Compton scattering from the proton: An analysis using the delta expansion up to $\mathrm{N}^3\mathrm{LO}$

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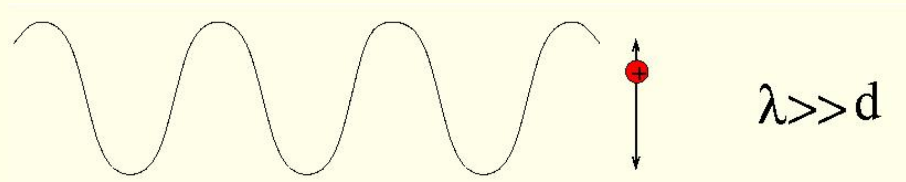
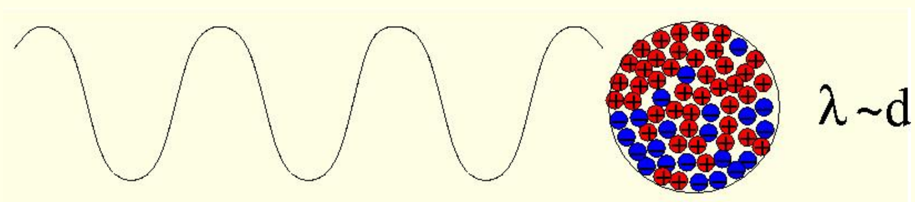
Work done in collaboration with Harald Grießhammer (GWU), Daniel Phillips (OU) and Deepshikha Shukla (GWU)

Compton Scattering

For large wavelengths, only sensitive to overall charge:



But for smaller wavelengths, the target is polarised by the electric and magnetic fields



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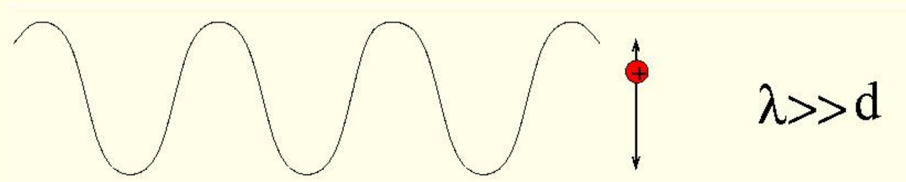

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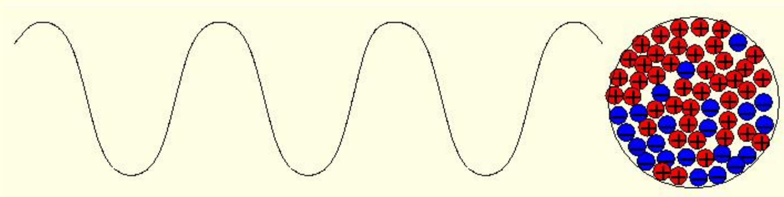
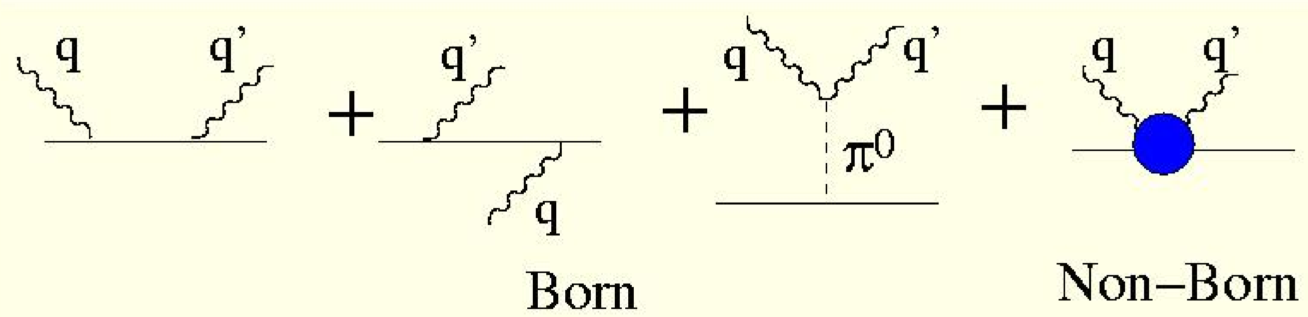


The low-energy Hamiltonian is:

$$H_{eff} = \frac{(\mathbf{p}-Q\mathbf{A})^2}{2m} + Q\phi - \frac{1}{2}4\pi\Big(\alpha\vec{E}^2 + \beta\vec{H}^2 + \gamma_{E1}\vec{\sigma}\cdot\vec{E}\times\dot{\vec{E}} + \gamma_{M1}\vec{\sigma}\cdot\vec{H}\times\dot{\vec{H}} - 2\gamma_{E2}E_{ij}\sigma_i H_j + 2\gamma_{M2}H_{ij}\sigma_i E_j\Big)$$

where $E_{ij} = \frac{1}{2}(\nabla_i E_j + \nabla_j E_i)$ and $H_{ij} = \frac{1}{2}(\nabla_i H_j + \nabla_j H_i)$

