Precise tests of ChPT from Ke4 decays by the NA48/2 experiment

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On behalf of the NA48/2 collaboration:

Cambridge, CERN, Chicago, Dubna, Edinburgh, Ferrara, Firenze, Mainz, Northwestern, Perugia, Pisa, Saclay, Siegen, Torino, Wien

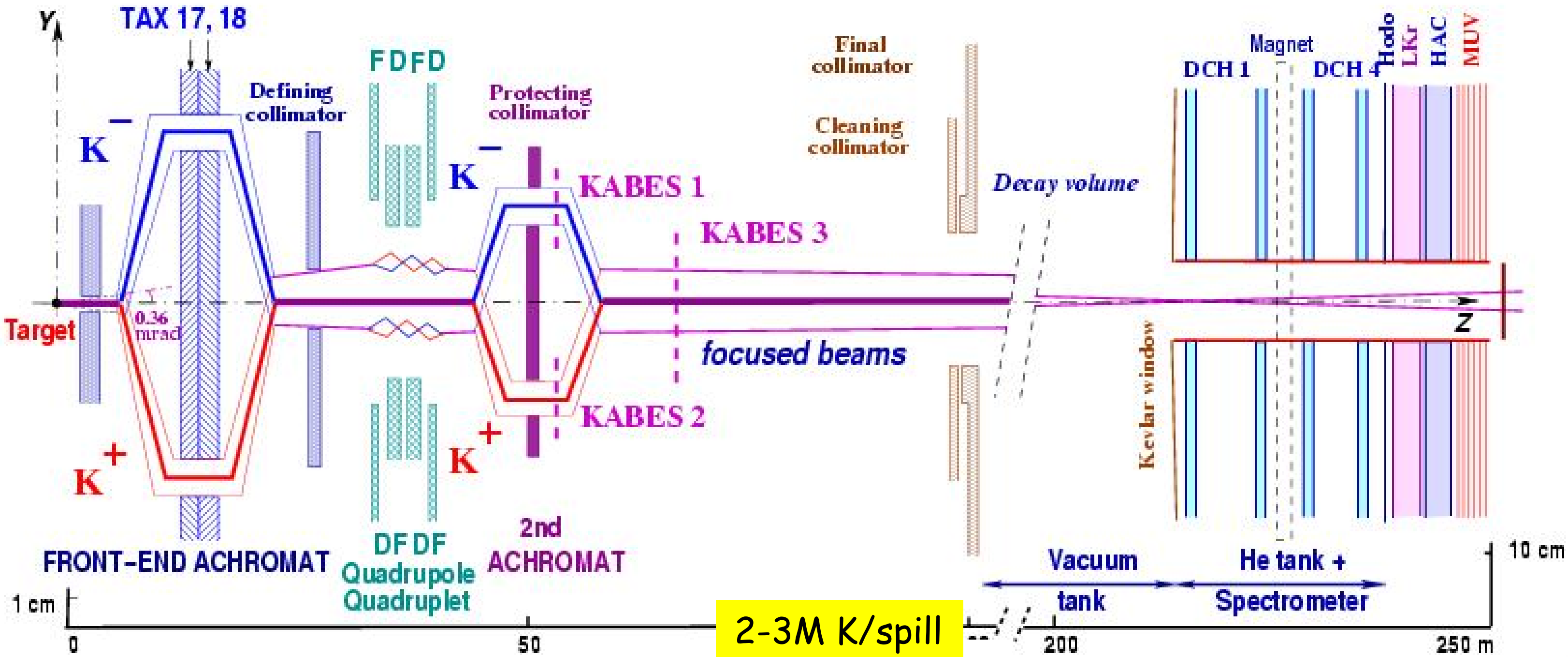
outline

- Brief introduction to NA48/2
- Ke4 decays ($K^{\pm} \rightarrow \pi^{+}\pi^{-} e^{\pm} \nu$) :

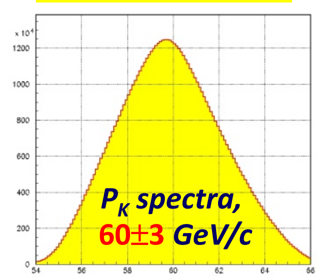
  Form Factors and $\pi\pi$ scattering lengths
- Combining NA48/2 results from cusp (S.Giudici's talk) and Ke4
- Summary

The NA48/2 experiment at the CERN-SPS: primarily designed for CP violating charge asymmetries studies in K3π decays

