Recent results on strange systems from QCD sum rules and lattice QCD

Makoto Oka

Tokyo Institute of Technology

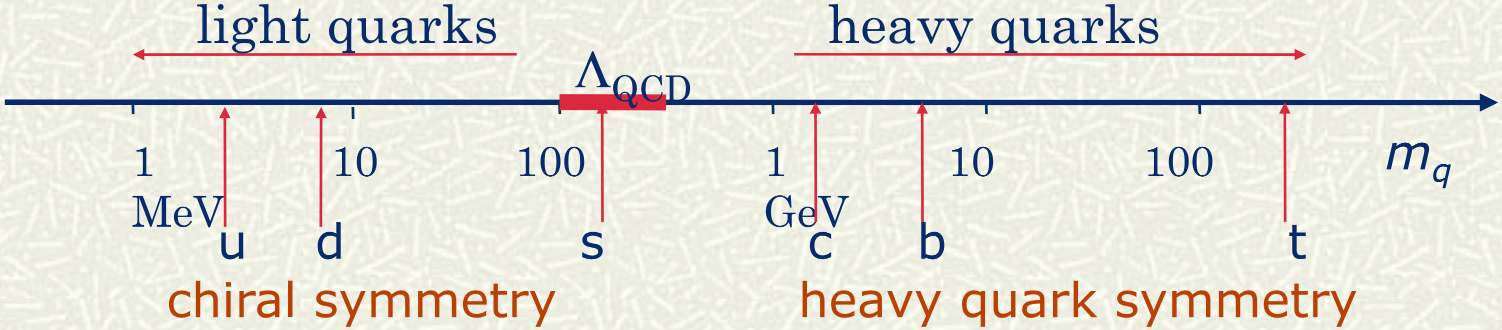
Chiral 09
06/07/09

Contents

Strangeness in QCD

$$\mathcal{L}_{QCD} = -\frac{1}{4}(G^a_{\mu\nu})^2 + \sum_q \bar{q}(i\, D_\mu \gamma^\mu - m_q)q\,,$$

If the quark masses of N_f flavors are equal, QCD has the exact flavor SU(N_f) symmetry.



light quarks

heavy quarks

Λ_{QCD}

1 MeV

10

100

1 GeV

10

100

m_q

u d s c b t

chiral symmetry

heavy quark symmetry

The dynamics of both the light and heavy quarks can be described by effective theories, but the strange quark is sensitive to the QCD details.

Strangeness in QCD

The quark model **with SU(3)/SU(6) flavor symmetry has played a major role in the history of hadron physics, but its limitation has become clear in the spectroscopy of excited and exotic hadrons.**

Technical developments in the direct QCD approaches start to allow reliable predictions to the hadronic observables from QCD.

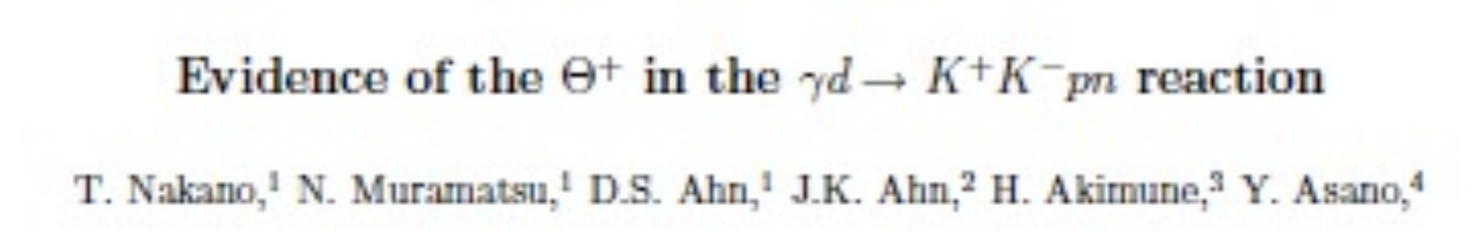
Contents

1. Introduction
2. **Pentaquark Θ^+ in (New) QCD Sum Rules**

   P. Gubler, D. Jido, T. Kojo, T. Nishikawa
3. Meson-Baryon Couplings

   QCD Sum Rules: T. Doi, H. Kim, Y. Kondo, G. Erkol

   Lattice QCD: T.T. Takahashi, G. Erkol
4. Conclusion and Outlook



Evidence of the Θ^+ in the $\gamma d \to K^+K^-pn$ reaction

T. Nakano,[1] N. Muramatsu,[1] D.S. Ahn,[1] J.K. Ahn,[2] H. Akimune,[3] Y. Asano,[4]

PRC79 (2009) 025210   arXiv:0812.1035 (4 Dec 2008)

