Hadron-Hadron Interactions with Lattice QCD

Chiral Dynamics 2009
Bern, Switzerland

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Preview

Numerical Redsults: NPLQCD

- 2-Hadron Interactions on the Lattice
- 2-meson interactions: precision predictions
  - $\mathcal{I}=2\ \pi\pi$ Scattering
  - $n \geq 3$ mesons and 3-body interactions
- 3-baryon interactions:

  $\Xi^0\Xi^0 n$

  nnp
- Conclusions

Preview

Numerical Redsults: NPLQCD

Silas Beane	University of NewHampshire
Will Detmold	College of William and Mary
Huey-Wen Lin	University of Washington
Tom Luu	LLNL
Kostas Orginos	College of William and Mary
Assumpta Parreno	University of Barcelona
Martin Savage	University of Washington
Aaron Torok	Univ. of NewHampshire → Indiana (Steve Gotlieb)

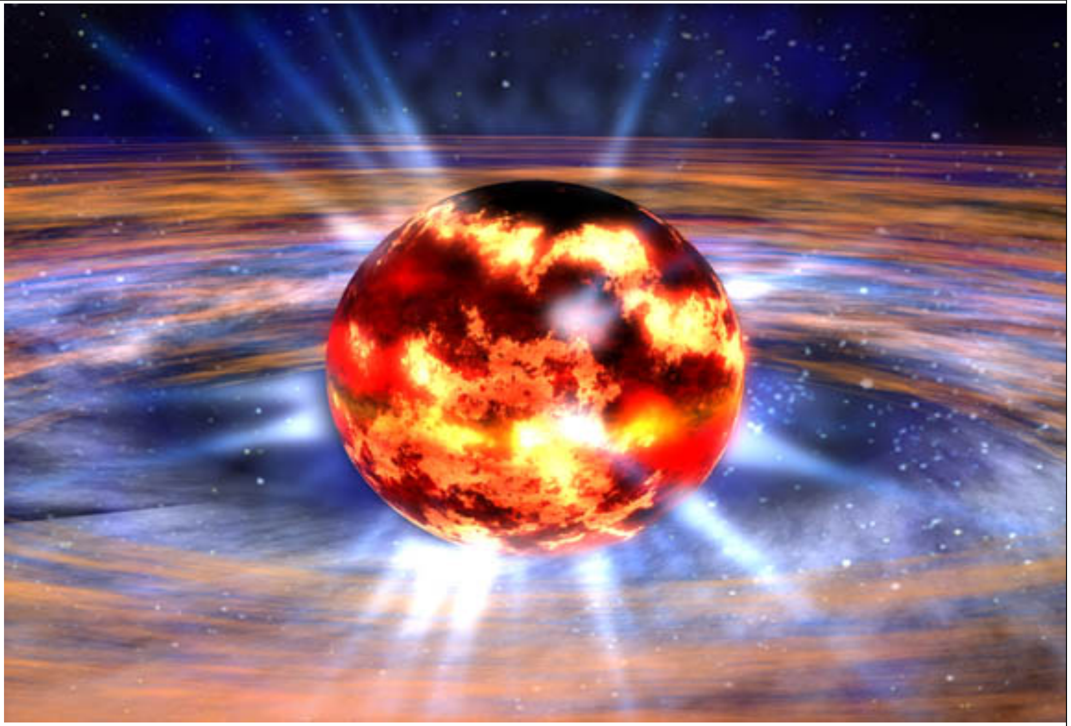
Analytic Work

Jiunn-Wei Chen	National Taiwan University
Donal O'Connell	IAS: Princeton
Paulo Bedaque	University of Maryland

Introduction

Why study nuclear interactions with lattice QCD?

much intrinsically interesting nuclear physics which is difficult/ impossible to access experimentally



for example, the nuclear equation of state in neutron stars

this requires an understanding of hyperon-nucleon interactions

we would like to connect our understanding of nuclear physics to the fundamental theory of QCD

2-Hadron Scattering on the Lattice

Minkowski vs Euclidean

In Minkowski space, scattering is performed by measuring the scattering phase shift of asymptotically separated, on-shell particles

In Euclidean space:
cuts moved off real axis
particles do not go on shell
➔ except at kinematic thresholds, can not reconstruct the Minkowski S-matrix elements

In FINITE Euclidean volume, particles can never escape eachother
the finite volume interaction energy can be related to the infinite volume scattering phase shift - Lüscher's Method

2-Hadron Scattering on the Lattice

two particle energy levels in a box:

for two identical particles:

$$\Delta E_2 = 2\sqrt{p^2 + m^2} - 2m$$

for non-interacting particles $\vec{p} = \frac{2\pi\vec{n}}{L}$

$$p\cot\delta = \frac{1}{\pi L} S\left(\frac{p^2 L^2}{16\pi^2}\right)$$

$$S(\eta) = \lim_{\Lambda\to\infty} \sum_{n<\Lambda} \frac{1}{n^2 - \eta} - 4\pi\Lambda$$

$$L/r >> 1 \qquad r \sim m_\pi^{-1}$$

$$\longrightarrow m_\pi L > 4$$

for low momenta

$$p\cot\delta = \frac{1}{a} + \frac{1}{2} r p^2 + \dots$$

the shift in energy due to interactions allows one to calculate the infinite volume scattering parameters (up to non-universal exponentially suppressed volume corrections).

2-Mesons

Why calculate 2-meson interactions with lattice QCD?

Scattering is *cool*

In particular, the interaction of two pseudo-Goldstone mesons is highly constrained by chiral dynamics

this allows for good check of the method

but more than that - can make precision predictions of meson-meson scattering parameters

for $\mathcal{I} = 2\ \pi\pi$ scattering, can make 1% predictions

clean system to study

SU(2) chiral dynamics

SU(3) symmetry/breaking in the meson sector

2-Mesons $\mathcal{I} = 2\ \pi\pi$

Coarse MILC ($b \sim 0.125$ fm)	Dimensions $L \times T \times L_5$	L $[fm]$	m_π [MeV]	m_K [MeV]	$N_{cfg} \times N_{source}$
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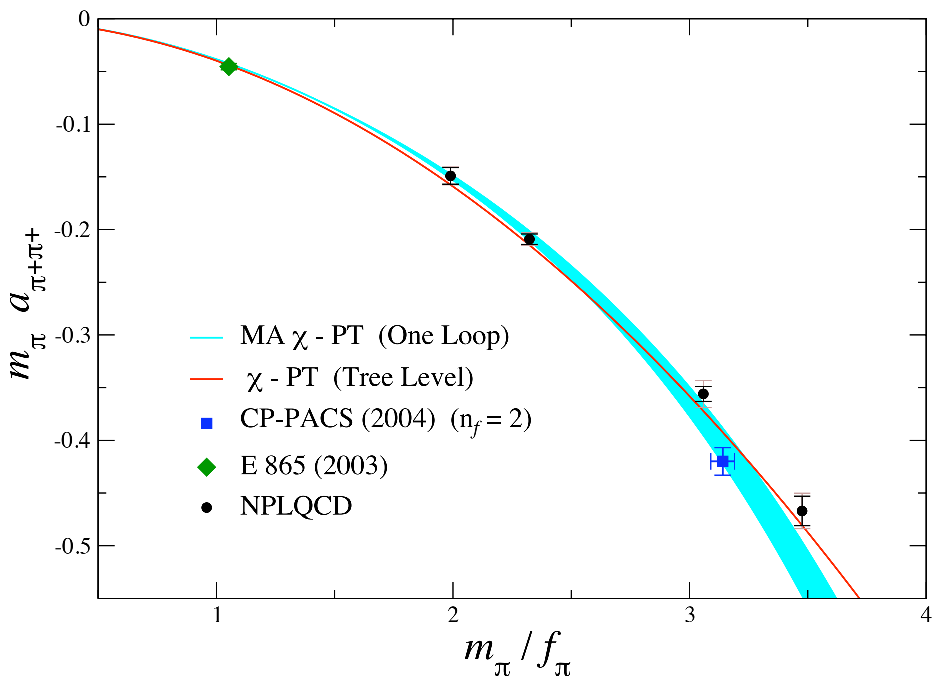
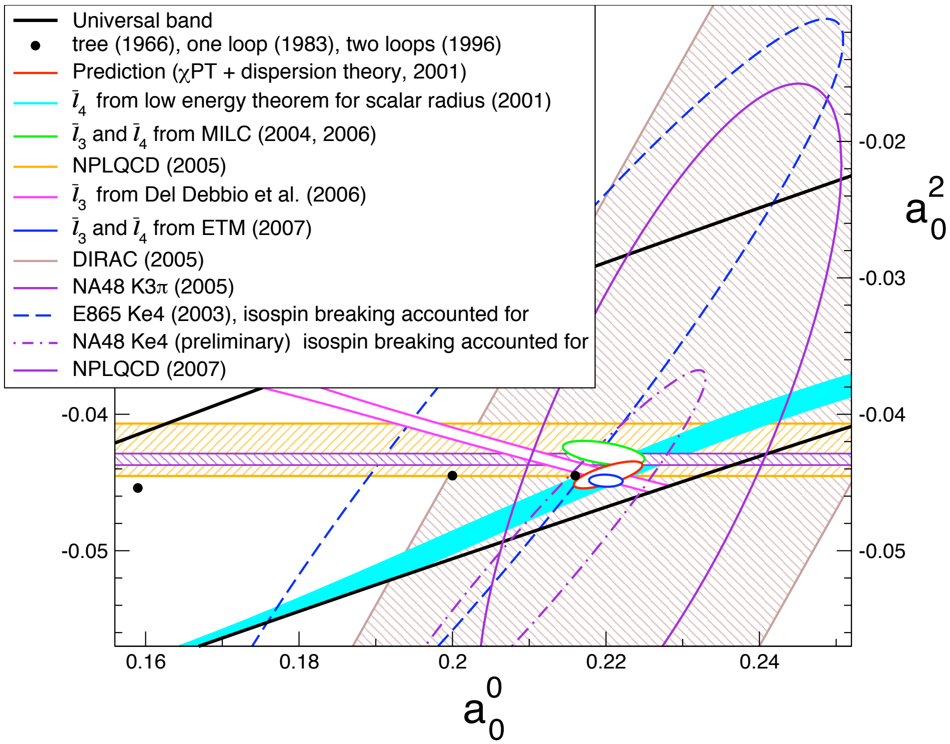
2-Mesons $\mathcal{I} = 2\ \pi\pi$

NPLQCD PRD 77 (2008)





$$m_\pi a_{\pi\pi}^{I=2} = -0.04330 \pm 0.00042$$ **2007**

$$m_\pi a_{\pi\pi}^{I=2}(LO) = -0.04438$$

2-Mesons $\mathcal{I} = 2\ \pi\pi$

NPLQCD PRD 77 (2008)

TABLE I: Calculated $I = 2\ \pi\pi$ scattering lengths and details of all uncertainties.

Quantity	$m_l = 0.007$	$m_l = 0.010$	$m_l = 0.020$	$m_l = 0.030$
m_π / f_π	1.990(11)(14)	2.323(6)(3)	3.059(5)(10)	3.476(10)(6)
$m_\pi a_{\pi\pi}^{I=2}$	-0.1458(78)(25)(14)	-0.2061(49)(17)(20)	-0.3540(68)(89)(16)	-0.465(14)(06)(05)
Δ_{MA}	0.0033(3)	0.0030(4)	0.0023(10)	0.0018(16)
Δ_{FV}	± 0.0055	± 0.0022	± 0.003	± 0.0001
$\Delta_{m_{res}}$	± 0.0032	± 0.0035	± 0.0036	± 0.0032
$m_\pi a_{\pi\pi}^{I=2}\, m_\pi r_{\pi\pi}^{I=2}\, \frac{\mathbf{p}^2}{2m_\pi^2}$	0.0004	0.0007	0.0014	0.0018

Can address all sources of systematic error (except for rooting of staggered action)

- Mixed Action Extrapolation formula (lattice spacing corrections)

  Chen, O'Connell, AWL PRD 75, 2007

- Exponential Corrections to Lüscher's formula (finite volume corrections)

  Bedaque, Sato, AWL PRD 73, 2006

- Residual chiral symmetry breaking from the domain-wall action
- Effective Range corrections

$$m_\pi a_{\pi\pi}^{I=2} = -0.04330 \pm 0.00042$$

Beane et al (NPLQCD) PRD 77 (2008)

For pion mass and decay constant, it is found that one-loop formulae get correct order of magnitude FV corrections, but two-loop formulae are needed for accurate corrections. G. Colangelo, S. Durr, C. Haefeli NPB 721 (2005)

2-Mesons $\mathcal{I} = 2\ \pi\pi$

Why are the lattice spacing corrections so small? The good chiral properties of the domain-wall valence quarks have a dramatic effect on the extrapolation formula to one-loop.

$$m_\pi a_{\pi\pi}^{I=2} = -\frac{m_{uu}^2}{8\pi f^2}\left\{1 + \frac{m_{uu}^2}{(4\pi f)^2}\left[4\ln\left(\frac{m_{uu}^2}{\mu^2}\right) + 4\frac{\tilde{m}_{ju}^2}{m_{uu}^2}\ln\left(\frac{\tilde{m}_{ju}^2}{\mu^2}\right) + l'_{\pi\pi}(\mu)\right.\right.$$
$$\left. - \frac{\tilde{\Delta}_{PQ}^2}{m_{uu}^2}\left[\ln\left(\frac{m_{uu}^2}{\mu^2}\right)\right] - \frac{\tilde{\Delta}_{PQ}^4}{6m_{uu}^4}\right]$$
$$\left. + \frac{\tilde{\Delta}_{PQ}^2}{(4\pi f)^2} l'_{PQ}(\mu) + \frac{b^2}{(4\pi f)^2} l'_{b^2}(\mu)\right\}$$

$$\tilde{\Delta}_{PQ}^2 = m_{jj}^2 + \Delta_{sea}(b) - m_{uu}^2$$
$$\tilde{m}_{ju}^2 = B_0(m_u + m_j) + b^2\Delta_{Mix}$$

Every sickness expected is apparent:

partial quenching $(\tilde{\Delta}_{PQ})$ lattice discretization effects (b)

2-Mesons $\mathcal{I} = 2\ \pi\pi$

lattice-physical parameters (mass and decay constant measured directly from correlators) the scattering length is given by

$$m_\pi a_{\pi\pi}^{I=2} = -\frac{m_\pi^2}{8\pi f_\pi^2}\left\{1 + \frac{m_\pi^2}{(4\pi f_\pi)^2}\left[3\ln\left(\frac{m_\pi^2}{\mu^2}\right) - 1 - l_{\pi\pi}^{I=2}(\mu)\right]\right\}$$

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The explicit dependence on the lattice spacing has exactly cancelled - up to a calculable effect from the hairpin interactions!!!