2003 run: ~ 50 days + 2004 run: ~ 60 days and >200TB Data on tape



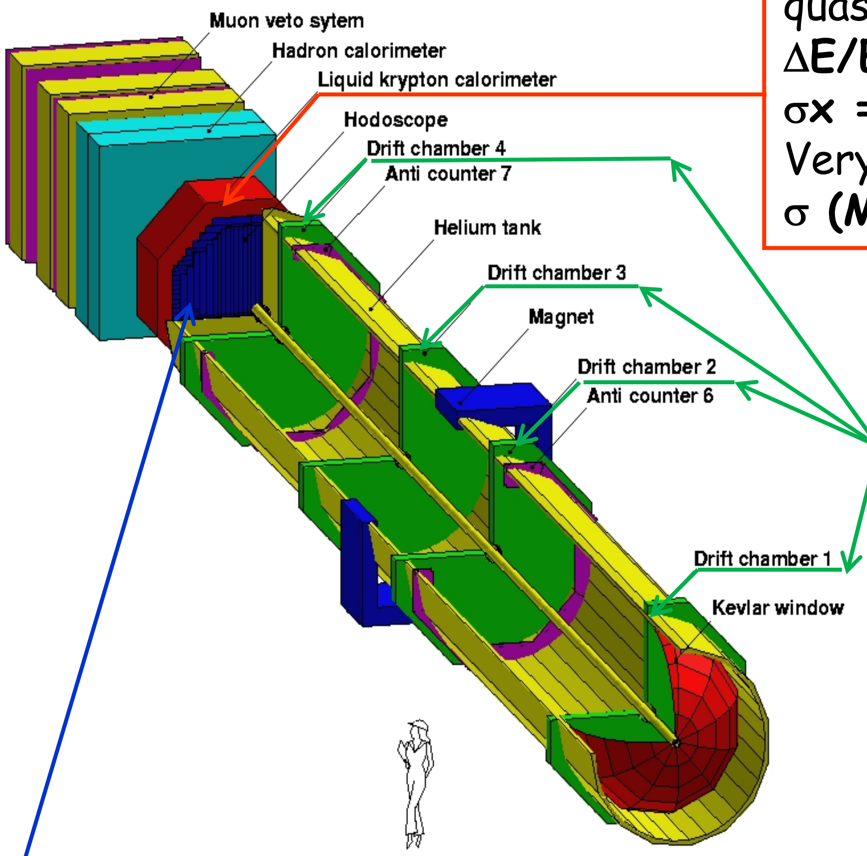
2-3M K/spill



Simultaneous K⁺ and K⁻ beams: large charge symmetrization of experimental conditions

Beams coincide within ~1mm all along the 114m decay volume flux ratio K^+/K^- ~1.8

The NA48/2 experiment: detector and performances



LKr electromagnetic calorimeter :

quasi-homogenous and high granularity

$\Delta E/E = (3.2/\sqrt{E} \oplus 9.0/E \oplus 0.42)\%$ (E in GeV)

$\sigma x = \sigma y \sim 1.5$ mm for E=10 GeV

Very good resolution for neutrals ($\pi^0 \rightarrow \gamma\gamma$)

$\sigma (M\pi\pi^0\pi^0) = 1.4$ MeV/c²

+

Magnetic spectrometer :

4 high-resolution DCH's + dipole magnet

$\Delta p/p = (1.0 \oplus 0.044\ p)\%$ (p in GeV/c)

Very good resolution for charged invariant masses: $\sigma (M3\pi^\pm) = 1.7$ MeV/c²

E/p ratio used for e /π discrimination

Hodoscope for charged fast trigger

$\sigma t = 150$ ps

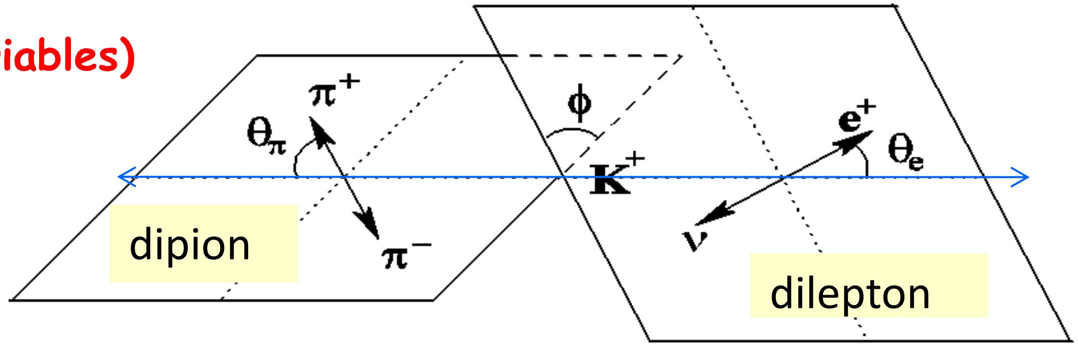
Ke4 decays : formalism

Five kinematic variables (Ca.Ma. variables)

(Cabibbo-Maksymowicz 1965)

S_π ($M^2_{\pi\pi}$), S_e ($M^2_{e\nu}$),

$\cos\theta_\pi$, $\cos\theta_e$ and ϕ.



Partial Wave expansion of the amplitude into s and p waves (Pais-Treiman 1968) + Watson theorem (T-invariance) for δ_ℓ^I

$\delta_0^0 \equiv \delta_s$ and $\delta_1^1 \equiv \delta_p$

F, G = 2 Axial Form Factors

$F = F_s e^{i\delta s} + F_p e^{id\delta p} \cos\theta_p$

$G = G_p e^{i\delta g}$

H = 1 Vector Form Factor

$H = H_p e^{i\delta h}$

F, G, H are complex

Map the distributions of the Ca.Ma. variables in the **five-dimensional space** with 4 Form factors and only one phase shift , assuming identical phases for the p-wave Form Factors F_p, G_p, H_p:

The fit parameters are :

F_s F_p G_p H_p and $\delta = \delta_s - \delta_p$

(F_s , F_p , $G_{p,}$, H_p are real)

Ke4 decays: event selection and background rejection

Signal ($\pi^+\pi^- e^{\pm}\nu$) topology:
- 3 charged tracks and a "good" vertex
- two opposite sign pions, 1 electron (E(LKr) /p ~ 1),
- some missing energy and p_T (ν)
- reconstruct PK (missing ν hypothesis)

Background main sources :

$K^+ \rightarrow \pi_1^{\ +} \pi^- \ \ \pi_2^{\ +}$ (dominant)
↳ $e^+ \nu$ or mis-ident as e^+

$K^+ \rightarrow \pi^+ \ \pi_1^{\ 0}$ ($\pi_2^{\ 0}$)
↳ ($e^- e^+ \gamma$) Dalitz decay
↳ mis-ident as π^- + γ (s) undetected

Control sample from data (assuming $\Delta S=\Delta Q$)

