(Generalized) Form Factors from Lattice QCD

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(Generalized) Form Factors from Lattice QCD

Outline of the Talk

- What are generalized form factors ?
- Why are they interesting ?
- Example n=1 form factor: Isovector Dirac Radius
- Example n=2 form factor: $A_{20}^{v}(t=0) = \langle x \rangle_{u-d}$
- Outlook/Summary

References:

- M. Dorati, T.A. Gail and TRH, Nucl. Phys. A798:96-131, 2008
- TRH, A. Schäfer et al., forthcoming work

What are generalized form factors ?

E.g. "Ordinary" nucleon vector form factors $F_1(t)$, $F_2(t)$

$$i\langle N|\bar{q}\gamma_\mu q|N\rangle = \bar{u}(p_2)\left\{F_1^q(t)\gamma_\mu + i\frac{F_2^q(t)}{2M_N}\sigma_{\mu\nu}(p_2-p_1)^\nu\right\}u(p_1)$$

- *u(p):* Dirac spinor of the baryon with momentum *p* and mass M_N
- *q ... q*: local operator in quark fields with flavor *q*

→ First generalization of nucleon vector form factors $A_{20}(t)$, $B_{20}(t)$, $C_{20}(t)$

$$i\langle N|\bar{q}\gamma_{\{\mu}\overleftrightarrow{D}_{\nu\}}q|N\rangle = \bar{u}(p_2)\left\{A_{20}^q(t)\gamma_{\{\mu}\bar{p}_{\nu\}} - i\frac{B_{20}^q(t)}{2M_N}\sigma_{\alpha\{\mu}\bar{p}_{\nu\}}(p_2-p_1)^\alpha + \frac{C_{20}^q(t)}{M_N}(p_2-p_1)_{\{\mu}(p_2-p_1)_{\nu\}}\right\}u(p_1)$$

$$\bar{p} = \frac{p_1+p_2}{2}$$

Why are GFFs interesting ?

Connection to Generalized Parton Distribution functions (GPDs)

- GPDs today are the universal framework to discuss nucleon structure (magnetic moments, spin, axial-couplings, radii, strangeness-content, ...)
- Within GPD-framework "ordinary" and "generalized" nucleon form factors are well defined x^{n-1}-moments of GPDs *H(t,x,ξ), E(t,x,ξ)*:

$$\int_{-1}^{+1} dx H^q(x,\xi,t) = F_1^q(t); \qquad \int_{-1}^{+1} dx\, x H^q(x,\xi,t) = A_{20}^q(t) + 4\xi^2 C_{20}^q(t);$$

$$\int_{-1}^{+1} dx E^q(x,\xi,t) = F_2^q(t); \qquad \int_{-1}^{+1} dx\, x E^q(x,\xi,t) = B_{20}^q(t) - 4\xi^2 C_{20}^q(t);$$

→ ordinary and generalized form factors are just different viewpoints on the same underlying nucleon structure parameterization in terms of GPDs

GFF Physics Program (within SFB TR55)

Generalized Form Factors of QCD with N_f=2 light flavors

- Step 1: Generalized form factors of the nucleon up to n=2 to N^2LO in Baryon ChPT in infinite volume
  - parity even √
  - parity odd (√)
  - tensor

  → Test of "convergence properties" of chiral expansion and error estimates
- Step 2: Extend N^2LO calculations to finite volume
  - expected timescale: 2010

<u>Note:</u>

- Use the same framework for n=1 and n=2 moments → "global fits"
- At the moment we only plan to utilize explicit Δ(1232) d.o.f.s for parity-odd GPD moments due to known convergence properties of chiral expansion

One page summary on covariant Baryon ChPT

1) Start from most-general chiral SU(N) meson-baryon Lagrangean
2) Choose matrix element you want to calculate
3) Decide on chiral power p^D up to which order you want to perform the calculation in perturbation theory

$$D = 2L + 1 + \sum_d (d-2) N_d^M + \sum_d (d-1) N_d^{MB}$$

→ see Gasser, Sainio, Svarc NPB 1988

4) Renormalize via $\overline{\text{MS}}$, augmented by infrared regular terms up to $O(p^D)$ which cure the power-counting violations observed in GSS 88. (→ exact HBChPT limit + correct analytic structure + scale-independent results for arbitrary dim.-reg. scale λ)
5) Study accuracy of the results via adding higher order estimates:

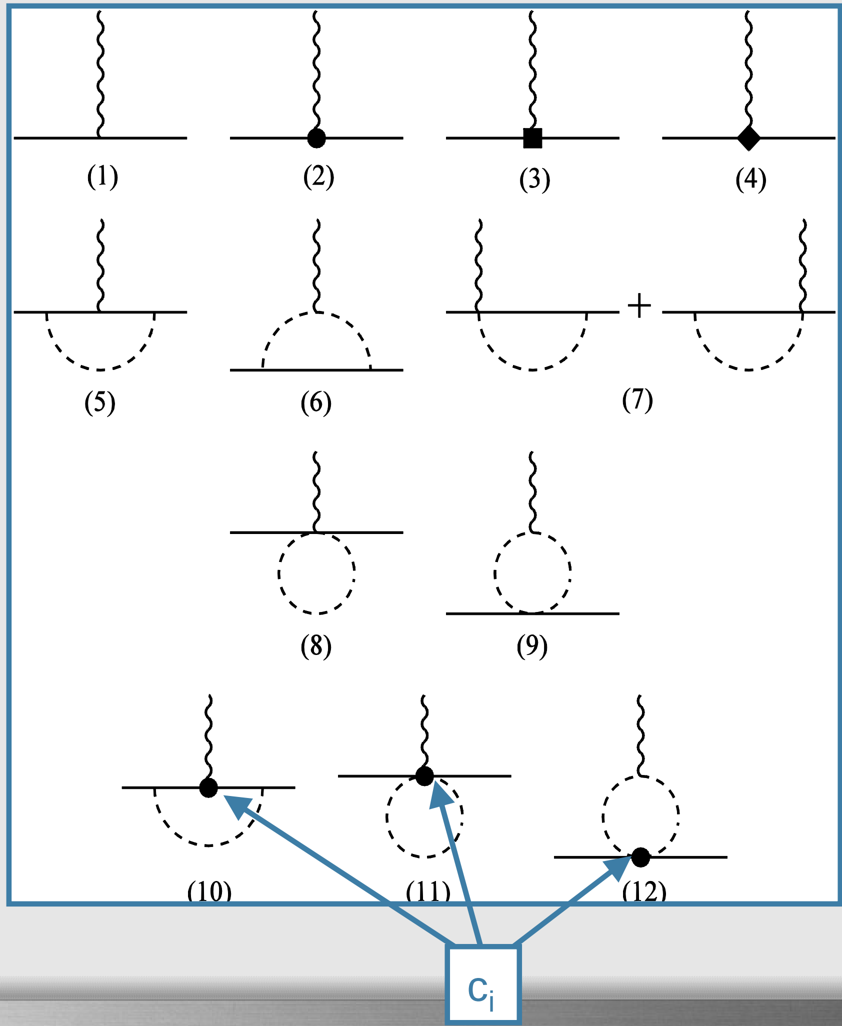
$$\pm\, \delta_B \left(\frac{m_{GB}}{\Lambda_{Gap}} \right)^{D+1} \quad \text{with } \delta_B \approx 1 \ldots 3$$

Example I: n=1 parity-even GFF

Isovector Dirac and Pauli form factors of the nucleon

- Calculate form factors up to N^2LO (i.e. $O(p^4)$) in Baryon ChPT for 2 light flavors
- Goal: Understand quark-mass dependence of these formfactors (t-dependence at physical point for N^2LO Baryon ChPT only valid up to t ~ 0.1 GeV2)

  → compare with Lattice QCD
- Main focus is isovector channel to avoid Lattice QCD approximations regarding "disconnected" diagrams
- "Gold-plated" observable is isovector Dirac-radius (free of uncertainties regarding value at t=0)



Example I: Isovector Dirac Radius

O(p³) Result

$$\left(r_1^v\right)^2 = -\frac{1}{16\pi^2 F_\pi^2}\left\{1+7g_A^2+2\left(5g_A^2+1\right)\log\left[\frac{m_\pi}{\lambda}\right]-15g_A^2\frac{m_\pi^2}{M^2}-g_A^2\frac{m_\pi^2}{M^2}\left(44-15\frac{m_\pi^2}{M^2}\right)\log\left[\frac{m_\pi}{M}\right]\right\}$$

$$-12B_{10}^r(\lambda)$$

$$+\frac{g_A^2}{32\pi^2F_\pi^2\sqrt{1-\frac{m_\pi^2}{4M^2}}}\frac{m_\pi}{M}\left[70-74\frac{m_\pi^2}{M^2}+15\frac{m_\pi^4}{M^4}\right]\arccos\left[\frac{m_\pi}{2M}\right]$$

O(p⁴) Result

$$\delta\left(r_1^v\right)^2 = +\frac{c_6 g_A^2}{64\pi^2F_\pi^2\sqrt{1-\frac{m_\pi^2}{4M^2}}}\frac{m_\pi^2}{M^2}\left[\frac{m_\pi}{M}\left(3-\frac{m_\pi^2}{M^2}\right)\arccos\left[\frac{m_\pi}{2M}\right]-2\sqrt{1-\frac{m_\pi^2}{4M^2}}\left(1+\left(1-\frac{m_\pi^2}{M^2}\right)\log\left[\frac{m_\pi}{M}\right]\right)\right]$$

$$+O\left(p^5\right)$$

Example I: Isovector Dirac Radius

Comparison with Lattice QCD data I

- Radii-data shown are obtained via dipole-fit to form factors at “low” t-values

→ systematic uncertainty in data !