This is independent of the type of sea-quarks

Chen, O'Connell, Van de Water, AWL PRD 73 (2006)

Chen, O'Connell, AWL PRD 75 (2007)

Chen, O'Connell, AWL JHEP 0904 (2009)

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SU(3): chiral symmetry dictates that any strange-quark mass dependence at NLO must be of the form $m_\pi^2 m_K^2$

→ there can not be any (local) strange-quark mass dependence in the on-shell renormalized scattering length in SU(3)

all strange (sea) quark mass dependence is renormalized in the on-shell renormalized values of m_π and f_π

For more details on mixed action EFT - see talk by Jack Laiho, Thur, 15:35

2-Mesons $\mathcal{I} = 2\ \pi\pi$

Bedaque, Sato, AWL   PRD 73, 2006

(exponential) finite volume corrections

$$\Delta(p\cot\delta_{\pi\pi}^{I=2}) = 8\pi\, m_\pi \left[\frac{\partial}{\partial m_\pi^2}\, i\Delta\mathcal{I}(m_\pi) + 2i\Delta\mathcal{J}_{exp}(4m_\pi^2)\right]$$

$$\Delta(ma_{\pi\pi}^{I=2}) = -\frac{m_\pi^2}{8\pi f_\pi^2}\left[\frac{m_\pi^2}{f_\pi^2}\frac{\partial}{\partial m_\pi^2} i\Delta\mathcal{I}(m_\pi) + \frac{2m_\pi^2}{f_\pi^2} i\Delta\mathcal{J}_{exp}(4m_\pi^2)\right]$$

$$i\Delta\mathcal{I}(m) = \int \frac{dq_0}{2\pi}\left[\frac{1}{L^3}\sum_{\vec{q}=\frac{2\pi\vec{n}}{L}} - \int\frac{d^3q}{(2\pi)^3}\right]\frac{i}{q^2-m^2}$$
$$= \frac{m}{4\pi^2 L}\sum_{\vec{n}\neq 0}\frac{1}{|\vec{n}|}K_1(|\vec{n}|mL).$$

$$i\Delta\mathcal{J}_{\rm exp}(4m^2) = \frac{1}{16\pi^2}\frac{1}{L\sqrt{m^2+p^2}}\sum_{\vec{n}\neq 0}\frac{1}{|\vec{n}|}\int_{-\infty}^{\infty} dy \frac{y\,{\rm Im}e^{i2\pi y|\vec{n}|}}{\sqrt{y^2+\frac{m^2L^2}{4\pi^2}}\left(\sqrt{y^2+\frac{m^2L^2}{4\pi^2}}+\sqrt{\frac{p^2L^2}{4\pi^2}+\frac{m^2L^2}{4\pi^2}}\right)}$$

$$\simeq -\frac{1}{16\pi}\sum_{\vec{n}\neq 0}\left[K_0(|\vec{n}|\,mL)\bar{L}_{-1}(|\vec{n}|\,mL) + K_1(|\vec{n}|\,mL)\bar{L}_0(|\vec{n}|\,mL) - \frac{1}{|\vec{n}|\,mL}\right], \quad \text{(A2)}$$

2-Mesons $\mathcal{I} = 2 \ \pi\pi$

domain-wall action at finite 5th dimension has residual chiral symmetry breaking

$$\begin{aligned}\bar{\mathcal{L}} &= 2B_0\,\bar{L}_4\,\mathrm{str}\left(\partial_\mu\Sigma\partial^\mu\Sigma^\dagger\right)\mathrm{str}\left(m_{res}\Sigma^\dagger + \Sigma m_{res}^\dagger\right)\\ &\quad +8B_0^2\,\bar{L}_6\,\mathrm{str}\left(m_q\Sigma^\dagger + \Sigma m_q^\dagger\right)\mathrm{str}\left(m_{res}\Sigma^\dagger + \Sigma m_{res}^\dagger\right) + \ldots\end{aligned}$$

naive dimensional analysis

A.V.Manohar and H.Georgi Nucl. Phys. B (1984)

$$\longrightarrow \qquad \Delta_{m_{res}}(m_\pi a_{\pi\pi}^{I=2}) = \frac{8\pi m_\pi^4}{(4\pi f_\pi)^4}\frac{m_{res}}{m_l}$$

2-Mesons $\mathcal{I} = 2\ \pi\pi$

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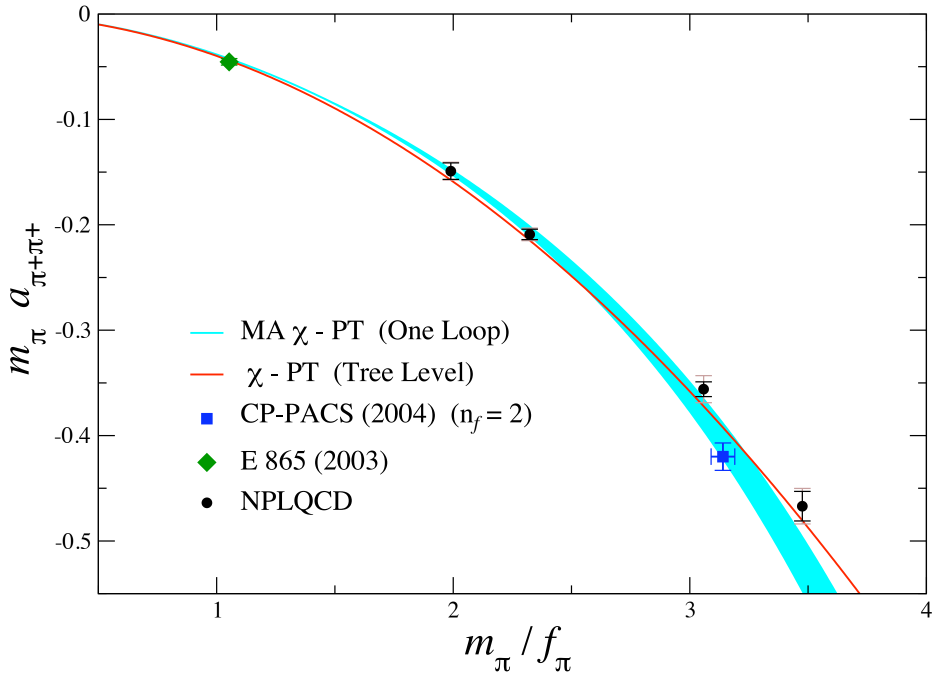
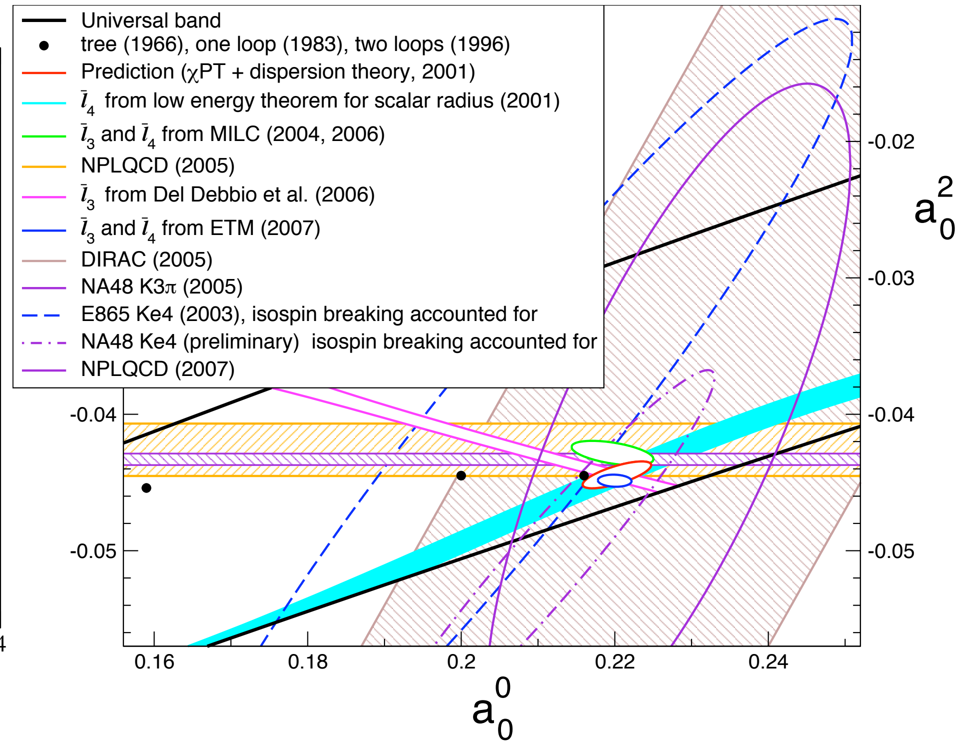
m_{res} is one of the dominant uncertainties in our calculation!?

2-Mesons $\mathcal{I} = 2\ \pi\pi$

NPLQCD PRD 77 (2008)





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SU(3) and SU(2) chiral extrapolation analysis in complete agreement

2-Mesons $\mathcal{I} = 2\ \pi\pi$

$$m_\pi a_{\pi\pi}^{I=2} = -\frac{m_\pi^2}{8\pi f_\pi^2}\left\{1 + \frac{m_\pi^2}{(4\pi f_\pi)^2}\left[3\ln\left(\frac{m_\pi^2}{\mu^2}\right) - 1 - l_{\pi\pi}^{I=2}(\mu)\right]\right\}$$

$l_{\pi\pi}^{I=2} = 5.7 \pm 1.3$ NPLQCD: PRD 77 (2007)

$$l_{\pi\pi}^{I=2} = 4(4\pi)^2\left(4l_1^r + 4l_2^r + l_3^r - l_4^r\right)$$

$$l_i^r(\mu) = \frac{\gamma_i}{32\pi^2}\left[\bar{l}_i + \ln\left(\frac{(m_\pi^{phys})^2}{\mu^2}\right)\right] \qquad \gamma_1 = \frac{1}{3}, \quad \gamma_2 = \frac{2}{3}, \quad \gamma_3 = -\frac{1}{2}, \quad \gamma_4 = 2$$

$$\longrightarrow \quad l_{\pi\pi}^{I=2}(\mu) = \frac{8}{3}\bar{l}_1 + \frac{16}{3}\bar{l}_2 - \bar{l}_3 - 4\bar{l}_4 + 3\ln\left(\frac{(m_\pi^{phys})^2}{\mu^2}\right)$$

combined phenomenology and lattice QCD determination of $\bar{l}_i$

$$-1.0 \le \bar{l}_1 \le 0.2 \qquad 4.2 \le \bar{l}_2 \le 4.4 \qquad 3.1 \le \bar{l}_3 \le 3.5 \qquad 4.0 \le \bar{l}_4 \le 4.2$$

$$\longrightarrow \quad 0.6 \le l_{\pi\pi}^{I=2}(f_\pi) \le 3.8 \qquad (5.7 \pm 1.3)$$

2-Mesons $\mathcal{I} = 2\ \pi\pi$

Coarse MILC ($b \sim 0.125$ fm)	Dimensions $L \times T \times L_5$	L $[fm]$	m_π [MeV]	m_K [MeV]	$N_{cfg} \times N_{source}$
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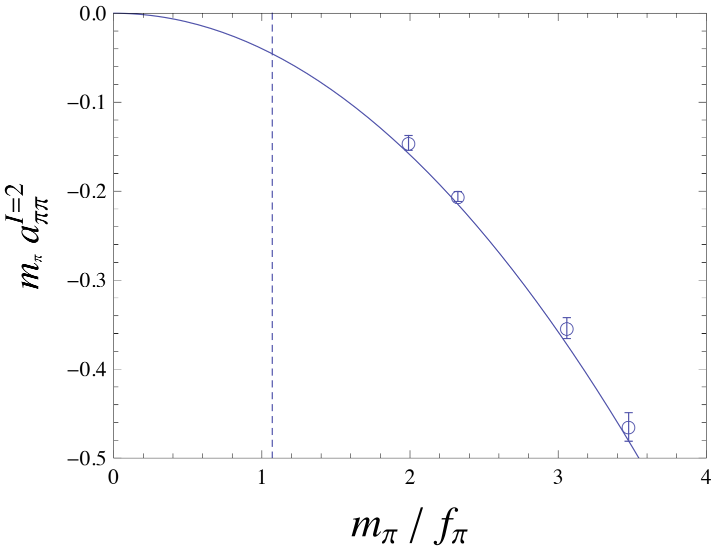
2064f21b676m010m050	$20^3 \times 32 \times 16$	2.5	350	595	$768 \times 24 = 18432$
2064f21b679m020m050	$20^3 \times 32 \times 16$	2.5	490	640	$486 \times 24 = 11664$
2064f21b681m030m050	$20^3 \times 32 \times 16$	2.5	590	675	$564 \times 24 = 13536$
Fine MILC ($b \sim 0.09$ fm)					
4096f2b7045m0062m031	$40^3 \times 96 \times 40$	2.5	230	539	$109 \times 1 = 109$
4096f2b7045m0062m031	$40^3 \times 96 \times 12$	2.5	234	540	$109 \times 1 = 109$
2896f2b709m0062m031	$28^3 \times 96 \times 12$	2.5	320	560	$1001 \times 7 = 7007$
2896f2b711m0124m031	$28^3 \times 96 \times 12$	2.5	446	578	$513 \times 3 = 1539$

New data set will allow us to address systematics more thoroughly

- two lattice spacings
- two volumes
- two $m_{res}(L_5)$
- can now perform 2-loop chiral extrapolation