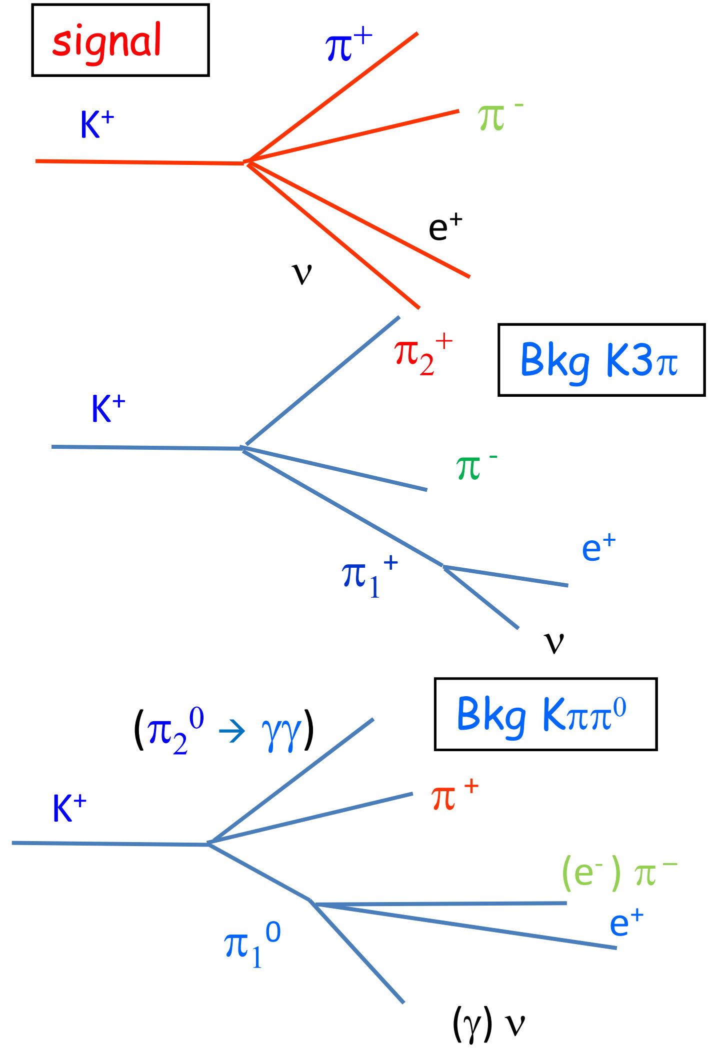
$K^{\pm} \rightarrow \pi^{\pm}\pi^{\pm} e^{\mp} \nu$ **"Wrong Sign" events**

(2 same sign pions, 1 electron)

Ratio (RS/WS) events:

2/1 if coming from K3π (dominant)

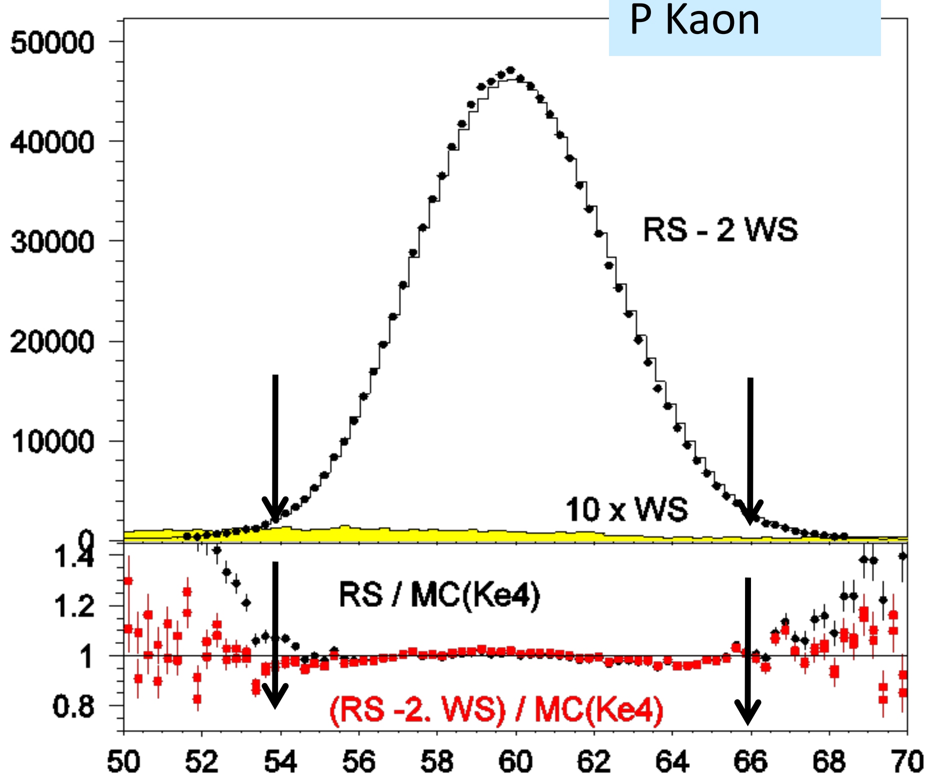
1/1 if coming from K2π (π^0)



Ke4 decays: background rejection

Total background level can be kept at ~ 2 x 0.3 % relative level
- estimated from WS events rate in Data
- checked from MC simulation of background processes

P Kaon



GeV/c

No BKG subtraction

subtraction = 2 x WS

Ke4 decays : fitting procedure

Total (2003+2004) 1.13 Million Ke4 decays

Using iso-populated boxes in the 5-dimension space of the Ca.Ma. variables, ($M_{\pi\pi}$, $M_{e\nu}$, $\cos\theta_\pi$, $\cos\theta_e$ and ϕ) one defines a grid of

10x5x5x5x12=15000 variable size boxes.

In each $M_{\pi\pi}$ "slice" (1500 boxes), a set of 4 fit parameters is found which minimizes the difference between the data and predicted populations

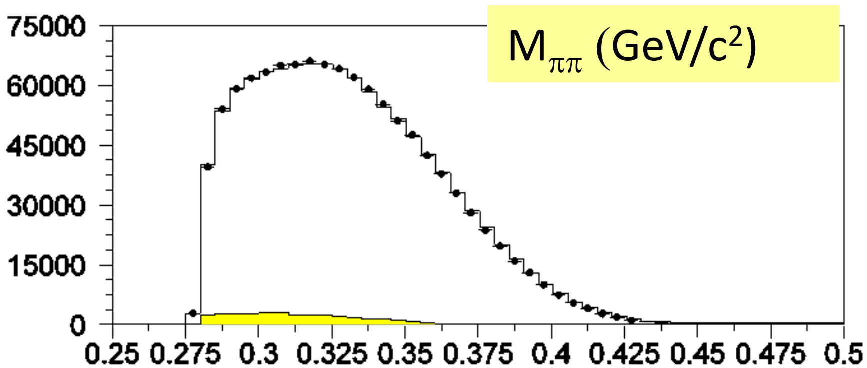
The normalisation F_s^2 is obtained in each bin/slice by the ratio $x_{slice} = \Sigma_{j \text{ in slice}} Nj / \Sigma_{j \text{ in slice}} MCj$

