Pion form factors from lattice QCD with exact chiral symmetry

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1. introduction

pion vector form factor $F_V(q^2)$

$$\langle\pi(p')|V_\mu|\pi(p')\rangle = (p'+p)_\mu F_V(q^2), \qquad F_V(q^2) = 1 + (\langle r^2\rangle_V/6)\, q^2 + O(q^4)$$

- well studied by expr't + ChPT $\Rightarrow$ precise estimate of $\langle r^2\rangle_V$, l_6 (L_9)
- a benchmark of LQCD calculation
  - at simulated quark mass m : chiral behavior $\Leftrightarrow$ ChPT predictions
  - at physical m : can reproduce $\langle r^2\rangle_V$?

pion scalar form factor $F_S(q^2)$

$$\langle\pi(p')|S|\pi(p')\rangle = F_S(q^2), \qquad F_S(q^2) = 1 + (\langle r^2\rangle_S/6)\, q^2 + O(q^4)$$

- chiral behavior of $\langle r^2\rangle_S$
  - determination of l_4 $\Leftrightarrow$ l_4 from F_π
  - $\times 6$ NLO chiral log : $-6/(4\pi F)^2 \ln[M_\pi^2]$ $\Leftrightarrow$ $\langle r^2\rangle_V$: $-1/(4\pi F)^2 \ln[\ldots]$
- direct determination in LQCD $\Leftarrow$ needs disconnected 3-pt. functions
  - only 2 previous studies ignoring disconnected diagram *(JLQCD, 2005; BGR, 2007)*

1. introduction

this work

JLQCD / TWQCD collaborations, arXiv:0905.2465

calculate pion form factors in $N_f = 2$ lattice QCD

- employ overlap quarks
  - exact chiral symmetry $\Rightarrow$ straightforward comparison w/ ChPT
- use all-to-all quark propagator
  - disconnected 3-pt. functions for $F_S(q^2)$
  - improved statistical accuracy $F_{V,S}(q^2)$

outline

- simulation method
- determination of $F_V(q^2)$ and $F_S(q^2)$
- parametrization of q^2 dependence of $F_{V,S}(q^2)$
- chiral extrapolation of $\langle r^2 \rangle_V$, $\langle r^2 \rangle_S$, ...

2.1 simulation method : configuration generation

set-up

- $N_f = 2$ QCD w/ degenerate u and d quarks
- improved gauge action *(Iwasaki, 1982)*
- overlap quark action *(Narayanan-Neuberger, 1995; Neuberger, 1998)*

  $\Rightarrow$ exact chiral symmetry on the lattice *(Hasenfratz, 1998; Lüscher, 1998)*

parameters

- $a = 0.1184(3)(21)$ fm $\Leftarrow$ overlap : no $O(a)$ errors

  (input : $r_0 = 0.49$ fm *(Sommer, 1994)*)
- $16^3 \times 32$: $L \sim 1.9$ fm $+$ NLO ChPT finite V correction (FVC)
- 4 m_{ud}'s : $m \simeq m_s/6 - m_s/2$, $M_\pi \simeq 290 - 520$ MeV
- 100 independent conf.s at each m (100 $\times$ 100 HMC trajectories)

2.2 simulation method : measurements

all-to-all quark propagator

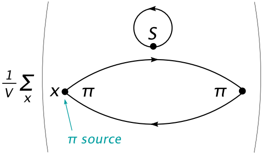
- propagation from *any* lattice site to *any* site *(TrinLat, 2005; JLQCD/TWQCD, 2009)*

$$D^{-1} = \sum_{k=1}^{12VT} \frac{1}{\lambda^{(k)}} u^{(k)} u^{(k)\dagger} = \sum_{k=1}^{N_{\rm eigen}} \frac{1}{\lambda^{(k)}} u^{(k)} u^{(k)\dagger} + (1 - P_{\rm low}) \sum_{r=1}^{N_r} \frac{x^{(r)}}{N_r} \eta^{(r)\dagger}$$

low-mode contributions $\Leftarrow$ evaluated *exactly* w/ eigenmodes of D

high mode contributions $\Leftarrow$ noise method (stochastic)