2-Mesons $\mathcal{I} = 2\ \pi\pi$



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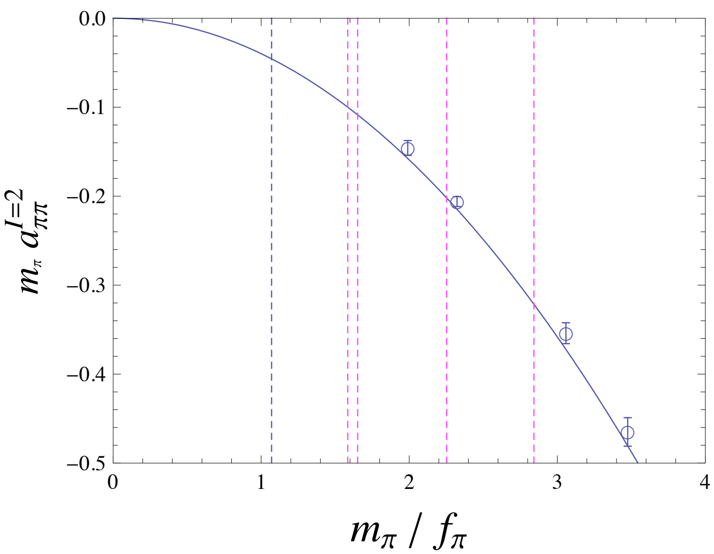
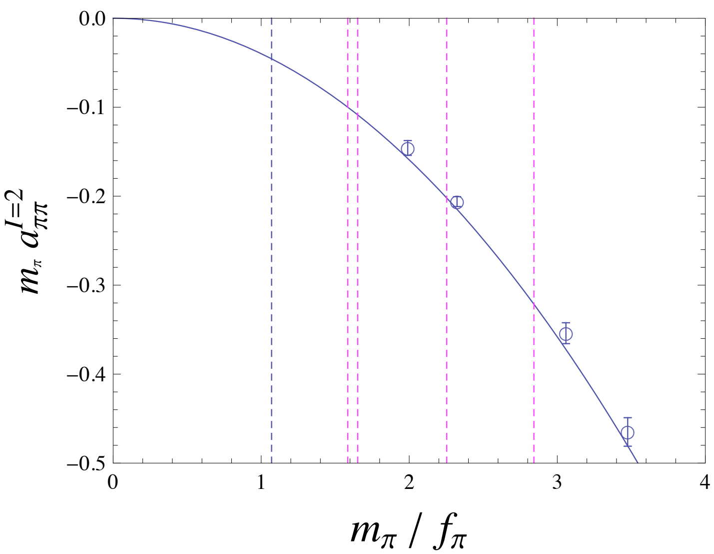
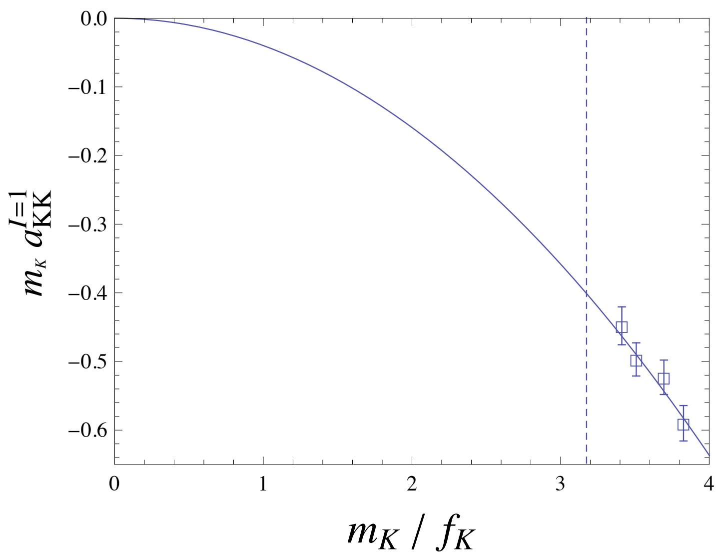


2-Mesons $\mathcal{I} = 2\ \pi\pi$





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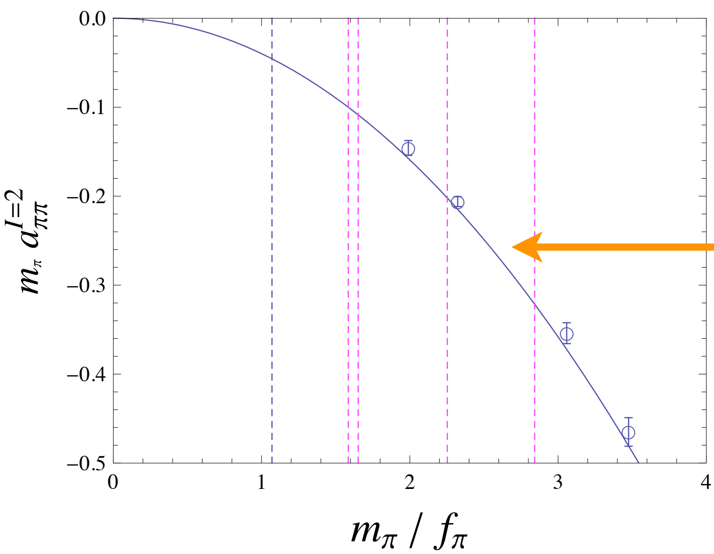
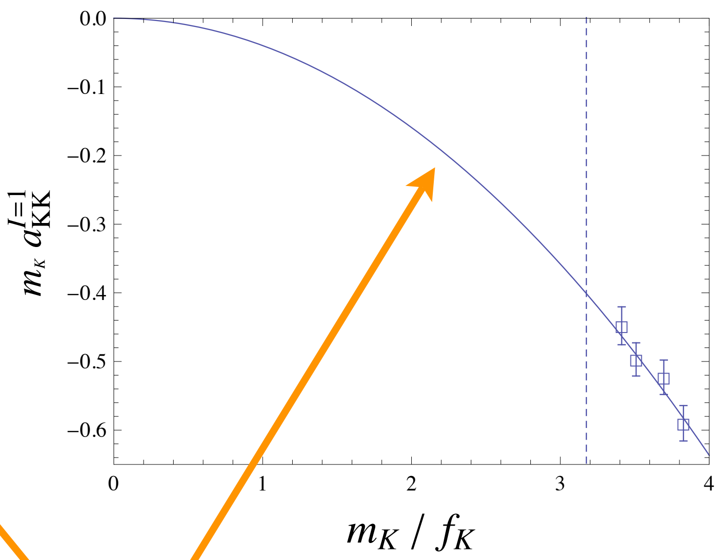
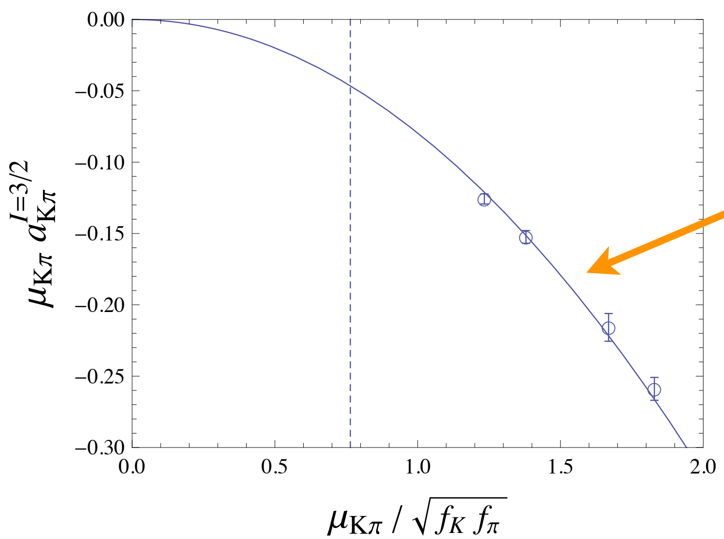


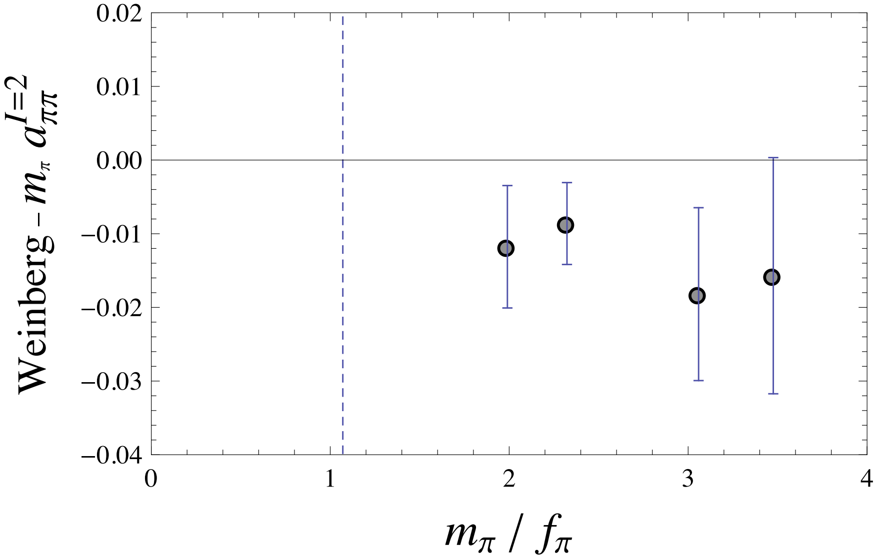


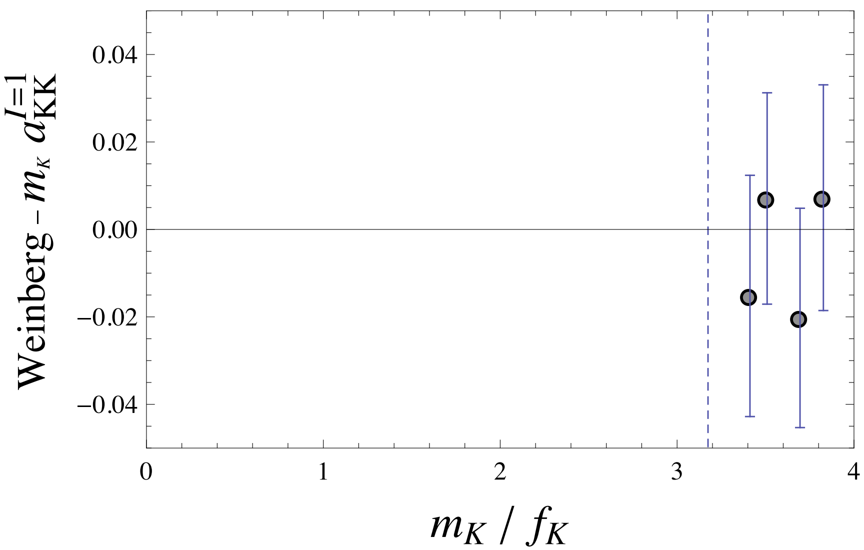


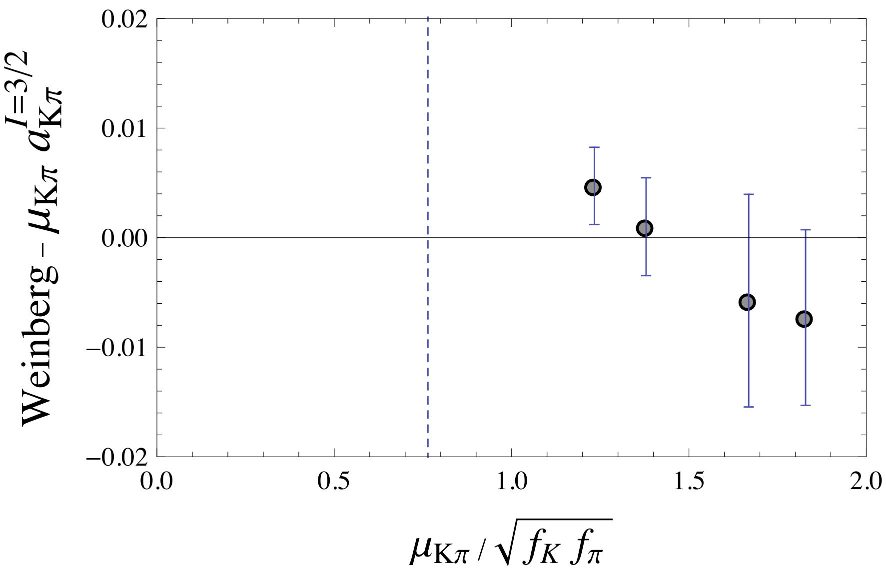
Weinberg's LO prediction

Where are the quantum loop corrections?

