Kaons on the Lattice

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Thanks

- Thanks to the organisers of the Workshop for the invitation to present this talk.
  - Close collaboration between the lattice and ChPT communities is very important for the development of our understanding of non-perturbative QCD effects in flavour physics and hadronic structure.
- Thanks to my colleagues from the RBC/UKQCD Collaboration with whom I have developed much of my understanding of the material of the talk.
  - The talk is designed to be more general than the work of RBC/UKQCD, although I will tend to use our data to illustrate the main ideas.
  - See the talks by Peter Boyle and Norman Christ at Kaon 2009.
- Thanks to my colleagues from Flavianet, and the Flavianet Lattice Averaging group in particular, whose preliminary results I use below.
  G. Colangelo, S. Dürr, A. Jüttner, L. Lellouch, H. Leutwyler, V. Lubicz, S. Necco, C. Sachrajda, S. Simula, A. Vladikas, U. Wenger, H. Wittig.
  - See the talk by Laurent Lellouch at Lattice 2008, arXiv:0902.4545.

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FLAG – Preliminary

- We have the following two precise experimental results:

$$\left|\frac{V_{us} f_K}{V_{ud} f_\pi}\right| = 0.27599(59) \quad \text{and} \quad |V_{us} f_+(0)| = 0.21661(47)$$

Flavianet – arXiv:0801.1817

- We can view these as two equations for the four unknowns f_K/f_π, $f_+(0)$, V_{us} and V_{ud}.
- Within the Standard Model we also have the unitarity constraint:

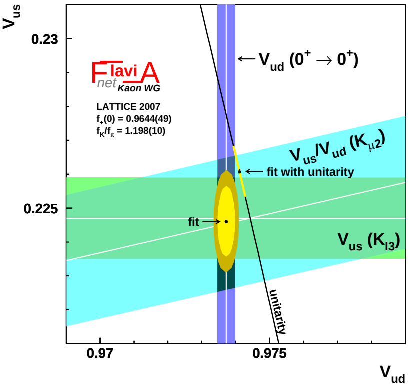
$$|V_{ud}|^2 + |V_{us}|^2 \cancel{+ |V_{ub}|^2} = 1$$

- Thus we now have 3 equations for four unknowns.
- There has been considerable work recently in updating the determination of V_{ud} based on 20 different superallowed transitions. Hardy and Towner, arXiV:0812.1202

$$|V_{ud}| = 0.97425(22)\,.$$

- If we accept this value then we are able to determine the remaining 3 unknowns:

$$|V_{us}| = 0.22544(95), \quad f_+(0) = 0.9608(46), \quad \frac{f_K}{f_\pi} = 1.1927(59)\,.$$



Flavianet Kaon WG

$$f_+^{K\pi}(0) \;=\; 0.9644(33)(34)$$
$$\Rightarrow \quad |V_{us}| = 0.2247(12)$$

$$\frac{f_K}{f_\pi} \;=\; 1.198(10)$$
$$\Rightarrow \quad |V_{us}| = 0.2241(24)$$

A.Jüttner, Lattice 2007

Our final result from the $K_{\ell 3}$ project is

$$f_+^{K\pi}(0) = 0.964(5)\,.$$

P.A.Boyle et al. [RBC&UKQCD – arXiv:0710.5136 [hep-lat]]

- Several collaborations are performing simulations in kaon physics, including

Collaboration	Quark Action	N_f	L (fm)	$m_\pi^{\min}$ MeV	a^{-1} GeV
JLQCD	Overlap	2+1	1.7	310	1.8
RBC/UKQCD	DWF	2+1	1.85, 2.8	330	1.7,2.3
ETMC	Twisted Mass	2	2.0	300	1.9, 2.2,2.8
PACS-CS	Clover	2+1	3.1	156	2.2
BMW	Smeared Clover	2+1	4.0	190	1.6,2.3,3.0
MILC	Staggered	2+1	2.4, 2.9, 3.4	320	1.1,1.3,1.6, 2.2,3.3,4.4

- This incomplete and rapidly-changing table illustrates the wide range of simulations being performed.
- The different actions have difference chiral and flavour properties. The Rolls-Royce Overlap and DWF formulations are considerably more expensive in computing resources.
- BMW and PACS-CS are very close to the physical point.

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- There are also a number of groups who use ensembles from others, sometimes with different formulations for the valence quarks (mixed-actions).

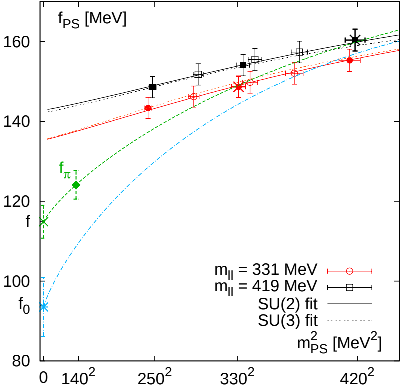
  The MILC ensembles in particular have been used by a number of other groups.