$\Rightarrow$ evaluate disconnect diagrams ; improve statistical accuracy



cf. conventional method

point-to-all prop:
a fixed site $\longrightarrow$ any site

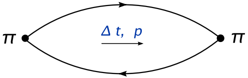
parameters

- $|q^2| \lesssim 1.7$ GeV^2 (w/ $|\mathbf{p}| \leq \sqrt{3}$ in units of $2\pi/L$)
- periodic boundary condition (different conditions $\Rightarrow$ re-calculation of D^{-1})

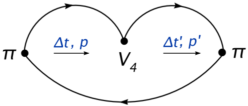
3.1 determination of form factors : $F_V(q^2)$

ratio method *(S. Hashimoto, et al., 2000)*

$C^{conn}_{\pi\,\pi}(\Delta t;\, p)$



$C^{conn}_{\pi\, V_4\, \pi}(\Delta t, \Delta t';\, p, p')$



$$C^{\rm conn}_{\pi V_4 \pi}(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}') \quad \to \quad \frac{\sqrt{Z_\pi(|\mathbf{p}|)\, Z_\pi(|\mathbf{p}'|)}}{4E(p)E(p')\, Z_V}\, e^{-E(p)\Delta t}\, e^{-E(p')\Delta t'}\, \langle \pi(p')\, |V_4|\, \pi(p)\rangle$$

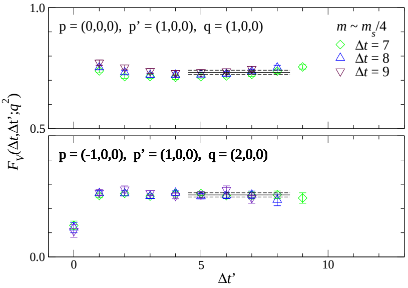
$$C^{\rm conn}_{\pi\pi}(\Delta t; \mathbf{p}) \to \frac{\sqrt{Z_\pi(|\mathbf{p}|)\, Z_\pi(|\mathbf{p}'|)}}{2E(p)}\, e^{-E(p)\Delta t}, \qquad \sqrt{Z_\pi(|\mathbf{p}|)} = \langle \pi(p)|O_\pi(\mathbf{p})^\dagger\rangle$$

$$R_4(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}') \quad = \quad \frac{C_{\pi V_4 \pi}(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}')}{C_{\pi\pi}(\Delta t; \mathbf{p})\, C_{\pi\pi}(\Delta t'; \mathbf{p}')} = \frac{\langle \pi(p')\, |V_4|\, \pi(p)\rangle}{\sqrt{Z_{\pi,\rm lcl}\, Z_{\pi,\rm lcl}}\, Z_V}$$

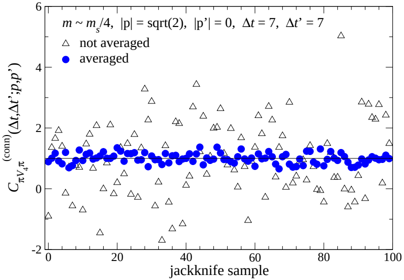
$$F_V(\Delta t, \Delta t'; q^2) \quad = \quad \frac{2M_\pi}{E(p)+E(p')}\, \frac{R_4(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}')}{R_4(\Delta t, \Delta t'; \mathbf{0}, \mathbf{0})} \quad (q^2 = (p'-p)^2)$$

