

Pion form factor from RBC and UKQCD

CHIRAL DYNAMICS 2009

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Institut für theoretische Kernphysik



This talk

- various talks on $f_{\pi\pi}(q^2)$ on the lattice

G. Herdoiza, WG1 Mon 14:40,

T. Kaneko, WG1 Mon 17:05,

S. Aoki, Plenary, Wed 09:00

- new quality of lattice simulations:

- $N_f \neq 0$ standard
- small quark masses
- small lattice spacings / large volume
- improved momentum resolution

- here:

- vector form factor $f_{\pi\pi}(q^2)$
- simulations by RBC+UKQCD
- very small space-like Q^2 region
- applicability of chiral perturbation theory?

The pion vector form factor - non-lattice

$$\langle \pi(p') | V_\mu | \pi(p) \rangle = f_{\pi\pi}(q^2)(p + p')_\mu$$

- charge radius $\langle r_\pi^2 \rangle_V = 6 \frac{d}{dq^2} f_{\pi\pi}(q^2)$
- chiral perturbation theory NLO (SU(2), SU(3)) J. Gasser & H. Leutwyler, *Ann. Phys.* 158 (1984) 142, *Nucl. Phys.*, B250 (1985) 465

$$\langle r_\pi^2 \rangle_{\text{SU}(3), \text{NLO}} = \frac{24L_9^r}{f_0^2} - \frac{1}{8\pi^2 f_0^2} \left(\log \frac{m_\pi^2}{\mu^2} + 1 \right) - \frac{1}{16\pi^2 f_0^2} \left(\log \frac{m_K^2}{\mu^2} + 1 \right)$$

chiral perturbation theory NNLO J. Bijnens, G. Colangelo and P. Talavera, *JHEP* 9805 (1998) 014,
J. Bijnens and P. Talavera, *JHEP*03(2002)046

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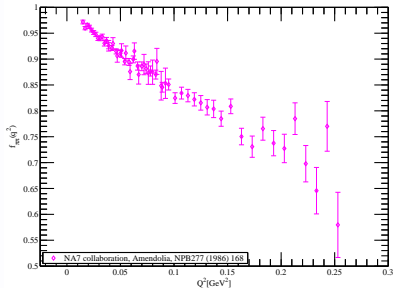
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- experimental situation:



PDG average: $\langle r_\pi^2 \rangle_V = 0.452(11) \text{fm}^2$

Eschrich, Liesenfeld, Amendolia, Dally, Bebek, Quenzer

The pion form factor - lattice

$$\langle O \rangle_{\text{QCD}} = \frac{1}{Z} \int D[\bar{q}, q, U] O e^{-S_{\text{QCD}}}$$

- two- and three-pt. correlation functions: e.g.

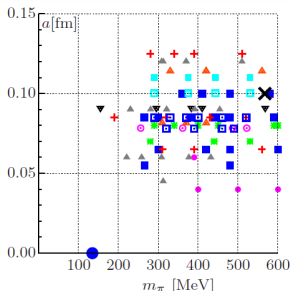
$$C_3(\vec{p}', \vec{p}, t) \rightarrow \frac{ZZ'}{2E(\vec{p}')E(\vec{p})} \langle \pi(\vec{p}') | V_\mu | \pi(\vec{p}) \rangle \{ e^{-E(\vec{p})t} \dots \}$$

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- ETMC $N_f = 2$ ■
- QCDSF $N_f = 2$ ■
- CERN-ToV $N_f = 2$ ○
- CLS $N_f = 2$ ●
- JLQCD $N_f = 2$ □
- JLQCD $N_f = 2 + 1$ □
- PACS-CS $N_f = 2 + 1$ ▼
- BMW $N_f = 2 + 1$ +
- RBC-UKQCD $N_f = 2 + 1$ ▲
- MILC $N_f = 2 + 1$ ▲
- ETMC $N_f = 2 + 1 + 1$ □
- JLQCD (2001) $N_f = 2$ ✕
- exp^t ●



$$L \rightarrow \vec{p} = \vec{n} \frac{2\pi}{L}$$

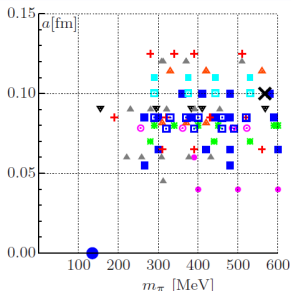
Kaon 09 talk by F. Mescia, plot by G. Herdoiza