- Here: Comparison with O(p^3) Baryon ChPT

Alternative:
Comparison with O(ε^3) SSE results
(see M. Göckeler et al., PRD 71, 034508 (2005))

Example I: Isovector Dirac Radius

Comparison with Lattice QCD data II

- Radii-data shown are obtained via dipole-fit to form factors at “low” t-values

→ systematic uncertainty in data !

- Here: Comparison with O(p^4) Baryon ChPT
- Preliminary result! Uncertainty of O(p^4) calculation still under investigation, but there is hope that contact can be achieved between lattice results and ChPT

Effect due to c6 !

Example II: n=2 GFF

SFB/ TR 55

Generalized form factor $A_{20}(t)$

Connection to (spin averaged) PDF-moment in forward limit!

$$A_{20}^{f}(t \rightarrow 0) = \langle x \rangle_{f} = \int_{0}^{1} dx\, x \left[q(x) + \bar{q}(x) \right]$$

Interpretation in Parton Model Picture (i.e. leading order QCD):

- *x*: momentum fraction carried by probed quark of flavor *f*
- *q*(*x*): probability of detecting a quark of flavor *f* with momentum fraction *x*

Phenomenology

- PDFs for many quark/anti-quark flavors well constrained from phenomenology (e.g. CTEQ)
- uncertainties at very small values of x → no effect on low moments
- Note: scale-/scheme-dependence in QCD ! Here: $\overline{MS}$ at $\Lambda = 2\,\mathrm{GeV}$
- E.g. $\langle x \rangle_{u-d} = 0.154 \pm 0.003$

Implementation in ChEFT I

ChEFT in the presence of external tensor background fields

- Need to find terms in the Lagrangean that involve $\gamma_\mu D_\nu$
  → check leading order nucleon Lagrangean

$$L_{\pi N}^{(1)} = \overline{\Psi}\left(i g_{\mu\nu}\gamma^\mu D^\nu - M_0 + \frac{g_A^0}{2}\gamma_\mu \gamma_5 u^\mu \right)\Psi$$

$$\Rightarrow L_{\pi N}^{(0)} = \frac{i}{2}\overline{\Psi}\left(\gamma^\mu g_{\mu\nu}\vec{D}^\nu - \overleftarrow{D}^\nu \gamma^\mu g_{\mu\nu}\right)\Psi$$

$$\text{now}: g_{\mu\nu} \Rightarrow \overline{g_{\mu\nu}}$$

with

$$\overline{g_{\mu\nu}} = g_{\mu\nu} + a_{20}^s V_{\mu\nu}^0 + \frac{a_{20}^v}{2} V_{\mu\nu}^+ + \frac{\Delta a_{20}^v}{2} V_{\mu\nu}^- \gamma_5 + \Delta a_{20}^s A_{\mu\nu}^0 \gamma_5$$

Implementation in ChEFT II

ChEFT in the presence of external tensor background fields

- Consider traceless, symmetric tensor field $t_{\mu\nu}$
- Choice of transformation properties: $h\, t_{\mu\nu}\, h^{+}$

$$V^{\pm}_{\mu\nu} = \frac{1}{2}\left(g_{\mu\alpha} g_{\nu\beta} + g_{\mu\beta} g_{\nu\alpha} - \frac{2}{d} g_{\mu\nu} g_{\alpha\beta} \right) \times \left(u^{+} V_{R}^{\alpha\beta} u \pm u V_{L}^{\alpha\beta} u^{+} \right)$$

$$V^{0}_{\mu\nu} = \frac{1}{2}\left(g_{\mu\alpha} g_{\nu\beta} + g_{\mu\beta} g_{\nu\alpha} - \frac{2}{d} g_{\mu\nu} g_{\alpha\beta} \right) \tilde{v}^{\alpha\beta} \frac{\mathbf{1}}{2}$$

$$A^{0}_{\mu\nu} = \frac{1}{2}\left(g_{\mu\alpha} g_{\nu\beta} + g_{\mu\beta} g_{\nu\alpha} - \frac{2}{d} g_{\mu\nu} g_{\alpha\beta} \right) \tilde{a}^{\alpha\beta} \frac{\mathbf{1}}{2}$$

and

$$V^{R/L}_{\alpha\beta} = \left(v^{i}_{\alpha\beta} \pm a^{i}_{\alpha\beta} \right) \frac{\boldsymbol{\tau}^{\mathbf{i}}}{2}; \quad i = 1,2,3$$

Implementation in ChEFT

Lagrangeans for external isovector tensor background fields

$$L_{\pi N}^{(0)} = \frac{1}{2}\bar{\Psi}\left(i\gamma^{\mu}\left[\frac{a_{20}^{\nu}}{2}V_{\mu\nu}^{+} + \frac{\Delta a_{20}^{\nu}}{2}V_{\mu\nu}^{-}\gamma_5 + \ldots\right]\vec{D}^{\nu} + h.c.\right)\Psi$$