  For a nice review see A.Bazavov et al., arXiv:0903.3598.

- The RBC/UKQCD results with $a^{-1}=2.3$ GeV are still preliminary and I will not present them here.

2. Comments on Chiral Behaviour

Comparison of Results obtained using SU(2) and SU(3) ChPT



RBC/UKQCD, arXiv:0804:0473

- Study is performed at NLO in the chiral expansion.
- black points - partially quenched results with $am_l = 0.01$ ($m_\pi^{\text{unitary}} \simeq 420\,$MeV).
- red points - partially quenched results with $am_l = 0.005$ ($m_\pi^{\text{unitary}} \simeq 330\,$MeV).
- We find:

$$f_\pi/f \simeq 1.08\,, \quad f/f_0 = 1.23(6)\,.$$

- The corresponding results from the MILC collaboration, who do an NNLO analysis in staggered chiral perturbation theory, with NNNLO analytic terms:

$$f_\pi/f = 1.052(2)\left(\begin{matrix}+6\\-3\end{matrix}\right)\,, \quad f/f_0\,{}_{\text{MILC}} = 1.15(5)\left(\begin{matrix}+13\\-3\end{matrix}\right)\,,$$

- The large value of f_π/f_0 (and even larger values of f_{PS}/f_0 of ~ 1.6 where we have data) lead RBC/UKQCD (and ETMC) to present results based on SU(2)$\times$ SU(2) ChPT.

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- PACS-CS however, find a larger value for f_0 (and smaller value of f/f_0).

Collaboration	f_0	$\frac{f}{f_0}$
MILC	106(5) MeV	1.15(5) $\binom{+13}{-3}$
RBC/UKQCD	93.5(7.3) MeV	1.229(59)
PACS-CS	118.5(9) MeV	1.078(59)

- As more collaborations do these analyses, the chiral behaviour of f_π will be further clarified.
- Of particular interest will be the MILC study with a light strange quark.

- NLO SU(3)$\times$ SU(3) chiral fits to the pseudoscalar masses and decay constants work well, but only at very light masses.
- Extending the analysis to higher orders may increase the range of the good fits.
  - The number of new LECs increases so that more lattice data is required.
  - MILC performs the analysis at NNLO together with NNNLO analytic terms “... we must include NNNLO analytic terms to get good fits."

A.Bazavov et al., arXiv:0903.3598

- In view of the large chiral corrections which we find for the pseudoscalar decay constant we (and some other collaborations) prefer to present our results using SU(2) ChPT where possible. (See also $K \to \pi\pi$ discussion below.)
- The dynamics of the interaction between the Lattice and ChPT communities has changed radically.
  - We continue of course to use ChPT to guide the chiral extrapolations.
  - The data is becoming sufficiently accurate that the Low Energy Constants of ChPT are being evaluated with unprecedented precision.
    (This is not surprising since we have the powerful tool of being able to vary the quark masses.)
  - For the ChPT results to be most useful to us, they should be presented in terms of the mass-independent LECs with the mass dependence explicitly exhibited.

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- Applying $SU(2)\times SU(2)$ χPT transformations to kaons, only the u and d quarks transform $\Rightarrow$ χPT formalism must be extended.
- Roessl has introduced the corresponding Lagrangian for the interactions of kaons and pions in order to study $K\pi$ scattering near threshold.

A.Roessl, hep-ph/9904230

- $\xi = \exp(i\phi/f) \Rightarrow L\xi U^\dagger = U\xi R^\dagger$ and $K=(K^+,K^0)^{\mathrm{T}} \to UK$.
- There are overlaps with Heavy Meson Chiral Perturbation Theory, but an important difference is that $m_{K^*} \neq m_K$, whereas in the heavy quark limit $m_{B^*} = m_B$.

M.B.Wise, Phys.Rev **D45** (1992) 2188; G.Burdman and J.Donoghue, Phys.Lett. **B280** (1992) 287

- We have derived the chiral behaviour of m_K^2, f_K and B_K in the unitary and partially quenched theories and have used the results in our phenomenological studies.

RBC/UKQCD, arXiv:0804:0473

  - (See also the discussion of $K_{\ell 3}$ decays below.)

