Chiral low-energy constants from τ-data

Chiral Dynamics **Bern 2009**

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- Approach: sum rules with the V-A correlator.
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M. G.-A., A. Pich & J. Prades,

Phys. Rev. D 78, 116012 (2008)

L_{10} & C_{87} : Motivation.

- ChPT Lagrangian:

$$\mathfrak{L}^{\chi PT} = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+ \rangle + \ldots + \frac{1}{4} L_{10}^r \langle f_{+\mu\nu} f_+^{\mu\nu} - f_{-\mu\nu} f_-^{\mu\nu} \rangle + \ldots + \\ + \ldots + C_{87}^r \langle \nabla_\rho f_{-\mu\nu} \nabla^\rho f_-^{\mu\nu} \rangle + \ldots + O(p^8)$$

- The motivation is two-fold:

  - **Phenomenology**:
    more precise LEC's → more precise predictions.
    For example… $\pi \to e \nu \gamma$

  - **Theoretical**:
    Our estimation (directly from the data) allows us to test the quality of the different theoretical models that have predicted these LEC's.

Approach: sum rule with $\Pi^{\mu\nu}_{V-A}(q)$

- V-A correlator:

$$\Pi^{\mu\nu}_{V-A}(q) \equiv i\int d^4x\, e^{iqx} \langle 0 | T(J_L^\mu(x) J_R^\nu(0)^\dagger) | 0 \rangle$$
$$= (q^\mu q^\nu - q^2 g^{\mu\nu}) \Pi^{(0+1)}(q^2) + q^2 g^{\mu\nu} \Pi^{(0)}(q^2)$$

$$\boxed{\begin{aligned} J_L^\mu &= \bar{u}\gamma^\mu(1-\gamma_5)d \\ J_R^\mu &= \bar{u}\gamma^\mu(1+\gamma_5)d \end{aligned}}$$

- Weight function:

$$\underline{w(z)}\Pi(z)$$

$$w(s) = \frac{1}{s}, \frac{1}{s^2}$$

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$$= (q^\mu q^\nu - q^2 g^{\mu\nu}) \Pi^{(0+1)}(q^2) + q^2 g^{\mu\nu} \Pi^{(0)}(q^2)$$

$$J^\mu_L = \bar{u}\gamma^\mu (1-\gamma_5) d$$
$$J^\mu_R = \bar{u}\gamma^\mu (1+\gamma_5) d$$

- Weight function:

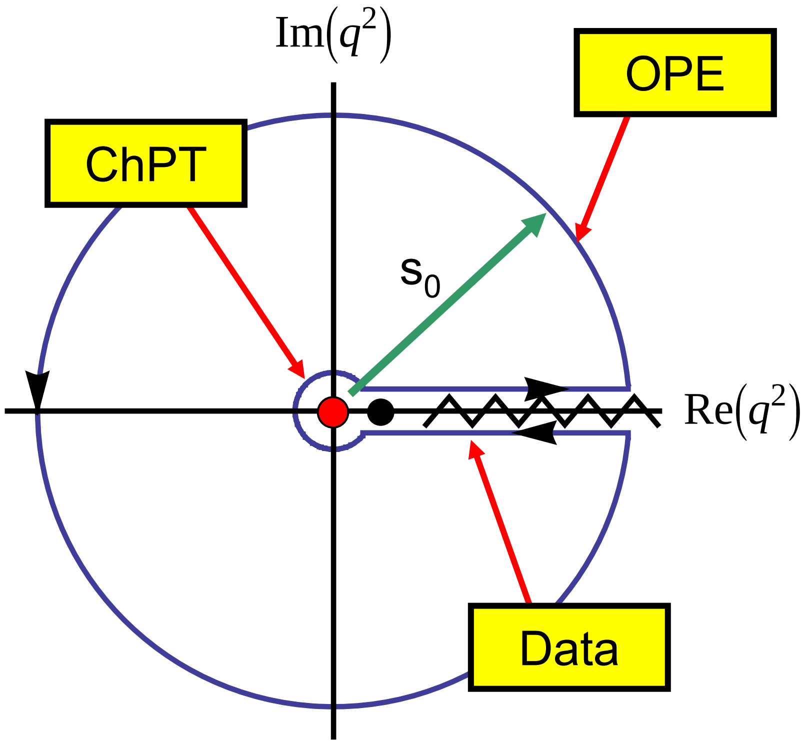
$$\underline{w(z)}\Pi(z)$$

$$w(s) = \frac{1}{s}, \frac{1}{s^2}$$

Analyticity of w(z)Π(z) relates different regions of the C-plane. → **SUM RULE**

Approach: sum rule with $\Pi^{\mu\nu}_{V-A}(q)$

Analytic properties of $w(z)\Pi(z)$



V-A spectral function

$$\operatorname*{Res}_{z=0}\frac{\Pi^{OPE}(z)}{z^n}-\operatorname*{Res}_{z=0}\frac{\Pi^{\chi PT}(z)}{z^n}=\frac{2F_\pi^2}{\left(m_\pi^2\right)^n}-\int_{s_{th}}^{s_0}\frac{\rho(s)}{s^n}ds$$

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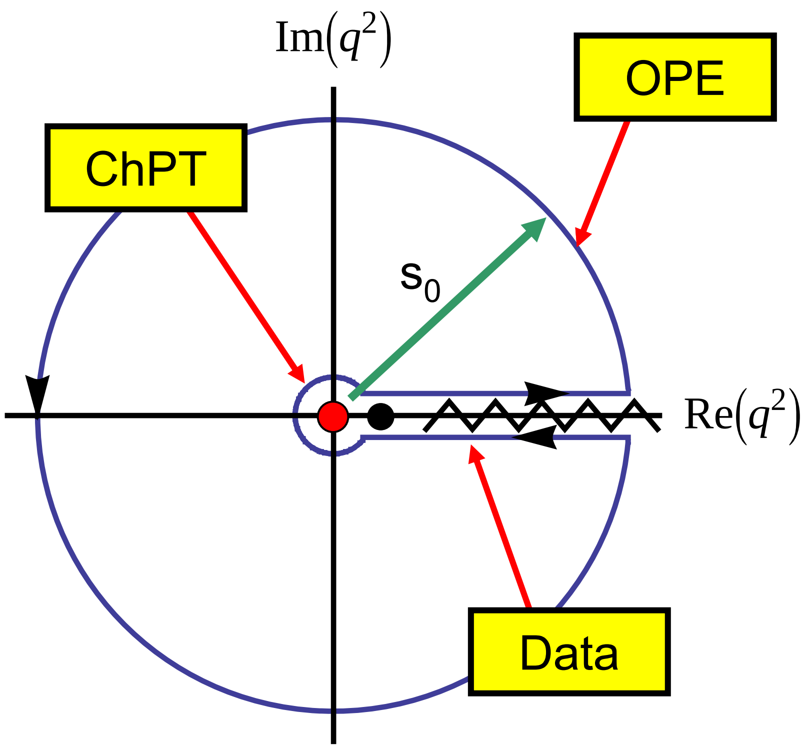