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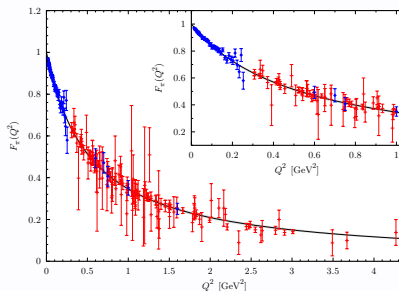
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- periodic boundary conditions for lattice fermions

$$q(\mathbf{x}_i + \hat{\mu}L) = q(\mathbf{x}_i) \rightarrow E(\vec{p}) = \sqrt{m_\pi^2 + (\vec{n} \frac{2\pi}{L})^2}$$

Form factors in a finite volume

- Lattice QCD results by **QCDSF** (2007) and **experiment**:



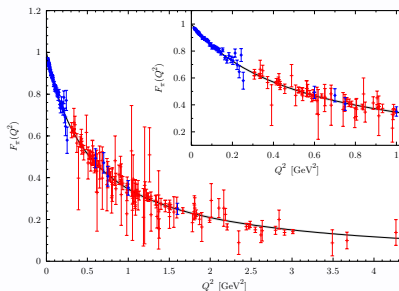
QCDSF Eur.Phys.J.C51:335-345,2007

- **lower bound** for accesible Q^2 -values due to **finite volume**:

$$Q_{\min}^2 = 2m_{\pi}(m_{\pi} - \sqrt{m_{\pi}^2 + (2\pi/L)})$$

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- it is now possible to go below this bound ...

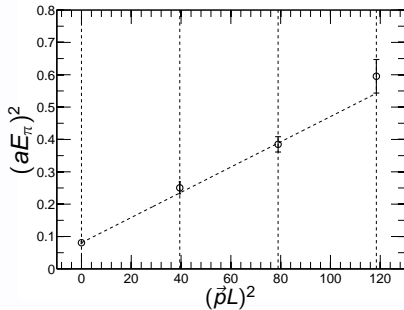
Twisted boundary conditions

periodic bc's

$$q(x_i + L) = q(x_i)$$

$$\vec{p}_{quark} = \vec{n} \frac{2\pi}{L}$$

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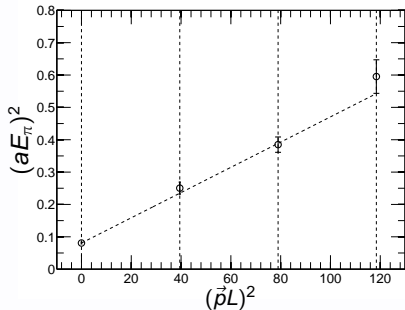
twisted bc's

Bedaque PLB539(2004)
Divitiis et al., PLB 595 (2004) 408
Bedaque, Chen, PLB 616:208-214,2005
Sachrajda, Viladoro, PLB 609:73-85,2005

$$q(x_i + L) = e^{i\theta_i} q(x_i)$$

$$\vec{p}_{quark} = \vec{n} \frac{2\pi}{L} + \frac{\vec{\theta}}{L}$$

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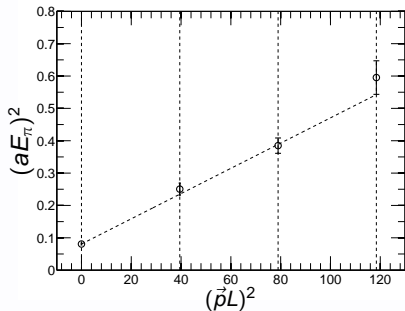
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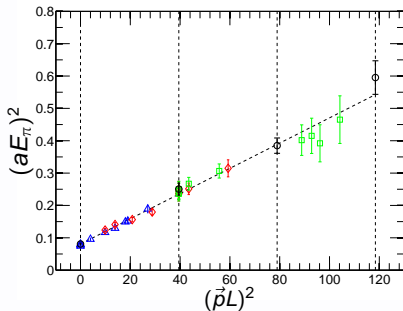
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J. Flynn, A. J., C. Sachrajda, PLB 632:313-318, 2006