3.1 determination of form factors : $F_V(q^2)$

$F_V(\Delta t, \Delta t'; q^2)$ at $m \sim m_s/4$

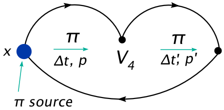


statistical fluctuation of $C_{\pi V_4 \pi}$



- statistical accuracy $\approx$ 3 – 5 %

  all-to-all prop $\Rightarrow$ can take average over source location x

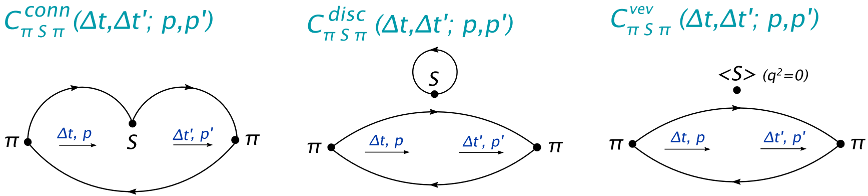


- constant fit to $F_V(\Delta t, \Delta t'; q^2)$
- include finite V correction from one-loop ChPT

  (Borasoy-Lewis, 2005; Bunton et al., 2006)

3.2 determination of form factors : $F_S(q^2)$

ratio method



$$R_S(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}') = \frac{C^{\rm sngl}_{\pi S \pi}(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}')}{C_{\pi\pi}(\Delta t; \mathbf{p})\, C_{\pi\pi}(\Delta t'; \mathbf{p}')} = \frac{\langle \pi(p') | S | \pi(p) \rangle}{\sqrt{Z_{\pi,\rm lcl} Z_{\pi,\rm lcl}}\, Z_S}$$

$$\frac{F_S(\Delta t, \Delta t'; q^2)}{F_S(\Delta t, \Delta t'; q^2_{\rm ref})} = \frac{R_S(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}')}{R_S(\Delta t, \Delta t'; \mathbf{p}_{\rm ref}, \mathbf{p}'_{\rm ref})} \quad (q^2_{\rm ref} = (p'_{\rm ref} - p_{\rm ref})^2)$$

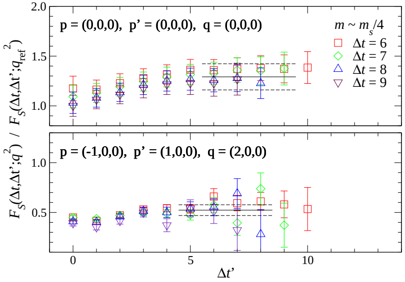
- normalize at smallest nonzero $|q^2_{\rm ref}|$ $(\mathbf{p}_{\rm ref} = 1,\ \mathbf{p}'_{\rm ref} = 0)$

  relatively large uncertainty in $F_S(\Delta t, \Delta t'; 0)$

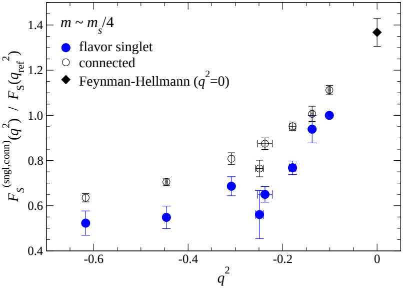
  $\Leftarrow$ VEV subtraction : $C^{\rm sngl}_{\pi S \pi}(q^2=0) = C^{\rm conn}_{\pi S \pi}(0) - \left(C^{\rm disc}_{\pi S \pi}(0) - C^{\rm vev}_{\pi S \pi}(0)\right)$