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$$\frac{d^{n-1}}{dz^{n-1}}\Pi^{\chi PT}(z)-\frac{2F_\pi^2}{\left(m_\pi^2\right)^n}=\int_{s_{th}}^{s_0}\frac{\rho(s)}{s^n}ds$$

$$\Pi^{\chi PT}(0)+2\frac{F_\pi^2}{m_\pi^2}=\int_{s_{th}}^{s_0}\frac{\rho(s)}{s}ds$$

$$\frac{d}{ds}\Pi^{\chi PT}(0)+2\frac{F_\pi^2}{m_\pi^4}=\int_{s_{th}}^{s_0}\frac{\rho(s)}{s^2}ds$$

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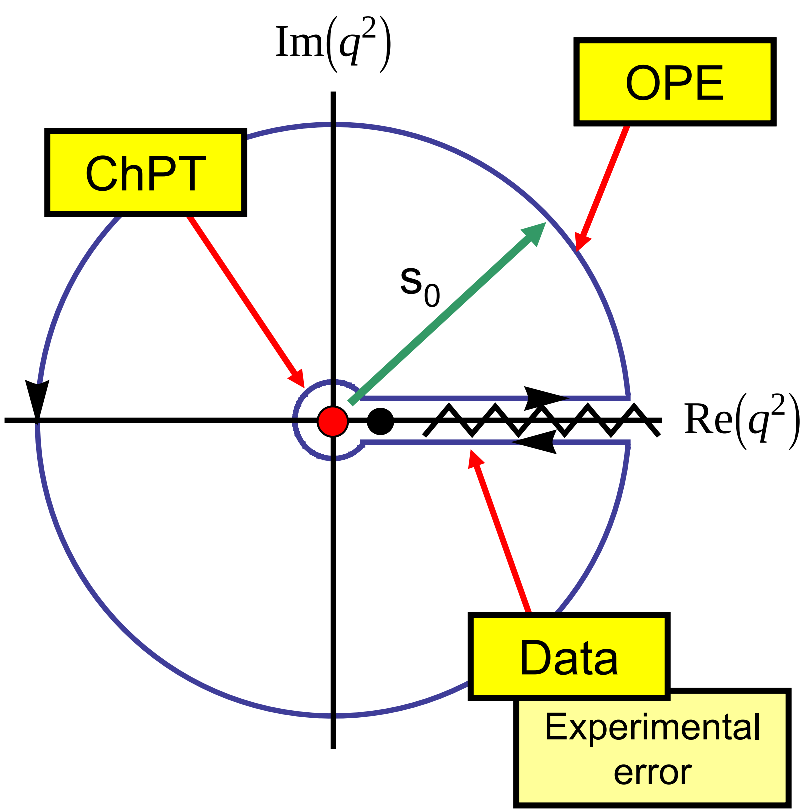

$$\cancel{\operatorname*{Res}_{z=0}\frac{\Pi^{OPE}(z)}{z^n}} - \operatorname*{Res}_{z=0}\frac{\Pi^{\chi PT}(z)}{z^n} = \frac{2F_\pi^2}{\left(m_\pi^2\right)^n} - \int_{s_{th}}^{s_0}\frac{\rho(s)}{s^n}ds$$

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$$\Pi^{\chi PT}(0) + 2\frac{F_\pi^2}{m_\pi^2} = \int_{s_{th}}^{s_0}\frac{\rho(s)}{s}ds$$

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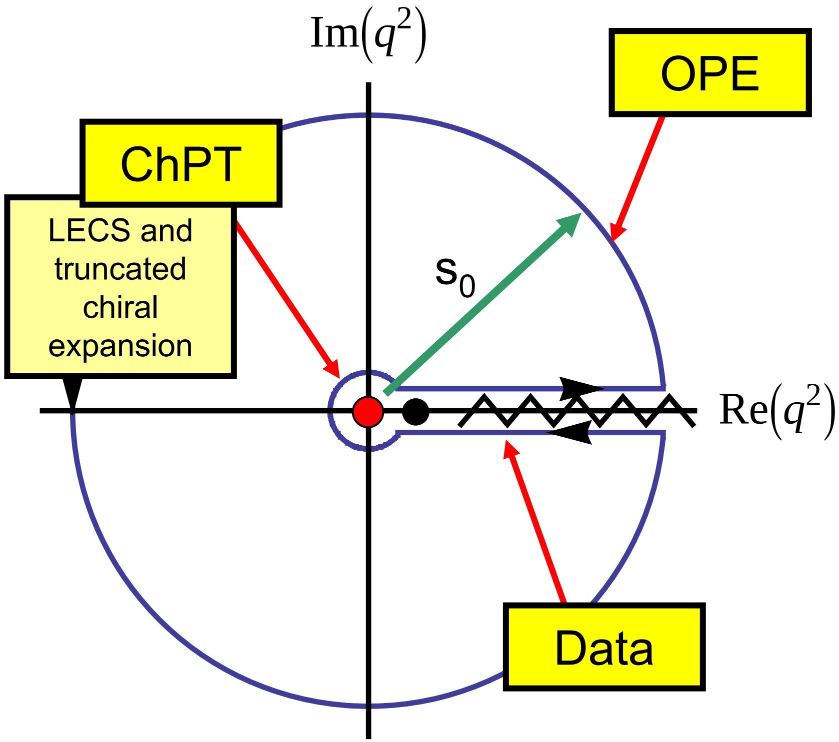

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$$\boxed{\begin{aligned} \Pi^{\chi PT}(0) + 2\frac{F_\pi^2}{m_\pi^2} &= \int_{s_{th}}^{s_0}\frac{\rho(s)}{s}ds \\ \frac{d}{ds}\Pi^{\chi PT}(0) + 2\frac{F_\pi^2}{m_\pi^4} &= \int_{s_{th}}^{s_0}\frac{\rho(s)}{s^2}ds \end{aligned}}$$