Twisted boundary conditions

- periodic fields $\tilde{q}(x)$: $\underbrace{q(x)}_{\text{twisted}} = \exp(i\Theta_i/Lx_i)\underbrace{\tilde{q}(x)}_{\text{periodic}} = V(x)\tilde{q}(x)$

$$\begin{aligned}\mathcal{L}_{\tilde{q}} &= \tilde{\bar{q}}(x)\left(\not{D} + (V^\dagger(x)\not{\partial}V(x)) + M\right)\tilde{q}(x) \\ &= \tilde{\bar{q}}(x)(\tilde{\not{D}} + M)\tilde{q}(x)\end{aligned}$$

where $\tilde{D}_\mu = D_\mu + iB_\mu$ with $B_i = \Theta_i/L$ and $B_4 = 0$.

- free quarks in a constant background vector field

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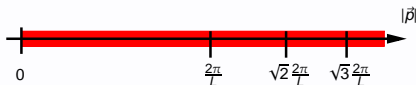
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- free quarks in a constant background vector field
- Free twisted quark propagator:

$$\tilde{S}^{\text{free}}(x, \theta) \equiv \langle \tilde{q}(x) \tilde{\bar{q}}(0) \rangle |_\theta = \int \frac{dp_4}{2\pi} \frac{1}{L^3} \sum_{\vec{p} = \frac{2\pi}{L} \vec{n}} \frac{e^{i\vec{p} \cdot x}}{i(\not{p} + \not{B}) + M}$$

quark mom. $\vec{p}_{\text{lat}} = \frac{2\pi}{L} \vec{n} + \frac{\vec{\theta}}{L}$



Twisted goldstone bosons

- apply twisting to finite volume chiral effective theory:

$$\mathcal{L}_{\chi\text{PT}}^{(2)}(\theta = 0) = \frac{f^2}{8} \text{tr} \left[(\partial_\mu \Sigma^\dagger) (\partial^\mu \Sigma) \right] - \frac{f^2}{8} \text{tr} \left[\Sigma^\dagger \chi + \chi^\dagger \Sigma \right]$$

Construction of the twisted chiral Lagrangian:

$$\Sigma(x + \hat{i}L) = U_i \Sigma(x) U_i^\dagger \quad \Sigma \text{ is } N_f \times N_f \text{ unitary}$$

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- reparametrize

$$\tilde{\Sigma}(x) = e^{-i\Theta \cdot x/L} \Sigma(x) e^{i\Theta \cdot x/L}$$

to get

$$\mathcal{L}_{\chi\text{PT}}^{(2)}(\theta) = \frac{f^2}{8} \text{tr} [\tilde{D}^\mu \tilde{\Sigma}^\dagger \tilde{D}_\mu \tilde{\Sigma}] - \frac{f^2}{8} \text{tr} [\tilde{\Sigma} \chi^\dagger + \chi \tilde{\Sigma}^\dagger]$$

where

$$\tilde{D}_\mu \tilde{\Sigma} = \partial_\mu \tilde{\Sigma} + i[B_\mu, \tilde{\Sigma}]$$

Twisted goldstone bosons

Effect of twist:

- neutral mesons:

$$[B_i, \pi^0] = 0 \rightarrow \text{no shift}$$

no effect on neutral pions

- charged mesons:

$$[B_i, \pi^\pm] = \pm \frac{\theta_{u,i} - \theta_{d,i}}{L} \pi^\pm$$

$$E_{\pi^\pm} = \sqrt{m_{\pi^\pm}^2 + \left(\vec{p}_{\text{lat}} - \frac{\vec{\theta}_u - \vec{\theta}_d}{L}\right)^2}$$