3.2 determination of form factors : $F_S(q^2)$

$F_S(\Delta t, \Delta t'; q^2)/F_S(\Delta t, \Delta t'; q^2_{\rm ref})$



$F_S(q^2)/F_S(q^2_{\rm ref})$, $F_S{}^{\rm conn}(q^2)/F_S(q^2_{\rm ref})$

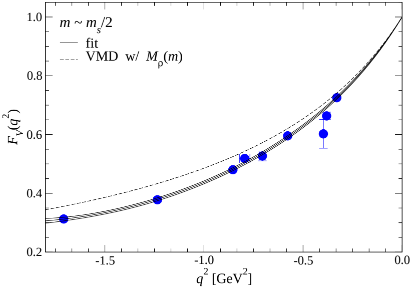


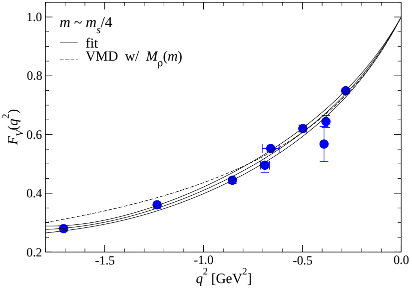
- statistical accuracy $\approx$ 5 – 10 % $\Leftarrow$ inclusion of $C^{\rm disc}_{\pi S \pi}$

  constant fit + NLO FVC $\Rightarrow$ $F_S(q^2 \neq 0)/F_S(q^2_{\rm ref})$

- disconnected diagram $\Rightarrow$ significant contribution

4.1 q^2 dependence : $F_V(q^2)$





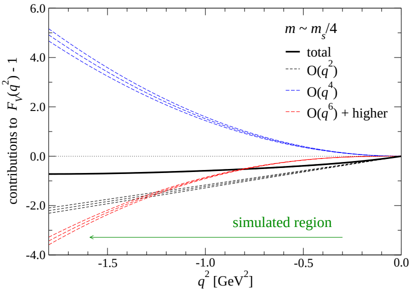
- close to VMD near $q^2 = 0$ $\Rightarrow$ include ρ meson pole into param. form
- approximate small deviation (higher poles/cuts) by generic polynomial form

$$F_V(q^2) = \frac{1}{1 - q^2/M_\rho^2} + c\,q^2 + d\,(q^2)^2 + e\,(q^2)^3 = 1 + \frac{\langle r^2 \rangle_V}{6}\,q^2 + c_V\,(q^2)^2 + \ldots$$

- fits up to $(q^2)^2$ and $(q^2)^3$ corrections $\Rightarrow$ reasonable χ^2 and consistent results
- employ fit with $(q^2)^3$ correction

4.1 q^2 dependence : $F_V(q^2)$

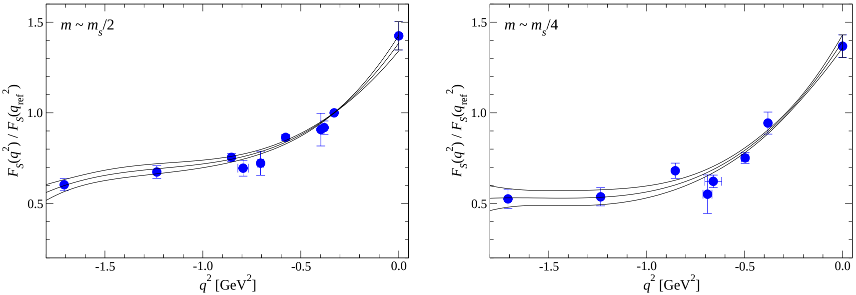
can be fitted by NNLO ChPT formula *(Gasser-Meißner, 1991; Bijens et al., 1998)* ?



- $O(q^4)$ contrib. (NNLO) $\lesssim 3\,\%$ at $|q^2| \lesssim 0.02\,\mathrm{GeV}^2$
- $O(q^6)$ contrib (NNNLO) $\lesssim 3\,\%$ at $|q^2| \lesssim 0.3\,\mathrm{GeV}^2$
- periodic boundary condition
  $\Rightarrow$ $|q^2| \gtrsim 0.3\,\mathrm{GeV}^2$ on our lattice

- in this work: do not parametrize q^2 dependence based on ChPT
- twisted boundary condition *(Bedaque, 2004)* can explore $q^2 \sim 0$
  (RBC/UKQCD → talk by Jüttner; ETM,2008)
  $\Rightarrow$ need to re-calculate all-to-all propagator
- $M_\pi^2 \lesssim 0.3\,\mathrm{GeV}^2$ $\Rightarrow$ NNLO ChPT fit for M_π^2 dependence of $\langle r^2 \rangle_{V,S}$

4.2 q^2 dependence : $F_S(q^2)$



with our statistical accuracy ...