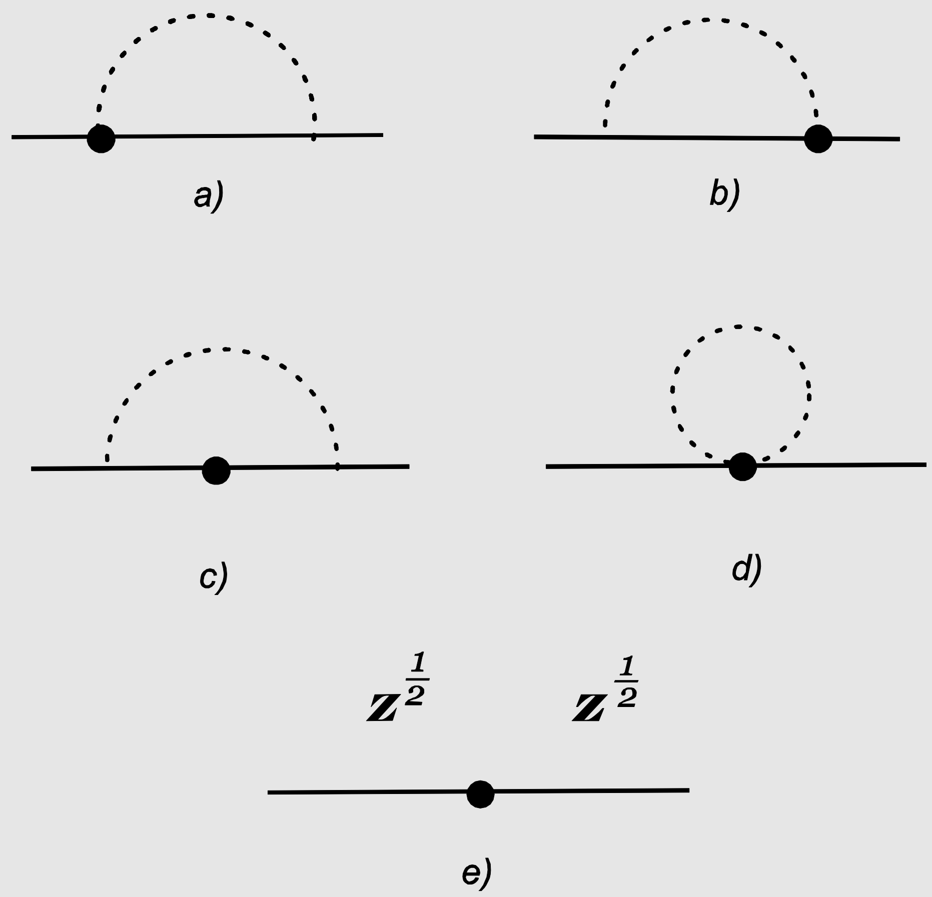
$$L_{\pi N}^{(1)} = \bar{\Psi}\left(\frac{1}{2}\left[i g_{\mu\nu}\gamma^{\mu}\vec{D}^{\nu} - M_0\right] + \frac{1}{4}\gamma^{\mu}\left[g_A^0 g_{\mu\nu} + h_A V_{\mu\nu}^{+} + \ldots\right]u^{\nu}\gamma_5 + \frac{1}{4}\gamma^{\mu}\left[\Delta h_A V_{\mu\nu}^{-} + \ldots\right]u^{\nu} + h.c.\right)\Psi$$

$$L_{\pi N}^{(2)} = \bar{\Psi}\left(c_1 Tr(\chi_+) + \ldots - \frac{c_7}{8}\sigma^{\mu\nu}Tr\left(F_{\mu\nu}^{+}\right) + \frac{1}{4}\left[Tr(\chi_+)\left(c_8 V_{\mu\nu}^{+}\gamma^{\mu} + \ldots + c_{14}V_{\mu\nu}^{-}\gamma^{\mu}\gamma_5 + \ldots\right)i\vec{D}^{\nu} + h.c.\right]\right)\Psi$$

For details see: M. Dorati, T.A. Gail and TRH, Nucl. Phys. A798:96-131, 2008

O(p^2) Diagrams

Calculation of $\langle x \rangle_{u-d}$ and $\langle \Delta x \rangle_{u-d}$ to leading one-loop order:



$O(p^2)$ Results

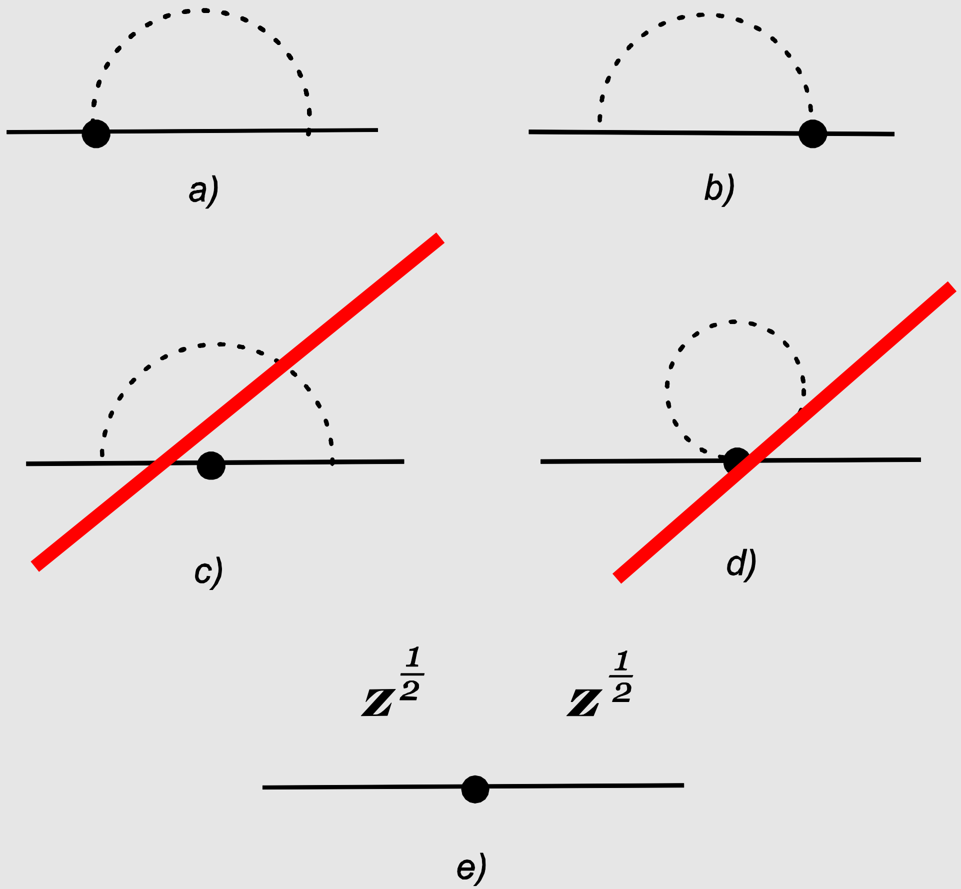
$<x>_{u-d}$ to leading one-loop order in $\overline{\text{IR}}$-regularization

$$A_{2,0}^{u-d}(0) = a_{20}^v + c_8 \frac{4m_\pi^2}{M_0^2} + a_{20}^v \frac{g_A^2 m_\pi^2}{16\pi^2 F_\pi^2}\Bigg[-\left(3+\frac{1}{g_A^2}\right)\ln\frac{m_\pi^2}{\lambda^2} + \frac{m_\pi^2}{M_0^2} - 2 + \frac{m_\pi^2}{M_0^2}\left(6-\frac{m_\pi^2}{M_0^2}\right)\ln\frac{m_\pi}{M_0}$$
$$+\frac{m_\pi}{\sqrt{4M_0^2-m_\pi^2}}\left(14 - 8\frac{m_\pi^2}{M_0^2} + \frac{m_\pi^4}{M_0^4}\right)\arccos\frac{m_\pi}{2M_0}\Bigg] \qquad (3.3)$$
$$+\Delta a_{20}^v \frac{g_A^2 m_\pi^2}{48\pi^2 F_\pi^2}\left[2\frac{m_\pi^2}{M_0^2} + \frac{m_\pi^2}{M_0^2}\left(6-\frac{m_\pi^2}{M_0^2}\right)\ln\frac{m_\pi^2}{M_0^2} + 2m_\pi \frac{(4M_0^2-m_\pi^2)^{3/2}}{M_0^4}\arccos\frac{m_\pi}{2M_0}\right]$$

- M. Dorati, T.A. Gail and TRH, NPA 798:96-131 (2008)
- **Note: λ-dependence compensated by c8 counter-term (implicit Λ-dependence hidden in couplings !)**
- **Leading non-analytic behaviour found in HBChPT by Chen+Ji and by Arndt+Savage is exactly contained/reproduced**

O(p³) Diagrams

Calculation of $\langle x \rangle_{u-d}$ and $\langle \Delta x \rangle_{u-d}$ to next-to-leading one-loop order:



$O(p^3)$ Results

$\langle x \rangle_{u-d}$ to next-to-leading one-loop order in $\overline{\text{I}}$R-regularization