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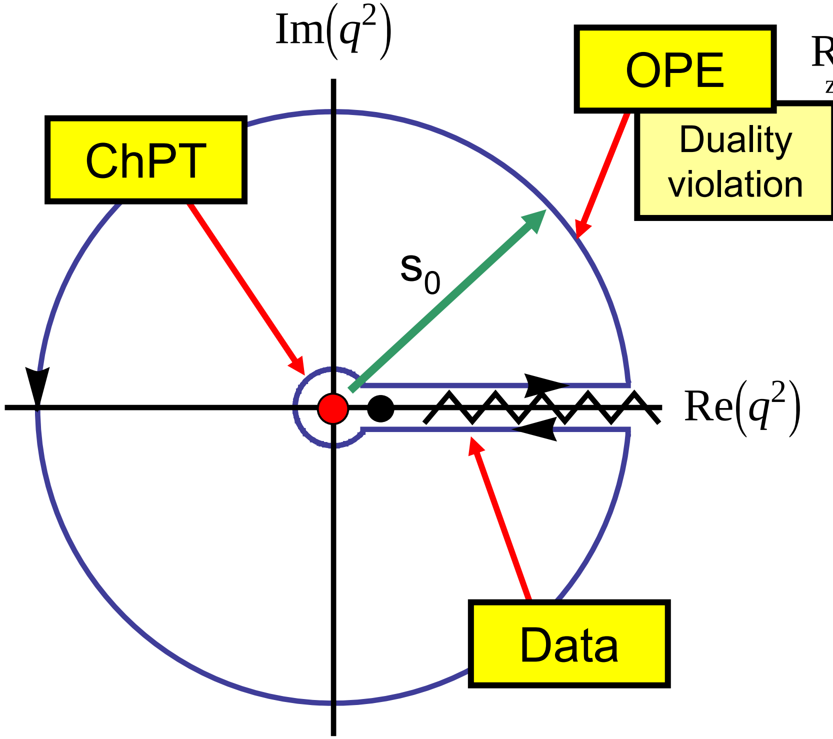

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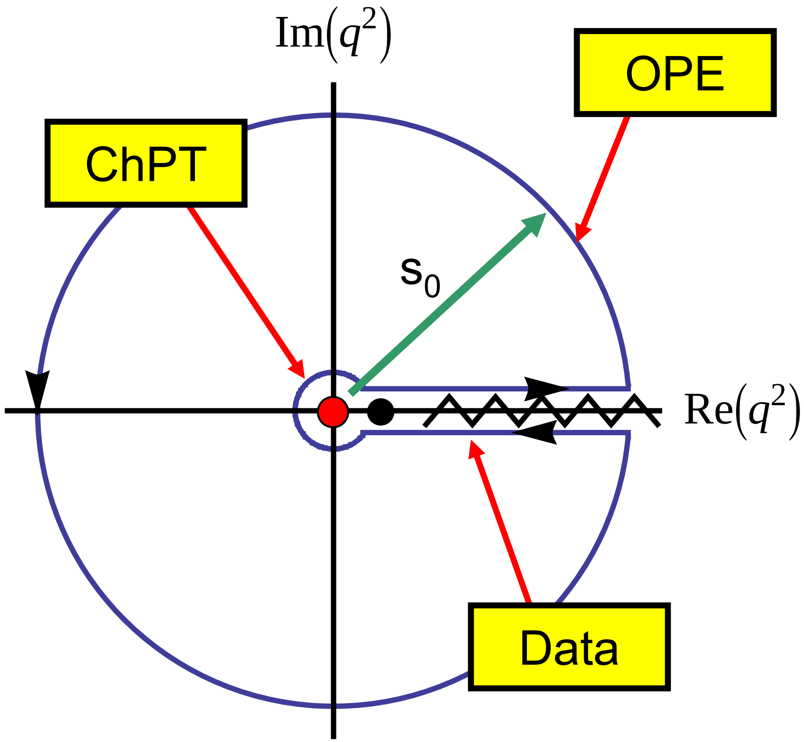

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$$\boxed{\Pi^{\chi PT}(0) + 2\frac{F_\pi^2}{m_\pi^2} = \int_{s_{th}}^{s_0}\frac{\rho(s)}{s}ds} \;\boxed{\equiv -8L_{10}^{eff}}$$

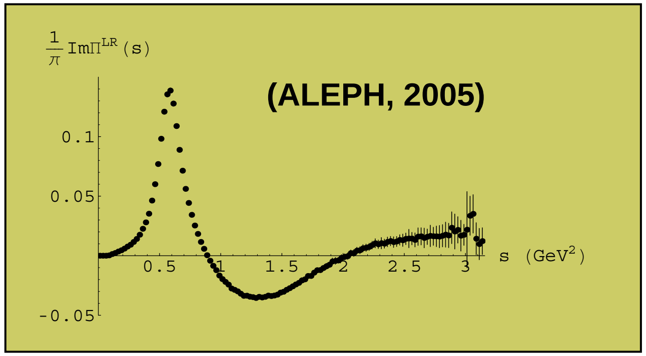
$$\boxed{\underbrace{\frac{d}{ds}\Pi^{\chi PT}(0) + 2\frac{F_\pi^2}{m_\pi^4}}_{\chi\text{PT side}} = \underbrace{\int_{s_{th}}^{s_0}\frac{\rho(s)}{s^2}ds \;\equiv 16C_{87}^{eff}}_{\text{Data side}}}$$