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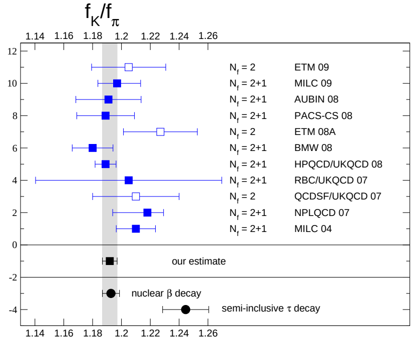
- The PQ formulae for the kaons take the form

$$\begin{aligned} m_{xy}^2 &= B^{(K)}(m_h)m_y\left(1+\frac{\lambda_1(m_h)}{f^2}\chi_l+\frac{\lambda_2(m_h)}{f^2}\chi_x\right) \\ f_{xy} &= f^K(m_h)\left(1+\frac{\lambda_3(m_h)}{f^2}\chi_l+\frac{\lambda_4(m_h)}{f^2}\chi_x - \right. \\ &\quad \left. \frac{1}{(4\pi f)^2}\left[\frac{\chi_x+\chi_l}{2}\log\frac{\chi_x+\chi_l}{2\Lambda_\chi^2}+\frac{\chi_l-2\chi_x}{4}\log\frac{\chi_x}{\Lambda_\chi^2}\right]\right) \end{aligned}$$

  where the χ are proportional to the quark masses ($\chi = 2Bm$), m_x and m_l (m_y and m_h) are the valence and sea light-quark (strange-quark) quark masses.
- The LECs depend on both m_h and m_y; for compactness of notation I exhibit only the m_h dependence explicitly.
- The formulae for B_K will be exhibited in the corresponding section.

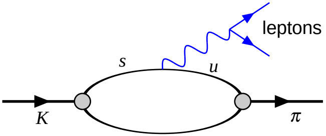


$$\text{MILC(09)}: \quad \frac{f_K}{f_\pi} = 1.197(3)\begin{pmatrix}+6\\-13\end{pmatrix} \quad \Rightarrow \quad V_{us} = 0.2247(7)\begin{pmatrix}+23\\-11\end{pmatrix}$$

A.Bazavov et al., arXiv:0903.3598

$$\text{HPQCD/UKQCD(08)}: \quad \frac{f_K}{f_\pi} = 1.189(2)(7) \quad \Rightarrow \quad V_{us} = 0.2261(6)(13)$$

E.Follana et al., arXiv:0706.1726



$$\langle \pi(p_\pi)\,|\bar{s}\gamma_\mu u\,|\,K(p_K)\,\rangle = f_0(q^2)\,\frac{M_K^2 - M_\pi^2}{q^2} q_\mu + f_+(q^2)\,\left[(p_\pi + p_K)_\mu - \frac{M_K^2 - M_\pi^2}{q^2} q_\mu\right]$$

where $q \equiv p_K - p_\pi$.

To be useful in extracting V_{us} we require $f_0(0) = f_+(0)$ to better than about 1% precision.

$$\chi\text{PT} \Rightarrow f_+(0) = 1 + f_2 + f_4 + \cdots \quad \text{where} \quad f_n = O(M^n_{K,\pi,\eta}).$$

Reference value $f_+(0) = 0.961 \pm 0.008$ where $f_2 = -0.023$ is well known from χPT and $f_4, f_6, \cdots$ are obtained from models. Leutwyler & Roos (1984)

1% precision of $f^+(0)$ is conceivable because it is actually $1-f^+(0)$ which is computed:

Bećirević et al. [hep-ph/0403217] based on S.Hashimoto et al. [hep-ph/9906376] for $B\to D$ Decays

- The starting point is the evaluation of the matrix elements at $q^2_{\rm max}$, i.e. with the pion and kaon at rest:

$$\frac{\langle\pi|\bar{s}\gamma_4 u|K\rangle\langle K|\bar{u}\gamma_4 s|\pi\rangle}{\langle\pi|\bar{u}\gamma_4 u|\pi\rangle\langle K|\bar{s}\gamma_4 s|K\rangle}=\left[f_0(q^2_{\rm max})\right]^2\frac{(m_K+m_\pi)^2}{4m_K m_\pi}\,.$$

$f_0(q^2_{\rm max})$ is obtained with excellent precision.

am_{ud}	m_π	$q^2_{\rm max}$ (GeV^2)	$f_0(q^2_{\rm max})$
0.03	670 MeV	0.00235(4)	1.00029(6)
0.02	555 MeV	0.01152(20)	1.00192(34)
0.01	415 MeV	0.03524(62)	1.00887(89)
0.005	330 MeV	0.06070(107)	1.02143(132)

RBC/UKQCD Collaborations, arXiv:0710.5136

$K_{\ell 3}$ Decays (Cont.)

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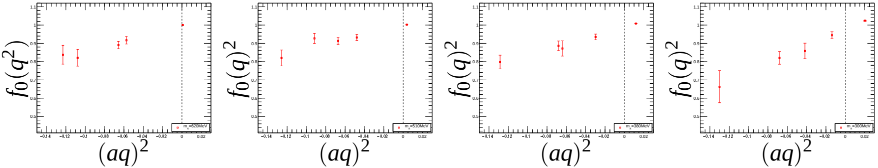
- Having obtained $f_0(q^2_{\rm max})$ we need to extrapolate in q^2 and m_{ud}.
- Conventionally the q^2 extrapolation is done by calculating the form factors with

$$|\vec{p}_K| \quad \text{or} \quad |\vec{p}_\pi| \quad = \quad p_{\rm min} \quad \text{or} \quad \sqrt{2}\,p_{\rm min}\,,$$

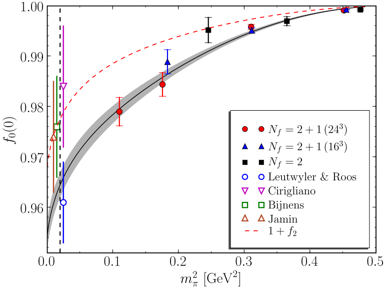
where $p_{\rm min} = 2\pi/L$ and L is the spatial extent of the lattice.