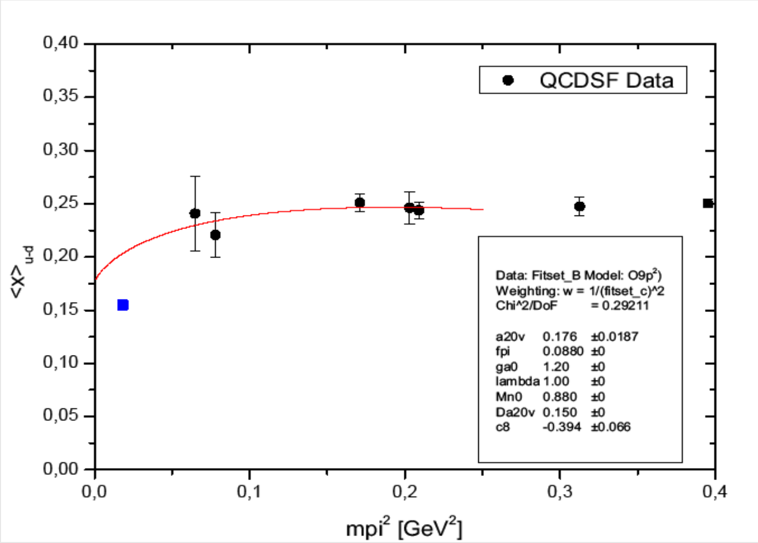
$$\langle x \rangle^{(3)}_{u-d} = -\frac{h_A g^0_A m^2_\pi}{24\pi^2 F^2_\pi}\mu_0 \times$$

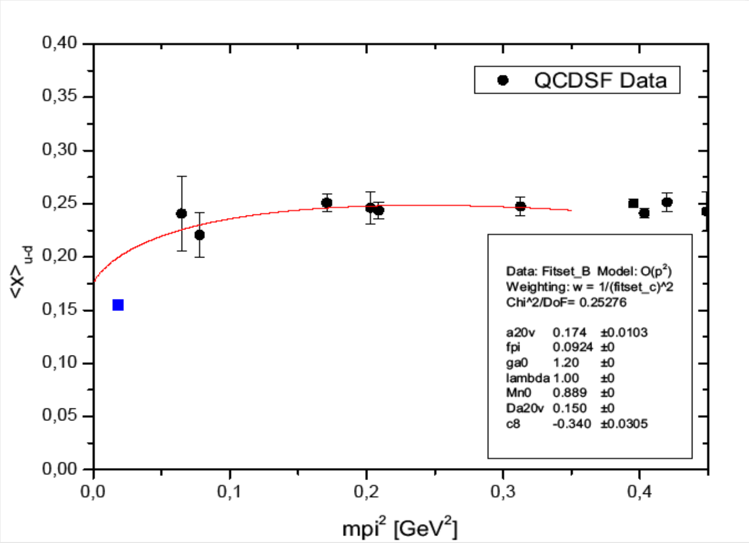
$$\left(2\left(1-\mu_0^2\right)\sqrt{1-\frac{\mu_0^2}{4}}\,\text{arcos}\left[\frac{\mu_0}{2}\right]+\mu_0+3\mu_0\log[\mu_0]-\mu_0^3\log[\mu_0]\right)$$

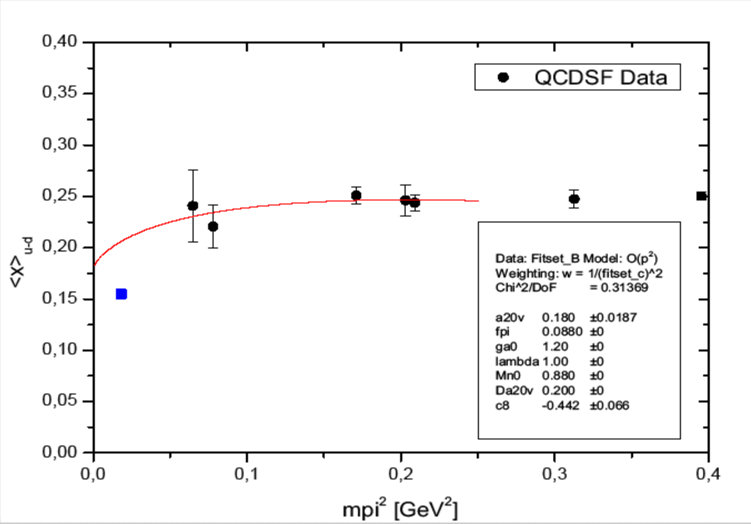
$$\text{with } \mu_0 = \frac{m_\pi}{M_0}$$

