Electromagnetic structure of the lowest-lying baryons in covariant chiral perturbation theory

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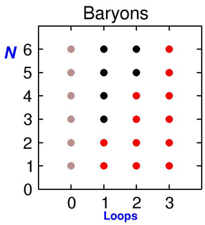
OUTLINE

Baryon ChPT and power counting: Problems & Solutions



- Baryon mass M_0: New large scale
- Diagrams with arbitrarily large number of loops contribute to lower orders
  −>**Power Counting is lost!**
  (Gasser et al.)

- Heavy Baryon χPT (Jenkins & Manohar):
  - Non-relativistic expansion: Considers $M_0 \simeq \Lambda_{ChSB}$;
  - Recovers the power counting pattern of meson χPT.
- Relativistic Baryon χPT:
  - Power counting breaking pieces: Analytical structure!;
  - Two remarkable schemes:
    - Infrared baryon χPT (Becher and Leutwyler);
    - EOMS-scheme (Gegelia, Japaridze and Scherer).

Inclusion of the Decuplet-resonances

- **Motivation**: We perform perturbations on $m_K/\Lambda_{\chi SB}$ ~0.5 that is over the scale for the onset of Decuplet resonances $\frac{M_D - M_B}{\Lambda_{\chi SB}}$ ~0.3.
- Problem of consistency
  - Rarita-Schwinger (RS) representation of relativistic 3/2-fermions: $\psi^\mu(x)$. RS is a field with 16 components of which only 8 (4 massless) are "physical".
  - How to introduce couplings to the RS spinor that don't activate 1/2 modes? Field-redefinition formalism: Consistent couplings "equivalent" to phenomenological ones Pascalutsa et al., 1999.
- Problem of higher-order divergencies

$$s^{\alpha\beta}(p) = \frac{\not{p}+m}{m^2-p^2}\left[g^{\alpha\beta} - \frac{1}{D-1}\gamma^\alpha\gamma^\beta - \frac{1}{(D-1)m}(\gamma^\alpha p^\beta - \gamma^\beta p^\alpha) - \frac{D-2}{(D-1)m^2}p^\alpha p^\beta\right];$$

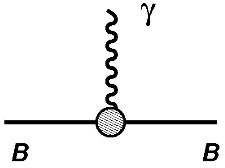
  - RS propagator has a problematic high-energy behavior.
  - Higher-order ∞'s regularized in $\overline{MS}$ $\rightarrow$Regularization-scale (μ) dependence.
- Problem of power-counting breaking
  - We use the EOMS-scheme and also obtain the HB limit (ϵ-expansion)

χPT and the baryon-octet MM: A historical introduction

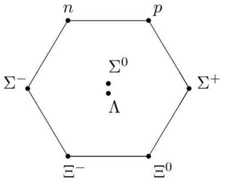
- Exact SU(3) symmetry: Coleman-Glashow relations (1961):
  - Relate the MM to 2 parameters, i.e. the measured proton and neutron MM;
  - Fit to data: Succesful although indicates sizable SU(3)-breaking effects.
- Leading order contributing to SU(3)-breaking fails to give a good description of data in different χPT approaches
  - The same 2 parameters of the SU(3)-symmetric description;
  - Pionnering works of Caldi and Pagels on leading chiral corrections (1974).
  - Systematic HBχPT calculations of Jenkins and Manohar (1993) and Steininger and Meissner (1997).
  - Infrared relativistic BχPT by Kubis and Meissner (2001).
- Calculations up to **next-to-leading** SU(3)-breaking have become standard:
  - Good description with 7 parameters (for 8 measured MM);
  - Inclusion of the decuplet explored Puglia and Musolf (2000).
- Relativistic EOMS-BχPT: improved **leading** SU(3)-breaking pattern$\Longrightarrow$ L.S.Geng, JMC, L. Alvarez-Ruso, M.J. Vicente Vacas, PRL **101**,222002 (2008)

Magnetic moments (MM) of the baryon octet: Introduction

- General γBB vertex



- SU(3)-flavor symmetry

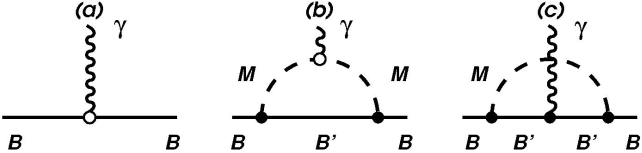


- The general vertex can be parameterized by two form factors: $\langle\psi(p')|J^\mu|\psi(p)\rangle = |e|\bar{u}(p')\left\{\gamma^\mu F_1(t) + \frac{i\sigma^{\mu\nu}q_\nu}{2M}F_2(t)\right\}u(p)$.
- At $q^2 = 0$: $F_1(0) = Q$ (charge), $F_2(0) = \kappa$ (anomalous magnetic moment).
- The fermion (baryon) MM: $\mu \equiv (Q+\kappa)\frac{|e|}{2M}$
- SU(3)-flavor symmetry$\Rightarrow$Coleman-Glashow relations:

$$\mu_{\Sigma^+} = \mu_p, \qquad \mu_\Lambda = \frac{1}{2}\mu_n, \qquad \mu_{\Xi^0} = \mu_n,$$
$$\mu_{\Sigma^-} = -(\mu_n+\mu_p), \qquad \mu_{\Xi^-} = \mu_{\Sigma^-}, \qquad \mu_{\Lambda\Sigma^0} = -\frac{\sqrt{3}}{2}\mu_n,$$

- The octet-baryons MM are very well measured quantities!

Baryon octet MM: Analytical results at NLO



- SU(3)-symmetric description parameterized by b_6^D and b_6^F:

$$\kappa_B^{(2)} = \alpha_B\, b_6^D + \beta_B\, b_6^F$$

- SU(3)-breaking provided by loop-functions $H^{(b)}(m)$ and $H^{(c)}(m)$:

$$\kappa_B^{(3)} = \frac{1}{8\pi^2 F_\phi^2}\left(\sum_{r=\pi,K} \xi_{BM}^{(b)} H^{(b)}(m_r) + \sum_{r=\pi,K,\eta} \xi_{BM}^{(c)} H^{(c)}(m_r)\right)$$

$$H^{(b)}(m) = -M^2 + 2m^2 + \frac{2m(m^4 - 4m^2M^2 + 2M^4)}{M^2\sqrt{4M^2-m^2}} \arccos\left(\frac{m}{2M}\right) + \frac{m^2}{M^2}(2M^2 - m^2)\log\left(\frac{m^2}{M^2}\right),$$

$$H^{(c)}(m) = M^2 + 2m^2 + \frac{2m^3(m^2 - 3M^2)}{M^2\sqrt{4M^2-m^2}} \arccos\left(\frac{m}{2M}\right) + \frac{m^2}{M^2}(M^2 - m^2)\log\left(\frac{m^2}{M^2}\right).$$

- α_B, β_B and $\xi_{BM}^{(b,c)}$ depend on known MBB and Clebsch-Gordan coefficients.

Baryon octet MM: Renormalization at NLO

- **EOMS**: Power counting breaking pieces M_B^2 absorbed by b_6^D and b_6^F:

$$b_6^D \longrightarrow \tilde{b}_6^D = b_6^D + \frac{3DFM_B^2}{2\pi^2 F_\phi^2}, \quad b_6^F \longrightarrow \tilde{b}_6^F = b_6^F$$

$$H^{(b)} \longrightarrow \tilde{H}^{(b)} = H^{(b)} + M_B^2, \quad H^{(c)} \longrightarrow \tilde{H}^{(c)} = H^{(c)} - M_B^2.$$

- **HB** results obtained setting $M_B \sim \Lambda_{\chi SB}$ in **EOMS** results:

$$\tilde{H}^{(b)}(m) \simeq \pi m M_B + \mathcal{O}(p^2), \qquad \tilde{H}^{(c)}(m) \simeq \mathcal{O}(p^2).$$

- **IR** results obtained substracting from $H^{(b,c)}$ the corresponding regular parts:

$$\begin{aligned} R^{(b)}(m) &= -M_B^2 + \frac{19m^4}{6M_B^2} - \frac{2m^6}{5M_B^4} - \frac{m^8}{21M_B^6} + \ldots , \\ R^{(c)}(m) &= M_B^2 + 2m^2 + \frac{5m^4}{2M_B^2} - \frac{m^6}{2M_B^4} - \frac{m^8}{15M_B^6} + \ldots \end{aligned}$$

- Loops depend on physical meson masses, $M_0 \simeq 0.940$ GeV and $F_\phi \simeq 1.17 F_\pi$.

Baryon-Octet MM: Numerical results at NLO

	p	n	Λ	Σ^-	Σ^0	Σ^+	Ξ^-	Ξ^0	$\Lambda\Sigma^0$	$\tilde{\chi}^2$
C-G	2.56	-1.60	-0.80	-0.97	0.80	2.56	-0.97	-1.60	1.38	0.46
HB	3.01	-2.62	-0.42	-1.35	0.42	2.18	-0.52	-0.70	1.68	1.01
IR	2.08	-2.74	-0.64	-1.13	0.64	2.41	-1.17	-1.45	1.89	1.83
EOMS	2.58	-2.10	-0.66	-1.10	0.66	2.43	-0.95	-1.27	1.58	0.18
Exp.	2.79	-1.91	-0.61	-1.16	—	2.46	-0.65	-1.25	1.61	—

$$\tilde{\chi}^2 = \sum (\mu_{th} - \mu_{expt})^2$$

- Study of the convergence of the chiral series (LO and NLO):