- can be fitted to cubic / quartic forms w/ reasonable χ^2

$$F_S(q^2) = 1 + \frac{\langle r^2 \rangle_S}{6} q^2 + c_S (q^2)^2 + d (q^2)^3 + e (q^2)^4$$

- cubic and quartic fits $\Rightarrow$ consistent results for $\langle r^2 \rangle_S$
  $\Rightarrow$ ill-determined c_S ($\gtrsim$ 100 % error) ...
- $\langle r^2 \rangle_S$ from cubic fit $\Rightarrow$ the following analysis

5.1 chiral extrapolation : w/ NLO ChPT formulae

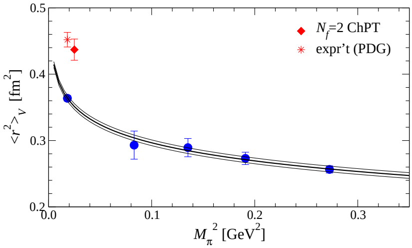
charge radius $\langle r^2 \rangle_V$

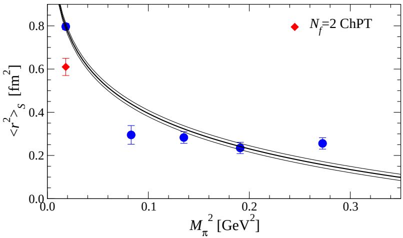
$$\langle r^2 \rangle_V = -(1/NF^2)(1 + N l_6^r) -(1/NF^2)\ln[M_\pi^2/\mu^2]$$

scalar radius $\langle r^2 \rangle_S$

$$\langle r^2 \rangle_S = (1/NF^2)(-13/2 + 6N l_4^r) -(6/NF^2)\ln[M_\pi^2/\mu^2]$$

$(N = (4\pi)^2; \quad \mu = 4\pi F; \quad$ use $F = 79.0(^{+5.0}_{-2.6})$ MeV from F_π *(JLQCD/TWQCD, 2008)*)





- acceptable χ^2/dof ~ 0.3
- $\langle r^2 \rangle_V = 0.364(1)$ fm^2 at m_{ud}

  $\Leftrightarrow$ expr't+ChPT : 0.437(16) fm^2

  (Bijnens et al., 1998)

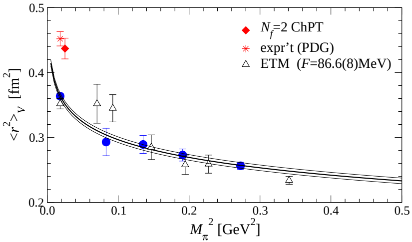
- unacceptable χ^2/dof ~ 9
- $\langle r^2 \rangle_S = 0.797(15)$ fm^2 at m_{ud}

  $\Leftrightarrow$ expr't+ChPT : 0.61(4) fm^2

  (Colangelo et al., 2001)

5.1 chiral extrapolation : w/ NLO ChPT formulae

- recent calculation of $\langle r^2 \rangle_V$ in $N_f = 2$ QCD by ETM *(ETM, 2008)*
  twisted mass quarks, $a = 0.09$ fm, $L = 2.2$ fm, twisted boundary



NLO analysis
$\Rightarrow \quad \langle r^2 \rangle_V = 0.352(8)$ fm

$\Rightarrow$ failure of NLO fit : not be due to $a \neq 0$, FVC, ... (due to $N_f = 2$?)