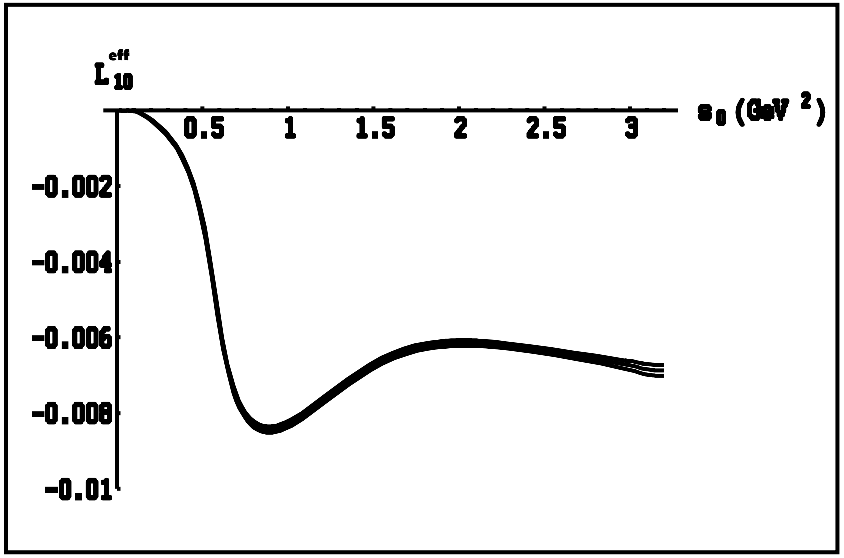
Data side

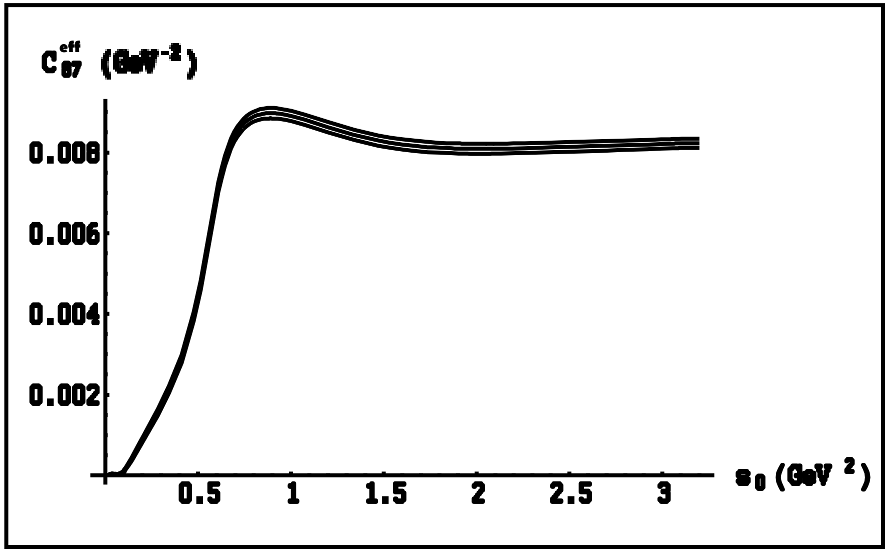
- Eff. parameters: determination.

$$\boxed{\begin{aligned} -8L_{10}^{eff} &\equiv \int_{s_{th}}^{s_0} \frac{\rho(s)}{s} ds \\ 16C_{87}^{eff} &\equiv \int_{s_{th}}^{s_0} \frac{\rho(s)}{s^2} ds \end{aligned}}$$

We can measure ρ(s) from the hadronic tau decays…







Data side

- Eff. parameters: determination.

$$-8L_{10}^{eff} \equiv \int_{s_{th}}^{s_0} \frac{\rho(s)}{s} ds$$

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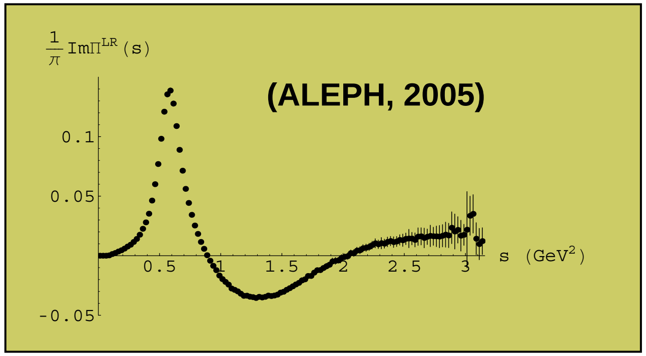