2-Mesons $\mathcal{I}=2\ \pi\pi \quad \mathcal{I}=1\ KK \quad \mathcal{I}=3/2\ K\pi$







2-Mesons $\mathcal{I}=2\ \pi\pi \quad \mathcal{I}=1\ KK \quad \mathcal{I}=3/2\ K\pi \quad f_K/f_\pi$

Counter Terms

Chen, O'Connell, AWL   PRD 75 (2007)

$$m_\pi a_{\pi\pi}^{I=2} : \quad \frac{4m_\pi^4}{\pi f_\pi^4} L_{\pi\pi}^{I=2}$$

$$m_K a_{KK}^{I=1} : \quad \frac{4m_K^4}{\pi f_K^4} L_{KK}^{I=1}$$

$$\mu_{K\pi} a_{K\pi}^{I=3/2} : \quad \frac{\mu_{K\pi}^2}{4\pi f_K f_\pi}\left[\frac{32 m_K m_\pi}{f_K f_\pi} L_{\pi\pi}^{I=2} - \frac{8(m_K - m_\pi)^2}{f_K f_\pi} L_5\right]$$

$$\mu_{\pi K} = \frac{m_\pi m_K}{m_\pi + m_K}$$

$$\frac{f_K}{f_\pi} : \quad \frac{8(m_K^2 - m_\pi^2)}{f_K f_\pi} L_5$$

$$L_{\pi\pi}^{I=2} = 2L_1 + 2L_2 + L_3 - 2L_4 - L_5 + 2L_6 + L_8$$

$$L_{KK}^{I=1} = L_{\pi\pi}^{I=2}$$

Excellent testing ground for SU(3) breaking!

meson baryon

Aaron Torok's Ph.D. Thesis work is on meson-baryon scattering

See Silas Beane's plenary talk - Thursday 11am

$n \geq 3$ mesons

multi-boson interaction energies in finite volume

$$\Delta E_n = \frac{4\pi\bar{a}}{ML^3}\,{}^nC_2\Big\{1-\left(\frac{\bar{a}}{\pi L}\right)\mathcal{I}+\left(\frac{\bar{a}}{\pi L}\right)^2\left[\mathcal{I}^2+(2n-5)\mathcal{J}\right]$$
$$-\left(\frac{\bar{a}}{\pi L}\right)^3\left[\mathcal{I}^3+(2n-7)\mathcal{I}J+(5n^2-41n+63)\mathcal{K}\right]$$
$$+\left(\frac{\bar{a}}{\pi L}\right)^4\Big[\mathcal{I}^4-6\mathcal{I}^2\mathcal{J}+(4+n-n^2)\mathcal{J}^2+4(27-15n+n^2)\mathcal{I}K$$
$$+(14n^3-227n^2+919n-1043)\mathcal{L}+16(n-2)(\mathcal{T}_0+n\mathcal{T}_1)\Big]\Big\}$$
$$+\,{}^nC_3\frac{\hat{\eta}_3^L}{L^6}+{}^nC_3\frac{6\pi\bar{a}^3}{M^3L^7}(n+3)\mathcal{I}+\mathcal{O}(L^-8)$$

$\mathcal{I},\mathcal{J},\mathcal{K},\mathcal{T}_0,\mathcal{T}_1$ known geometric constants

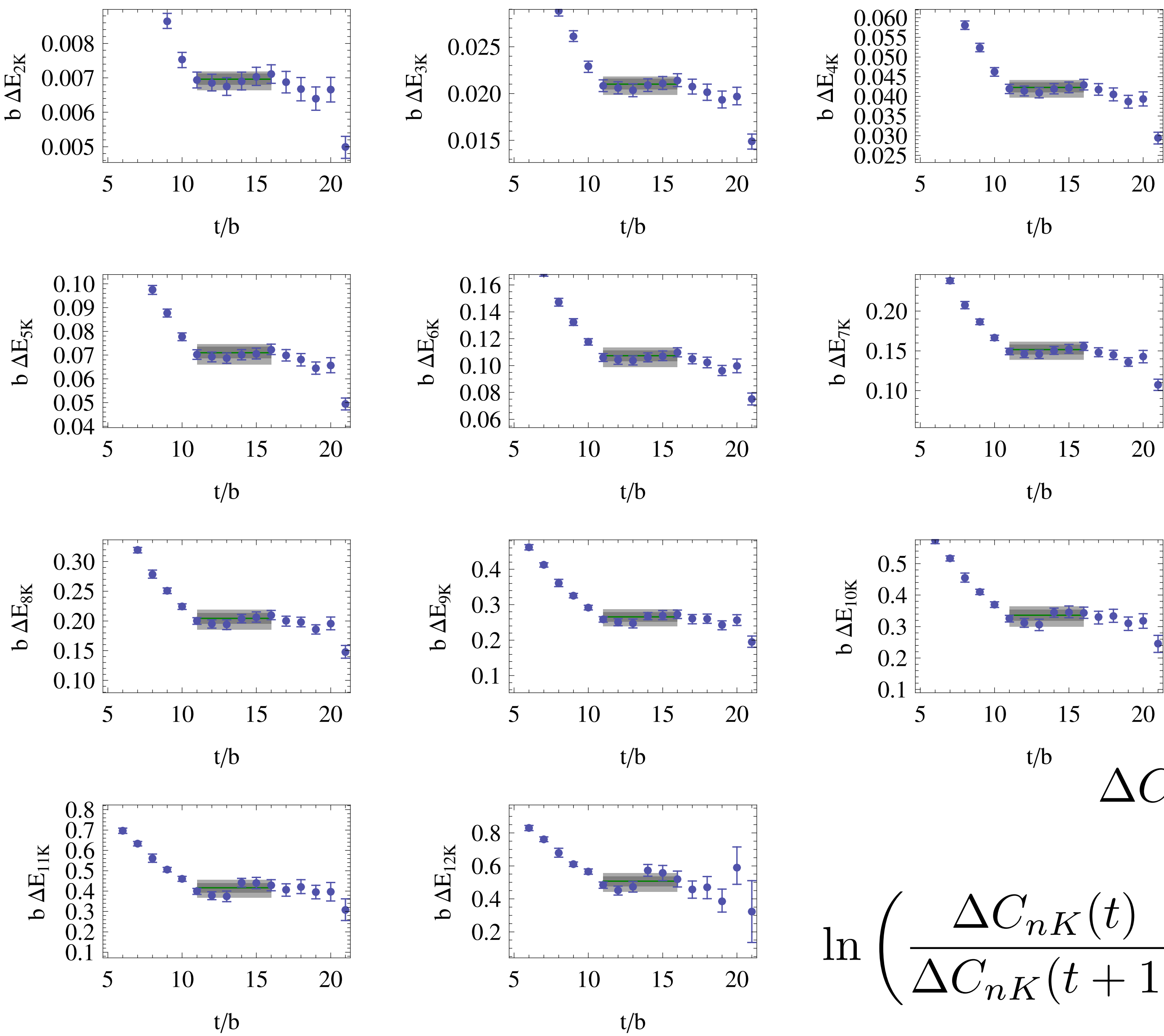
$$\bar{a}=a+\frac{2\pi}{L^3}a^3\,r \qquad\qquad \hat{\eta}_3^L=\bar{\eta}_3^L\left[1-\frac{6\bar{a}}{\pi L}\mathcal{I}\right]+\frac{72\pi\bar{a}^4\,r}{ML}\mathcal{I}$$

S. Beane, W. Detmold and M.J. Savage
PRD 76, 2007

W. Detmold and M.J. Savage
PRD 77, 2008

Multi-Kaon

NPLQCD PRD 78 (2008)

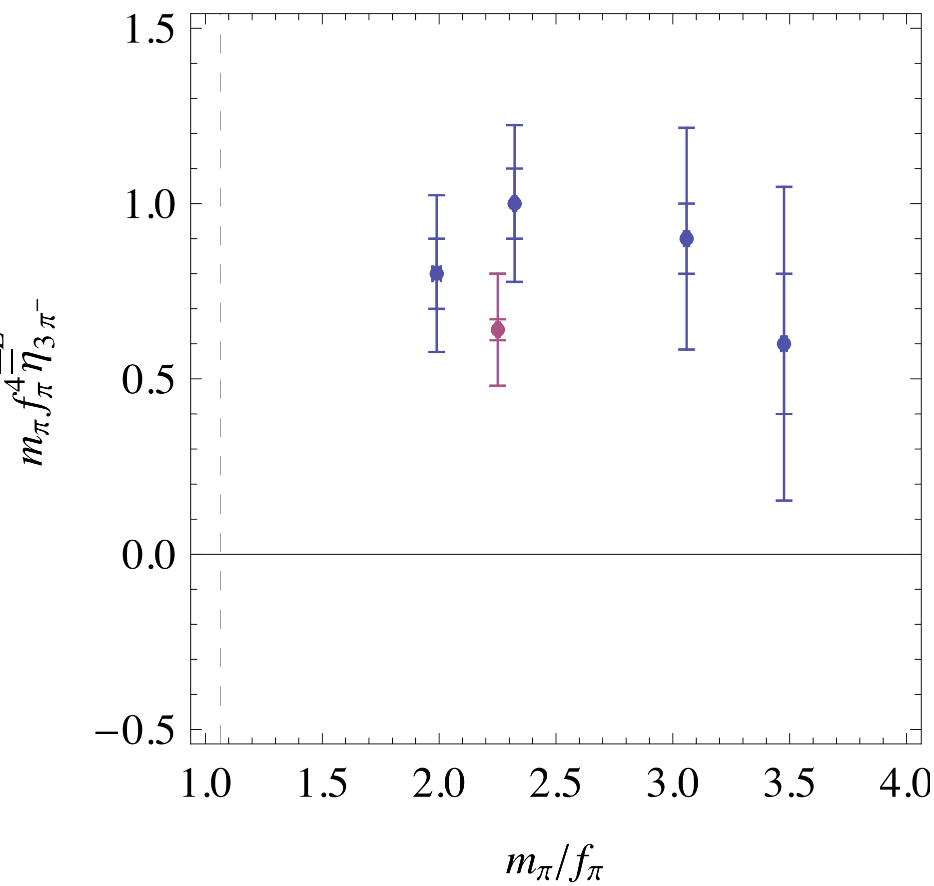


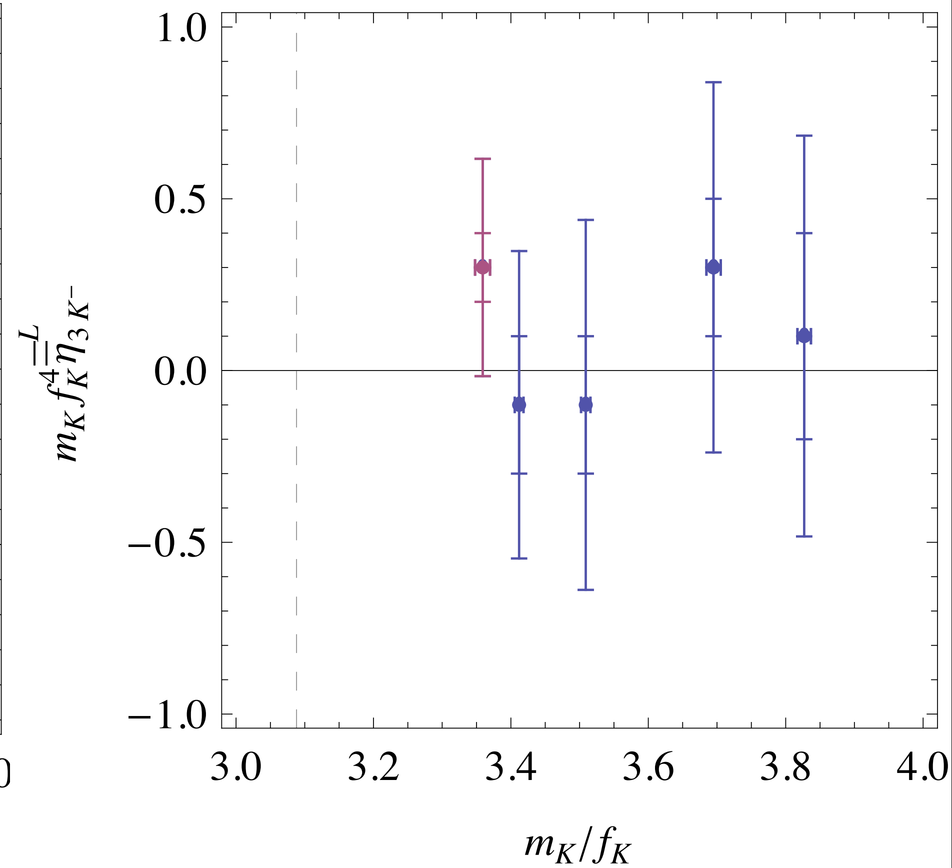
$$\Delta C_{nK}(t) \equiv \frac{C_{nK}(t)}{(C_K(t))^n}$$

$$\ln\left(\frac{\Delta C_{nK}(t)}{\Delta C_{nK}(t+1)}\right) \longrightarrow b\,\Delta E_{nK}$$

Multi-Kaon

NPLQCD PRD 78 (2008)





NPLQCD PRL 100, 2008

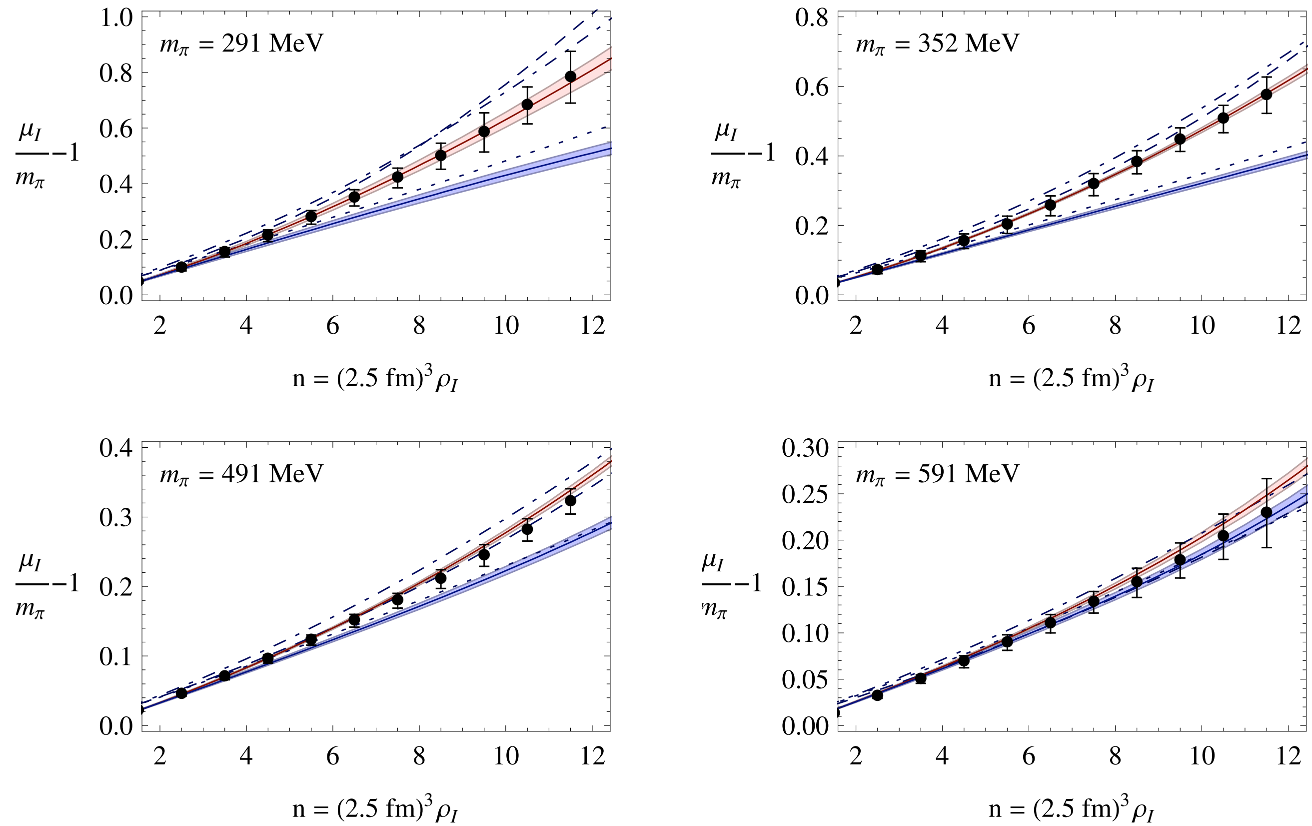
NPLQCD PRD 78 (2008)