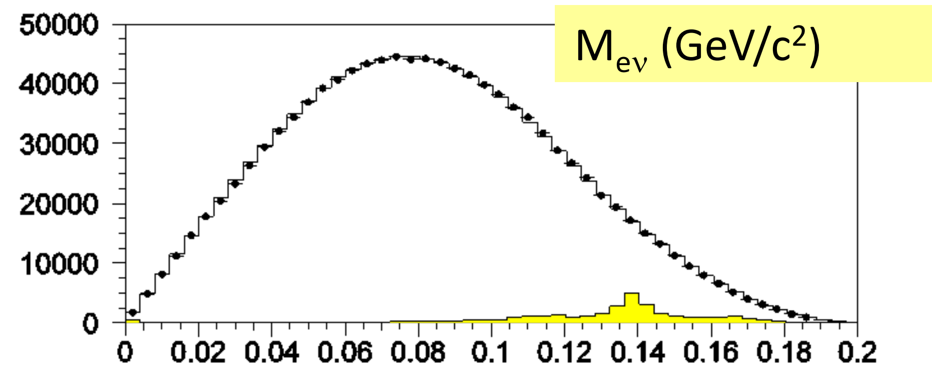
Data sample	MC sample
K^+ sample (726 400 events) 48 events/box	K^+ MC (17.4 Million events) 1160 events/box
K^- sample (404 400 events) 27 events/box	K^- MC (9.7 Million events) 650 events/box

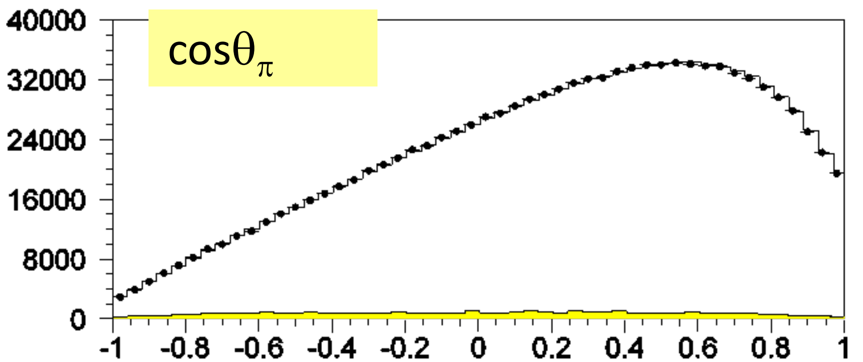
K^+ and K^- samples fitted separately in 10 independent $M\pi\pi$ bins/slices, then combined in each slice according to their statistical error.

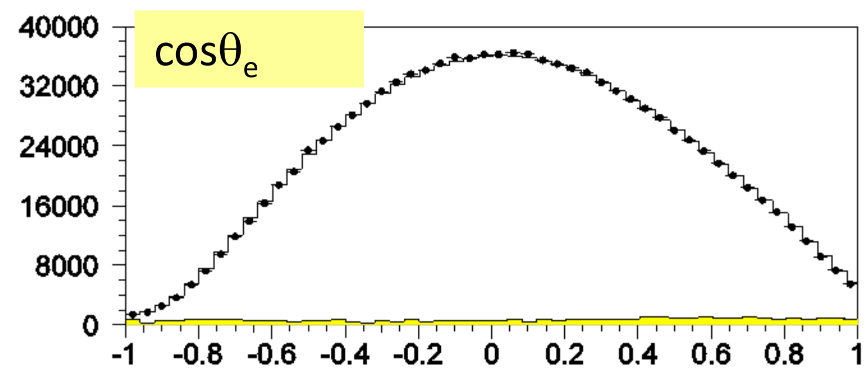
No assumption is made on the shape of the variation of the phase δ (and FF) from one $M_{\pi\pi}$ slice to the next (i.e. "model independent" analysis)

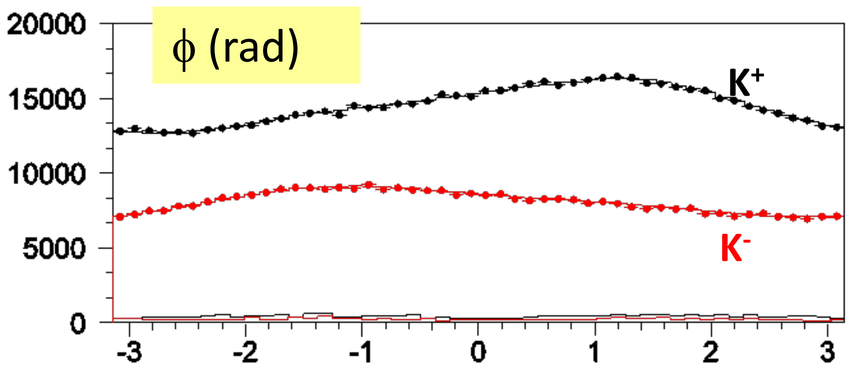
Ke4 decays : Data/MC comparison after fit











= Data

histogram = Simulation after fit

Yellow hist = WS Background (x 10 to be visible)

CP symmetry :

(K^+) ϕ distribution is opposite of (K^-) ϕ distribution

Ke4 Form Factors : fit results

Series expansion with q^2 ($q^2 = S_\pi/4m_\pi^2 - 1$) and $S_e/4m_\pi^2$ used to describe the FF variations, in the isospin symmetry limit (Amoros Bijnens 1999)

$$F_s^2 = f_s^2(1 + f_s'/f_s\, q^2 + f_s''/f_s\, q^4 + f_e'/f_s\, S_e/4m_\pi^2)^2$$