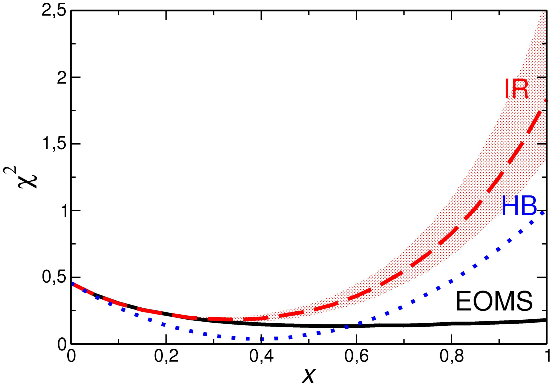
$$\mu_p = 3.47\,(1 - 0.257)\,, \quad \mu_n = -2.55\,(1 - 0.175)\,, \quad \mu_\Lambda = -1.27\,(1 - 0.482)\,,$$

$$\mu_{\Sigma^-} = -0.93\,(1 + 0.187)\,, \quad \mu_{\Sigma^+} = 3.47\,(1 - 0.300)\,, \quad \mu_{\Sigma^0} = 1.27\,(1 - 0.482)\,,$$

$$\mu_{\Xi^-} = -0.93\,(1 + 0.025)\,, \quad \mu_{\Xi^0} = -2.55\,(1 - 0.501)\,, \quad \mu_{\Lambda\Sigma^0} = 2.21\,(1 - 0.284)\,.$$

- Reasonable convergence of chiral series: Corrections $\lesssim m_K/\Lambda_{ChSB} \sim 50\%$.
- The EOMS NLO-calculation improves the C-G relations!

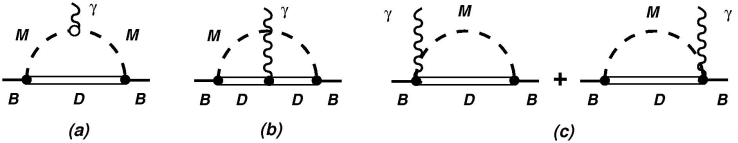
Baryon octet MM: Graphical comparison



$x \equiv m/m_{phys}$ with m the meson masses

- The three approaches agree in the vicinity of the chiral limit.
- IR and EOMS coincide up to $x \sim 0.4$. IR description then get worse.
- Shaded area(s) represent the variation 0.8GeV$\leq M_0 \leq$ 1.1GeV.
- EOMS provides a realistic SU(3)-breaking mechanism for MM!

Baryon octet MM:Inclusion of the Decuplet-resonances



	p	n	Λ	Σ^-	Σ^0	Σ^+	Ξ^-	Ξ^0	$\Lambda\Sigma^0$	$\tilde{\chi}^2$
C-G	2.56	-1.60	-0.80	-0.97	0.80	2.56	-0.97	-1.60	1.38	0.46
HB-O	3.01	-2.62	-0.42	-1.35	2.18	0.42	-0.70	-0.52	1.68	1.01
HB-OD	3.47	-2.84	-0.17	-1.42	1.77	0.17	-0.41	-0.56	1.86	2.58
C-O	2.60	-2.16	-0.64	-1.12	0.64	2.41	-0.93	-1.23	1.58	0.18
C-OD	2.61	-2.23	-0.60	-1.17	0.59	2.37	-0.92	-1.22	1.65	0.22
Exp.	2.79	-1.91	-0.61	-1.16	—	2.46	-0.65	-1.25	1.61	—

- **Par.** $\mathcal{C}$=1.0, M_B =1.151 GeV, M_D = 1.428 GeV, $F_\phi \simeq 1.17 F_\pi$, μ=1 GeV
- The problem of consistency has also been investigated:

  L.S.Geng, JMC, M.J. Vicente Vacas, PLB **676**,63 (2009)

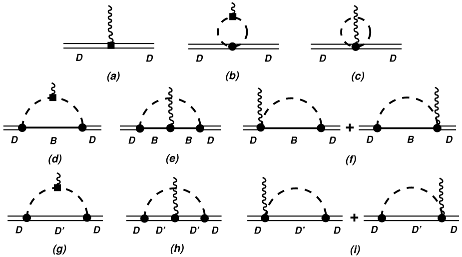
EM structure of decuplet-baryons at NLO in BχPT