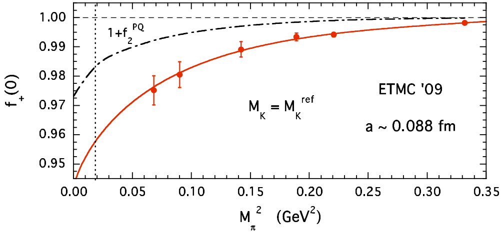
$m_\pi = 670\,\text{MeV}$ $\quad m_\pi = 555\,\text{MeV}$ $\quad m_\pi = 415\,\text{MeV}$ $\quad m_\pi = 330\,\text{MeV}$

RBC/UKQCD, arXiv:0710.5136

- The form factors can be calculated directly at $q^2 = 0$ (or any specified value of q^2) by using *twisted boundary conditions*. (See below.)



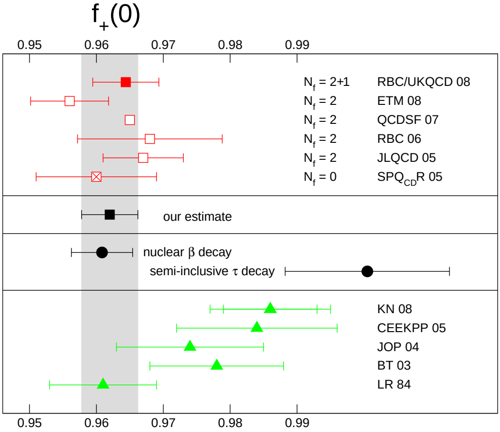
RBC/UKQCD, arXiv:0710.5136



ETMC ($N_f = 2$), Courtesy of S.Simula

- $1+f_2$ is not a good approximation to $f_0(0)$.
- We could use all possible guidance for the chiral extrapolation.
- RBC/UKQCD: $f_0(0) = 0.964(5)\,.$ arXiv:0710.5136
- ETMC: $f_0(0) = 0.9560(57)(62)\,.$ arXiv:0906.4728

FLAG Compendium – Preliminary



- The momentum resolution with conventional methods is very poor, e.g. on the RBC/UKQCD lattice

$$L = 24a \quad \text{with} \quad a^{-1} = 1.73\,\text{GeV} \quad \Rightarrow \quad \frac{2\pi}{L} = .45\,\text{GeV}$$

- Using twisted boundary conditions, $q(x_i + L) = e^{i\theta_i} q(x_i)$, the momentum spectrum is modified:

$$p_i = n_i \frac{2\pi}{L} + \frac{\theta_i}{L}\,.$$

- For many quantities *partially twisted boundary conditions*, in which the sea quarks satisfy periodic BC's but the valence quarks satisfy twisted BC's, can be used. CTS & G. Villadoro (2004); Bedaque & Chen (2004)

We do not need to perform new simulations for every choice of $\{\theta_i\}$.

- By tuning the twisting angles appropriately it is possible to calculate the matrix element at $q^2 = 0$ directly (or at any other required value of q^2) .
P.A.Boyle, J.M.Flynn, A.Jüttner, CTS, and J.M.Zanotti, [hep-lat/0703005]

- The feasibility of this method was demonstrated.

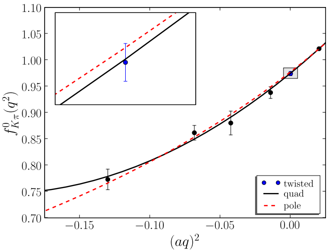
- We are currently using partially twisted boundary conditions to get $f_0(0)$ for our lightest quark mass ($ma = 0.005$) directly at $q^2 = 0$.

$$f_0(0) = 0.9774(35)\ \text{(pole fit)} \quad \text{and} \quad f_0(0) = 0.9749(59)\ \text{(quadratic fit)}$$

quoted in the paper. Our preliminary result computing the form-factor directly at $q^2 = 0$ is

$$f_0(0) = 0.974(4), \qquad f_-(0) = -0.113(12)$$

J.M.Flynn et al, arXiV:0812.6265 [hep-lat]



- In our final result for the physical $f_0(0) = 0.9644(33)(34)(14)$, 34 was due to the model dependence.