Correlation	f_s''/f_s	f_e'/f_s
f_s'/f_s	-0.95	0.08
f_s''/f_s		0.02

$$G_p/f_s = g_p/f_s + g_p'/f_s\, q^2$$

Correlation (g_p/f_s, g_p'/f_s) -0.91

Total statistics (2003+2004)

	value ± stat ± syst
f_s'/f_s	0.152 ± 0.007 ± 0.005
f_s''/f_s	-0.073 ± 0.007 ± 0.006
☆ f_e'/f_s	0.068 ± 0.006 ± 0.007
☆ f_p/f_s constant	-0.048 ± 0.003 ± 0.004
g_p/f_s	0.868 ± 0.010 ± 0.010
g_p'/f_s	0.089 ± 0.017 ± 0.013
h_p/f_s constant	-0.398 ±0.015 ± 0.008

systematics

- mostly from background + acceptance control
- ~ same size as statistical error or smaller

☆ first evidence by NA48

Ke4 decays: from phase shifts to scattering lengths (a_0,a_2)

$\pi\pi$ phases near threshold can be predicted from data above 0.8 GeV using Roy equations (unitarity, analyticity and crossing symmetries) and 2 subtraction constants a_0 and a_2

Numerical solutions have been developed (ACGL, DFGS) valid in the Isospin symmetry limit (Universal Band in the [a_2,a_0] plane), but broken in the experimental world.

•Take into account radiative effects before going from phases to scattering lengths

•Use factorization of electromagnetic and mass effects :

Gamow-Sommerfeld factor x PHOTOS generator × Isospin corrections

Radiative effects (except mass effects) included in the simulation,

Gamow factor : "classical" Coulomb attraction between the 2 charged pions

PHOTOS generator: real photon(s) are emitted and tracked in the simulation

(→ effect on event selection + possible bias on reconstructed quantities)

Mass effects:

• recently computed as a correction to the measurements

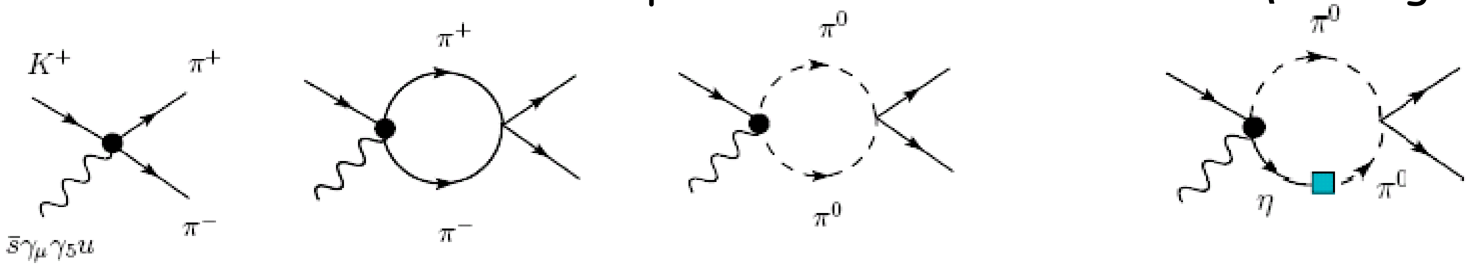
• even larger than current experimental precision !

(CGR EPJ C59 (2009) 777,
BDK preliminary June 2008 …in progress)

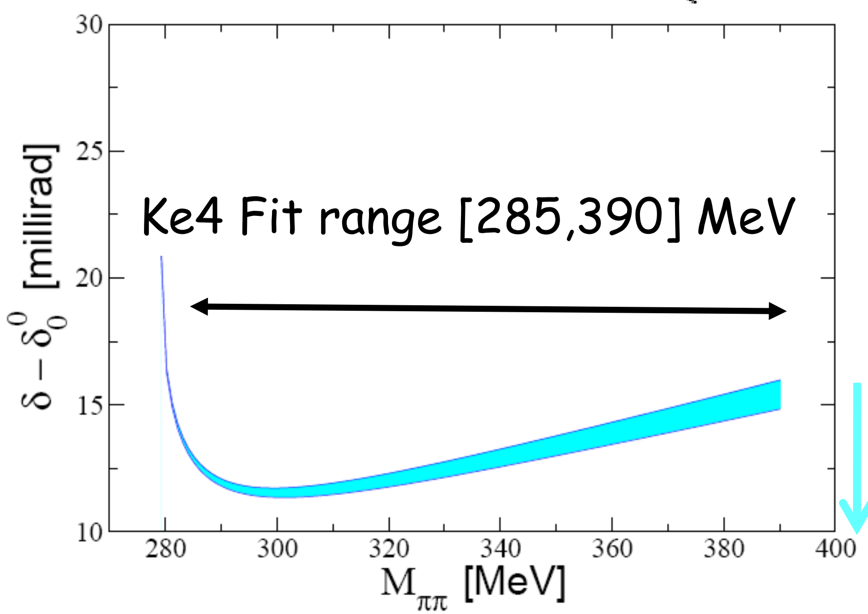
Ke4 charged decays : isospin corrections to δ

CGR EPJ C59 (2009) 777 formulation developed in close contact with NA48

tree | one loop | π^0–η mixing



$\delta_0^0 \rightarrow$ $$\delta = \frac{1}{32\pi F^2}\left\{(4\Delta_\pi + s)\sigma + (s - M_{\pi^0}^2)\left(1 + \frac{3}{2R}\right)\sigma_0\right\}$$