- q^2 dep. of $F_V(q^2)$: NNLO contribution is not small at $|q^2| \gtrsim (150\,\text{MeV})^2$
  $\Rightarrow$ significant NNLO contribution in m_q dep. of $\langle r^2 \rangle_V$ at $M_\pi \gtrsim 150$ MeV (?)
- $O(q^4)$ dep. of F_V $\Rightarrow$ c_V $\Rightarrow$ NNLO ChPT

5.2 chiral extrapolation : w/ NNLO ChPT formulae

NNLO formulae *(Gasser-Meißner, 1991; Bijnens-Colangelo-Talavera, 1998)*

$$\langle r^2\rangle_V = -\frac{1}{NF^2}\left(1+6N{l_6}^r\right) - \frac{1}{NF^2}\ln\left[\frac{M_\pi^2}{\mu^2}\right]$$

$$\frac{1}{N^2F^4}\left(\frac{13N}{192}-\frac{181}{48}+6N^2 r_{V,1}^r\right)M_\pi^2 + \frac{1}{N^2F^4}\left(\frac{19}{6}-12Nl_{1,2}^r\right)M_\pi^2\ln\left[\frac{M_\pi^2}{\mu^2}\right]$$

$$\langle r^2\rangle_S = \frac{1}{NF^2}\left(-\frac{13}{2}+6Nl_4^r\right) - \frac{6}{NF^2}\ln\left[\frac{M_\pi^2}{\mu^2}\right]$$

$$\frac{1}{N^2F^4}\left(-\frac{23N}{192}+\frac{869}{108}+88Nl_{1,2}^r+80Nl_2^r+5Nl_3-24N^2l_3^rl_4^r+6N^2r_S^r\right)M_\pi^2$$

$$+\frac{1}{N^2F^4}\left(-\frac{323}{36}+124Nl_{1,2}^r+130Nl_2^r\right)M_\pi^2\ln\left[\frac{M_\pi^2}{\mu^2}\right] - \frac{65}{3N^2F^4}M_\pi^2\ln\left[\frac{M_\pi^2}{\mu^2}\right]^2$$

$$c_V = \frac{1}{60NF^2}\frac{1}{M_\pi^2} + \frac{1}{N^2F^4}\left(\frac{N}{720}-\frac{8429}{25920}+\frac{N}{3}l_{1,2}^r+\frac{N}{6}l_6^r+N^2r_{V,2}^r\right)$$

$$+\frac{1}{N^2F^4}\left(\frac{1}{108}+\frac{N}{3}l_{1,2}^r+\frac{N}{6}l_6^r\right)\ln\left[\frac{M_\pi^2}{\mu^2}\right]+\frac{1}{72N^2F^4}\ln\left[\frac{M_\pi^2}{\mu^2}\right]^2$$

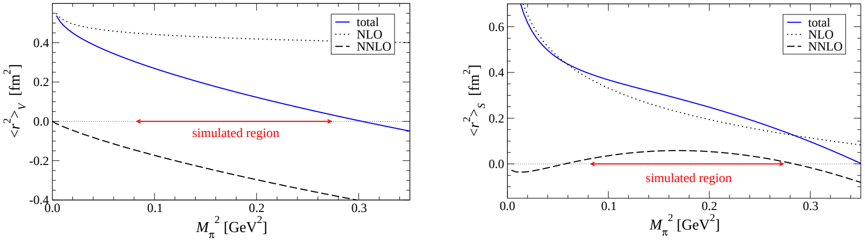
$$l_{1,2}^r = l_1^r - l_2^r/2$$

5.2 chiral extrapolation : w/ NNLO ChPT formulae

an exercise

$\langle r^2 \rangle_{V,S}$ at NNLO w/ phenomenological estimates of LECs

- $F = F_\pi / 1.067$ from *Colangelo-Dürr, 2004*
- $O(p^4)$ couplings l_i^r from *Bijnens et al., 1998*, or *Colangelo et al., 2001*
  $\bar{l}_6 = 16.0, \quad \bar{l}_4 = 4.39, \quad \bar{l}_1 = -0.36, \quad \bar{l}_2 = 4.31, \quad \bar{l}_3 = 4.39$
- $O(p^6)$ couplings $r_{X,i}^r$ from *Bijnens et al., 1998* ($\Leftarrow$ resonance saturation)
  $r_{V,1} = 2.5 \times 10^{-4}, \quad r_{V,2} = 2.6 \times 10^{-4}, \quad r_S = -3.0 \times 10^{-5}$