J.Flynn and CTS, arXiv:0809.1229 [hep-ph]

We studied the chiral behaviour of $K_{\ell 3}$ form factors at:

- $q^2 = q^2_{\rm max} = (m_K - m_\pi)^2$.

  The Callan-Trieman point is the (unphysical) $q^2 = m_K^2 - m_\pi^2$.Thus as we approach the $SU(2)$ chiral limit, $m_u = m_d = 0$

  $$f_0(q^2_{\rm max}) \to \frac{f^{(K)}}{f} \simeq 1.26\,,$$

  where $f^{(K)}$ and f are the kaon and pion decays constants in the SU(2) chiral limit.

- $q^2 = 0$.

  In the kaon rest frame, the external pion has energy $m_K/2$. Nevertheless, “because the chiral logarithms arise from soft regions of phase-space for the *internal* pions they are calculable" (J.Flynn & CTS)

  $\Rightarrow$ Hard-Pion Chiral Perturbation Theory.

J.Bijnens - Preceding talk.

am_{ud}	m_π	$q^2_{\max}$ (GeV^2)	$f_0(q^2_{\max})$
0.03	670 MeV	0.00235(4)	1.00029(6)
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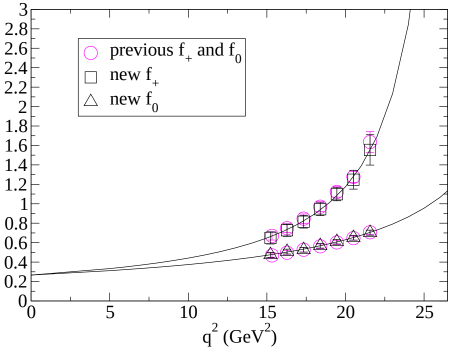
0.005	330 MeV	0.06070(107)	1.02143(132)

- In the SU(2) chiral limit, $m_{ud} = 0$, we have the Callan-Treiman Relation

$$f_0(q^2_{\max}) = \frac{f_K}{f_\pi} \simeq 1.26.$$

- We have investigated whether the difference of the numbers in the table and 1.26 can be understood using SU(2) ChPT. J.Flynn & CTS
  - The one-loop chiral logarithms have a large coefficient and are of the correct size to account for the difference. However they have the wrong sign!
  - There are linear and quadratic terms in m_π.
    They cannot be calculated in SU(2) ChPT, but estimating the LECs by converting results from SU(3) ChPT suggests that these terms have the correct sign and magnitude to account for the difference.

Callan-Treiman relation for heavy flavours



HPQCD, $B \to \pi$ Form Factors in the Chiral Limit.

E.Gulez et al. hep-lat/0601021

- The Callan-Treiman relation can be generalised to the f_0 form factor for other flavours (and in the static theory) in the SU(2) Chiral limit.

$$f_0^{D\to\pi}(q^2_{\rm max}) \underset{m_\pi^2\to 0}{\longrightarrow} \frac{f^{(D)}}{f} \qquad \text{and} \qquad f_0^{B\to\pi}(q^2_{\rm max}) \underset{m_\pi^2\to 0}{\longrightarrow} \frac{f^{(B)}}{f}\,.$$

- Lattice results show that the chiral behaviour at $q^2_{\rm max}$ must also be steep for heavy quarks.

- We also argue that information can be obtained at values of q^2 where the external pion is not soft, such as at the reference point $q^2 = 0$. J.Flynn, CTS, arXiV:0809.1229.

$$\begin{aligned} f^0(0) = f^+(0) &= F_+\left(1 - \frac{3}{4}\frac{m_\pi^2}{16\pi^2 f^2}\log\left(\frac{m_\pi^2}{\mu^2}\right) + c_+ m_\pi^2\right) \\ f^-(0) &= F_-\left(1 - \frac{3}{4}\frac{m_\pi^2}{16\pi^2 f^2}\log\left(\frac{m_\pi^2}{\mu^2}\right) + c_- m_\pi^2\right). \end{aligned}$$

- "Leading" higher-order contributions to the vector current can be reduced to the lowest order term by the use of the equations of motion, for example:

$$0 = \partial^2 \langle \pi(p_\pi) \,|\, (D_\mu K)^\dagger (\xi - \xi^\dagger) h \,|\, \bar{K}(p_K) \rangle$$

$$= \langle \pi(p_\pi) \,|\, (D^2 D_\mu K)^\dagger (\xi - \xi^\dagger) h + 2(D_\nu D_\mu K)^\dagger D^\nu (\xi - \xi^\dagger) h + (D_\mu K)^\dagger D^2 (\xi - \xi^\dagger) h \,|\, \bar{K}(p_K) \rangle$$

- It is possible to calculate the chiral logarithm because this comes from a soft internal loop.
- The approach can be applied at other values of q^2.
- This idea has recently been extended to $K \to \pi\pi$ decays.