$$\Delta_\pi = M_{\pi^+}^2 - M_{\pi^0}^2,$$

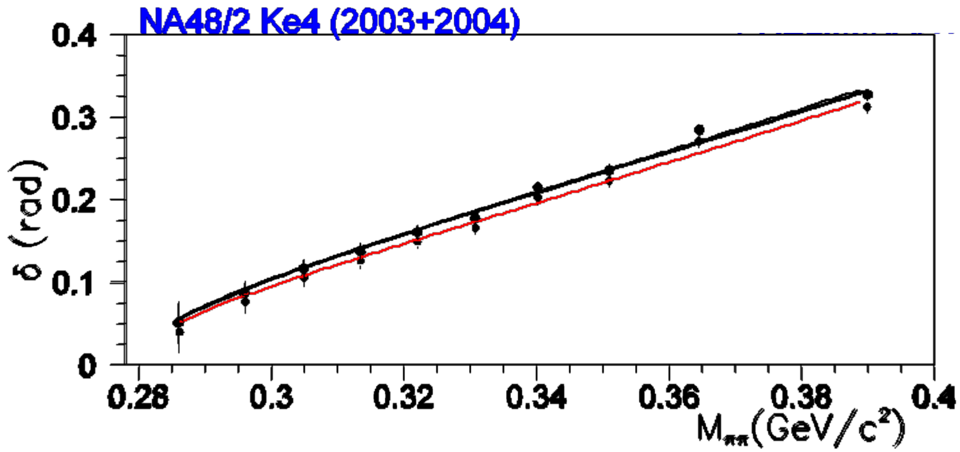
$$\sigma = \sqrt{1 - \frac{4M_\pi^2}{s}},$$

$$R = \frac{m_s - \hat{m}}{m_d - m_u}$$

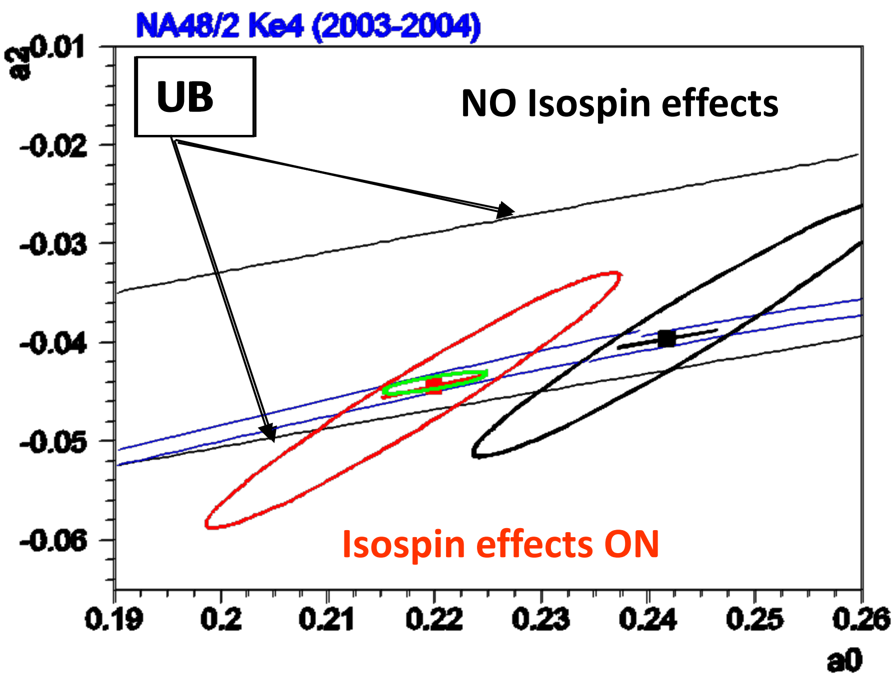
Correction is ~ 10-15 mrad

Exp. stat precision (δ) is ~ 7-8 mrad

Ke4 decays: from phase shifts to scattering lengths (a_0,a_2)



a tiny effect from theory…. a big change in now precise experimental measurement !



This induces a large **change** on (a_0,a_2) values

from a 2p fit			from a 1p fit	
Δa_0 = -0.025, Δa_2 = -0.007			Δa_0 = -0.022	
error	stat	syst	stat	syst
$\sigma(a_0)$:	± 0.0128	± 0.0050	± 0.005	± 0.002
$\sigma(a_2)$:	± 0.0084	± 0.0034		

Ellipses are 68% CL contours in 2p fits
Errors shown are statistical only

Ke4 decays: comparison with theoretical predictions

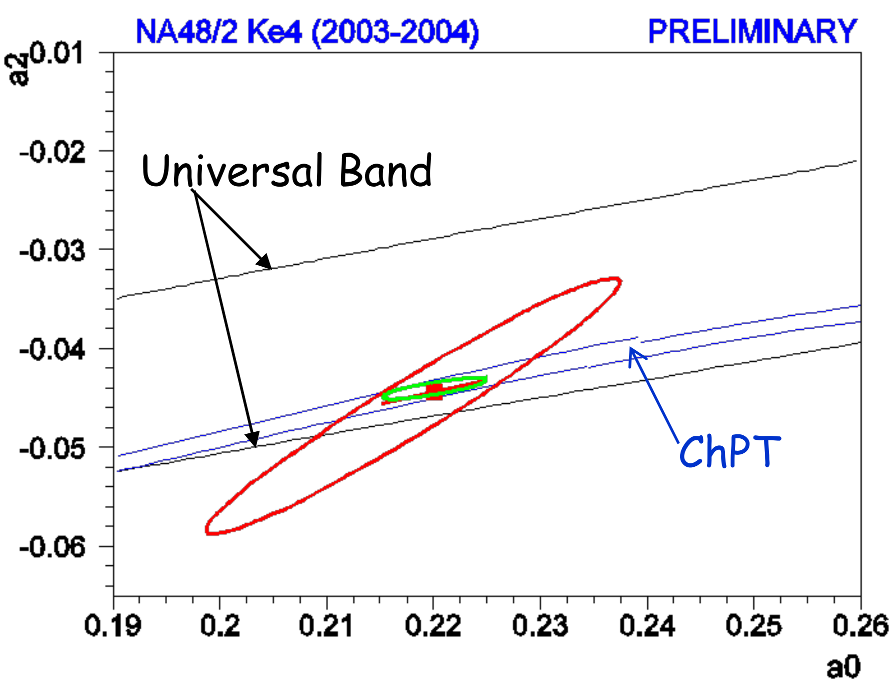
Total statistics (2003+2004)