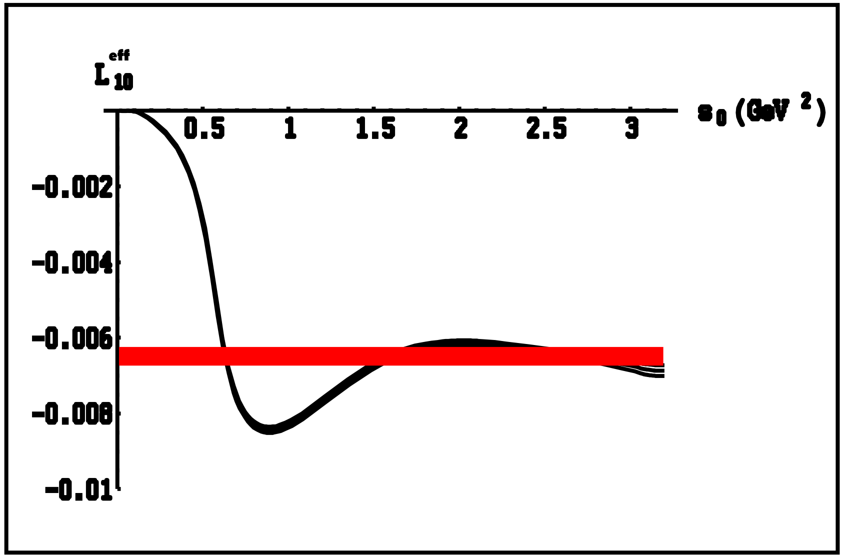

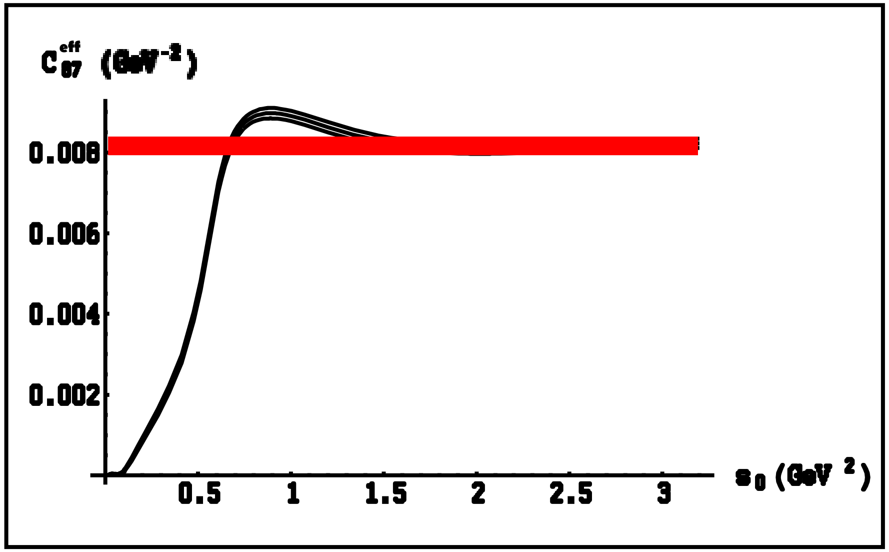

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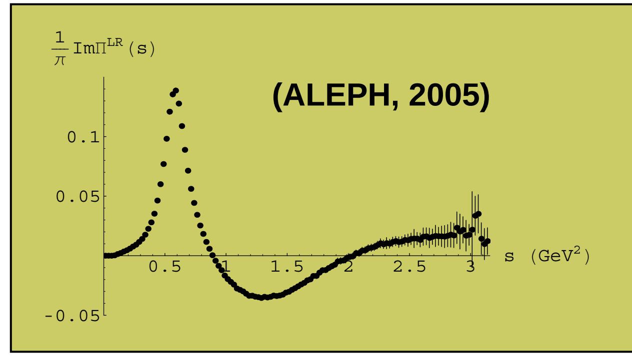
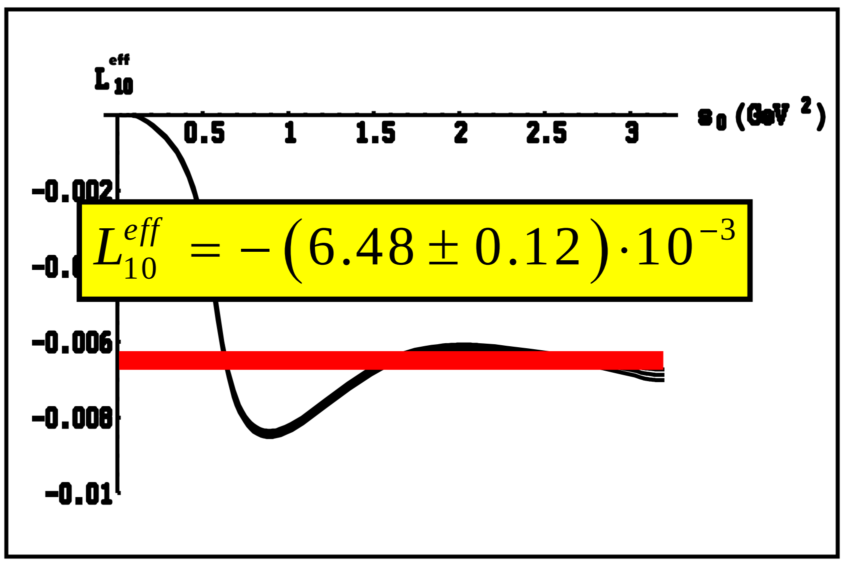
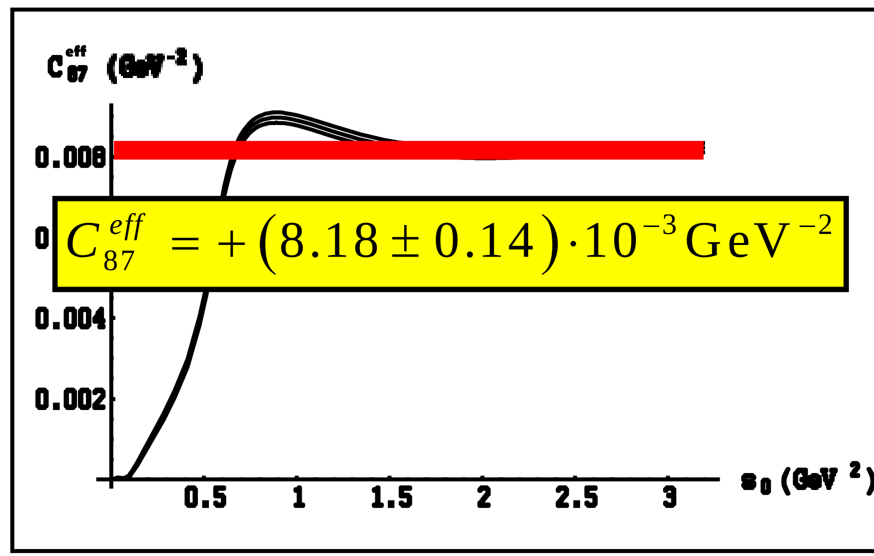




$$L_{10}^{eff} = -(6.48 \pm 0.12) \cdot 10^{-3}$$



$$C_{87}^{eff} = +(8.18 \pm 0.14) \cdot 10^{-3}\,\mathrm{GeV}^{-2}$$

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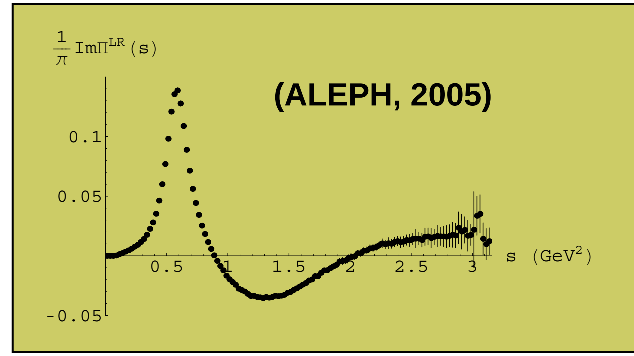

