tilde{a}^+ = a^+ + \frac{m_\mathrm{p}}{4\pi(m_\mathrm{p}+M_\pi)}\left\{\frac{4\Delta_\pi}{F_\pi^2}c_1 - 2e^2 f_1\right\}$$

Outlook: extraction of a^+ and a^- from πH and πD



Outlook: extraction of a^+ and a^- from πH and πD



- IV at order $\mathcal{O}(p^3)$ for width and level shift of πH and single-nucleon sector of πD
- Complete picture requires few-body corrections to πD including IV   Baru, Hanhart, Hoferichter, Kubis, Nogga, Phillips, in preparation

Spares

Deser-formulae

- Strong energy shift of the ground state of pionic hydrogen

$$\epsilon_{1s} = -2\alpha^3\mu_{\rm h}^2\big(a^+ + a^-\big)(1+\delta_\epsilon) \qquad \delta_\epsilon = \frac{\Delta a_{\pi^- p}}{a^+ + a^-} + K_\epsilon + \delta_\epsilon^{\rm vac}$$

$$K_\epsilon = 2\alpha(1-\log\alpha)\mu_{\rm h}(a^+ + a^-) \quad \delta_\epsilon^{\rm vac} = 2\frac{\delta\Psi_{\rm h}(0)}{\Psi_{\rm h}(0)} = 0.48\,\%$$

- Width of pionic hydrogen

$$\Gamma_{1s} = 8\alpha^3\mu_{\rm h}^2 p_1\left(1+\frac{1}{P}\right)\big(a^-(1+\delta_\Gamma)\big)^2\big(1+K_\Gamma+\delta_\epsilon^{\rm vac}\big) \qquad \delta_\Gamma = \frac{\Delta a^{\rm cex}_{\pi^- p}}{-\sqrt{2}\,a^-}$$

$$K_\Gamma = 4\alpha(1-\log\alpha)\mu_{\rm h}(a^+ + a^-) + 2\mu_{\rm h}\big(m_{\rm p} + M_\pi - m_{\rm n} - M_{\pi^0}\big)(a^+)^2$$

p_1: CMS momentum of $\pi^0 n$, P: Panofsky ratio

- Strong energy shift of the ground state of pionic deuterium

$$\epsilon_{1s}^{\rm d} = -2\alpha^3\mu_{\rm d}^2\,\text{Re}\,a_{\pi d}\,\big(1+K_{\rm d}+\delta_{\epsilon{\rm d}}^{\rm vac}\big) \quad K_{\rm d} = 4\alpha(1-\log\alpha)\mu_{\rm d}a^+ \quad \delta_{\epsilon{\rm d}}^{\rm vac} = 0.51\,\%$$

Pion–proton scattering lengths

$$\Delta a^{\text{cex}}_{\pi^- p} = a^{\text{cex}}_{\pi^- p} + \sqrt{2}\, a^- = \frac{\sqrt{2}\, m_{\text{p}}}{4\pi(m_{\text{p}} + M_\pi)} \Bigg\{ \frac{e^2 f_2}{2} - \frac{M_\pi \Delta_{\text{N}}}{4F_\pi^2 m_{\text{p}}} (1 + 2g_{\text{A}}^2)$$

$$+ \frac{g_{\text{A}}^2 \Delta_\pi}{4F_\pi^2 m_{\text{p}}} + \frac{M_\pi \Delta_\pi}{4m_{\text{p}}^2} B^-_{\text{thr}} - \frac{3M_\pi \Delta_\pi}{16F_\pi^2 m_{\text{p}}^2} + \frac{8M_\pi \Delta_\pi}{F_\pi^2} d_5^{\text{r}}$$

$$+ \frac{M_\pi \Delta_\pi}{192\pi^2 F_\pi^4} \left(2 - 7g_{\text{A}}^2 + (2 - 5g_{\text{A}}^2) \log \frac{M_\pi^2}{\mu^2} \right) + \frac{e^2 M_\pi}{32\pi^2 F_\pi^2} \left(5 + 3 \log \frac{M_\pi^2}{\mu^2} \right)$$

$$+ \frac{e^2 M_\pi}{2F_\pi^2} \left(F_\pi^2 g_7^{\text{r}} - 2k_3^{\text{r}} + k_4^{\text{r}} + \frac{20}{9} (k_1^{\text{r}} + k_2^{\text{r}}) \right) \Bigg\}$$

$$\Delta a_{\pi^0 p} = a_{\pi^0 p} - a^+ = -\frac{\Delta_\pi}{M_\pi^2} a^+ + \frac{m_{\text{p}}}{4\pi(m_{\text{p}} + M_\pi)} \Bigg\{ \frac{3g_{\text{A}}^2 M_\pi \Delta_\pi}{128\pi F_\pi^4}$$

$$- \frac{M_\pi^2 \sqrt{\Delta_\pi + 2M_\pi \Delta_{\text{N}}}}{8\pi F_\pi^4} + \frac{2c_5 B(m_{\text{d}} - m_{\text{u}})}{F_\pi^2} \Bigg\}$$