J.Bijnens and A Celis, arXiV:0906.0302; J.Bijnens, Preceding talk.

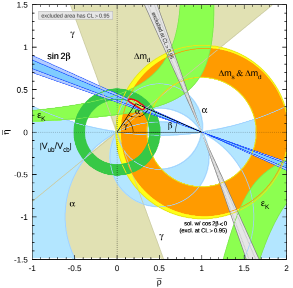
$$
\begin{aligned}
f_0(0)=f_+(0) &= F_+\left(1-\frac{3}{4}\frac{m_\pi^2}{16\pi^2 f^2}\log\left(\frac{m_\pi^2}{\mu^2}\right)+c_+m_\pi^2\right)\\
f_-(0) &= F_-\left(1-\frac{3}{4}\frac{m_\pi^2}{16\pi^2 f^2}\log\left(\frac{m_\pi^2}{\mu^2}\right)+c_-m_\pi^2\right).
\end{aligned}
$$

- Since the chiral extrapolation is a major source of systematic uncertainty for the lattice determination of V_{us} from $K_{\ell 3}$ decays, it is important to have all the possible theoretical information to guide us.

  It would be useful to know the result at NNLO.

- It would be reassuring to confirm that it is possible to develop an effective theory in which hard and soft pions are separated.

5. B_K

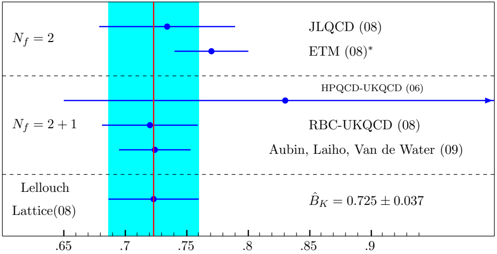


- B_K contains the non-perturbative QCD effects in K - $\bar{K}$ mixing:

$$\langle \bar{K}^0 \,|\, (\bar{s}\gamma_\mu(1-\gamma_5)d)\,(\bar{s}\gamma^\mu(1-\gamma_5)d)\,|K^0\rangle = \frac{8}{3} f_K^2 m_K^2 B_K(\mu)$$

- Lattice calculations of B_K have been performed for 20 years or so.
- If the lattice formulation has chiral (and flavour) symmetry then the $\Delta S = 2$ operator renormalizes multiplicatively and can be renormalized nonperturbatively.
- $SU(2)_L \times SU(2)_R$ (PQ)ChPT $\Rightarrow$

$$B_K = B_0^{(K)} \left\{ 1 + \frac{b_1 \chi_l}{f^2} + \frac{b_2 \chi_x}{f^2} - \frac{\chi_l}{32\pi^2 f^2} \log \frac{\chi_x}{\Lambda_\chi^2} \right\}$$



I will briefly comment on the two computations:

- RBC-UKQCD $\hat{B}_K = 0.720\,(13)\,(37)$. arXiv:hep-ph/0702042, 0804.0473
  (This corresponds to $B_K^{\overline{\rm MS}}(2\,\mathrm{GeV}) = 0.524(10)(28)$.)
- Aubin, Laiho, Van de Water, $\hat{B}_K = 0.724\,(8)\,(28)$, (DWF/Staggered Mixed Action)
  arXiv:0905.3947

- We now have two well-normalised $N_f = 2+1$ calculations (in remarkable agreement).

RBC/UKQCD, (2007,2008) $\hat{B}_K = 0.720(13)(37)$		Aubin, Laiho, Van de Water (2009) $\hat{B}_K = 0.724(8)(28)$	
Discretization	4%	Nonperturbative Renormalization	3.3%
Nonperturbative Renormalization	2%	Chiral/continuum Extrapolation	1.9%
Chiral Extrapolation	2%	Statistical	1.2%
Statistical	1.8%	Scale and Masses	0.8%
Sea strange mass adjustment	1%	Finite Volume	0.6%
Finite Volume	1%		
Total	5.4%	Total	4.1%

- RBC/UKQCD have only presented results at a single lattice spacing (dominant error).
  Calculation at a second spacing is well advanced.
- ALVdW use DWF valence quarks on a staggered (MILC) sea at two lattice spacings. The *taste* unitarity violations are removed using SU(3) ChPT at NLO (and partly at NNLO).

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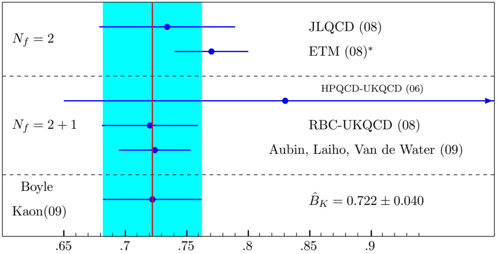


- Peter Boyle (Kaon 2009) proposes the (cautious?) “average" value

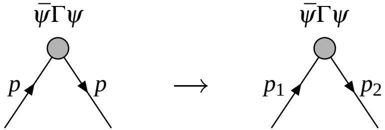
$$\hat{B}_K = 0.722(40)\,,$$

which I am happy to take as the current *best value*.