NPLQCD PRD 78 (2008)

Multi-Kaon

NPLQCD PRD 78 (2008)

Pion condensate



$$\chi\text{−PT}: \quad \rho_I = \frac{1}{2} f_\pi^2 \mu_I \left(1 - \frac{m_\pi^4}{\mu_I^4}\right)$$

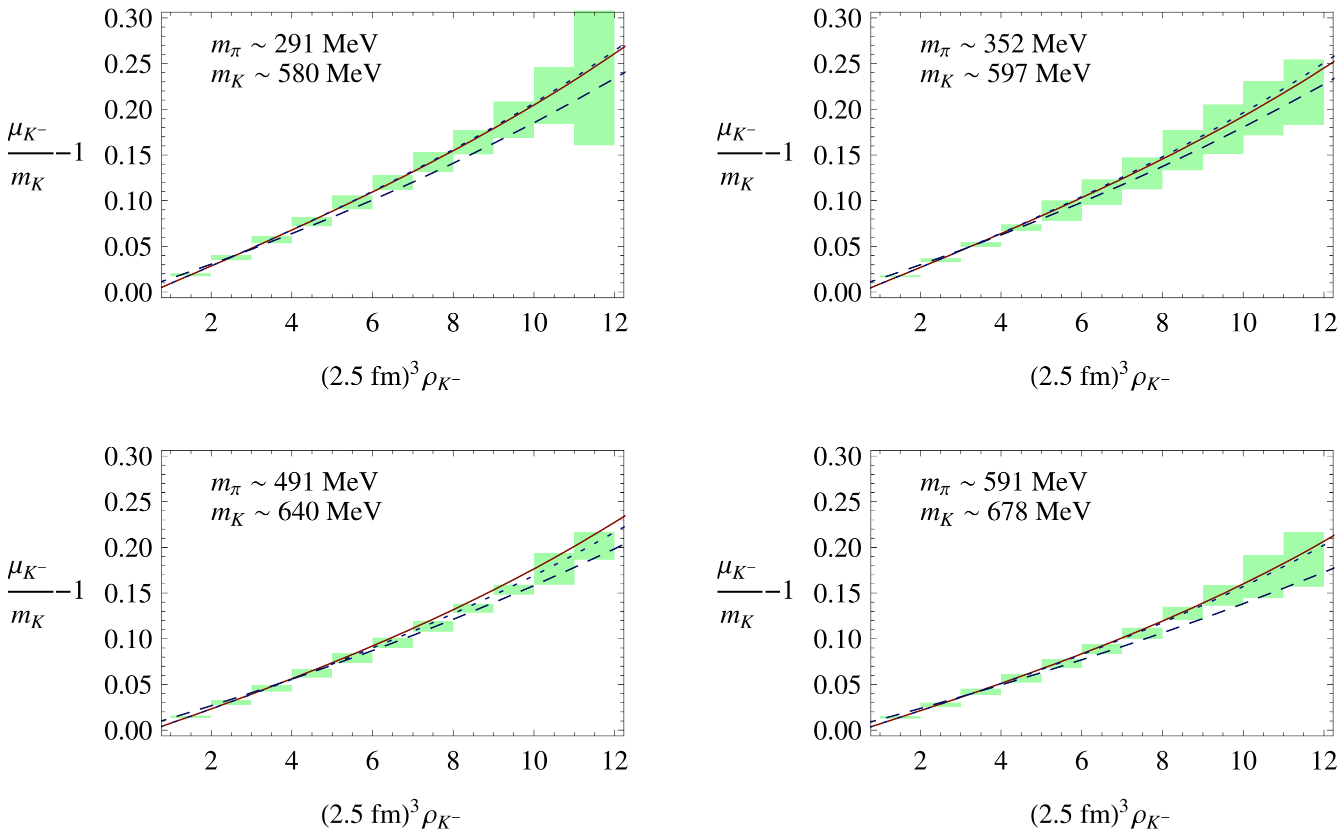
three body force is important!!

Silas Beane

Multi-Kaon

NPLQCD PRD 78 (2008)

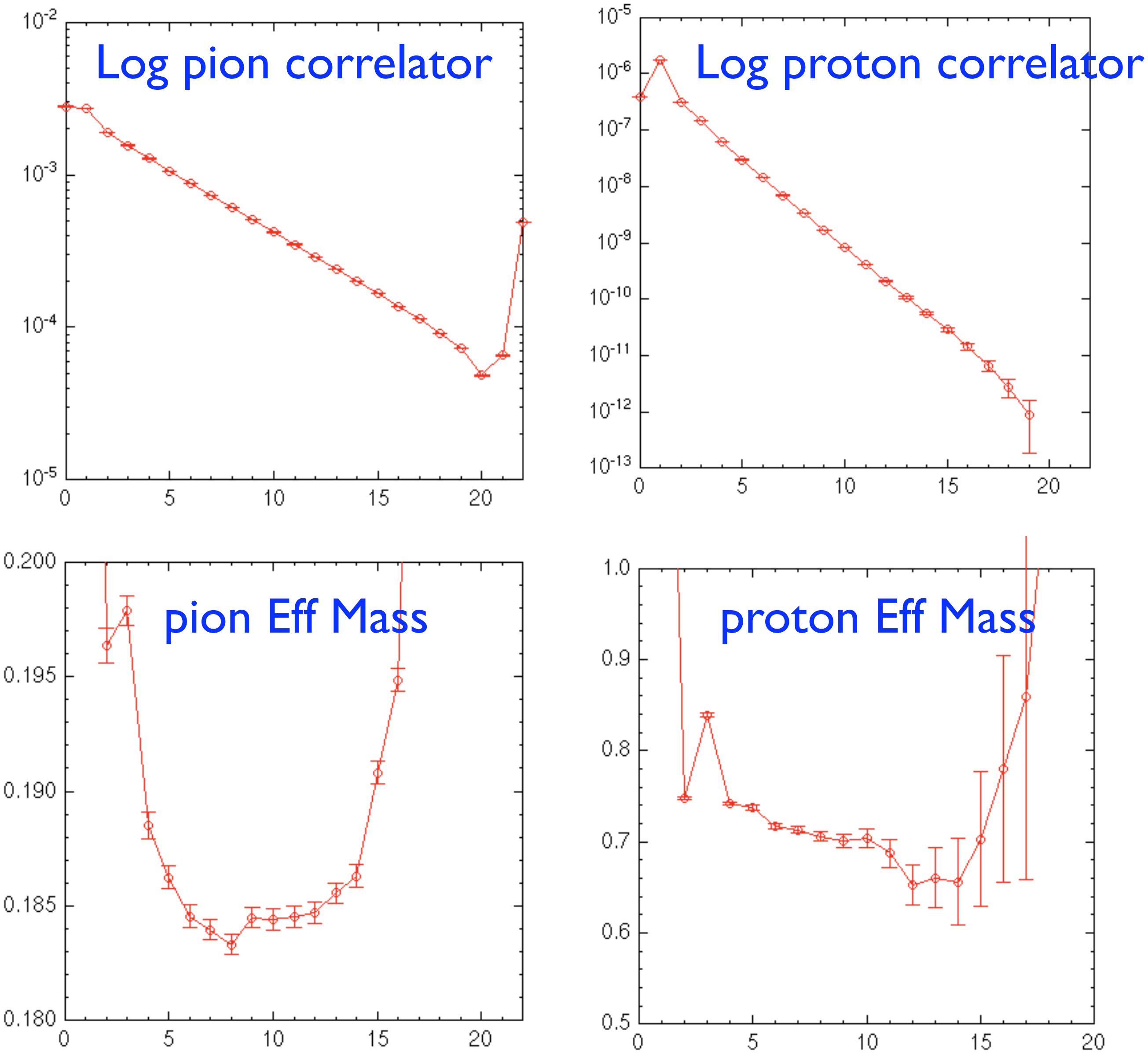
Kaon condensate   NPLQCD/arXiv:0807.1924



$$\chi\text{−PT}: \quad \rho_K = \frac{1}{2} f_K^2 \left(\mu_K - \frac{m_K^4}{\mu_K^3} \right)$$

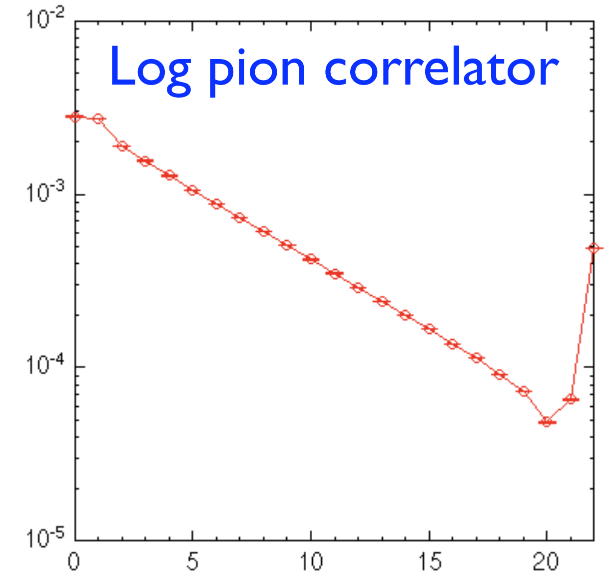
Silas Beane

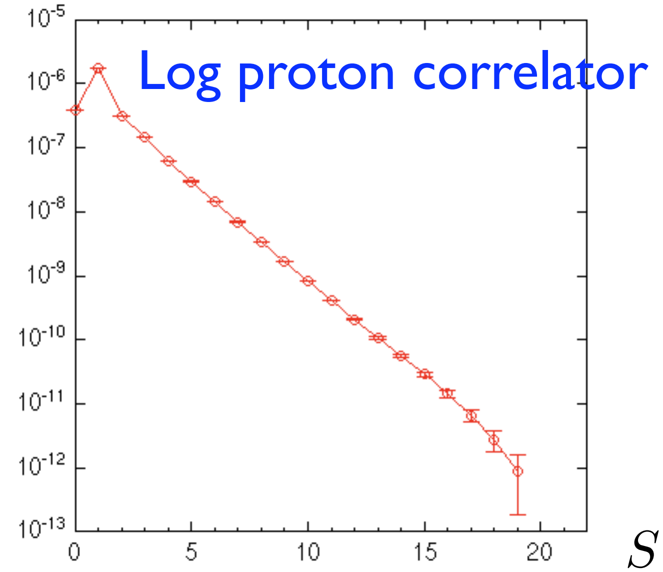
2-Baryons



30,000 propagators!

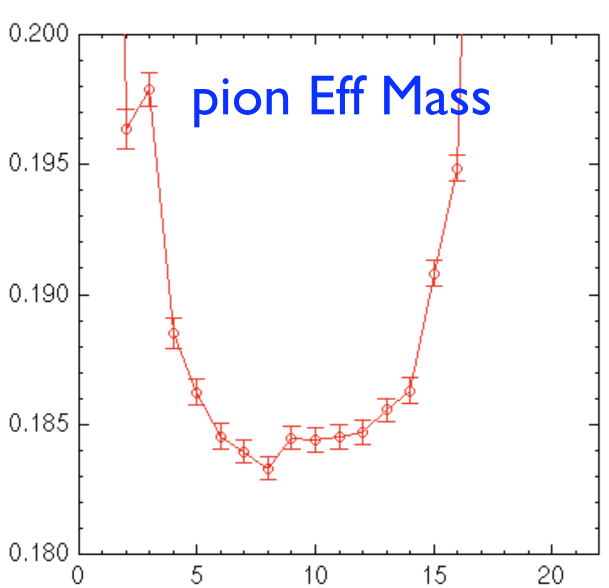
2-Baryons

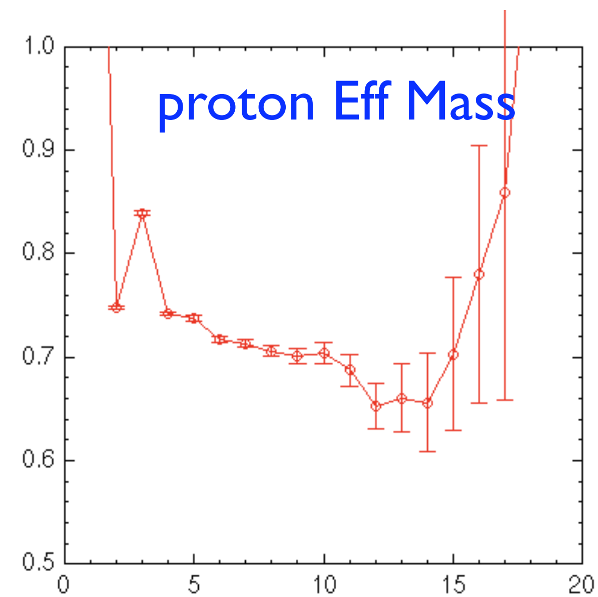




30,000 propagators!