Finite volume effects, partial twisting

- finite volume effects with twist

Sachrajda, Viladoro, PLB 609:73-85,2005

$$\xi_s^\theta(L, M) = \frac{1}{L^3} \sum_{\vec{k}} \frac{1}{(\vec{k} + \vec{\theta}/L)^2 + M^2)^s} - \int \frac{d^3k}{(2\pi)^3} \frac{1}{(\vec{k}^2 + M^2)^s} = \xi_s^0(L, M) \frac{\sum_i \cos(\theta_i)}{3}$$

$\xi_s^0(L, M) \approx e^{-LM}$ for ME with max. one particle in either initial or final state

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- partial quenching/twisting in QCD

$$\begin{aligned} \mathcal{Z}_{\text{QCD}}^{\text{PQ}} &= \int DU \prod_{Q=\{q_v, q_s, q_g\}} D\bar{Q}DQ e^{-S_{\text{YM}} - \sum_{Q=\{q_v, q_s, q_g\}} \bar{Q}(\not{D} + m_Q)Q} \\ &= \int DU \prod_{N_V} \frac{(\det \not{D}(\theta_v) + m_v)}{(\det \not{D}(\theta_v) + m_v)} (\det \not{D}(\theta_s) + m_s) e^{-S_{\text{YM}}} \end{aligned}$$

- field theoretic frame work for correlation functions computed with sea quark mass m_s /twist θ_s and valence quark mass $m_v = m_g$ /twist $\theta_v = \theta_g$
- partial quenching/twisting can be studied in chiral perturbation theory: standard $SU(N)$ -flavour group $\rightarrow SU(N_V + N|N_V)$

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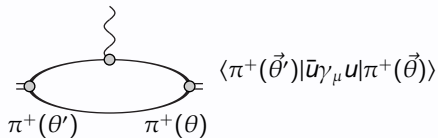
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- partial quenching/twisting can be studied in chiral perturbation theory: standard $SU(N)$ -flavour group $\rightarrow SU(N_v + N|N_v)$
- expressions for FVE change only slightly

momentum can be tuned continuously in practical simulations

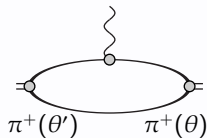
Partially twisted bc's and the pion's vector form factor

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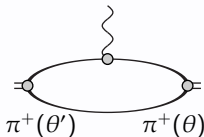
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F.-J. Jiang, B.C. Tiburzi PLB 645:314-321, 2007

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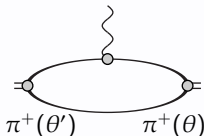
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UKQCD JHEP 0705:016,2007

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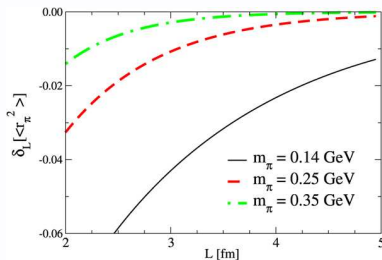
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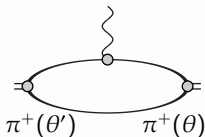
- finite volume effects (one pion at rest, the other one twisted)



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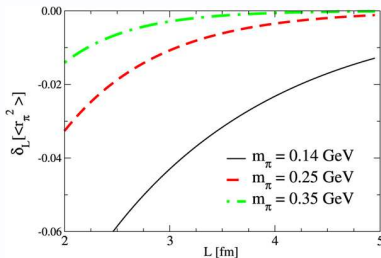
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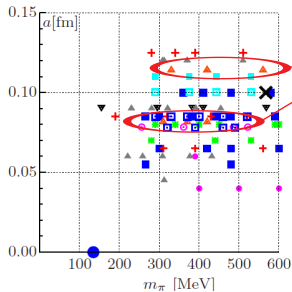
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F.-J. Jiang, B.C. Tiburzi PLB 645:314-321, 2007

- choose q^2 arbitrarily small \rightarrow compute charge radius without any phenomenological fit