m_Θ = 1.524 (2)(3) GeV/c^2 with statistics 5.1 σ

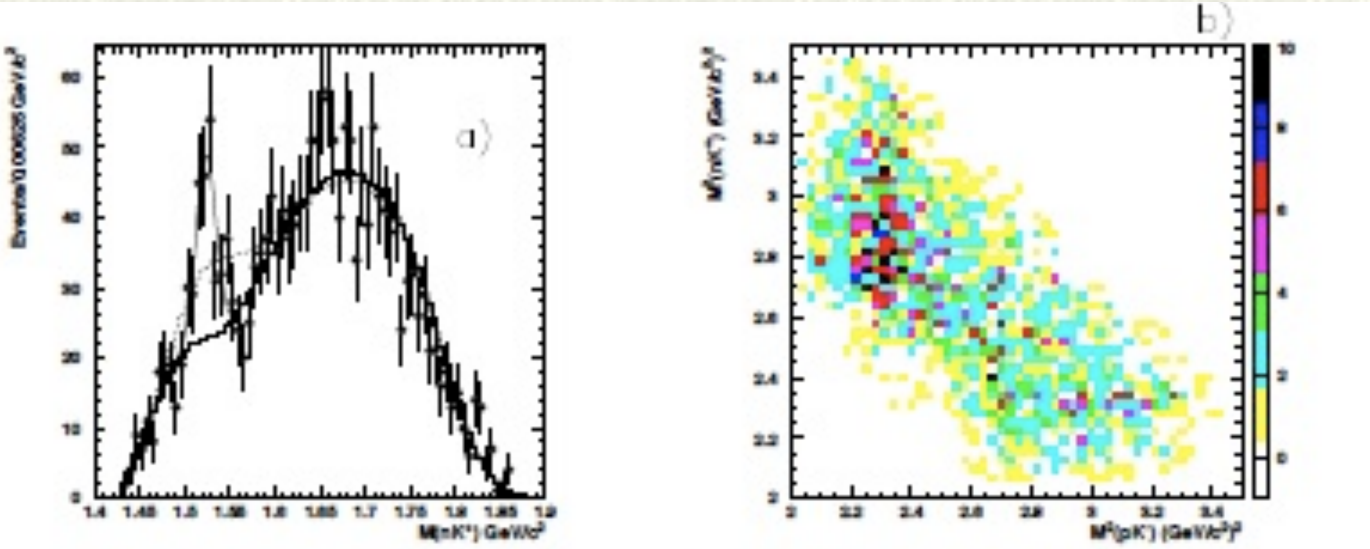


FIG. 12: (a) $M(nK^+)$ distribution with a fit to the RMM background spectrum only (dashed line) and with a Gaussian function (solid line). The dotted line is the background. (b) Dalitz plot of $M^2(nK^+)$ vs. $M^2(pK^-)$.

Lattice QCD for Θ^+

- Lattice QCD for Θ^+ (so far *quenched*)

  F. Csikor, S. Sasaki, T.-W. Chiu, N. Mathur

  N. Ishii, T. Takahashi, B. Lasscock

  - No low-lying $1/2^+$ ($I = 0$) resonance state.
  - May exists a $1/2^-$ ($I = 0$) resonance near the NK threshold, but it requires careful study to distinguish from NK scattering states.
  - Some contradictory results for the $3/2^+$ or $3/2^-$ states.
- Anticipate full-QCD calculations with physical quark masses in which scattering states and resonances are clearly distinguished.