Scattering Amplitudes



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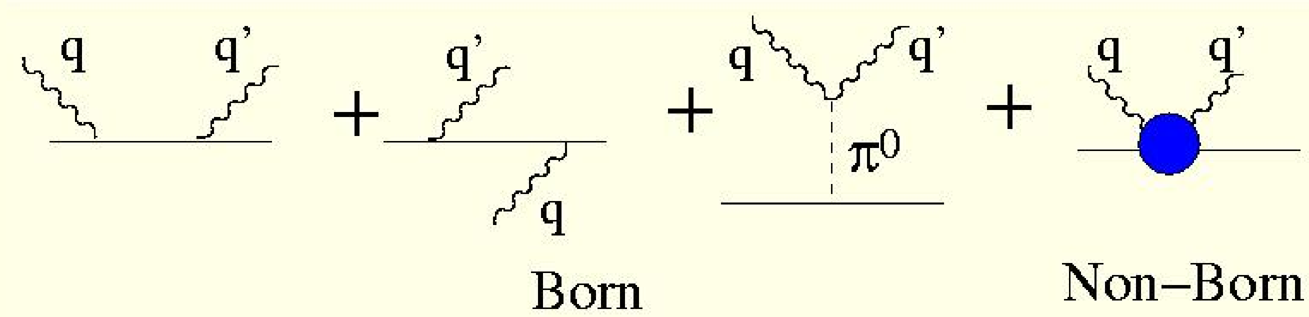
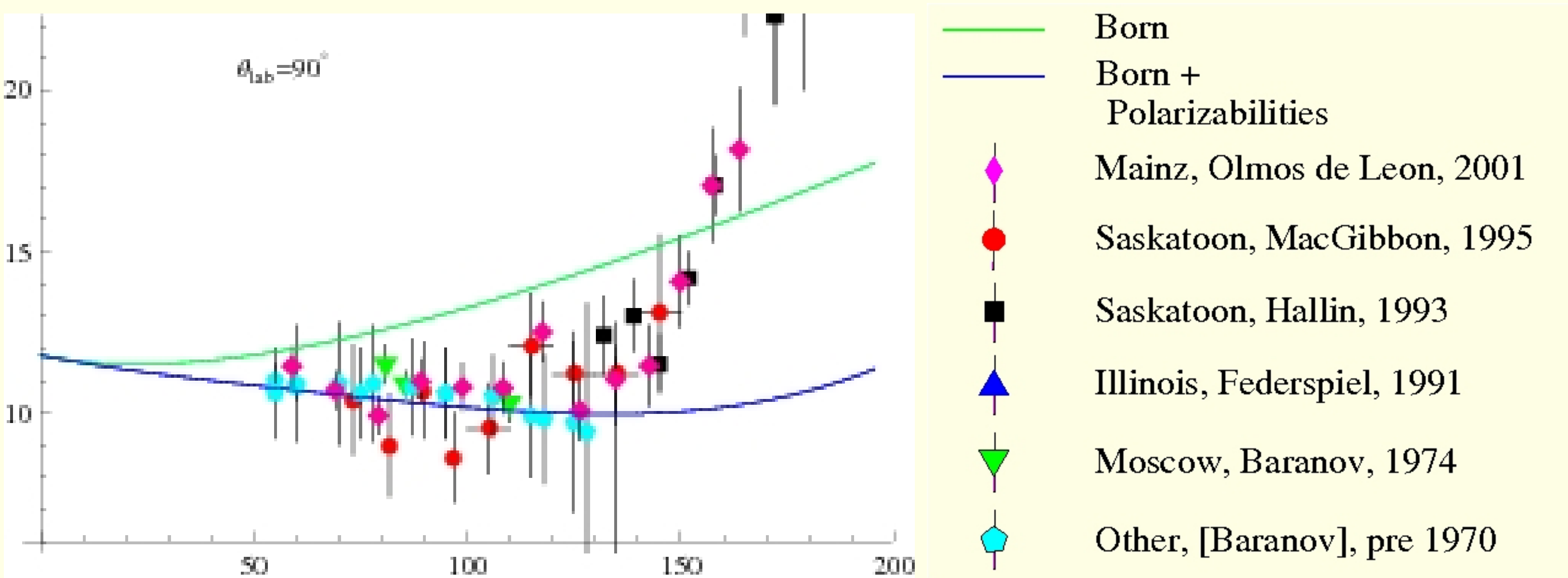
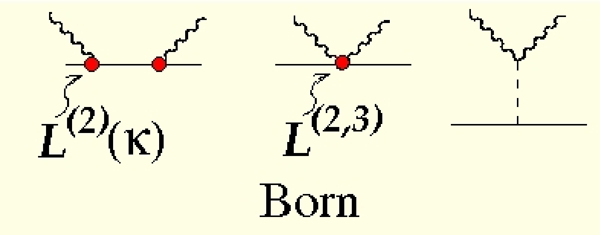




There is only a narrow window where polarisabilities are significant but expansion in powers of photon energy ω is still valid. Need theory which covers wider energy range - χPT.

HBχPT for Compton Scattering



$O(q^2)$: Thomson term

$O(q^3)$: LET and pion-pole terms terms for spin-dependent for amplitudes,

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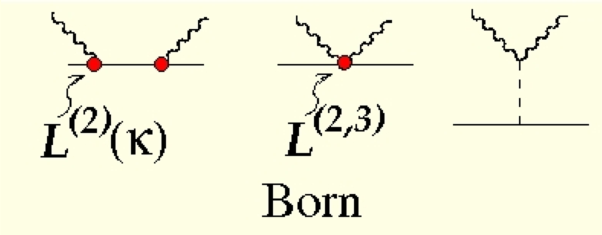

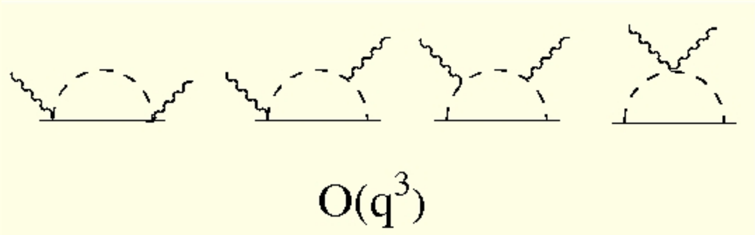

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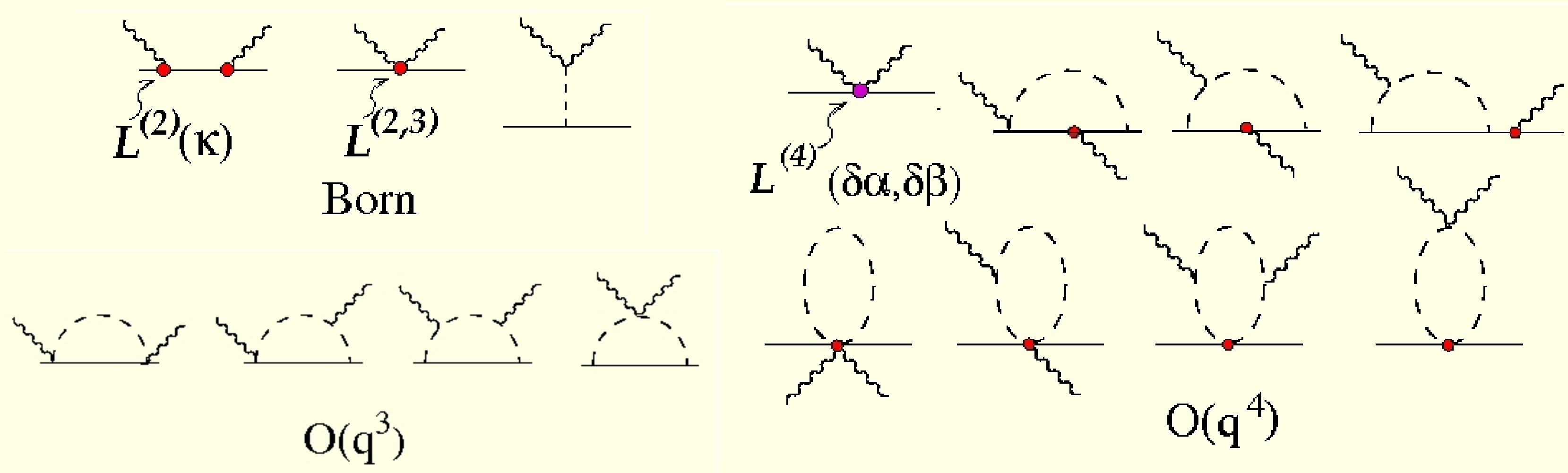