THEORY prediction

Using more inputs from ChPT and low energy constants, the prediction is better constrained (CGL NPB603(2001),PRL86(2001)):

a_0 = 0.220 ± 0.005

a_2 = -0.0444 ± 0.0008



Experimental measurement

a_0 ChPT 1p fit	0.2206 ± 0.0049 stat ± 0.0018 syst* ± 0.0064 theo**
a_0 free	0.2220 ± 0.0128 stat ± 0.0050 syst* ± 0.0037 theo**
a_2 free 2p fit	-0.0432 ± 0.0086 stat ± 0.0034 syst* ± 0.0028 theo**
	Correlation 96.7%

* syst. error mostly from Bkg and e-id

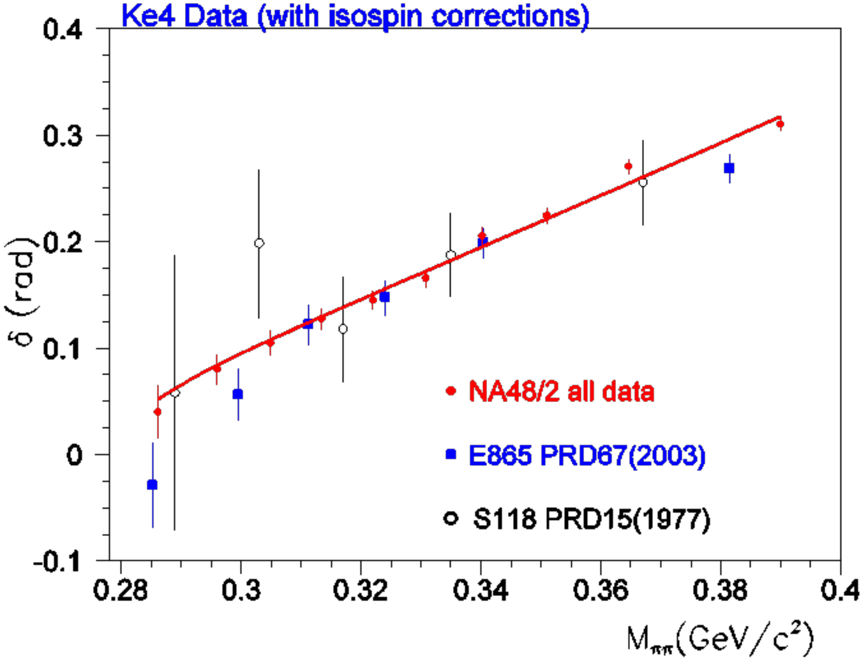
**Theory error evaluated from control of isospin corrections & inputs to Roy equation numerical solutions (CGR EPJ C59 (2009)777)

Comparison of Ke4 phase shift experimental measurements

Apply Isospin corrections (10-15 mrad) to all published points :

S118 (Geneva-Saclay): typical error 40-50 mrad

E865: typical error 15-20 mrad



NA48/2 typical error 7-8 mrad

improved precision due to both
- -larger statistics ~3 x E865
- -larger acceptance at high $\pi\pi$ mass

- All Phase points corrected for isospin mass effects
- Correction may be small wrt experimental error but coherent shift downwards for all data points
- Independent experiments
- Errors = stat + syst

Line from a 2p fit to NA48 data alone

Fit to all data points (21 points) : dominated by NA48/2 measurements

2p fit:

$a_0 = 0.2199 \pm 0.0125_{exp} \pm 0.0037_{theo}$

$a_2 = -0.0430 \pm 0.0083_{exp} \pm 0.0028_{theo}$

1p fit :

$a_0 = 0.2168 \pm 0.0048_{exp} \pm 0.0064_{theo}$

(theory error common to all expts)

Cusp and Ke4 scattering lengths results

- Two statistically independent measurements by NA48/2

60 Millions K3π decays ⟺ 1.13 Million Ke4 decays

- systematic uncertainties from different origins : mostly independent

calorimeter + trigger ⟺ background + electron identification

- different theoretical inputs

(1 & 2-loop) re-scattering models ⟺ Roy equations + isospin mass effects

- Large overlap of results in the (a_0,a_2) plane or (a_0-a_2,a_2) plane with different correlations suggests to combine the two results in a single one with improved uncertainties

Cusp:		Δstat	Δsyst+ext	Δtheo
a_0-a_2 =	0.2571	± 0.0048	± 0.0029	(± 0.0088)
a_2 =	−0.0241	± 0.0129	± 0.0096	(± 0.0149)

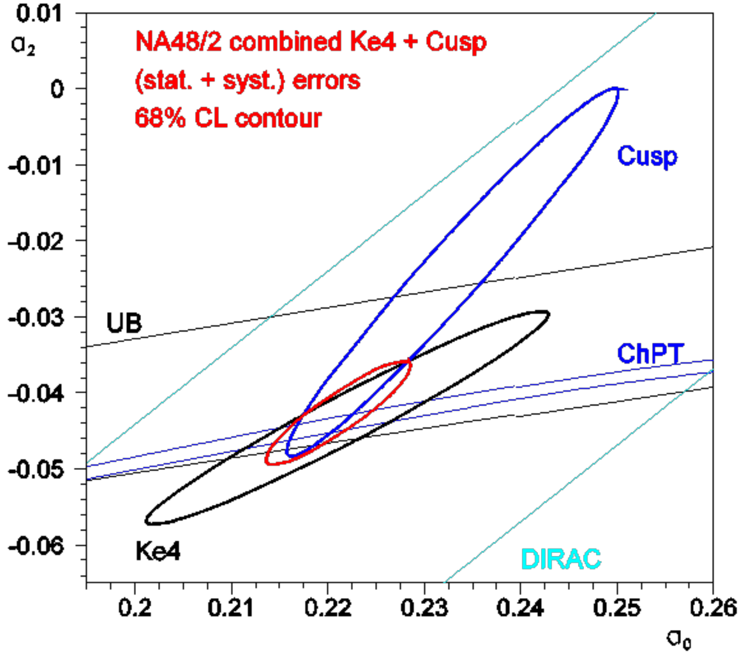
ρ = -0.839 (stat only) = -0.879 (all errors)