Pion–neutron scattering lengths

- Calculation simplified by charge symmetry $\Rightarrow$ only virtual photons need to be calculated explicitly
- Results

$$\Delta a_{\pi^+ n} = a_{\pi^+ n} - (a^+ + a^-) = \Delta a_{\pi^- p} + \frac{m_{\rm p}}{4\pi(m_{\rm p} + M_\pi)} \Big\{ e^2 f_2 - 2e^2 M_\pi \left(2g_6^{\rm r} + g_8^{\rm r}\right)$$

$$+ i \frac{M_\pi^2}{8\pi F_\pi^4} \left(\sqrt{\Delta_\pi + 2M_\pi \Delta_{\rm N}} - \sqrt{\Delta_\pi - 2M_\pi \Delta_{\rm N}} \right) \Big\}$$

$$\Delta a_{\pi^- n} = a_{\pi^- n} - (a^+ - a^-) = \Delta a_{\pi^+ p} + \frac{m_{\rm p}}{4\pi(m_{\rm p} + M_\pi)} \Big\{ e^2 f_2 + 2e^2 M_\pi \left(2g_6^{\rm r} + g_8^{\rm r}\right) \Big\}$$

$$\Delta a^{\rm cex}_{\pi^+ n} = a^{\rm cex}_{\pi^+ n} + \sqrt{2}\, a^- = \Delta a^{\rm cex}_{\pi^- p} + \frac{\sqrt{2}\, m_{\rm p}}{4\pi(m_{\rm p} + M_\pi)} \Big\{ \frac{M_\pi \Delta_{\rm N}}{2F_\pi^2 m_{\rm p}} (1 + 2g_{\rm A}^2) - e^2 f_2 \Big\}$$

$$\Delta a_{\pi^0 n} = a_{\pi^0 n} - a^+ = \Delta a_{\pi^0 p} + \frac{m_{\rm p}}{4\pi(m_{\rm p} + M_\pi)} \Big\{ - \frac{4c_5 B(m_{\rm d} - m_{\rm u})}{F_\pi^2}$$

$$+ \frac{M_\pi^2}{8\pi F_\pi^4} \left(\sqrt{\Delta_\pi + 2M_\pi \Delta_{\rm N}} - \sqrt{\Delta_\pi - 2M_\pi \Delta_{\rm N}} \right) \Big\}$$

Imaginary parts

- The imaginary parts at threshold can be calculated exactly in terms of scattering lengths and electric dipole amplitudes E_{0+} using Cutkosky rules

$$\text{Im}\begin{Bmatrix} a_{\pi^- p} \\ a^{\text{cex}}_{\pi^- p} \end{Bmatrix} = \frac{a^- \sqrt{2m_\text{p}}}{\sqrt{m_\text{p}+M_\pi}} \sqrt{\Delta_\pi - 2M_\pi \Delta_\text{N}} \begin{Bmatrix} \sqrt{2}\, a^- \\ -a^+ \end{Bmatrix}$$

$$+ \frac{M_\pi E_{0+}(\pi^- p)}{m_\text{p}+M_\pi} \big(M_\pi + 2m_\text{p}\big) \begin{Bmatrix} E_{0+}(\pi^- p) \\ E_{0+}(\pi^0 n) \end{Bmatrix}$$

$\Rightarrow$ Im $a^{\text{cex}}_{\pi^- p}$ suppressed by one chiral order

- Numerically

$$\text{Im}\, a_{\pi^- p} = (4.77 \pm 0.15) \cdot 10^{-3} M_\pi^{-1}$$

$$\text{Im}\, a^{\text{cex}}_{\pi^- p} = (-0.16 \pm 0.06) \cdot 10^{-3} M_\pi^{-1}$$

Effective Lagrangian (1)

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_\pi^{(p^2)} + \mathcal{L}_\pi^{(e^2)} + \mathcal{L}_\pi^{(p^4)} + \mathcal{L}_\pi^{(e^2p^2)} + \mathcal{L}_{\text{N}}^{(p)} + \mathcal{L}_{\text{N}}^{(p^2)} + \mathcal{L}_{\text{N}}^{(e^2)} + \mathcal{L}_{\text{N}}^{(p^3)} + \mathcal{L}_{\text{N}}^{(e^2p)} + \mathcal{L}_\gamma$$

$$\mathcal{L}_\pi^{(p^2)} + \mathcal{L}_\pi^{(e^2)} = \frac{F^2}{4}\langle d^\mu U^\dagger d_\mu U + \chi^\dagger U + U^\dagger \chi\rangle + ZF^4\langle \mathcal{Q}U\mathcal{Q}U^\dagger\rangle$$