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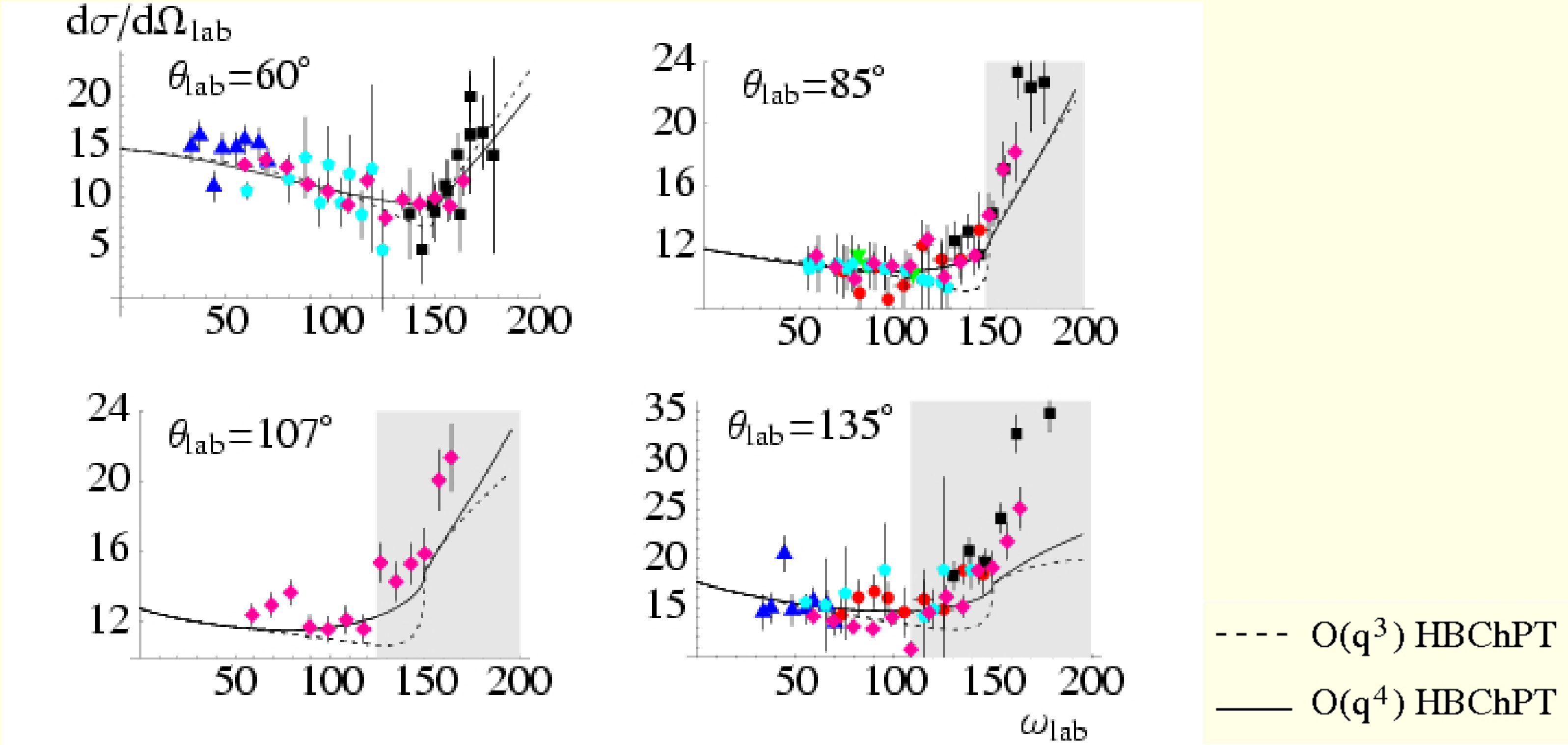
$O(q^4)$: $1/M$ corrections and further contribution to energy-dependent amplitude
BUT four undetermined LECs $\delta\alpha_p$, $\delta\beta_p$, $\delta\alpha_n$ and $\delta\beta_n$. The γ_i are still predicted

Fitting α_p and β_p in $O(q^4)$ χPT



source: Beane *et al*, Phys. Lett. B567 200 (2003); Nucl. Phys. A747 311 (2005)

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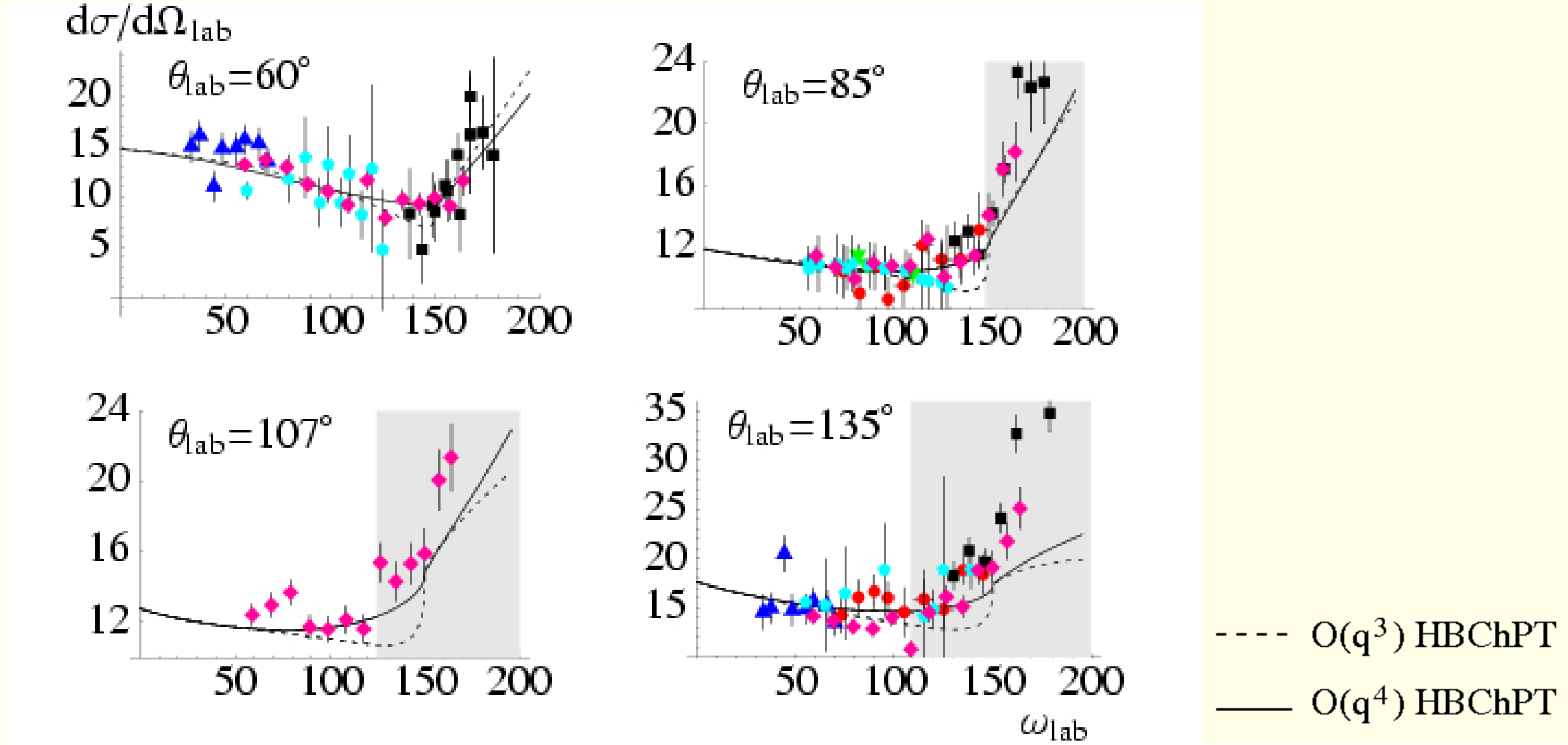