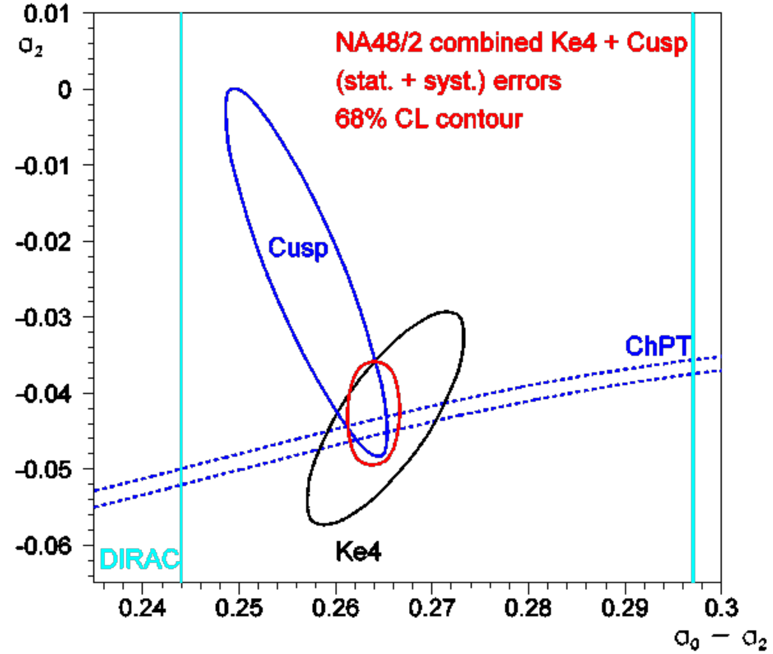
Ke4:		Δstat	Δsyst	Δtheo
a_0 =	0.2220	± 0.0128	± 0.0050	± 0.0037
a_2 =	-0.0432	± 0.0086	± 0.0034	± 0.0028

ρ = 0.967 (stat only) = 0.969 (all errors)

- similar combination can be done for the results using the same ChPT constraint

Combined results from cusp and Ke4





free (a_0, a_2) stat syst (theo)

a_0 = 0.2210 ± 0.0047 ± 0.0015 (± 0.0049)

a_2 = - 0.0429 ± 0.0044 ± 0.0016 (± 0.0030)

Correlation 0.910

Total errors Δa_0: ± 0.0050 (2.3% rel.precision)

Δa_2: ± 0.0046 (11% rel. precision)

free $(a_0 - a_2, a_2)$ stat syst (theo)

$a_0 - a_2$ = 0.2639 ± 0.0020 ± 0.0004 (± 0.0021)

a_2 = - 0.0429 ± 0.0044 ± 0.0016 (± 0.0030)

Correlation 0.277

Total errors $\Delta (a_0 - a_2)$: ± 0.0021 (0.8% rel.precision)

Δa_2: ± 0.0046 (11% rel. precision)

Including the ChPT constraint: stat syst (theo)

a_2 = -0.0444 ± 0.0007 ± 0.0005 (± 0.0012)

a_0 = 0.2195 ± 0.0027 ± 0.0021 (± 0.0048) or **$a_0 - a_2$ = 0.2640 ± 0.0020 ± 0.0017 (± 0.0035)**

Total error Δa_0 : ± 0.0034 Δa_2 : ± 0.0009 $\Delta (a_0 - a_2)$: ± 0.0026

Summary

NA48/2 has analyzed ~1.13 M Ke4 events recorded in (2003+2004)

- Ke4 Form Factors measured with an improved precision + first evidence for fp and f'e
- Scattering lengths obtained from precise experimental results are powerful tests of theoretical predictions both for free (a_0, a_2) and constrained (a_0, $a_2 = f(a_0)$) ChPT fits :

(free 2p Ke4) $a_0 = 0.2220 \pm 0.0128 \pm 0.0050 \pm 0.0037$, $a_2 = -0.0432 \pm 0.0086 \pm 0.0034 \pm 0.0028$

(ChPT Ke4) $a_0 = 0.2206 \pm 0.0049_{stat} \pm 0.0018_{syst} \pm 0.0064_{th}$

- Other experimental measurements (NA48/2 K3π Cusp, DIRAC pionium life time)

(ChPT NA48/2 cusp) $a_0 - a_2 = 0.2633 \pm 0.0024_{stat} \pm 0.0014_{syst} \pm 0.0019_{ext}$ ($\pm 0.0053_{th}$)
(DIRAC pionium, PLB619(2005)) $|a_0 - a_2| = 0.264 \,^{+0.033}_{-0.020}$

- Lattice calculations (LEC's calculations l3bar,l4bar,...)
- Most precise predictions of ChPT, $a_0 = 0.220 \pm 0.005$, $a_2 = -0.0444 \pm 0.0008$, $a_0 - a_2 = 0.264 \pm 0.004$

The achieved experimental precision (stat + syst) of NA48/2 combined result for a_0 and $a_0 - a_2$ is now competitive with the best theoretical precision .

NA48 Ke4 + Cusp	free 2p fit stat syst theo	ChPT 1p fit stat syst theo
a_0	$0.2210 \pm 0.0047 \pm 0.0015$ (± 0.0049)	$0.2196 \pm 0.0027 \pm 0.0021$($\pm 0.0048$)
a_2	$-0.0429 \pm 0.0044 \pm 0.0016$ (± 0.0030)	$-0.0444 \pm 0.0007 \pm 0.0005$($\pm 0.0012$)
$a_0 - a_2$	$0.2639 \pm 0.0020 \pm 0.0004$(± 0.0021)	$0.2640 \pm 0.0020 \pm 0.0017$($\pm 0.0035$)

The collaboration with many theory groups was/still is invaluable in understanding how to extract scattering lengths from NA48 Data. Thanks to all groups in Bern, Bonn, Dubna, Orsay, Madrid, Marseille, Rome .. !