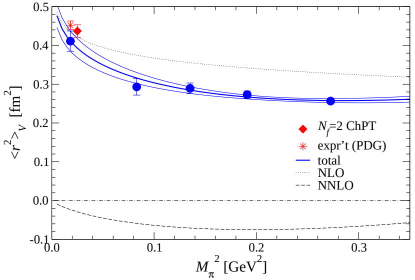


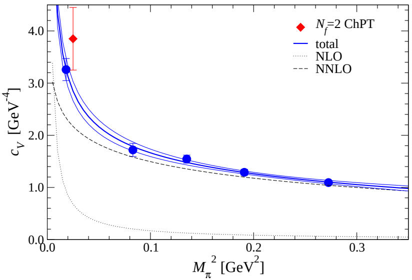
NNLO contribution may modify M_π^2 dependence significantly

5.2 chiral extrapolation : w/ NNLO ChPT formulae

simultaneous fit to $\langle r^2 \rangle_V$ and c_V

(only) 4 parameters for 8 data ; l_6^r, $l_{1,2}^r$, $r_{V,1}$, $r_{V,2}$ $(l_{1,2}^r = l_1^r - l_2^r/2)$





- describe our data w/ $\chi^2/\text{dof} = 0.7$
- consistent with expr't (with larger errors than NLO analysis...)

$$\langle r^2 \rangle_V = 0.411(26)\ \text{fm}^2, \quad c_V = 3.26(21)\text{GeV}^{-4}$$

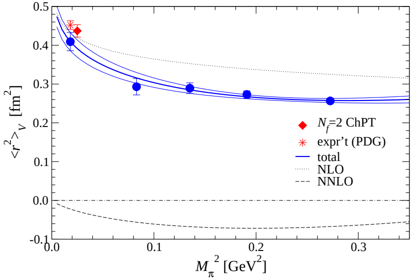
$\Leftrightarrow$ $c_V\,[\text{GeV}^{-4}] = 3.85(60)$ *(Bijnens et al., 1998)*; 4.0(5) *(Guo et al., 2008)*;
3.5 – 4.0 *(Ananthanarayan-Ramaman, 2008)*

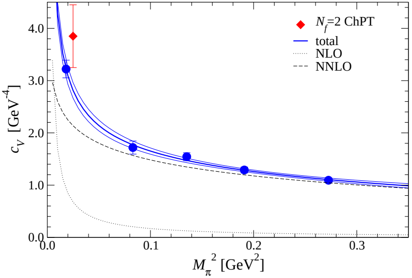
- w/o phenomenological inputs

5.2 chiral extrapolation : w/ NNLO ChPT formulae

simultaneous fit to $\langle r^2 \rangle_V$, $\langle r^2 \rangle_S$ and c_V

- inclusion of $\langle r^2 \rangle_S$ $\Rightarrow$ additional parameters : l_4^r, l_1^r (or l_2^r), l_3^r, r_S^r
  - fix $\bar{l}_2 = 4.31(11)$ *(Colangelo et al., 2001)*, $\bar{l}_3 = 3.38(56)$ *(JLQCD/TWQCD, 2008)*
  - free parameters : l_4^r ($\Leftrightarrow$ l_4^r from F_π) and (poorly known) r_S^r
- 6 fit parameters for 12 data : l_6^r, l_4^r, $l_{1,2}^r$, $r_{V,1}^r$, $r_{V,2}^r$ r_S^r