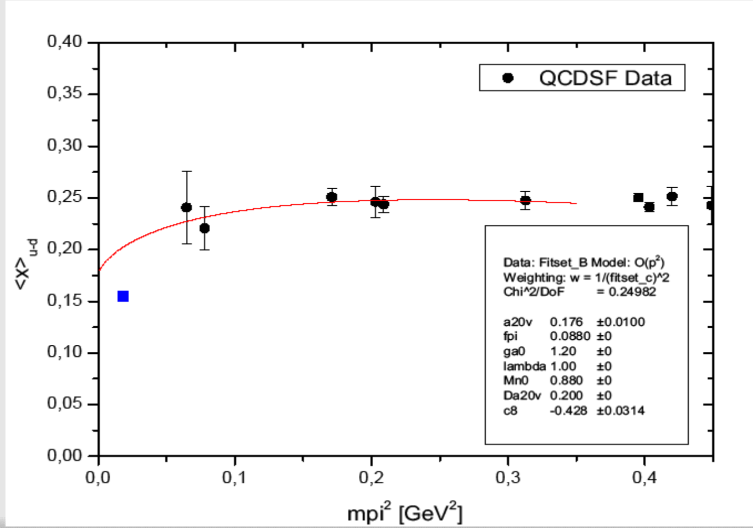
- M. Dorati, A. Schäfer and TRH, forthcoming
- $O(p^3)$ contribution itself is finite
- Extra effect: Nucleon mass terms shown in $O(p^2)$ result start to acquire quark-mass dependence