RBC+UKQCD calculations of $f_{\pi\pi}(q^2)$



RBC+UKQCD simulation

UKQCD+RBC PRD76:014504,2007,PRD, 78:114509,2008

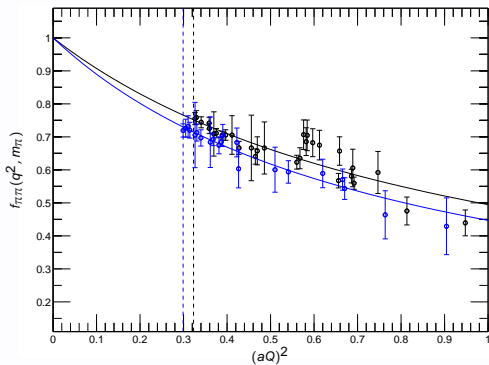
- DWF
- $a \approx 0.11$ fm & 0.09 fm
- $m_\pi \gtrsim 300$ MeV

from Kaon 09 talk by F. Meschia, plot by G. Herdoiza

First try:

$$\langle \pi(p') | V_\mu | \pi(p) \rangle = f_{\pi\pi}(q^2) (p + p')_\mu$$

- exploratory study by UKQCD $m_\pi = 590\text{MeV}$ $m_\pi = 480\text{MeV}$ $a = 0.11\text{fm}$, $L = 2\text{fm}$

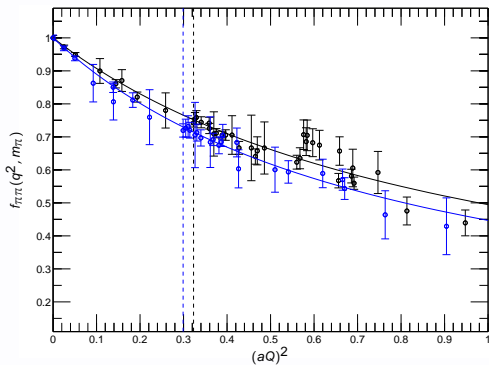


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First try:

$$\langle \pi(p') | V_\mu | \pi(p) \rangle = f_{\pi\pi}(q^2)(p + p')_\mu$$

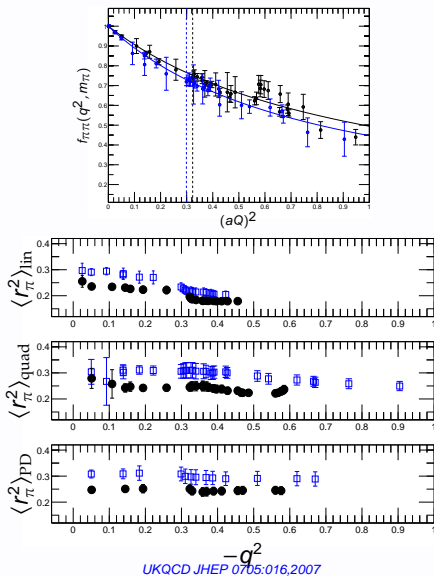
- exploratory study by UKQCD $m_\pi = 590\text{MeV}$ $m_\pi = 480\text{MeV}$ $a = 0.11\text{fm}$, $L = 2\text{fm}$



UKQCD JHEP 0705:016,2007

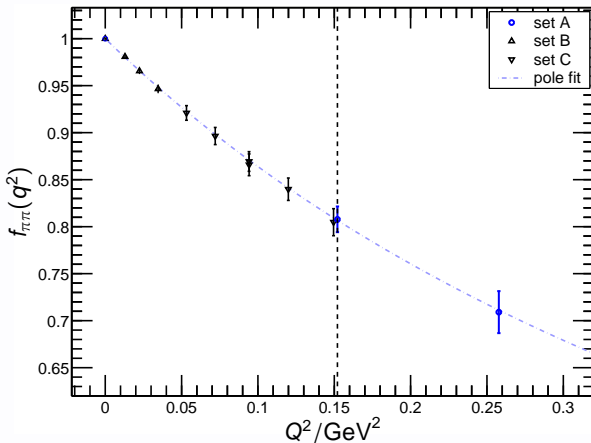
First try:

- exploratory study by UKQCD $m_\pi = 590\text{MeV}$ $m_\pi = 480\text{MeV}$ $a = 0.11\text{fm}$, $L = 2\text{fm}$



Closer to the real world ...