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Best fit (excluding grey regions): $\alpha_p = (12.4 \pm 1.1)^{+0.5}_{-0.5}$, $\beta_p (= 3.4 \pm 1.1)^{+0.1}_{-0.1}$

Baldin Sum Rule constrained fit: $\alpha_p = (11.0 \pm 0.2)^{+0.5}_{-0.5}$, $\beta_p = (2.8 \pm 0.5 \mp 0.2)^{+0.1}_{-0.1}$

Units: $10^{-4}\mathrm{fm}^3$

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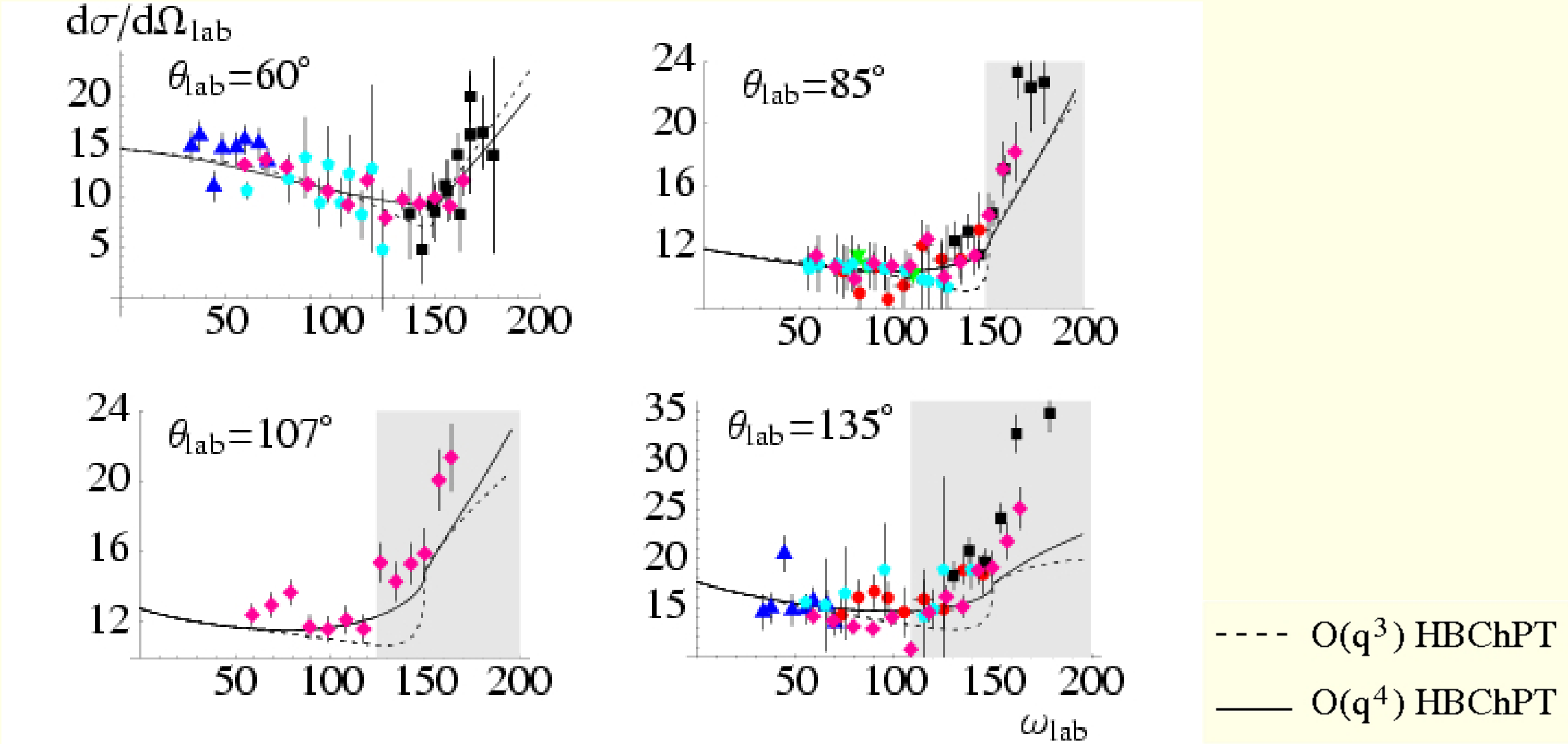

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Obviously something is missing though. The Delta!

Including the Δ

$\Delta \equiv M_\Delta - M_N \approx 271$ MeV is a rather small scale. Traditionally it is counted as $\Delta/\Lambda_\chi \sim m_\pi/\Lambda_\chi$ ("SSE"). But in Compton scattering the pion is clearly important at lower energies than the Delta.

Alternative: count $$\frac{m_\pi}{\Delta} \sim \frac{\Delta}{\Lambda_\chi} \quad \Rightarrow \quad \delta^2 \equiv \left(\frac{\Delta}{\Lambda_\chi}\right)^2 \sim \frac{m_\pi}{\Lambda_\chi}$$

Then graphs with one Δ propagator are one order of δ higher than the corresponding nucleon graphs.

Pascalutsa and Phillips, Phys. Rev. C67 (2003) 055202

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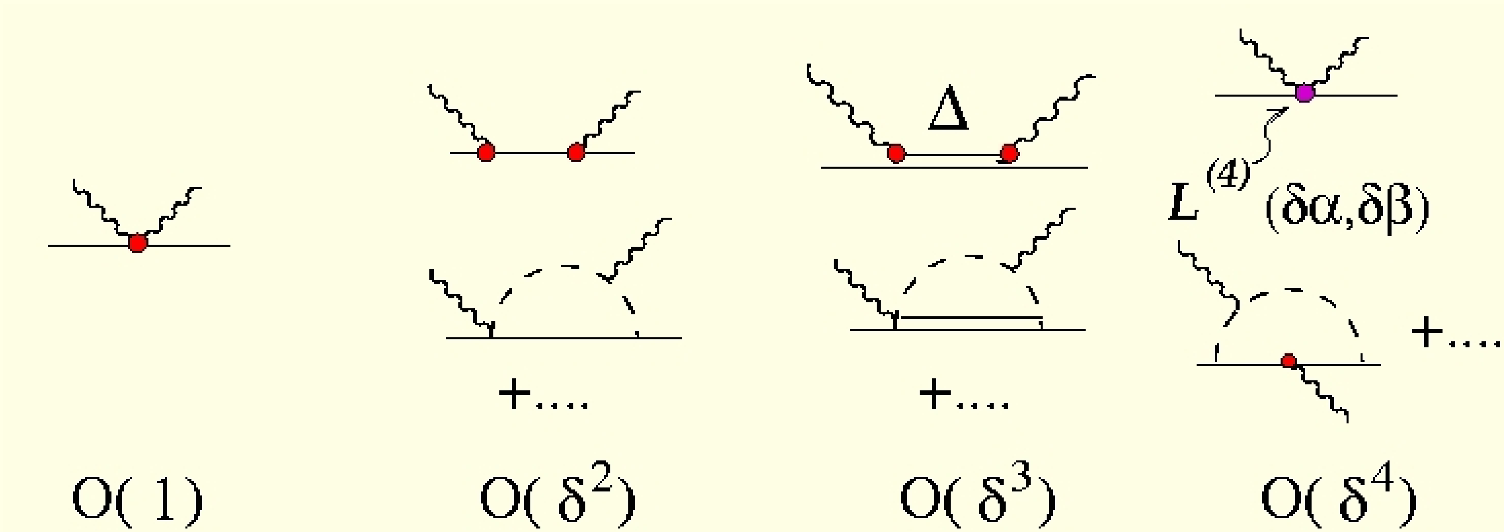

Third order with Delta (SSE or δ)

Problem: Including Delta pole and loops gives $\alpha = 17,\ \beta = 13 \quad (\times 10^{-4}\mathrm{fm}^3)$

Solution: include counterterms $\delta\alpha$ and $\delta\beta$ at this order.