QCD SR for Θ^+

P. Gubler, D. Jido, T. Kojo, T. Nishikawa, M.O., PRD 79 (2009) 114011, arXiv:0902.2049

- **QCD SR predicts (J, I) = ($3/2^+$,0) and ($3/2^+$,1) penta-quarks in the mass region of 1.4-1.6 GeV/ c^2.**
- **They decay into NK P-wave states. The decay widths can be relatively small. No QCD prediction of the decay width is available.**

Contents

1. Introduction
2. Pentaquark Θ^+ in (New) QCD Sum Rules

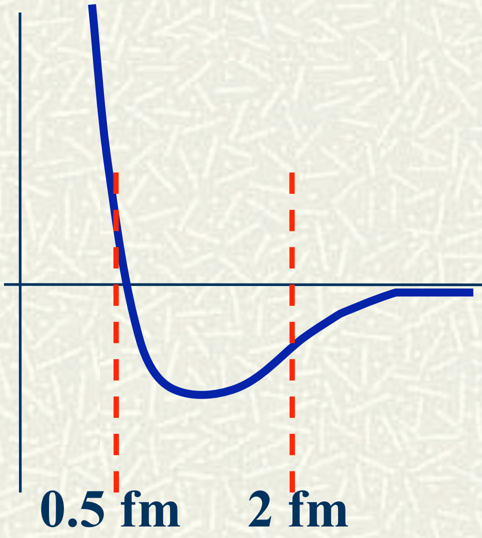
   P. Gubler, D. Jido, T. Kojo, T. Nishikawa
3. **Meson-Baryon Couplings**

   QCD Sum Rules: T. Doi, H. Kim, Y. Kondo, G. Erkol

   Lattice QCD: T.T. Takahashi, G. Erkol
4. Conclusion and Outlook

Nuclear force

- Three Regions of Nuclear Force
  - N.F. = OPE + Med. Attr. + SR Repulsion
  - Energy scales ~ 100 ~ 500 MeV
    - ⇔ B.E. of the deuteron ~ 2 MeV
  - Range of the OBE ~ 1 fm
    - ⇔ Size of the deuteron ~ 4 fm



- Are the other baryonic potential similar?
  - Use of SU(3) symmetry to generalize meson exchanges
  - How can we extrapolate the SR part? Its origin?

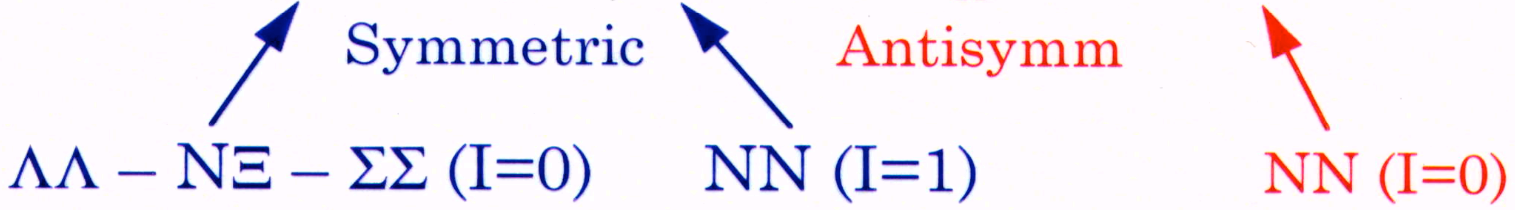
Generalized Nuclear Force

SU(6) ⊃ SU(3) flavor x SU(2) spin

$$[3]\ \underline{56} \left\langle \begin{array}{l} \mathbf{8}\ (S=1/2):\ N\ \Lambda\ \Sigma\ \Xi \\ \mathbf{10}\ (S=3/2):\ \Delta\ \Sigma^*\ \Xi^*\ \Omega \end{array} \right.$$