- UKQCD $m_\pi = 330\text{MeV}$ $a = 0.11\text{fm}$, $L = 2.7\text{fm}$



UKQCD JHEP 0807:112,2008

Fits

- currently we have results for only one pion mass $m_\pi = 330\text{MeV}$
- we fit the NLO ansatz $f_{\pi\pi}^{\text{SU}(2),\text{NLO}}(q^2) = 1 + \frac{1}{f^2} \left[-2l_6^r q^2 + 4\tilde{\mathcal{H}}(m_\pi^2, q^2, \mu^2) \right]$

J. Gasser & H. Leutwyler Ann. Phys. 158 (1984) 142

further input: pion decay constant in the $SU(2)$ chiral limit $f = 115(8)$ MeV
(normalisation s.t. $f_\pi^{\text{phys}} = 130.7(4)\text{MeV}$)

UKQCD+RBC Phys. Rev. Lett. 2008

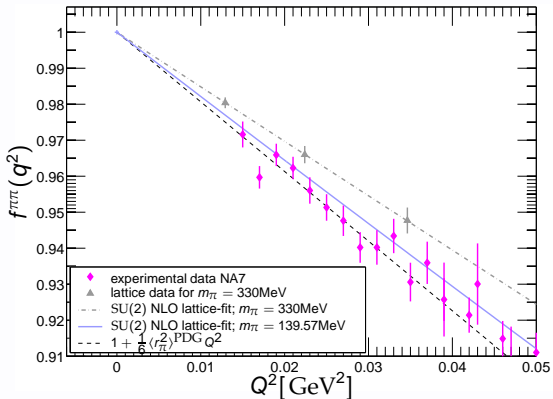
- this gives

$$l_6(m_\rho) = -0.0093(10)$$

$$\langle r_\pi^2 \rangle|_{m_\pi=330\text{MeV}} = 0.354(31)\text{fm}^2 \rightarrow \langle r_\pi^2 \rangle = 0.418(31)\text{fm}^2$$

UKQCD JHEP 0807:112,2008

Summary of UKQCD results

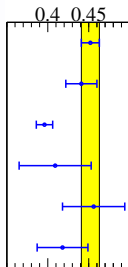


UKQCD arXiv:0804.3971

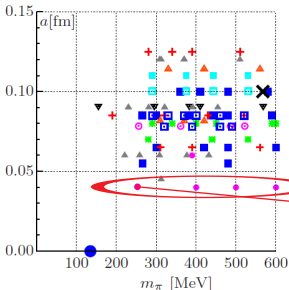
experimental data from NA7 Nucl. Phys. 1986

Results for the charge radius

collaboration	technique		$\langle r_\pi^2 \rangle_\chi$ [fm ²]
PDG			0.452(11)
QCDSF/UKQCD <i>EPJ C51 (2007) 335345</i>	$N_f = 2$	Clover	0.441(19)
JLQCD <i>hep-lat/0510085</i>	$N_f = 2$	Clover	0.396(10)
JLQCD <i>arXiv:0905.2465</i>	$N_f = 2$	Overlap	0.409(44)
ETM <i>arXiv:0812.4042</i>	$N_f = 2$	twisted mass	0.456(38)
RBC+UKQCD <i>JHEP 0807:112,2008</i>	$N_f = 2 + 1$	Domain Wall	0.418(31)



Mainz/CLS $N_f = 2$ effort



from Kaon 09 talk by F. Mescia, plots by G. Herdoiza

Coordinated Lattice Simulations

(CLS) *Berlin, CERN, DESY-Zeuthen, Madrid, Mainz, Rome, Valencia*

- improved Wilson
- $a \gtrsim 0.04\text{fm}$
- $m_\pi \gtrsim 250\text{MeV}$ (planned)

Discussion

- discussion (RBC+UKQCD)
 - 2nd (finer) lattice spacing under way
 - no statement on applicability of NLO χ PT possible
RBC+UKQCD: three/four unitary simulation points $am_\pi \gtrsim 300\text{MeV}$ on 0.1/0.09 fm lattices
 - not enough data points for NNLO (cf. ETM [arXiv:0812.4042](https://arxiv.org/abs/0812.4042))
- interesting question
 - what is the influence of the vector resonance? - is χ PT the right theory for the simulated quark masses?