$$\frac{S}{e} \sim \sqrt{N} e^{\{-(M_N - 3/2 m_\pi)t\}}$$

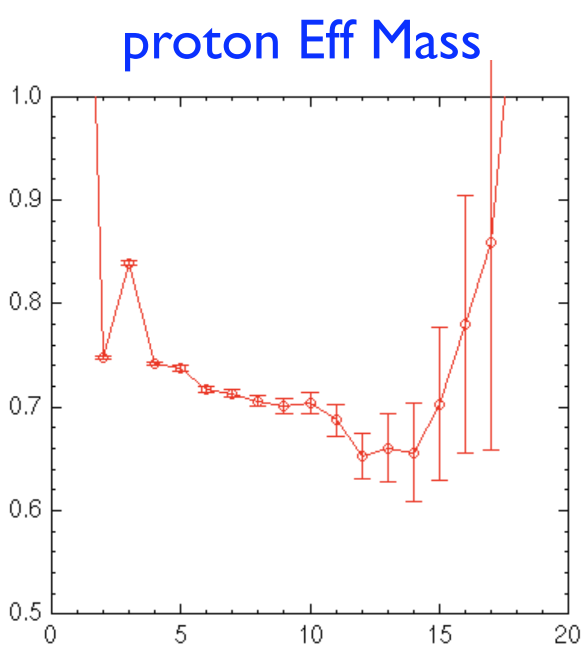


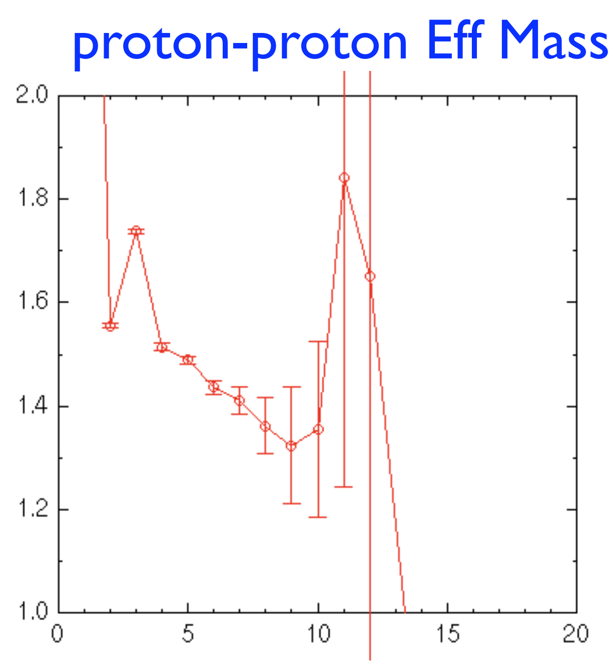


LePage

2-Baryons

30,000 propagators! $m_\pi \simeq 290$ MeV





$$\frac{S}{e} \sim \sqrt{N} e^{\{-(M_N - 3/2 m_\pi)t\}} \qquad \frac{S}{e} \sim \sqrt{N} e^{\{-(2M_N - 3m_\pi)t\}} \qquad \frac{S}{e} \sim \sqrt{N} e^{\{-A(M_N - 3/2 m_\pi)t\}}$$

need strategy to alleviate this signal to noise problem

- more statistics!
- modify boundary conditions to eliminate pion-zero mode
  - restless pions - parity-orbifold boundary conditions
    - still needs numerical implementation
- improved two-body interpolating operators

P.Bedaque and AWL
arXiv:0708.0207
arXiv:0811.2127

2-Baryons: High Statistics on Anisotropic Clover Lattices

we have switched our production to using the anisotropic clover lattices produced by R. Edwards et.al.

- clover propagators are ~10 times faster
- with EigCG inverter, we get an extra factor of ~7

A.Stathopoulos and K.Orginos
arXiv:0707.0131

In the last year, we have performed ~284 light/strange quark propagator calculations on each of 1194 configurations on the $20^3 \times 128$ anisotropic gauge ensembles

$m_\pi = 390\ \mathtt{MeV}$

$a_s/a_t = 3.5$

$$284 \times 1194 \simeq 340,000\ measurements!$$

This is an order of magnitude increase in our previous statistics

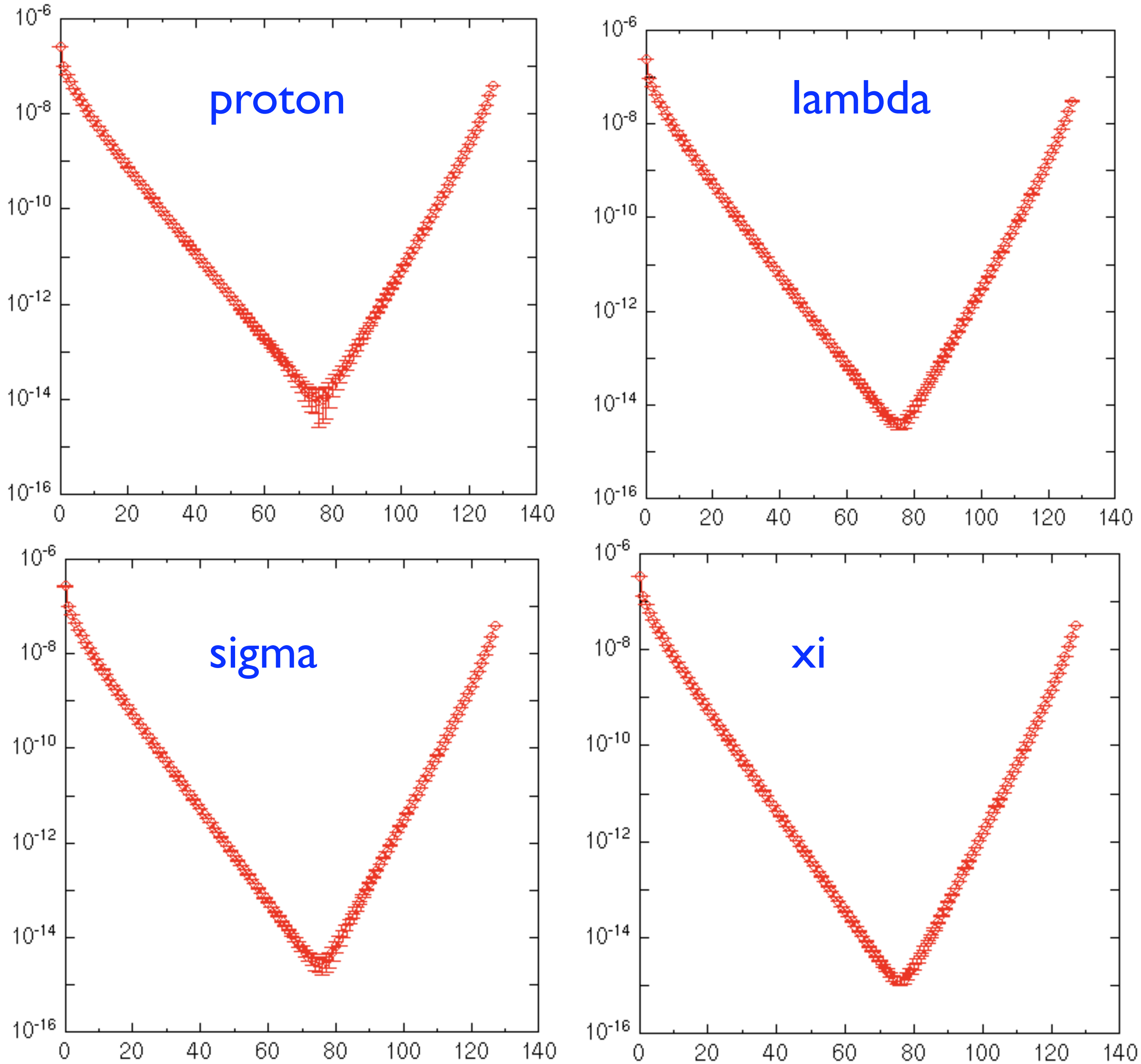
The principle goal was to perform a scaling study in the extraction of the hadron spectrum/ two-body energies as a function of

- number of sources per configuration
- number of configurations

NPLQCD
arXiv:0903.2990
arXiv:0905.0466

2-Baryons: High Stat. on Aniso Clover Lattices

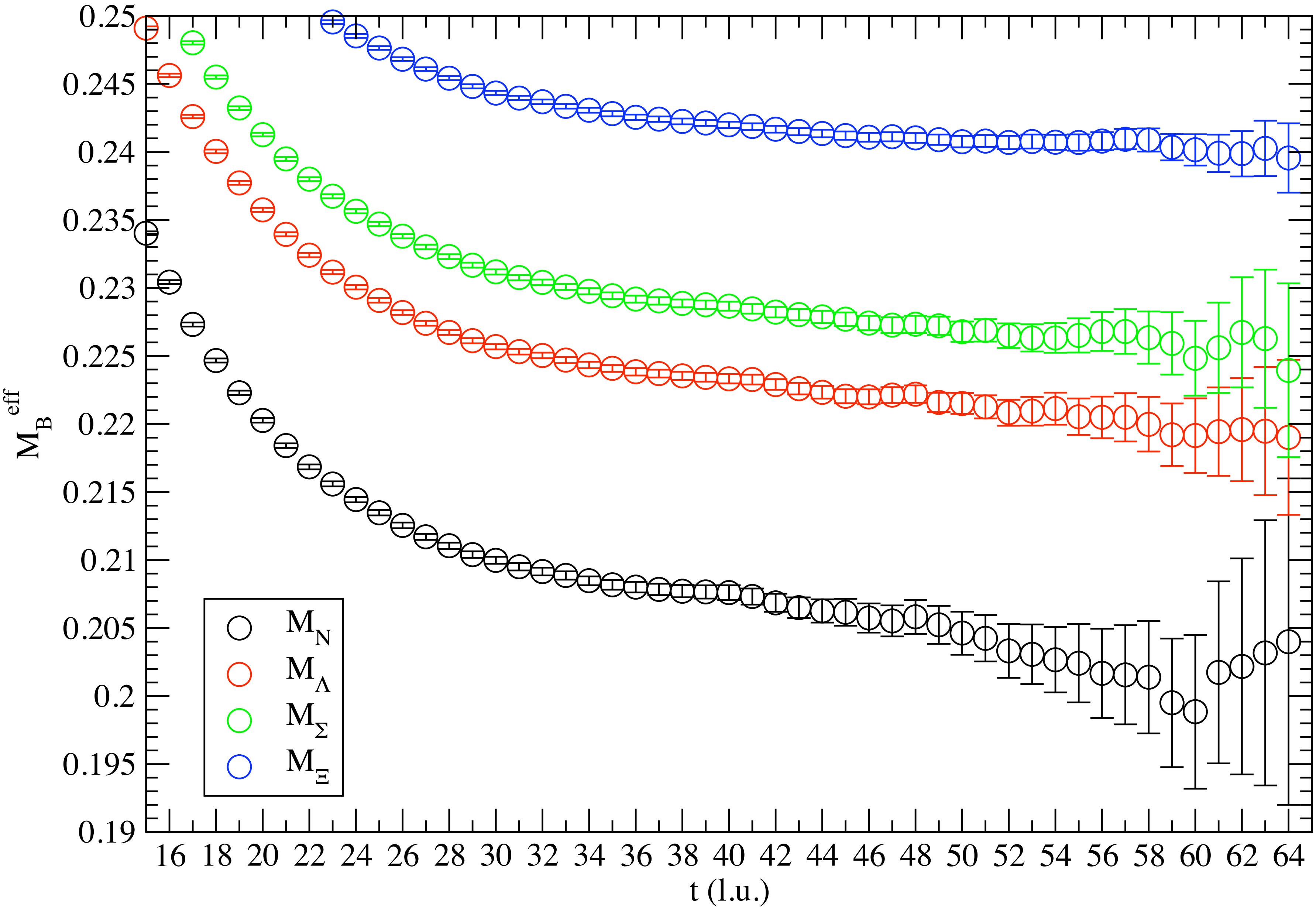
NPLQCD
arXiv:0903.2990
arXiv:0905.0466



quantitatively useful information from all time slices

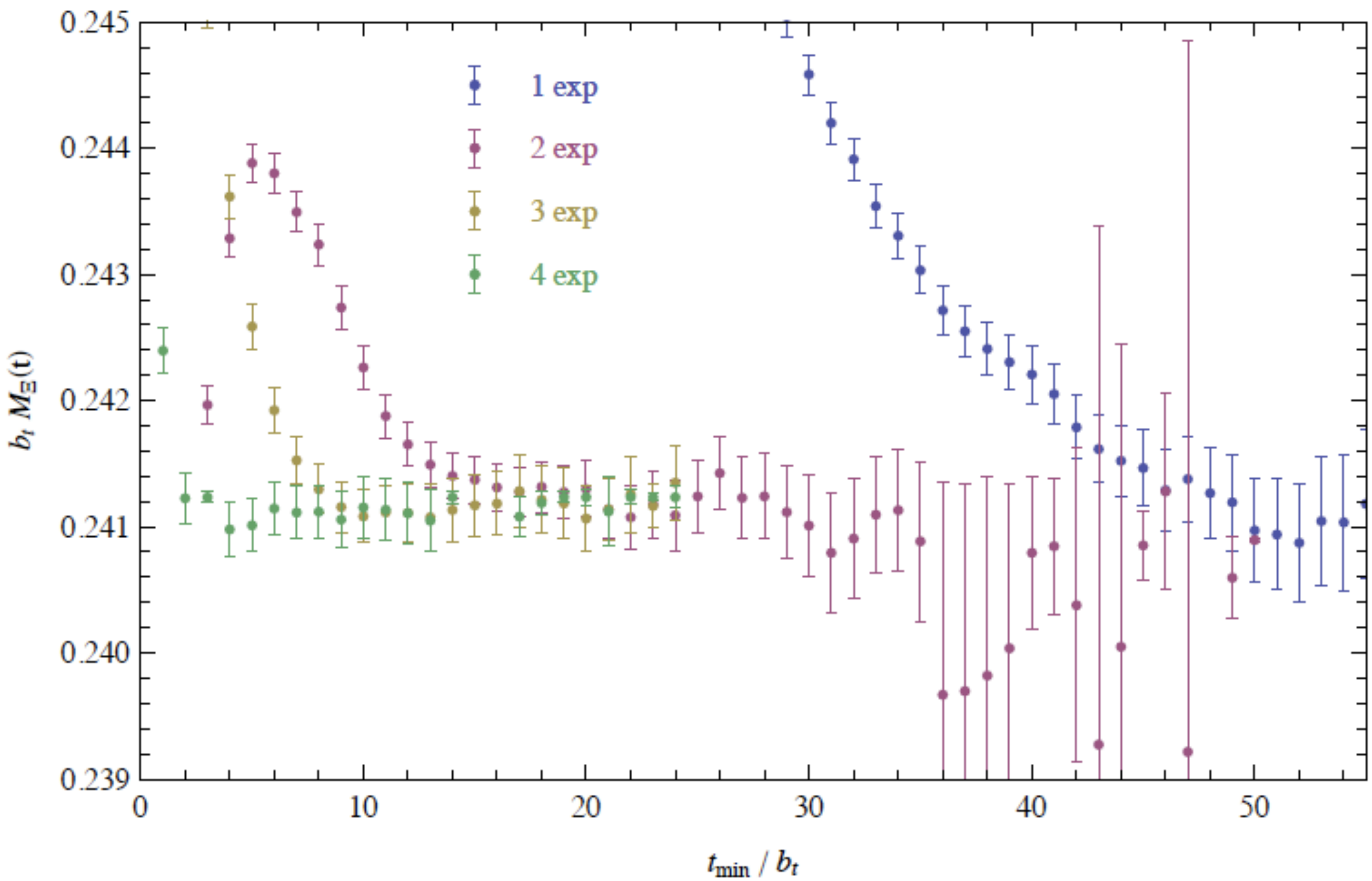
2-Baryons: High Stat. on Aniso Clover Lattices