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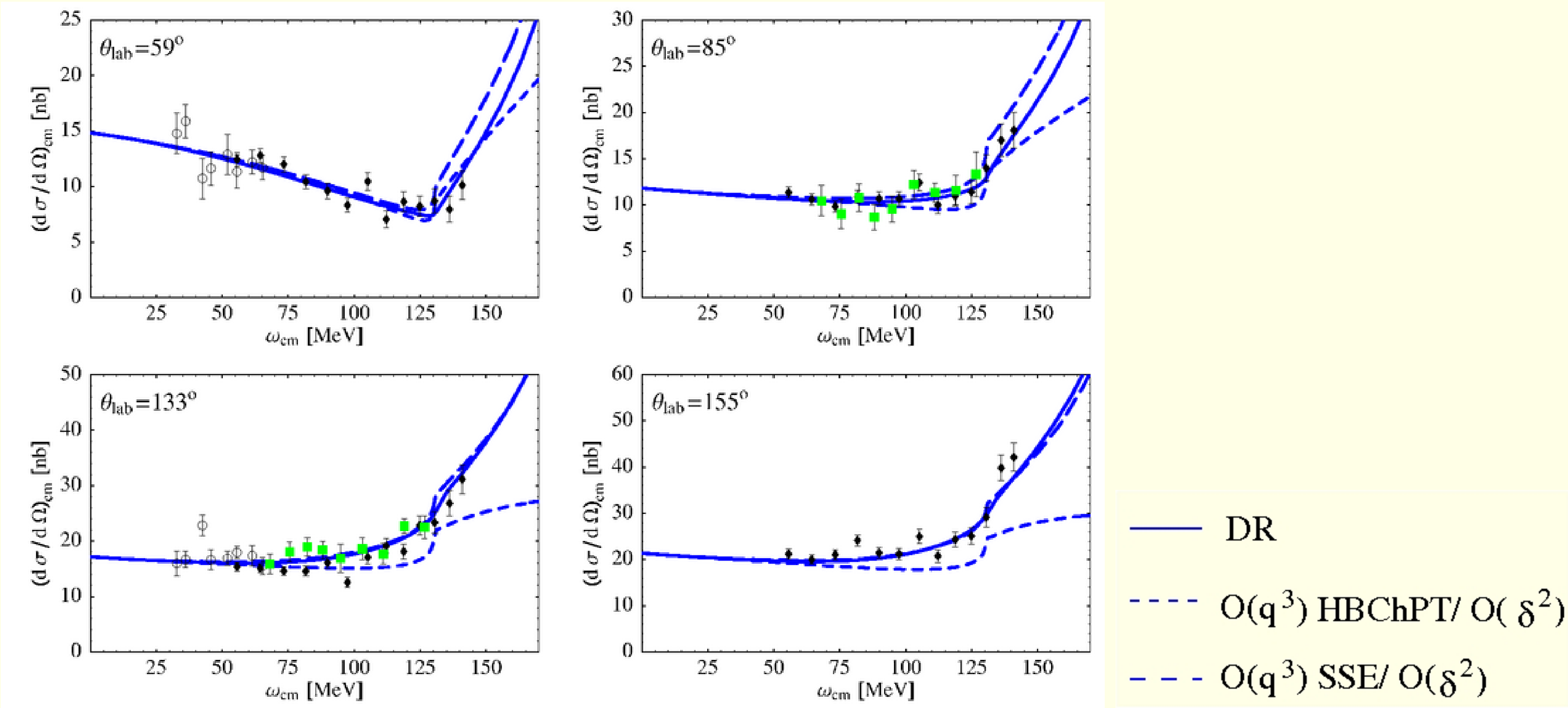

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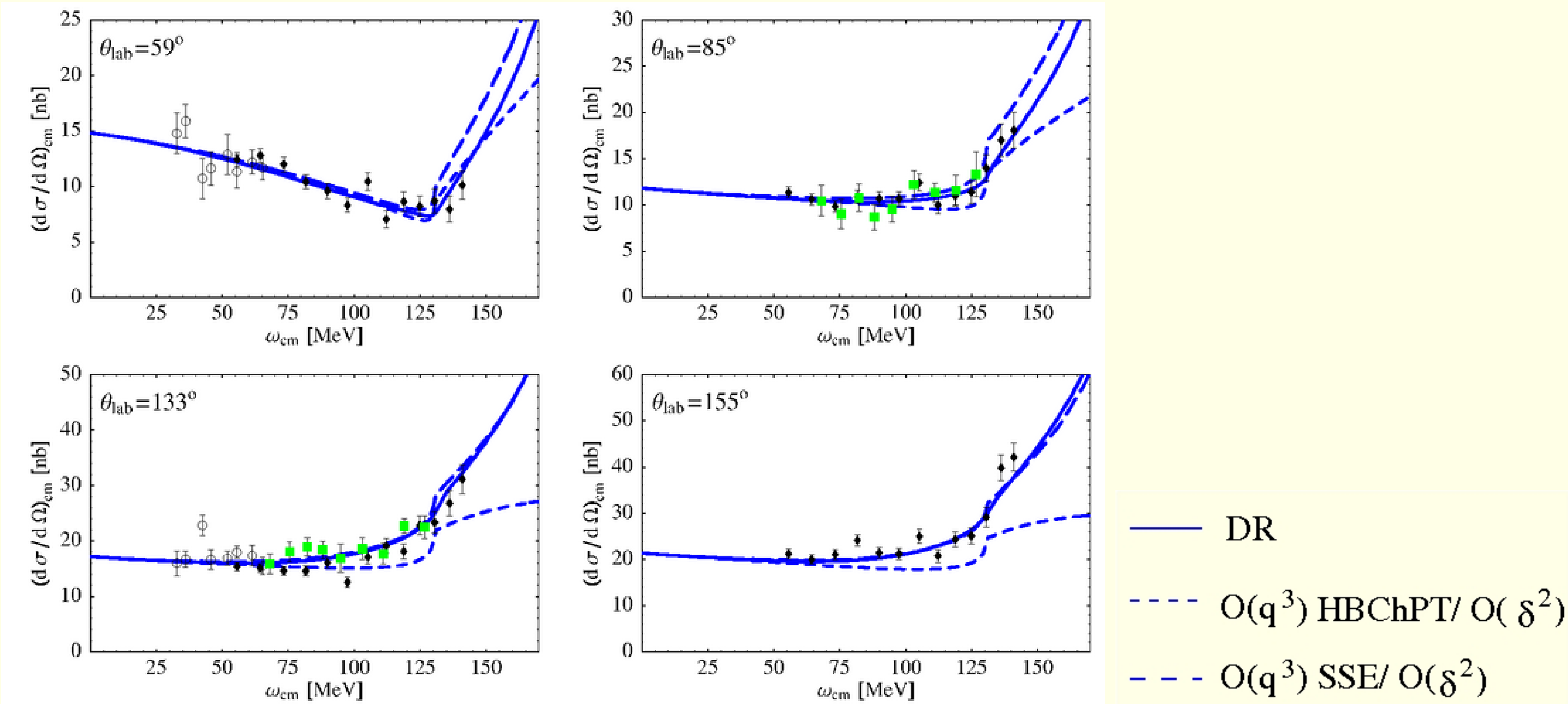