$$\mathcal{L}_\pi^{(e^2p^2)} = F^2\Big\{\langle d^\mu U^\dagger d_\mu U\rangle\big(k_1\langle\mathcal{Q}^2\rangle + k_2\langle\mathcal{Q}U\mathcal{Q}U^\dagger\rangle\big) + k_4\langle d^\mu U^\dagger \mathcal{Q}U\rangle\langle d_\mu U\mathcal{Q}U^\dagger\rangle$$
$$+ k_3\big(\langle d^\mu U^\dagger\mathcal{Q}U\rangle\langle d_\mu U^\dagger\mathcal{Q}U\rangle + \langle d^\mu U\mathcal{Q}U^\dagger\rangle\langle d_\mu U\mathcal{Q}U^\dagger\rangle\big)\Big\}$$

$$\mathcal{L}_\pi^{(p^4)} = \frac{l_4}{4}\langle d^\mu U^\dagger d_\mu\chi + d^\mu\chi^\dagger d_\mu U\rangle \qquad \mathcal{L}_\gamma = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\big(\partial_\mu A^\mu\big)^2$$

$$\mathcal{L}_{\text{N}}^{(p)} = \bar\Psi\Big\{i\not{D} - m + \frac{1}{2}g\not{u}\gamma_5\Big\}\Psi \qquad \mathcal{L}_{\text{N}}^{(e^2)} = F^2\bar\Psi\Big\{f_{1/3}\langle\hat{Q}_+^2 \mp Q_-^2\rangle + f_2\langle Q_+\rangle\hat{Q}_+\Big\}\Psi$$

$$\mathcal{L}_{\text{N}}^{(p^2)} = \bar\Psi\Big\{c_1\langle\chi_+\rangle - \frac{c_2}{4m^2}\langle u_\mu u_\nu\rangle D^\mu D^\nu + \text{h.c.} + \frac{c_3}{2}\langle u_\mu u^\mu\rangle + \frac{i}{4}c_4\sigma^{\mu\nu}[u_\mu, u_\nu] + c_5\hat\chi_+\Big\}\Psi$$

$$\mathcal{L}_{\text{N}}^{(p^3)} = \bar\Psi\Big\{ - \frac{d_1}{2m}[u_\mu,[D_\nu,u^\mu]]D^\nu - \frac{d_2}{2m}[u_\mu,[D^\mu,u_\nu]]D^\nu$$
$$+ \frac{d_3}{12m^3}[u_\mu,[D_\nu,u_\lambda]]\big(D^\mu D^\nu D^\lambda + \text{sym}\big) + \frac{i}{2m}d_5[\chi_-,u_\mu]D^\mu\Big\}\Psi + \text{h.c.}$$

$$\mathcal{L}_{\text{N}}^{(e^2p)} = \frac{iF^2}{2m}\bar\Psi\Big\{g_6\langle Q_+\rangle\langle Q_-u_\mu\rangle D^\mu + g_{7/8}\langle Q_\pm u_\mu\rangle Q_\mp D^\mu\Big\}\Psi + \text{h.c.}$$

Effective Lagrangian (2)

- $\langle A \rangle$ denotes the trace of a matrix A, $\hat{A} = A - \langle A \rangle/2$ its traceless part and $\bar{\Psi}(\mathcal{O} + \mathrm{h.c.})\Psi \equiv \bar{\Psi}\mathcal{O}\Psi + \mathrm{h.c.}$ for an operator $\mathcal{O}$
- building blocks

$$
\begin{aligned}
d_\mu U &= \partial_\mu U - iA_\mu[\mathcal{Q}, U], \quad \chi = 2B\,\text{diag}(m_\mathrm{u}, m_\mathrm{d}), \quad U = u^2, \\
F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu, \quad \mathcal{Q} = \frac{e}{3}\,\text{diag}(2, -1), \quad Q = e\,\text{diag}(1, 0), \\
D_\mu &= \partial_\mu + \Gamma_\mu, \quad \Gamma_\mu = \frac{1}{2}\Big(u^\dagger(\partial_\mu - iQA_\mu)u + u(\partial_\mu - iQA_\mu)u^\dagger\Big), \\
\chi_\pm &= u^\dagger\chi u^\dagger \pm u\chi^\dagger u,\ u_\mu = i\Big(u^\dagger(\partial_\mu - iQA_\mu)u - u(\partial_\mu - iQA_\mu)u^\dagger\Big), \\
Q_\pm &= \frac{1}{2}(uQu^\dagger \pm u^\dagger Qu), \quad [D_\mu, u_\nu] = \partial_\mu u_\nu + [\Gamma_\mu, u_\nu].
\end{aligned}
$$

- $\Psi = (p, n)^T$ contains the nucleon fields and the matrix U collects the pion fields in the usual way.