NPLQCD
arXiv:0903.2990
arXiv:0905.0466



2-Baryons: High Stat. on Aniso Clover Lattices

NPLQCD
arXiv:0903.2990
arXiv:0905.0466



	Exponential Fitting				Matrix-Prony		
state	$b_t M$	range	χ^2/dof	Q	$b_t M$	range	χ^2/dof
N	0.20693(33)(07)	7–64	0.72	0.99	0.20682(32)(10)	11–40	1.50
Λ	0.22265(25)(16)	9–64	0.89	0.78	0.22255(28)(5)	10–47	1.21
Σ	0.22819(25)(07)	8–64	0.85	0.86	0.22811(28)(18)	12–47	0.77
Ξ	0.24112(21)(06)	7–64	0.84	0.87	0.24097(25)(3)	11–50	0.81

2-Baryons: High Stat. on Aniso Clover Lattices

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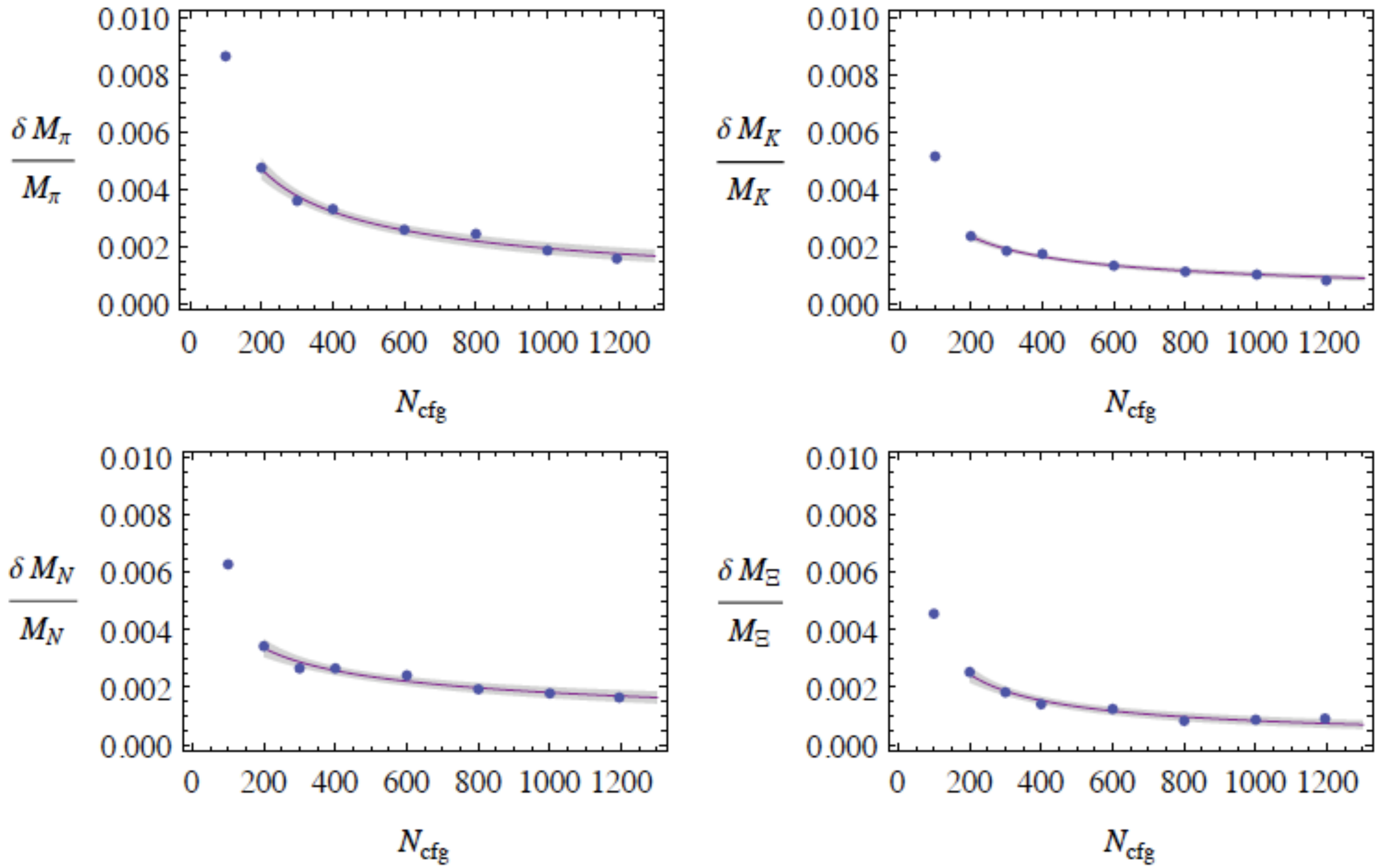
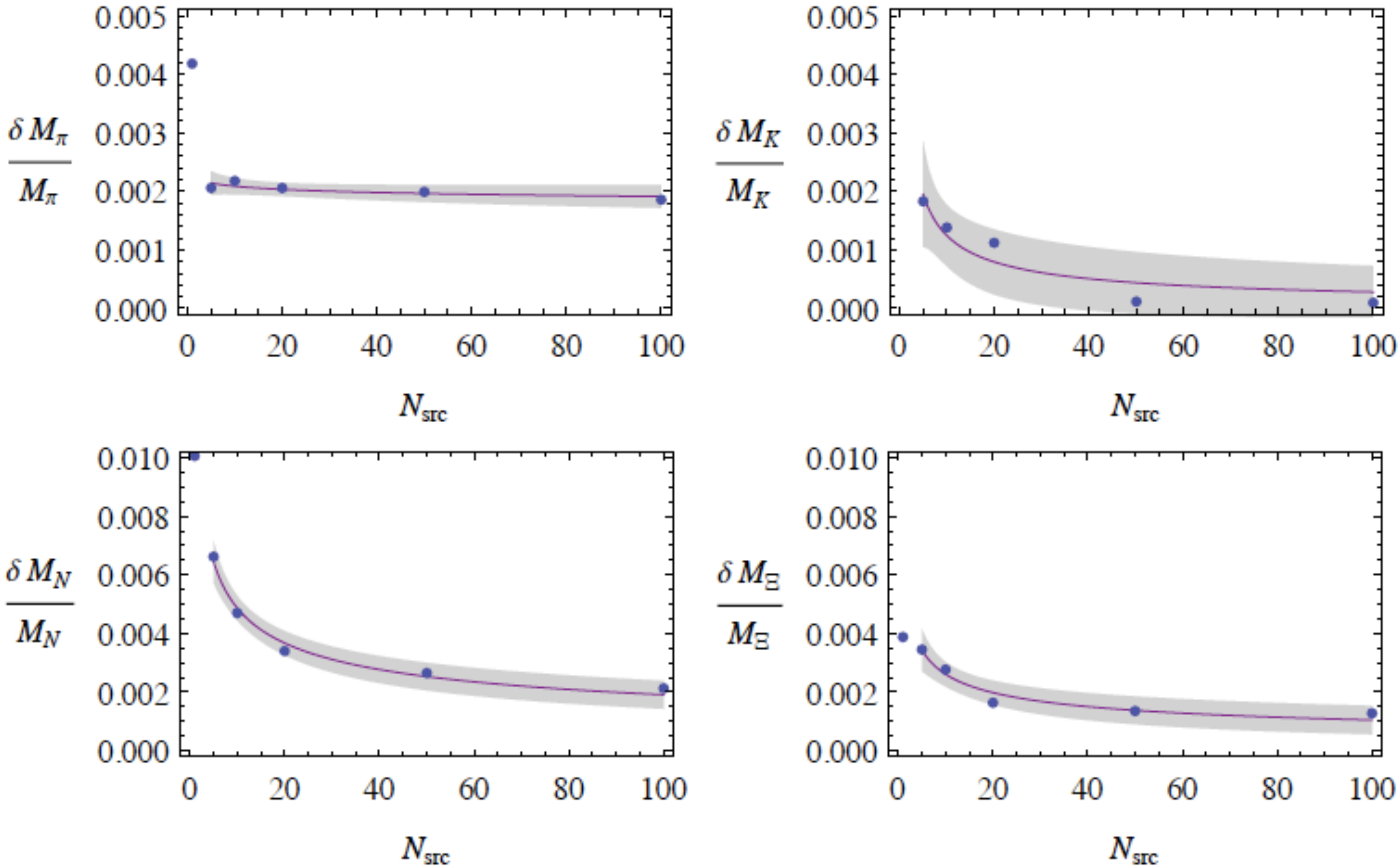


$$\frac{\delta M}{M} = A N_{cfg}^{b} \qquad b = \{-0.55(4), -0.51(3), -0.38(4), -0.67(6)\}$$
$$\{\pi, K, N, \Xi\}$$

2-Baryons: High Stat. on Aniso Clover Lattices

NPLQCD
arXiv:0903.2990
arXiv:0905.0466

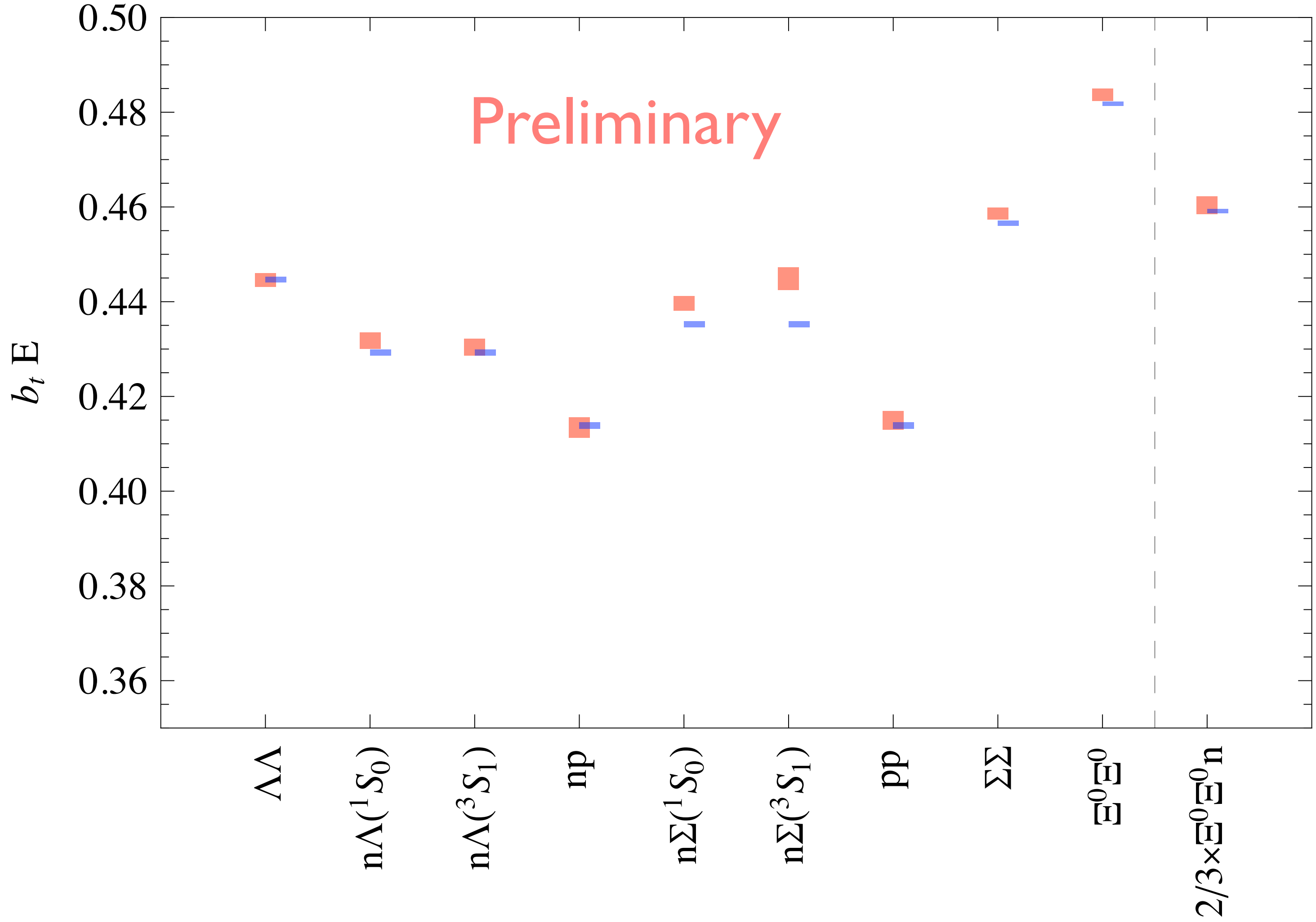


$$\frac{\delta M}{M} = A N_{src}^b \qquad b = \{-0.03(2), -0.65(19), -0.41(3), -0.40(6)\}$$