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Better fit for backward angles. Best fit: $\alpha_p = 11.52 \pm 2.43$, $\beta_p = 3.42 \mp 1.70$

V. similar central values, but a number of differences in approach and data set...

Fourth order with Delta

Problem with naive addition of $\mathcal{O}(\delta^3)$ and $\mathcal{O}(\delta^4)$ amplitudes. Both raise cross section for intermediate energies and backward angles. Combination is too much. Trace to γ_{M1} which has large contributions from both NLO πN and Delta-pole graphs. Drop the latter (as already required for spin-independent polarisabilities).

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LOW: Polarisabilities are a low-energy phenomenon. At high energies the response is much more complex.
Also, effects of Delta width start to be visible.

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LOW: Polarisabilities are a low-energy phenomenon. At high energies the response is much more complex.
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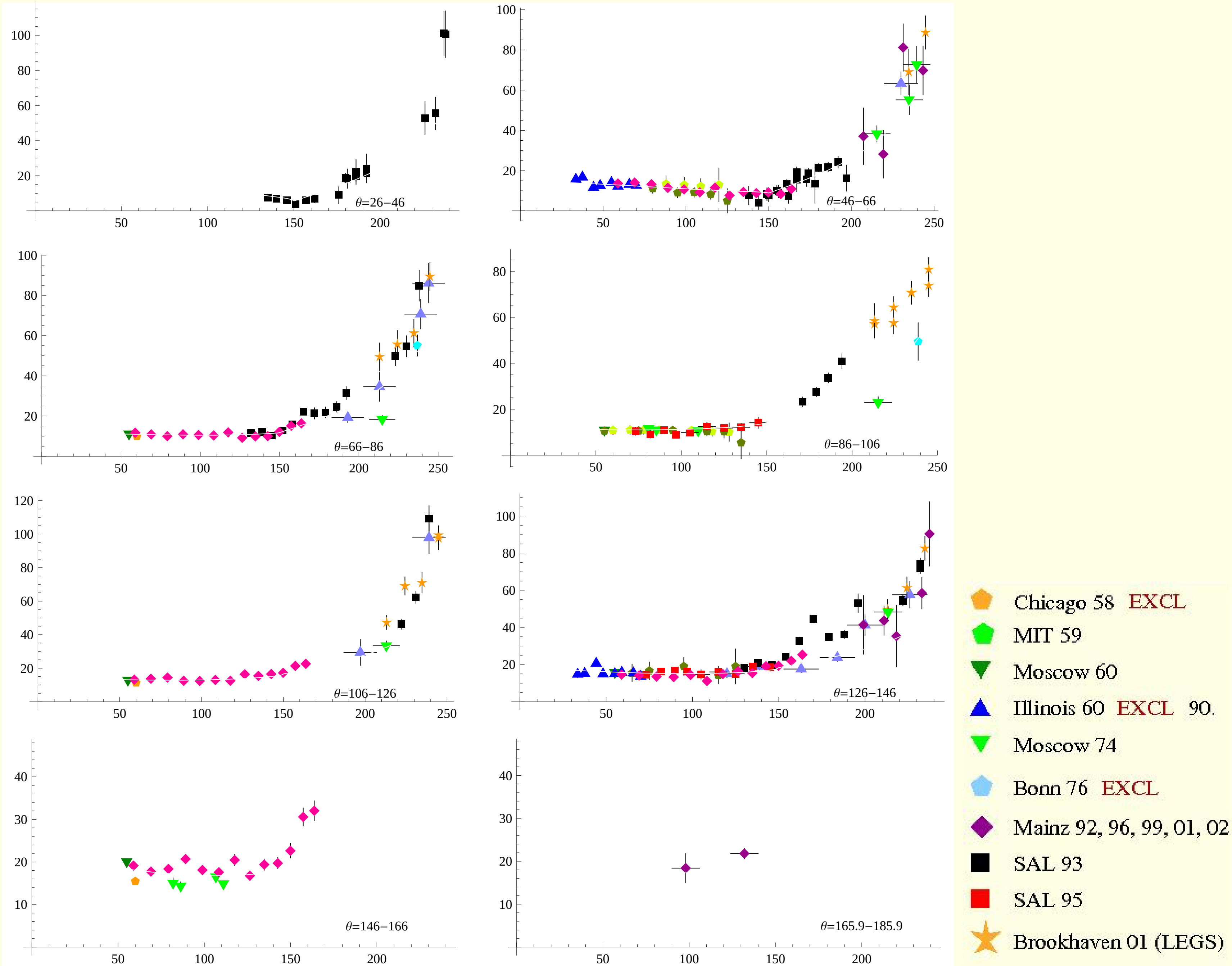
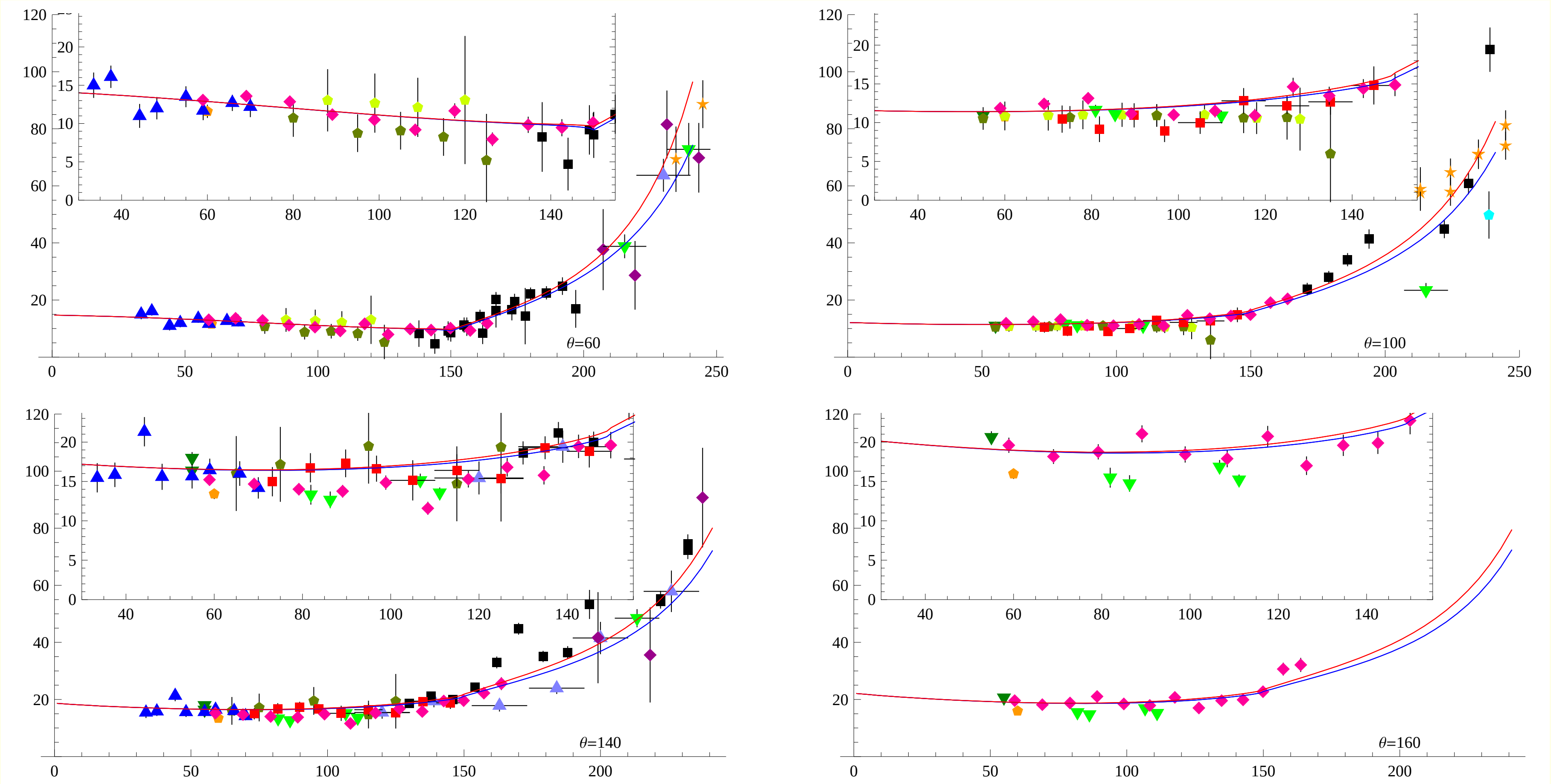
HIGH: The fit is very sensitive to the value of the $\gamma N\Delta$ coupling b1. At low energies may get unrealistic values, so distorting other fit parameters.
Also, terms of higher order become important - eg choice of frame for calculations