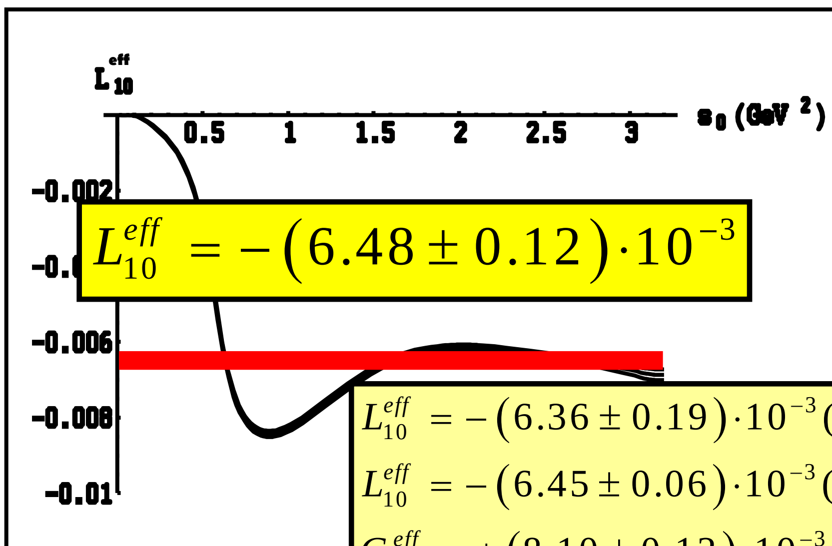

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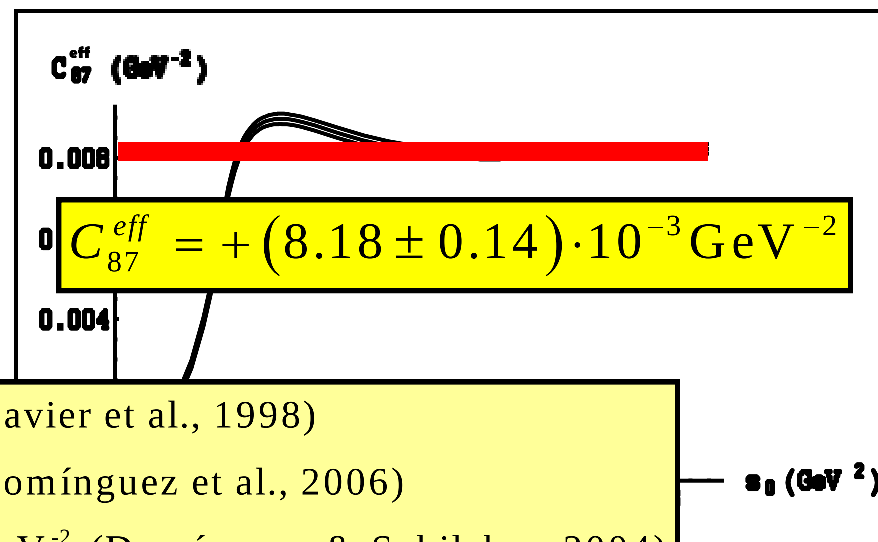

$$C_{87}^{eff} = +(8.18 \pm 0.14) \cdot 10^{-3}\,\mathrm{GeV}^{-2}$$

$L_{10}^{eff} = -(6.36 \pm 0.19) \cdot 10^{-3}$ (Davier et al., 1998)

$L_{10}^{eff} = -(6.45 \pm 0.06) \cdot 10^{-3}$ (Domínguez et al., 2006)

$C_{87}^{eff} = +(8.10 \pm 0.12) \cdot 10^{-3}\,\mathrm{GeV}^{-2}$ (Domínguez & Schilcher, 2004)

χPT side

- Theoretical calculation of the effective parameters

$$\boxed{\begin{aligned} L_{10}^{eff} &= \frac{-1}{8}\left(\Pi^{\chi PT}(0) + 2\frac{F_\pi^2}{m_\pi^2}\right) \equiv \frac{-1}{8}\bar{\Pi}^{\chi PT}(0) = f(L_{10}^r(\mu),\mu) \\ C_{87}^{eff} &= \frac{1}{16}\left(\Pi^{\chi PT}\,'(0) + 2\frac{F_\pi^2}{m_\pi^4}\right) \equiv \frac{1}{16}\bar{\Pi}^{\chi PT}\,'(0) = g(C_{87}^r(\mu),\mu) \end{aligned}}$$

- Using ChPT…

$$\bar{\Pi}^{\chi PT}(s) = -8L_{10}^r - 8B_V^{\pi\pi}(s) - 4B_V^{KK}(s) + O(p^6)$$

χPT side

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- Using ChPT…

$$\begin{aligned} \overline{\Pi}^{\chi PT}(s) = &-8L_{10}^r - 8B_V^{\pi\pi}(s) - 4B_V^{KK}(s) \\ &+16C_{87}^r s - 32m_\pi^2\left(C_{61}^r - C_{12}^r - C_{80}^r\right) - 32\left(m_\pi^2 + 2m_K^2\right)\left(C_{62}^r - C_{13}^r - C_{81}^r\right) \\ &+\frac{16}{f_\pi^2}\left(\left(2\mu_\pi + \mu_\pi\right)\left(L_9^r + 2L_{10}^r\right) - \left(2B_V^{\pi\pi}(s) + B_V^{KK}(s)\right)sL_9^r\right) \\ &+G_{2L}(s,\mu) + O(p^8) \end{aligned}$$

(Amorós et al., 2000)

(Golowich & Kambor, 1995, 1998)

Chiral Dynamics'09

χPT side

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- Therefore at O(p⁴)…

$$L_{10}^{eff} = \tfrac{-1}{8}\overline{\Pi}^{\chi PT}(0) = L_{10}^r(\mu) + \left(\frac{\log\frac{m_K^2}{m_\pi^2}}{384\pi^2} + \frac{1+\log\frac{m_\pi^2}{\mu^2}}{128\pi^2}\right)$$

$$L_{10}^r(m_\rho) = -(5.22\pm 0.12)\cdot 10^{-3}$$

$$L_{10}^r(m_\rho) = -(5.13\pm 0.19)\cdot 10^{-3}$$

(Davier et al., 1998)

χPT side

$$L_{10}^{eff} = \frac{-1}{8}\overline{\Pi}^{\chi PT}(0)$$
$$C_{87}^{eff} = \frac{1}{16}\overline{\Pi}^{\chi PT}{}'(0)$$

- At O(p⁶)...

$$L_{10}^{eff} = L_{10}^{r} + \left(\frac{\log \frac{m_K^2}{m_\pi^2}}{384\pi^2} + \frac{1+\log \frac{m_\pi^2}{\mu^2}}{128\pi^2} \right) + $$
$$+4m_\pi^2\left(C_{61}^r - C_{12}^r - C_{80}^r\right) + 4\left(m_\pi^2 + 2m_K^2\right)\left(C_{62}^r - C_{13}^r - C_{81}^r\right)$$
$$-\frac{2}{f_\pi^2}\left(2\mu_\pi + \mu_\pi\right)\left(L_9^r + 2L_{10}^r\right)$$
$$-\frac{1}{8}G_{2L}(0,\mu)$$

$$C_{87}^{eff} = \frac{1}{7680\pi^2}\left(\frac{1}{m_K^2} + \frac{2}{m_\pi^2}\right) +$$
$$+C_{87}^r(\mu)$$
$$-\frac{2L_9^r(\mu)}{f_\pi^2}\left(\frac{\log \frac{m_K^2}{m_\pi^2}}{384\pi^2 f_\pi^2} + \frac{1+\log \frac{m_\pi^2}{\mu^2}}{128\pi^2} \right)$$
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χPT side

$$L_{10}^{eff} = \frac{-1}{8}\overline{\Pi}^{\chi PT}(0)$$
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- At O(p⁶)…