<u>SU(3) flavor</u>

$$8 \times 8 = 1 + 8_S + 27 \quad + \quad 8_A + 10 + 10^*$$



Symmetric

Antisymm

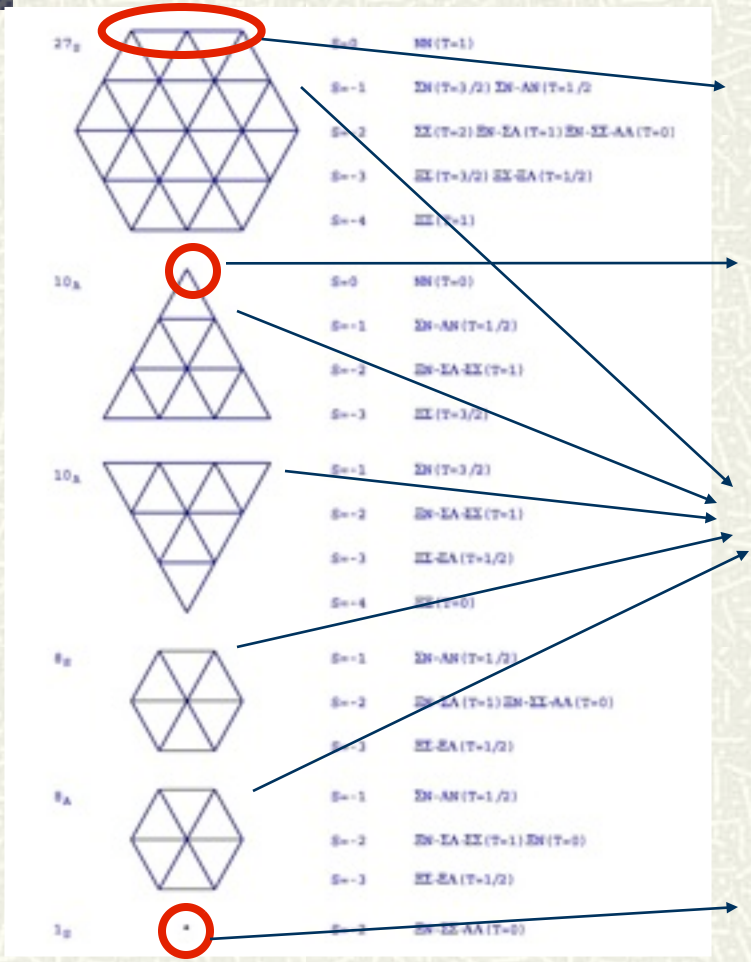
ΛΛ – NΞ – ΣΣ (I=0)

NN (I=1)

NN (I=0)

M. O. , K. Shimizu, K. Yazaki, PLB130 (1983) 365, NPA464 (1987) 700

Generalized Nuclear Force



27

10*

10

8_S

8_A

1

NN (I=1) $^3S_1$

NN (I=0) $^1S_0$

NΛ, NΣ

Hypernucleus

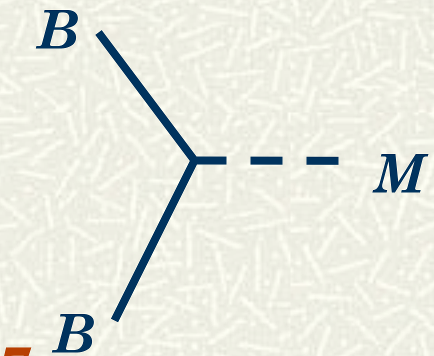
H dibaryon ΛΛ - NΞ - ΣΣ (I=0)
6-quark states?

Generalized Nuclear Force

- Understanding generalized nuclear force from the first principle is a key to disentangle origins and mechanisms of hadronic interactions.
- As the first step, we explore the SU(3) structure of the meson exchange part of the potential.

SU(3) Symmetry of M-B-B vertex

Baryon octet	8	N Λ Σ Ξ
Meson octet	8	π η_8 K
singlet	1	η_1



$$8 \times 8 = 1 + 8_D + 8_F + 10 + \overline{10} + 27$$

Two independent couplings for 8 x 8 = 8

F Tr[[B , B] M]	F coupling
D Tr[{ B , B } M]	D coupling

$\alpha_F = F/(F+D)$ is a free parameter in SU(3)

Spin-Flavor SU(6) Symmetry

Based on the NR quark model $(u\uparrow\ u\downarrow\ d\uparrow\ d\downarrow\ s\uparrow\ s\downarrow)$

Baryons $56 = (8, 1/2) + (10, 3/2)$

$N\ \Lambda\ \Sigma\ \Xi \qquad \Delta\ \Sigma^*\ \Xi^*\ \Omega$

Mesons $1 = (1,0) \qquad \eta_1$

$35 = (8,0) + (8,1) + (1,1)$

$\pi\ \eta_8\ K \qquad \rho\ \omega\ K^* \qquad \varphi$

B B M

$56 \times 56 = 1 + 35 + 405 + 2695$

$\underline{(8,0)}$ (unique) $(8,0)$

F/D ratio is fixed by the SU(6) symmetry

$\rightarrow F/D = 2/3$ or $\alpha_F = 2/5$

Spin-Flavor SU(6) Symmetry

But,

QCD has no reason to adopt the SU(6) symmetry.