LO-Fit to QCDSF-RBC Data I



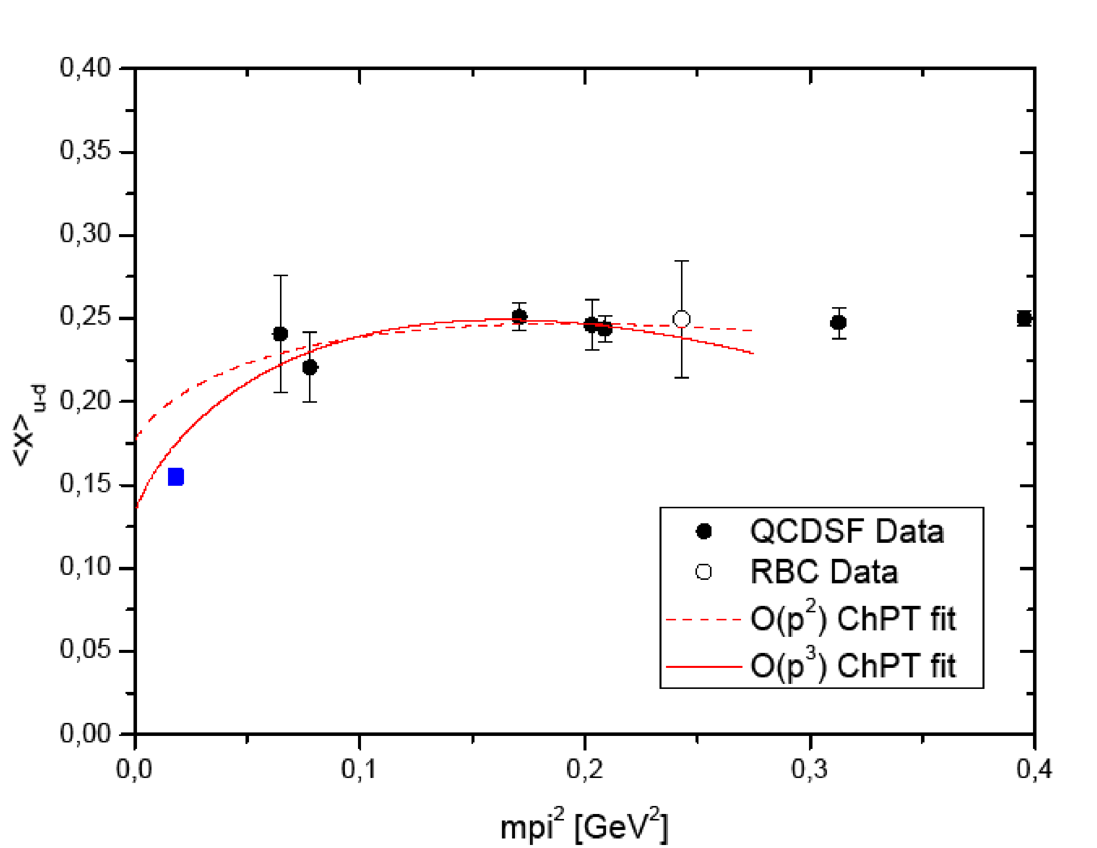






NLO-Fit to QCDSF-RBC Data II

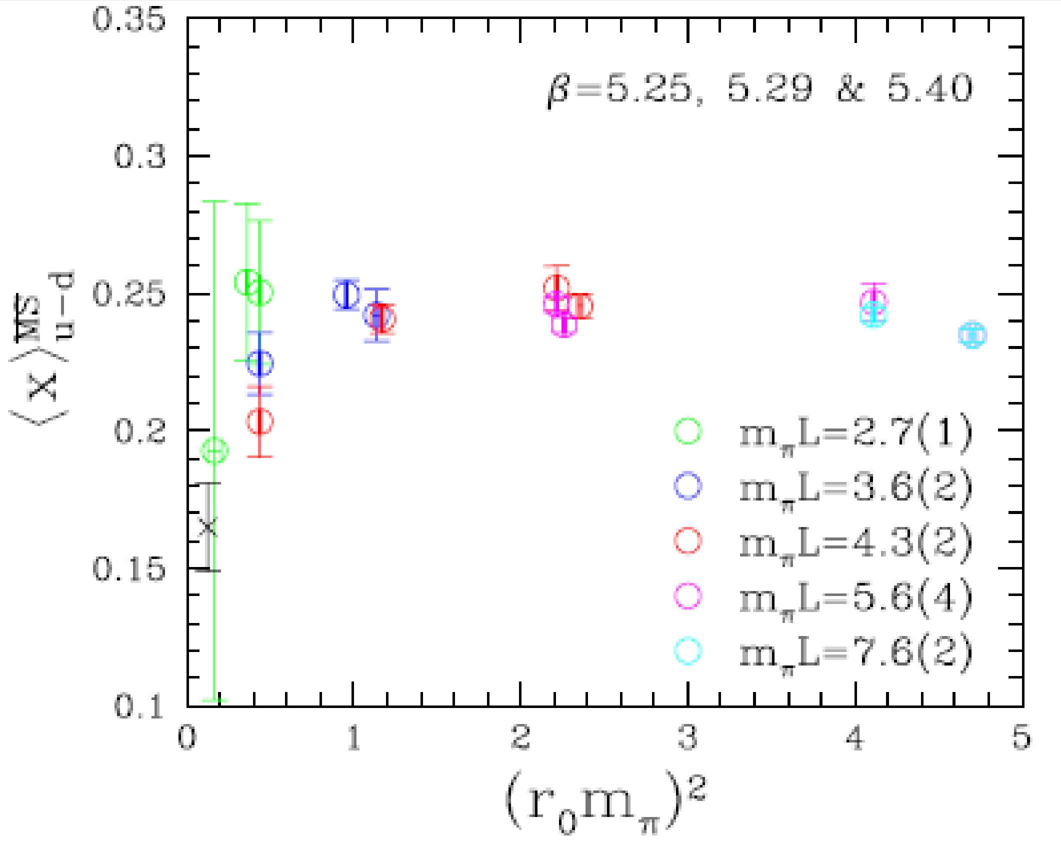
TRH, Dorati, Schäfer, forthcoming



- 3 parameter fit (a_{20v}, c_8, h_A) to 6 lattice points
- Preliminary Result !
- predicted chiral curvature of O(p^3) Baryon ChPT improves the leading-one loop result significantly
- So far:
  - reasonable central values (e.g. h_A=3.5), albeit with large error bars (poor statistics)
  - effect of "running" M_N still needs to be properly implemented

Update on QCDSF Data

2 flavor improved Wilson data on several volumes



- Preliminary Result !
- Data now show the bending downward for small quark-masses as expected by ChPT and phenomenology
- In principle our next-to-leading one-loop Baryon ChPT calculations must be able to predict the observed volume-dependence

Outlook/Summary

Moments of Parton Distribution Functions from Lattice QCD

- SFB TR55 (Bern+Graz+Regensburg+Wuppertal) set to calculate GPD-moments (=generalized form factors) up to n=2 at next-to-leading one-loop order in Baryon ChPT with 2 light flavors

  → goal is consistent picture of nucleon structure from lattice QCD within one ChPT formalism with check of convergence properties + errors
- Examples:
  - Progress in quark-mass dependence of isovector Dirac radius. NLO curve has the possibility to make contact with lattice data. Uncertainties still need to be evaluated (TRH et al., in preparation)
  - Long-standing problem of missing chiral curvature in $\langle x \rangle_{u-d}$ at low quark-masses for $O(p^2)$ BChPT extrapolation of QCDSF data towards the physical point now appears to be an effect of missing $O(p^3)$ contributions.

    (TRH, Dorati, Schäfer, forthcoming)
- Outlook: finite volume effects essential (see e.g. recent QCDSF results for $\langle x \rangle_{u-d}$), work (including TBCs) is under way (L. Greil,TRH, A. Schäfer)

A note on SFB/TR 55

A new SFB in Hadron Physics in Germany

- Joint SFB proposal ("Transregio") by Universities of Regensburg and Wuppertal (+ U Bern and U Graz)
  - Coordinator: Prof. Dr. Andreas Schäfer (U Regensburg)
  - FP1: Fall 2008 – Fall 2012
- Title "Hadron Physics from Lattice QCD"
- Close interplay/communication between Lattice QCD and ChEFT will be crucial for success → Project A8
  - SP 1: Generalized Parton Distributions (GPDs)
  - SP 2: Hadron Resonances and 2(+1) flavor QCD
  - SP 3: a) Spin-Polarizabilities from Lattice QCD
    b) Non-perturbative methods in ChEFT
- Successful pass of scientific review in Dec. 2007
- Final approval by DFG-Senat given on May 20, 2008