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$$-\frac{2}{f_\pi^2}\left(2\mu_\pi + \mu_\pi\right)\left(L_9^r + 2L_{10}^r\right)$$
$$-\frac{1}{8}G_{2L}(0,\mu)$$

Experimental error

LEC's error

$$L_{10}^r(m_\rho) = -\left(4.06 \pm 0.08 \pm 0.39\right)\cdot 10^{-3}$$
$$= -\left(4.06 \pm 0.40\right)\cdot 10^{-3}$$

$$C_{87}^{eff} = \frac{1}{7680\pi^2}\left(\frac{1}{m_K^2} + \frac{2}{m_\pi^2}\right) +$$
$$+C_{87}^r(\mu)$$
$$-\frac{2L_9^r(\mu)}{f_\pi^2}\left(\frac{\log\frac{m_K^2}{m_\pi^2}}{384\pi^2 f_\pi^2} + \frac{1+\log\frac{m_\pi^2}{\mu^2}}{128\pi^2}\right)$$
$$+\frac{1}{16}G'_{2L}(0,\mu)$$

$$C_{87}^r(m_\rho) = +\left(4.89 \pm 0.14 \pm 0.13\right)\cdot 10^{-3}\,\mathrm{GeV}^2$$
$$= +\left(4.89 \pm 0.19\right)\cdot 10^{-3}\,\mathrm{GeV}^2$$

Results & Comparisons at $\mathcal{O}(p^6)$

M.G-A., Pich & Prades'08

$\chi\mathrm{PT}_3$

$$\boxed{L_{10}^r(m_\rho) = -\left(4.06 \pm 0.39\right)\cdot 10^{-3}}$$

$$L_{10}^r(m_\rho) = -\left(5.7 \pm 1.4\right)\cdot 10^{-3}\,\text{(Ecker et al.'89)}$$

$$L_{10}^r(m_\rho) = -\left(4.4 \pm 0.9\right)\cdot 10^{-3}\,\text{(Pich et al.'08)}$$

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$$\boxed{(L_9^r + L_{10}^r)(m_\rho) = -\left(1.44 \pm 0.08\right)\cdot 10^{-3}}$$

Unterdorfer & Pichl'08

$$\boxed{L_9^r(m_\rho) = +\left(5.50 \pm 0.40\right)\cdot 10^{-3}}$$

$L_9^r(m_\rho) = +\left(5.93 \pm 0.43\right)\cdot 10^{-3}$

(Bijnens & Talavera'02)

Results & Comparisons $\boxed{\text{at } \mathcal{O}(p^6)}$

M.G-A., Pich & Prades'08

J. Gasser et al., 2007

$\chi PT_3 \longrightarrow \chi PT_2$

Talk by Ivanov

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Results & Comparisons at $\mathcal{O}(p^6)$

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$\chi PT_3 \longrightarrow \chi PT_2$

J. Gasser et al., 2007

Talk by Ivanov

$$L_{10}^r(m_\rho) = -\left(4.06 \pm 0.39\right)\cdot 10^{-3} \quad\Longrightarrow\quad \bar{l}_5 = +12.24 \pm 0.21$$

$$L_{10}^r(m_\rho) = -\left(5.7 \pm 1.4\right)\cdot 10^{-3} \text{(Ecker et al.'89)}$$

$$L_{10}^r(m_\rho) = -\left(4.4 \pm 0.9\right)\cdot 10^{-3} \text{(Pich et al.'08)}$$

$$(L_9^r + L_{10}^r)(m_\rho) = -\left(1.44 \pm 0.08\right)\cdot 10$$

Unterdorfer & Pichl'08

$$\bar{l}_6 - \bar{l}_5 = -\left(2.98 \pm 0.33\right)\cdot 10^{-3}$$

Bijnens & Talavera'97

$$L_9^r(m_\rho) = +\left(5.50 \pm 0.40\right)\cdot 10^{-3} \quad\Longrightarrow\quad \bar{l}_6 = +15.22 \pm 0.39$$

$$L_9^r(m_\rho) = +\left(5.93 \pm 0.43\right)\cdot 10^{-3}$$

(Bijnens & Talavera'02)

Ph.

$\bar{l}_6 = +16.0 \pm 0.5 \pm 0.7$ (Bijnens et al., 1998)

Lattice

$\bar{l}_6 = +14.9 \pm 1.2 \pm 0.7$ (ETM Coll., 2009) Talk by Herdoiza

$\bar{l}_6 = +11.9 \pm 0.7 \pm 1.0$ (JLQCD-TWQCD, 2009)

Talk by Kaneko

Results & Comparisons $\boxed{\text{at } \mathcal{O}(p^4)}$

M.G-A., Pich & Prades'08

χPT_3

$$\boxed{L_{10}^r(m_\rho) = -(5.22 \pm 0.06) \cdot 10^{-3}}$$

$\Longrightarrow$

χPT_2

$$\boxed{\bar{l}_5 = +13.30 \pm 0.11}$$

Phen. $L_{10}^r(m_\rho) = -(5.13 \pm 0.19) \cdot 10^{-3}$ (τ, Davier et al.'98)

Latt. $L_{10}^r(m_\rho) = -(5.2 \pm 0.2 \pm 0.4) \cdot 10^{-3}$ (JLQCD Coll.'08)

Talk by Hashimoto

Th. $L_{10}^r(m_\rho) = -(5.7 \pm 1.4) \cdot 10^{-3}$ (Ecker et al.'89)

$L_{10}^r(m_\rho) = -(4.4 \pm 0.9) \cdot 10^{-3}$ (Pich et al.'08)

Results & Comparisons $\boxed{\text{at } \mathcal{O}(p^4)}$

M.G-A., Pich & Prades'08

χPT_3 ## χPT_2

$$\boxed{L_{10}^r(m_\rho) = -\left(5.22 \pm 0.06\right)\cdot 10^{-3}} \longrightarrow \boxed{\bar{l}_5 = +13.30 \pm 0.11}$$

$L_{10}^r(m_\rho) = -\left(5.13 \pm 0.19\right)\cdot 10^{-3}$ (τ, Davier et al.'98)

$L_{10}^r(m_\rho) = -\left(5.2 \pm 0.2 \pm 0.4\right)\cdot 10^{-3}$ (JLQCD Coll.'08)

Talk by Hashimoto

$L_{10}^r(m_\rho) = -\left(5.7 \pm 1.4\right)\cdot 10^{-3}$ (Ecker et al.'89)

$L_{10}^r(m_\rho) = -\left(4.4 \pm 0.9\right)\cdot 10^{-3}$ (Pich et al.'08)

$$\boxed{\bar{l}_6 - \bar{l}_5 = -\left(2.57 \pm 0.35\right)\cdot 10^{-3}}$$

Bijnens & Talavera'97

$$\boxed{(L_9^r + L_{10}^r)(m_\rho) = -\left(1.44 \pm 0.06\right)\cdot 10^{-3}}$$

PIBETA exper.