$$\{\pi, K, N, \Xi\}$$

2-Baryons: High Stat. on Aniso Clover Lattices

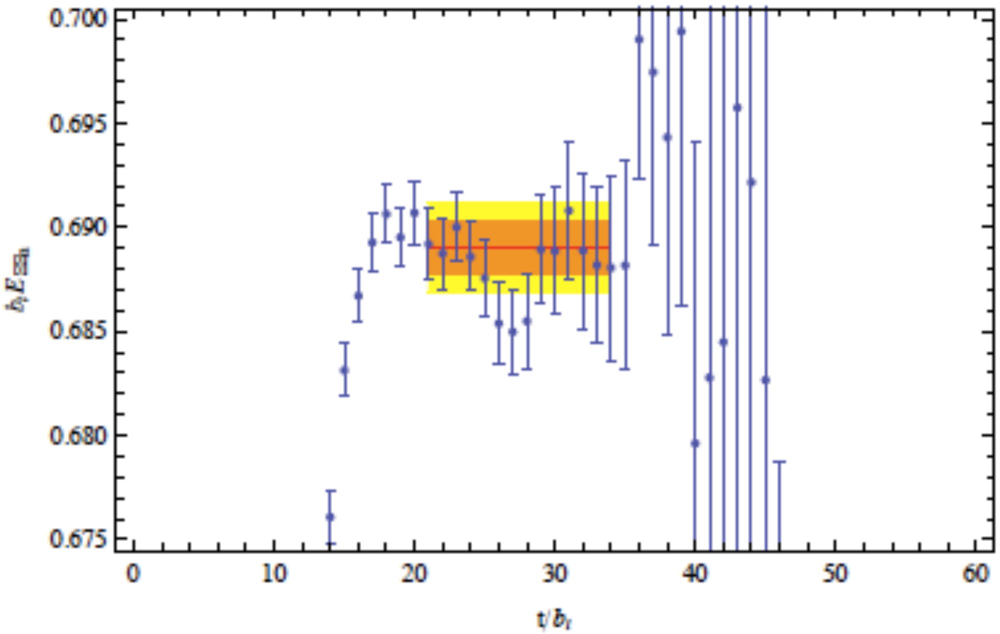
NPLQCD
arXiv:0903.2990
arXiv:0905.0466



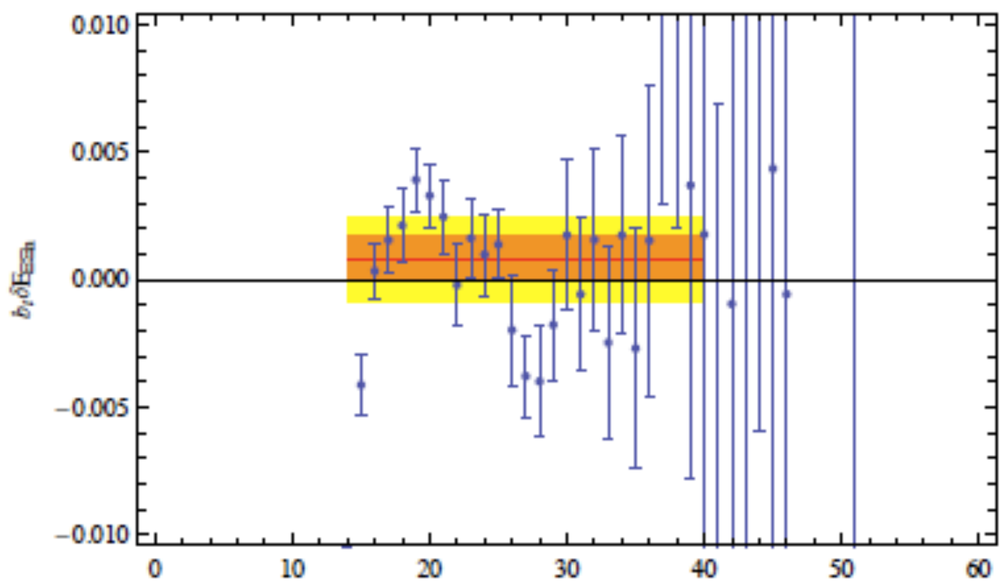
Two Baryons - see talk by Assumpta Parreno, Thursday 15:40

2-Baryons: High Stat. on Aniso Clover Lattices

NPLQCD
arXiv:0903.2990
arXiv:0905.0466



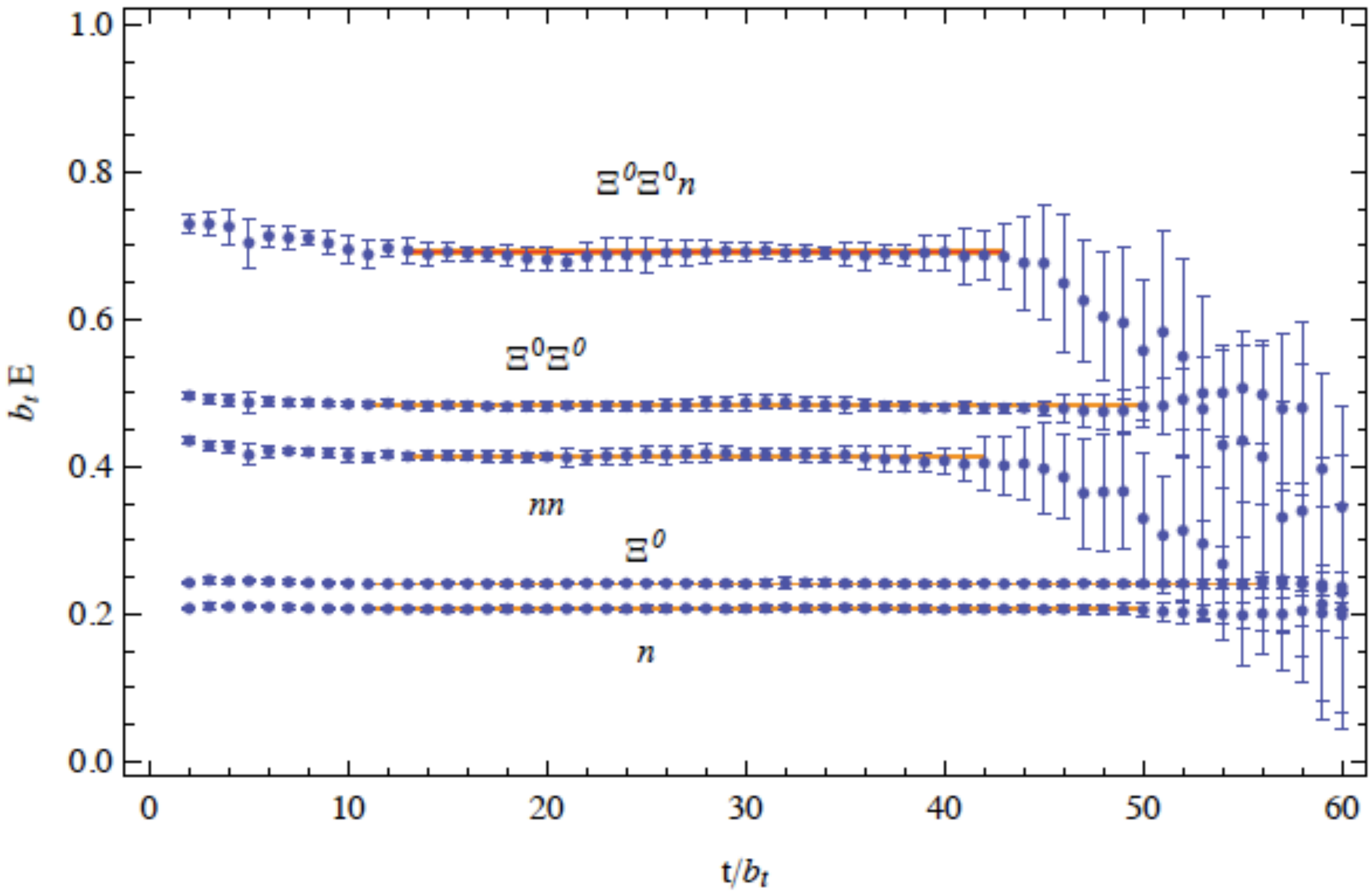
$$E_{\Xi^0\Xi^0 n}$$



$$E_{\Xi^0\Xi^0 n} - 2m_{\Xi} - m_N$$

2-Baryons: High Stat. on Aniso Clover Lattices

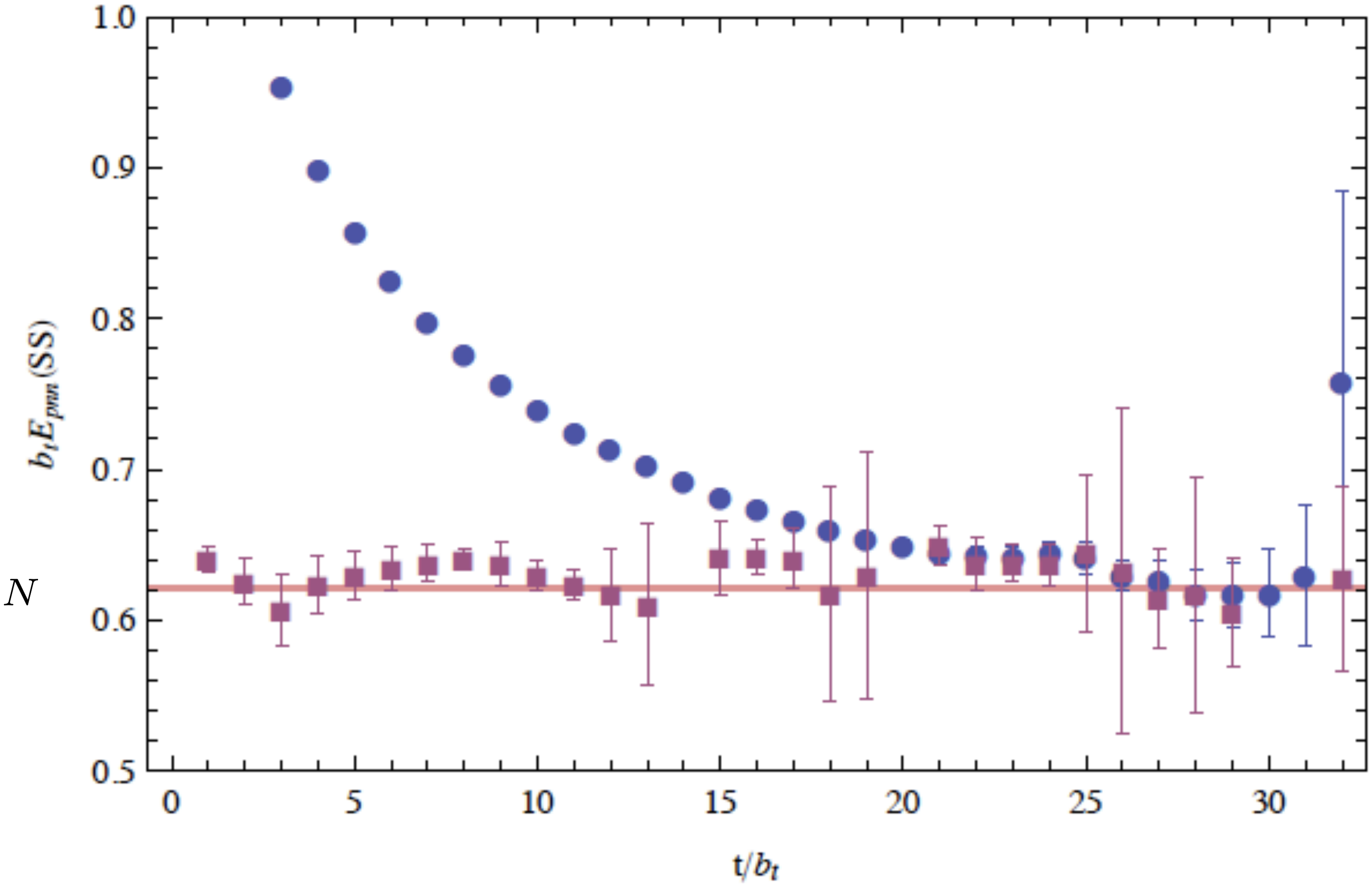
NPLQCD
arXiv:0903.2990
arXiv:0905.0466



2-Baryons: High Stat. on Aniso Clover Lattices

NPLQCD
arXiv:0903.2990
arXiv:0905.0466

$3m_N$



$\Xi^0\Xi^0 n$ 288 contractions triton 2880 contractions

2-Baryons: High Stat. on Aniso Clover Lattices

NPLQCD
arXiv:0903.2990
arXiv:0905.0466

computational cost - 292,500 sets

	cost per set	total cost
gauge generation (12000 trajectories ~ 1200 configurations)	13.7 cpu hours	4 M cpu hours
propagator calculation	20.5 cpu hours	6 M cpu hours
block production	23.9 cpu hours	7 M cpu hours
other	3.4 cpu hours	1 M cpu hours
1- 2- body contractions	3.4 cpu hours	1 M cpu hours
$\Xi^0\Xi^0 n$ contractions	16 cpu hours	4.7 M cpu hours
triton contractions	160 cpu hours	47 M cpu hours (we have not done this)

for nuclear physics - gauge generation is not the dominant cost

we need to figure out how to make contractions faster

Conclusions

two meson scattering (lengths) from lattice QCD is a precision science

- requires chiral perturbation theory
- provides stringent constraints/tests of chiral perturbation theory

why stop at two mesons?

- can calculate 3-body interactions for maximal isospin pions/kaons
  - 3-body pion interaction is not consistent with zero
  - 3-body kaon interaction is consistent with zero
- can study the chemical potential of a gas of pions and kaons
  mixed pion-kaon interactions under way

same calculation which gives 1% uncertainty in $\mathcal{I} = 2\ \pi\pi$ scattering, does not provide useful information for proton-proton (baryon-baryon) scattering - 30,000 propagator calculations!

- we have switched to using anisotropic clover lattices/propagators
  - this provides a factor of ~100 in statistics for same cpu hours
  - we have performed a high statistics study 1- 2- and 3- body baryon correlation functions