Evaluating the data

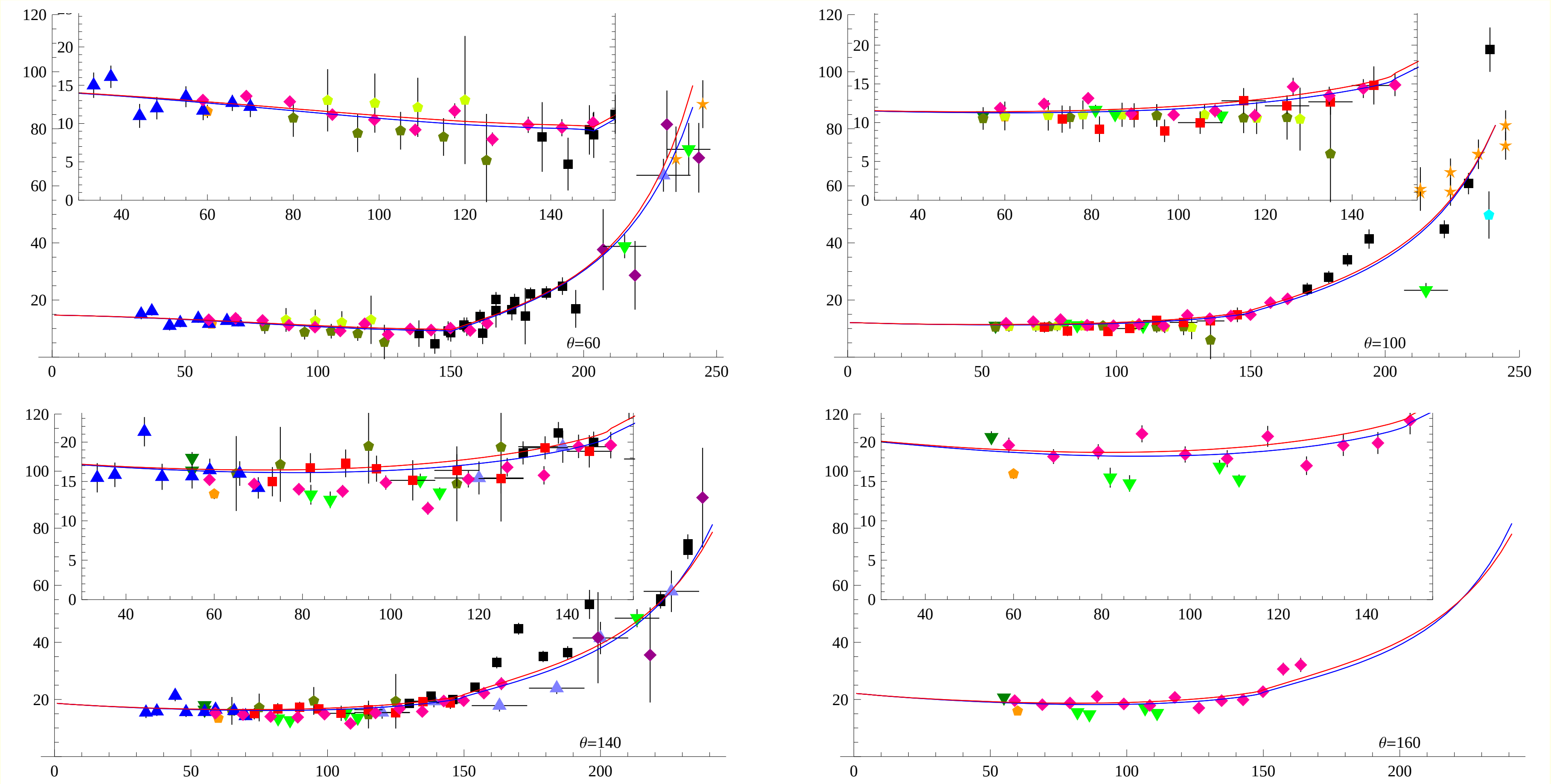


Comparison of cm and Breit frames



Here the red curve is the Breit frame, and the blue curve the center of mass frame, for the same parameters.