Talk by Počanić

$$\boxed{L_9^r(m_\rho) = +\left(6.67 \pm 0.08\right)\cdot 10^{-3}} \longrightarrow \boxed{\bar{l}_6 = +15.87 \pm 0.37}$$

$L_9^r(m_\rho) = +\left(6.9 \pm 0.7\right)\cdot 10^{-3}$ (Ecker'07)

Comparisons

$$\boxed{C_{87}^{r}(m_\rho) = +\left(4.89 \pm 0.19\right)\cdot 10^{-3}\,\mathrm{GeV}^{-2}}$$

M.G-A., Pich & Prades'08

- Theoretical (model dependent) estimations:

μ-sensitive (NLO)

$C_{87}^{r}(m_\rho) = +\left(3.6 \pm 1.3\right)\cdot 10^{-3}\,\mathrm{GeV}^{-2}$ (Pich et al., 2008)

Not μ-sensitive (LO)

$C_{87}^{r}(m_\rho) = +\left(5.7 \pm 0.5\right)\cdot 10^{-3}\,\mathrm{GeV}^{-2}$ (Masjuan & Peris, 2008)

$C_{87}^{r}(m_\rho) = +\left(5.8 \pm 1.5\right)\cdot 10^{-3}\,\mathrm{GeV}^{-2}$ (Cirigliano et al. 2004)

$C_{87}^{r}(m_\rho) = +\left(4.7 \pm 1.2\right)\cdot 10^{-3}\,\mathrm{GeV}^{-2}$ (Knecht & Nyffeler, 2001)

$C_{87}^{r}(m_\rho) = +\left(7.6 \pm 1.9\right)\cdot 10^{-3}\,\mathrm{GeV}^{-2}$ (Amoros et al., 2000)

25% assigned error

Conclusions

- We have calculated L_{10} with recent inclusive tau data including $O(p^6)$ terms in the chiral expansion, obtaining

$$L_{10}^r(m_\rho) = -\left(4.06 \pm 0.40\right) \cdot 10^{-3}$$
$$\bar{l}_5 = +12.24 \pm 0.29$$

$$\xrightarrow[\bar{l}_6 - \bar{l}_5]{L_9^r + L_{10}^r}$$

$$L_9^r(m_\rho) = +\left(5.50 \pm 0.41\right) \cdot 10^{-3}$$
$$\bar{l}_6 = +15.22 \pm 0.44$$

Most precise determinations!

Good agreement with previous ones and with lattice!

- The same approach has allowed us to calculate the first determination from data of C_{87} (up to $O(p^6)$ terms) obtaining

$$C_{87}^r(m_\rho) = +\left(4.89 \pm 0.19\right) \cdot 10^{-3}\,\mathrm{GeV}^{-2}$$

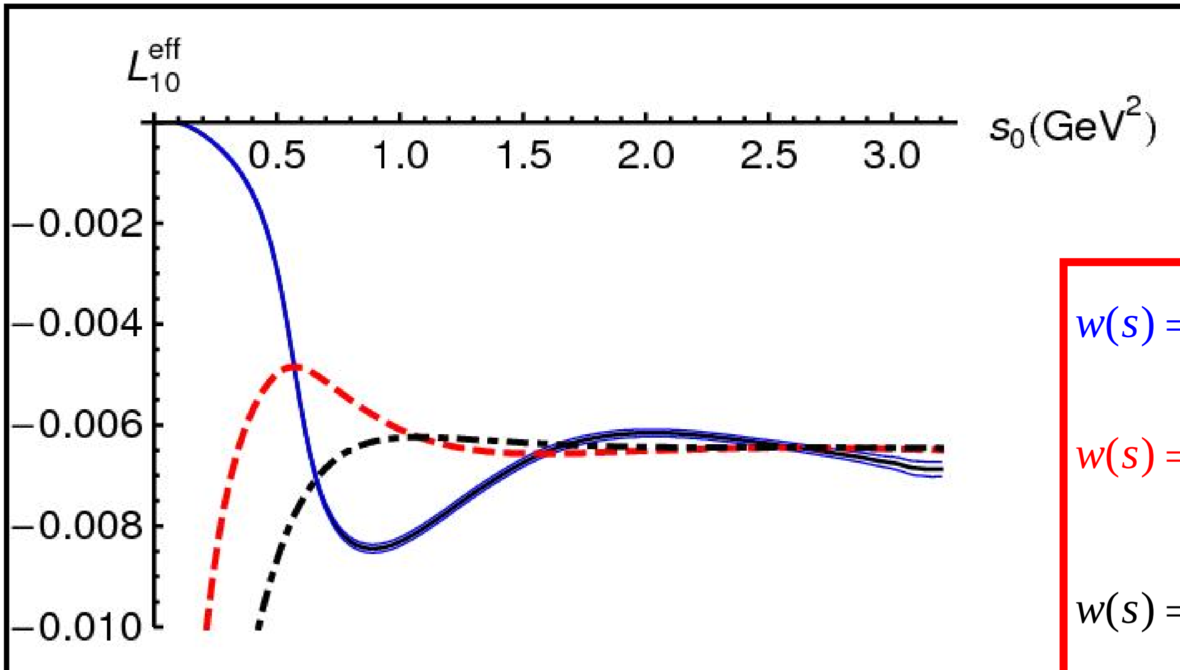
... in good agreement with different theoretical predictions.

Thanks!

Backup slides

Data side

- Eff. parameters: determination.



$$w(s) = \frac{1}{s}$$

$$w(s) = \frac{1}{s}\left(1 - \frac{s}{s_0}\right)$$

$$w(s) = \frac{1}{s}\left(1 - \frac{s}{s_0}\right)^2$$

(F. Le Diberder & A. Pich, 1992,
K. Maltman, 1998,
Domínguez & Schilcher, 1999, ...)

OPE

Operator Product Expansion

(Wilson, 1967):

$$\Pi_{V-A}(q^2) = \Pi_{V-A}^{pert}(q^2) + \frac{\mathcal{O}_2}{-q^2} + \frac{\mathcal{O}_4}{(-q^2)^2} + \frac{\mathcal{O}_6}{(-q^2)^3} + \dots$$