- (This is indistinguishable from Laurent Lellouch's estimate from Lattice 2008 ($\hat{B}_K = 0.723(37)$), which was dominated by the RBC/UKQCD result.)

- We are almost completed the full analysis of B_K on our finer lattice and hence to be able to compute the continuum extrapolation.
  We are waiting to complete the full chiral analysis and determine the lattice spacing and physical quark masses.
- We are currently repeating the procedure for all the possible dimension 6 $\Delta S = 2$ operators which contribute in extensions of the standard model.
- We have been generalizing the Rome-Southampton Non-Perturbative Renormalization method (RI-MOM) to non-exceptional momenta.

C.Sturm et al., arXiv:0901.2599



$$p_1^2 = p_2^2 = (p_1 - p_2)^2$$

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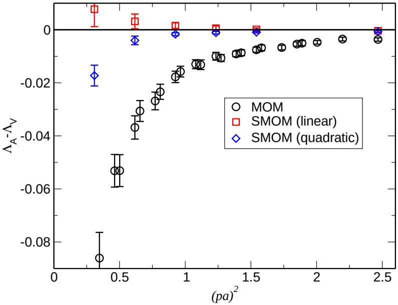
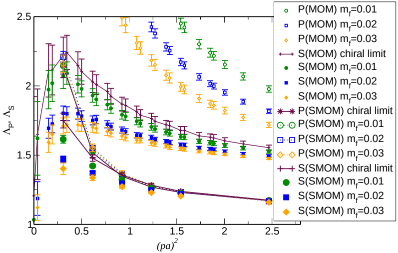




- $\Lambda_A - \Lambda_V$.
- Λ_S and Λ_P.

Y.Aoki arXiv:0901.2595 [hep-lat]

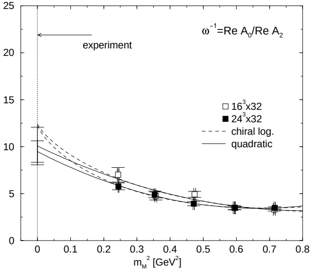
- We have also renormalized $O^{\Delta S=2}$ using non-exceptional momentum configurations.

- In 2001, two collaborations published some very interesting (quenched) results on non-leptonic kaon decays in general and on the $\Delta I = 1/2$ rule and $\varepsilon\,'/\varepsilon$ in particular:

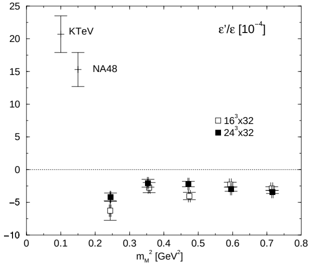
Collaboration(s)	Re A_0/Re A_2	ε'/ε
RBC	25.3 ± 1.8	$-(4.0 \pm 2.3) \times 10^{-4}$
CP-PACS	9÷12	(-7 ÷ -2)$\times 10^{-4}$
Experiments	22.2	$(17.2 \pm 1.8) \times 10^{-4}$

- At Lowest Order in the Chiral Expansion one can obtain the $K \to \pi\pi$ decay amplitude from $K \to \pi$ and $K \to$ vacuum matrix elements.
- This required the control of the *ultraviolet* problem, the subtraction of power divergences and renormalization of the operators – highly non-trivial.

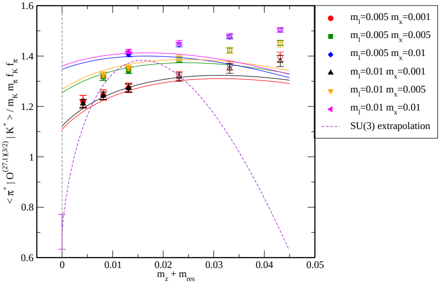
- Re A_0/Re A_2 as a function of the meson mass.



- ε'/ε as a function of the meson mass.



- The RBC and CP-PACS simulations were quenched, and relied on the validity of lowest order χPT in the region of approximately 400-800 MeV.



$$O^{3/2}_{(27,1)} = (\bar{s}d)_L\left\{(\bar{u}u)_L - (\bar{d}d)_L\right\} + (\bar{s}u)_L\,(\bar{u}d)_L$$

- RBC/(UKQCD) have repeated the calculation with the 24^3 DWF ensembles in the pion-mass range 240-415 MeV.
- For illustration consider the determination of α_{27}, the LO LEC for the (27,1) operator. Satisfactory fits were obtained, but again the corrections were found to be huge, casting serious doubt on the approach.
- Soft pion theorems are not sufficiently reliable $\Rightarrow$ need to compute $K \to \pi\pi$ matrix elements.
- To arrive at this important conclusion required a major effort.