- **Motivation**: Electromagnetic structure of $\Delta(1232)$
  - Experiments to measure MDMs of Δ^{++} and Δ^{+} Kotulla, Pr.Nuc.Phys (2008)
  - Increasing effort to calculate in lQCD
    Leinweber et al. (1992), Lee et al. (2005), Aubin et al. (2008), Alexandrou et al. (2009), Boinepalli et al. (2009),...
  - Theoretical predictions: Quark models, QCD sum rules, large N_c, EFT,...
- **Goal**: Predict the EM structure of decuplet ($\Delta(1232)$) with covariant BχPT

$$\langle T(p')|J^\mu|T(p)\rangle = -\bar{u}_\alpha(p')\left\{\left[F_1^*(\tau)\gamma^\mu + \frac{i\sigma^{\mu\nu}q_\nu}{2M_D}F_2^*(\tau)\right]g^{\alpha\beta} + \left[F_3^*(\tau)\gamma^\mu + \frac{i\sigma^{\mu\nu}q_\nu}{2M_D}F_4^*(\tau)\right]\frac{q^\alpha q^\beta}{4M_D^2}\right\}u_\beta(p)$$

- $F_i^*(0) \Rightarrow$EM static observables: μ (MDM), $\mathcal{Q}$ (EQM) and O (MOM)
- Up-to $\mathcal{O}(p^3)$: one LEC appears for the MDMs and one for the EQMs
  - LEC for MDMs fixed with $\mu_{\Omega^-} = -2.02\ \mu_N$
  - LEC for EQMs could be fixed with lQCD result for $\mathcal{Q}_{\Omega^-}$
  - MOMs come as a prediction

Predictions of BχPT on the MDMs of the decuplet at NLO



	Δ^{++}	Δ^{+}	Δ^{0}	Δ^{-}	Σ^{*+}	Σ^{*0}	Σ^{*-}	Ξ^{*0}	Ξ^{*-}
EOMS	6.04	2.84	-0.36	-3.56	3.07	0	-3.07	0.36	-2.56

Values in the table in μ_N

- The results for the Δ^{++} and Δ^{+} are compatible with PDG values:

$$\mu_{\Delta^{++}} = 5.6 \pm 1.9\mu_N, \qquad \mu_{\Delta^{+}} = 2.7^{+1.0}_{-1.3} \pm 1.5 \pm 3\mu_N$$

- The HB result for Δ^{++} is $\mu_{\Delta^{++}} = 7.94\mu_N$
- More details and results for EQMs, MOMs and charge radii:

  L.S.Geng, JMC, M.J. Vicente Vacas arXiv:0907.0631 [hep-ph]

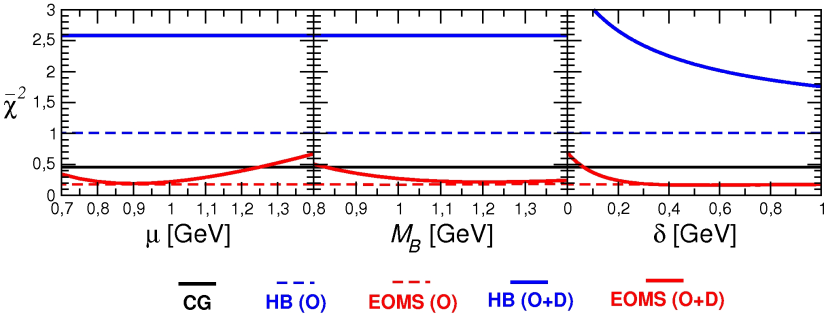
Conclusions

- NLO fully relativistic BχPT (EOMS) calculation of Baryon octet MMs:
  - Incorporates (higher-order) **relativistic corrections**.
  - Consistent with **analyticity**
  - Provides a realistic SU(3)-breaking pattern of baryon-octet MM.
  - The NLO improvement prevails when decuplet resonances are included.
  - The comparison with HB and IR suggest that in SU(3)-BχPT to keep proper analytic properties and full covariance is of the most importance
- NLO calculation of the decuplet MDMs
  - One LEC fixed with μ_{Ω^-} $\Rightarrow$Predictions on μ_{Δ^+} and $\mu_{\Delta^{++}}$:

$$\mu_{\Delta^{++}} = 6.04\mu_N; \qquad \mu_{\Delta^+} = 2.84\mu_N$$

  - The LEC ruling the EQM can be fixed using lQCD result for Ω^-
  - MOMs come as prediction

Baryon octet MM: Uncertainties in decuplet contributions



- Improvement over CG for 0.7 GeV$\leq \mu \leq$1.3 GeV
- Smooth dependence on the average baryon mass ($\delta = 0.231$ GeV)
- The decuplet contributions vanish at $\delta \to \infty$: Decoupling of the decuplet
- Covariant O.+D. NLO-calculation improves the C-G relations!