Comparison of cm and Breit frames II

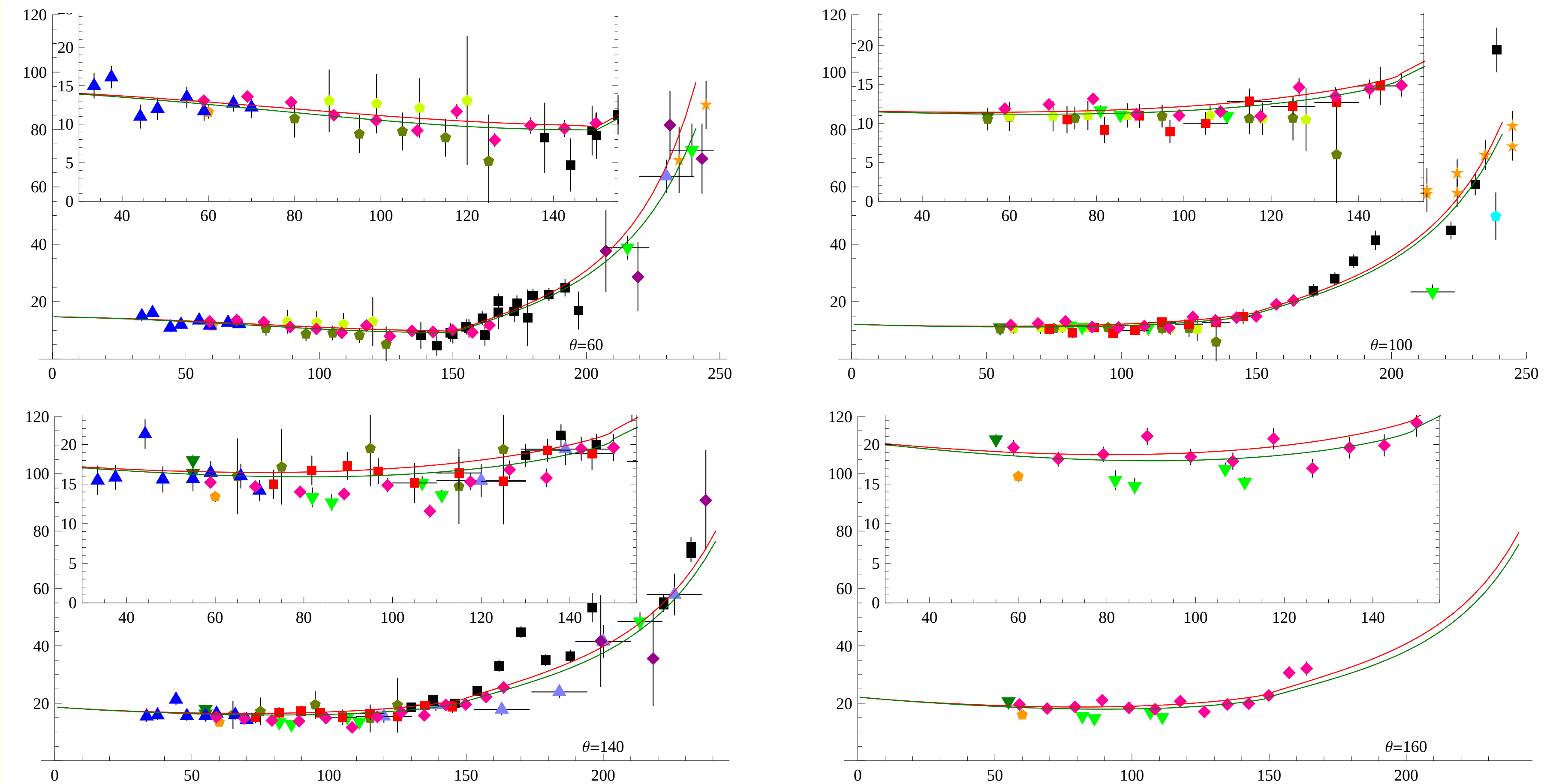


Best fit parameters (up to 240 MeV)

CM: $\alpha = 11.1$, $\beta = 4.2$, $b_1 = 4.7$

Breit: $\alpha = 9.9$, $\beta = 4.3$, $b_1 = 4.2$

Including Delta width



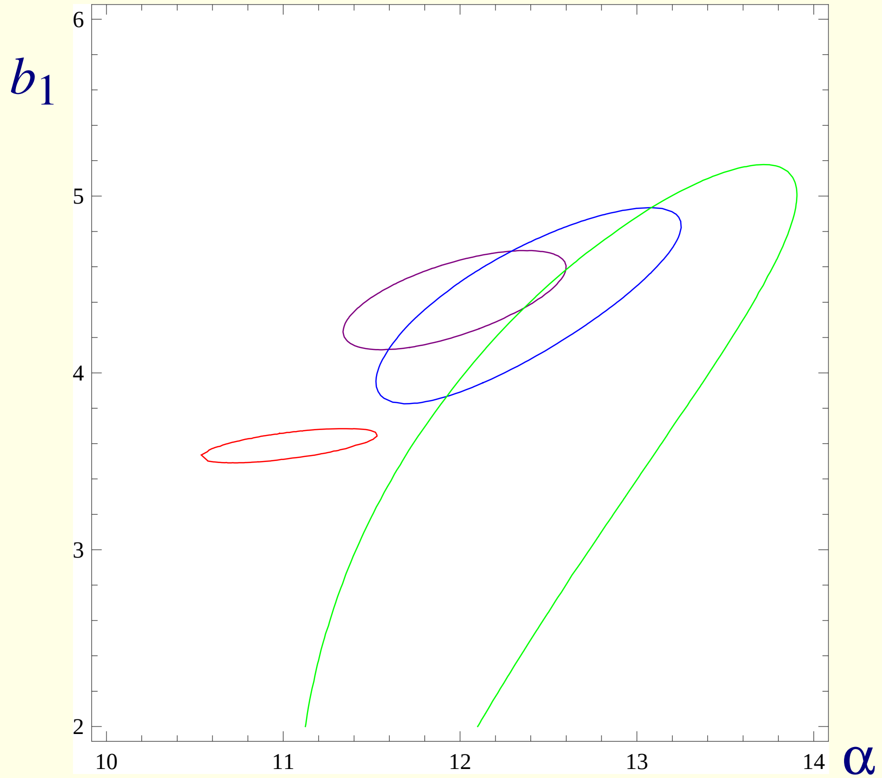
Inclusion of Delta width à la Pascalutsa and Phillips; also include higher-order Δ/M_N and ω/M_N corrections to vertex. Reduces frame dependence.

PP: $\alpha = 12.0$, $\beta = 4.1$, $b_1 = 3.5$. $\chi^2 = 282$ for 233 points and 18 parameters.

Varying the cutoff

$\omega_{max} = 130$ MeV: $\alpha = 13.1$, $\beta = 3.6$, $b_1 = 2.1$, $\chi^2 = 96$ for $110 - 13$ dof
$\omega_{max} = 160$ MeV: $\alpha = 13.3$, $\beta = 2.9$, $b_1 = 4.2$, $\chi^2 = 132$ for $156 - 14$ dof
$\omega_{max} = 200$ MeV: $\alpha = 12.7$, $\beta = 3.0$, $b_1 = 4.3$, $\chi^2 = 189$ for $194 - 14$ dof
$\omega_{max} = 240$ MeV: $\alpha = 12.0$, $\beta = 4.1$, $b_1 = 3.5$, $\chi^2 = 283$ for $235 - 18$ dof

Baldin-constrained: $\alpha + \beta \approx 14$: Tentative results $\alpha = 11.5$, $\beta = 2.5$ - errors?



Still to do

- Estimates of higher-order (N^4LO) effects including electric $\gamma N\Delta$ coupling
- Understanding of mechanism that determines promotion of CTs.
- Understanding what new data would be useful
- Deuteron (next talk!).