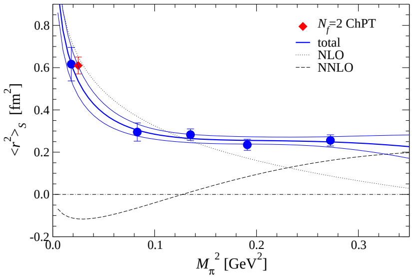




- results for vector channel : $\langle r^2 \rangle_V$, c_V, l_6^r, $l_{1,2}^r$, $r_{V,1}$, $r_{V,2}$

  inclusion of r_S $\Rightarrow$ does not change significantly

5.2 chiral extrapolation : w/ NNLO ChPT formulae



- $\chi^2/\mathrm{dof} = 1.3$
- $r_{V,S}$, c_V
  - reasonable accuracy
  - consistent w/ expr't

systematic uncertainties

- chiral extrap. : repeat whole analysis w/o data at largest m
- input for l_2^r, l_3^r : shifted by their uncertainty
- input to fix scale : test $r_0 = 0.47$ fm *(MILC, 2004)*
- discretization error : $O((a\Lambda)^2) \sim 3\,\%$

5.2 chiral extrapolation : w/ NNLO ChPT formulae

$\langle r^2\rangle_V = 0.409(23)(37)\ \mathrm{fm}^2$, $\langle r^2\rangle_S = 0.617(79)(66)\ \mathrm{fm}^2$, $c_V = 3.22(17)(36)\ \mathrm{GeV}^{-4}$

- consistent w/ experiment w/ 10 – 15 % accuracy
- largest uncertainties : i) input to fix scale (r_0), ii) chiral fit

$\bar{l}_6 = 11.9(0.7)(1.0)$

$\Leftrightarrow$ $\bar{l}_6 = 16.0(0.9)$ *(Bijnens et al., 1998; $F_{V,S}$)*, $15.22(39)$ *(Gonz'alez-Alonso et al., 2008; τ)*

$\bar{l}_4 = 4.09(50)(52)$

$\Leftrightarrow$ $\bar{l}_4 = 4.12(56)$ *(JLQCD/TWQCD, 2008; F_π)*, $\bar{l}_4 = 4.39(22)$ *(Colangelo et al., 2001)*

$\bar{l}_1 - \bar{l}_2 = -2.9(0.9)(1.3)$ $\Leftrightarrow$ $\bar{l}_1 - \bar{l}_2 = -4.67(60)$ *(Colangelo et al., 2001)*

- largest uncertainty : chiral fit
- consistent w/ lattice / phenomenological estimates

  except l_6 $\Leftrightarrow$ $F = 79$ MeV slightly smaller than phenomenology

$O(p^6)$ couplings at $\mu = 4\pi F$

$r^r_{V,1} = -1.0(1.0)(2.5) \times 10^{-5}$, $r^r_{V,2} = 4.00(17)(64) \times 10^{-5}$, $r_S = 1.74(36)(78) \times 10^{-4}$

6. summary

pion form factors in $N_f = 2$ lattice QCD

- exact chiral symmetry
  - direct comparison w/ (continuum) ChPT
- all-to-all propagators
  - accurate determination of $F_{V,S}(q^2)$
  - (the 1st) calculation of $F_S(q^2)$ w/ disconnected diagrams
- q^2 dependence
  - $O(q^6)$ contribution is not small at $|q^2| \gtrsim (550\,\mathrm{MeV})^2$
  - generic polynomial form (and pole contribution for $F_V(q^2)$)
- chiral fit
  - $O(p^2)$ ChPT fails to reproduce $\langle r^2 \rangle_S$ at $300 \lesssim M_\pi[\mathrm{MeV}] \lesssim 500$
  - chiral fit based on $O(p^4)$ ChPT $\Rightarrow$ $\langle r^2 \rangle_{V,S}$, c_V w/ 10 – 15% accuracy
- future directions
  - extension to $N_f = 3$: on-going
  - better control of q^2 interpolation : twisted boundary condition; dispersive bound; model indep. information of scalar resonance(s) at simulated m