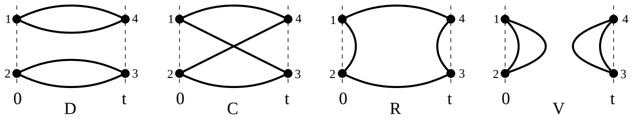
- The RBC/UKQCD Collaboration is undertaking a major $K \to \pi\pi$ study.
  T.Blum, P.Boyle, D. Broemmel, J. Flynn, E. Goode, T. Izubuchi, C. Kim, M. Lightman, Qi Liu, R. Mawhinney, C. Sachrajda, A. Soni.
- Two-pion quantization condition in a finite-volume

$$\delta(q^*) + \phi^P(q^*) = n\pi\,,$$

  where $E^2 = 4(m_\pi^2 + q^{*2})$, δ is the s-wave $\pi\pi$ phase shift and ϕ^P is a kinematic function. M.Lüscher, 1986, 1991, $\cdots$.
- The relation between the physical $K \to \pi\pi$ amplitude A and the finite-volume matrix element M

$$|A|^2 = 8\pi V^2 \frac{m_K E^2}{q^{*2}} \left\{\delta'(q^*) + \phi^{P\prime}(q^*)\right\} |M|^2\,,$$

  where $\prime$ denotes differentiation w.r.t. q^*.
  L.Lellouch and M.Lüscher, hep-lat/0003023; C.h.Kim, CTS and S.Sharpe, hep-lat/0507006; N.H.Christ, C.h.Kim and T.Yamazaki hep-lat/0507009
- For $I = 2$ final states, by using twisted boundary conditions it is possible to calculate $E_{\pi\pi}$ and hence the scattering phase-shift for a range of momenta and hence obtain the derivative of the phase-shift and compute the finite-volume corrections. C.h.Kim and CTS (in preparation)



- We are starting a major project to calculate the $\Delta I = 3/2\ K \to \pi\pi$ Decay Amplitudes.
- We are also exploring whether it will be feasible to compute the $\Delta I = 1/2\ K \to \pi\pi$ Decay Amplitudes.
- For I=2 $\pi\pi$ states the correlation function is proportional to D-C.
- For I=0 $\pi\pi$ states the correlation function is proportional to D+C-6R+3V.

The major practical difficulty is to subtract the vacuum contribution with sufficient precision.

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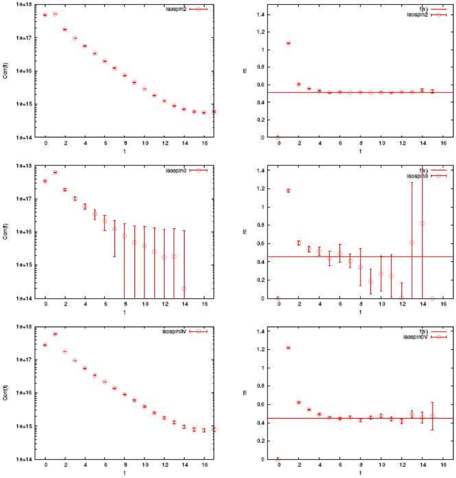

- RBC/UKQCD, Very Preliminary, Qi Liu et al.
- $I = 2$ (Correlator and Effective Mass)
- $I = 0$ (Correlator and Effective Mass)
- $I = 0$ (Correlator - V and Effective Mass)

- The theoretical and numerical conclusions specific to each section are presented on the corresponding slides. Here I just make some very general remarks.
- These are exciting times. Unquenched lattice simulations are being performed with very light quarks.
- The consistency of results from simulations with very different actions is impressive and important and adds hugely to our confidence.
  - The chiral regime is being mapped out for the spectrum and decay constants. Some simulations are now being performed with light quarks very close to the physical ones.
  - More expensive simulations using formulations with good chiral and flavour properties will continue to extend the range of physical quantities in flavour physics which can be studied.
  - The close collaboration of the ChPT and Lattice communities is an important element in extracting the maximum physics information from the data.
- I imagine that by the next Chiral Dynamics workshop, the chiral behaviour of many fundamental quantities will be well understood (regions of validity of SU(3) and SU(2) ChPT), and that the Low Energy Constants of ChPT will be determined with excellent precision.

- I believe that much effort will now have to be devoted to thinking how lattice calculations can help clarify any new physics discovered at the LHC (and elsewhere).
  - You don't have to be a *lattice physicist* to contribute.
  - **Challenge:** The b-factories have produced a huge amount of data on $B \to M_1M_2$ decays, where $M_{1,2}$ are mesons. We do not know how to formulate the lattice projects so that they quantify the non-perturbative QCD effects in these processes and hence yield the fundamental physics.

    (For $K \to \pi\pi$ decays this is possible because we can neglect inelastic channels.)
- A huge amount has been achieved; an even larger challenge is ahead of us.