The SU(6) symmetry is broken!!

by mass difference of quarks

by spin-dependent interactions of quarks

ex N (J=1/2) – Δ (J=3/2) mass difference

(8,1/2) (10, 3/2) 56-dim. SU(6) rep.

QCD calculation of coupling constants

- The SU(3) invariance for the coupling constants is not established, although the phenomenological models often assume the invariance. The F/D ratios of the coupling constants are the fitting parameters in the models.
- How good is the SU(3) symmetry in the coupling constants?
- What does QCD predict for F/D ratio, if SU(3) is valid?
- Direct computation of the coupling constants from QCD

  QCD sum rule T. Doi, H. Kim, Y. Kondo, G. Erkol, M. O.

  Lattice QCD T.T. Takahashi, G. Erkol, M.O.

QCD Sum rule for coupling constants

T. Doi, Y. Kondo, M.O., Phys. Rept. 398 (2004) 253

- **πNN coupling constant**

$$\Pi^{\alpha\beta}(q,p) = i\int d^4x\, e^{iq\cdot x}\langle 0|\mathrm{T}\left[J_N^{\alpha}(x)\bar{J}_N^{\beta}(0)\right]|\pi(p)\rangle$$

$$\Pi(q,p) \;=\; i\gamma_5\not{p}\,\Pi^{\rm PV} + i\gamma_5\,\Pi^{\rm PS} + \boxed{\gamma_5\sigma^{\mu\nu}q_\mu p_\nu\,\Pi^{\rm T}} + i\gamma_5\not{q}\,\tilde{\Pi}^{\rm PV}$$

tensor structure

- **nucleon interpolation field**

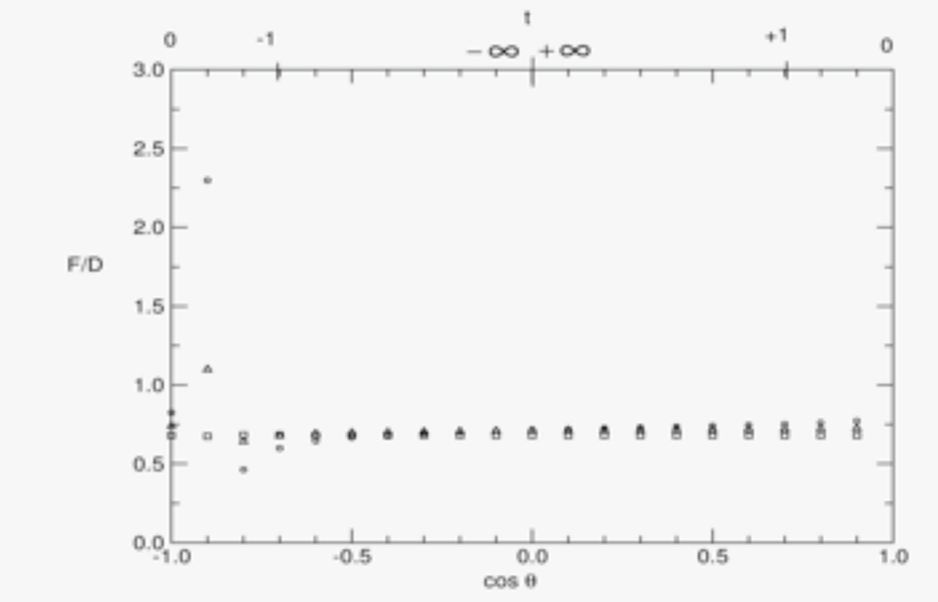
$$J_N(x;t) \;=\; 2\epsilon_{abc}[\,(u_a^T(x)Cd_b(x))\gamma_5 u_c(x) + t\,(u_a^T(x)C\gamma_5 d_b(x))u_c(x)\,]$$

two independent terms mixed by $\tan\theta = t$

F/D ratio *v.s.* cosθ for T sum rule

Tensor sum rule

T. Doi, H. Kim, M.O., Phys.Rev. C62 (2000) 055202



F/D = 0.65 ± 0.10

= 2/3 for SU(6)

= 0.57 from g_A (exp)

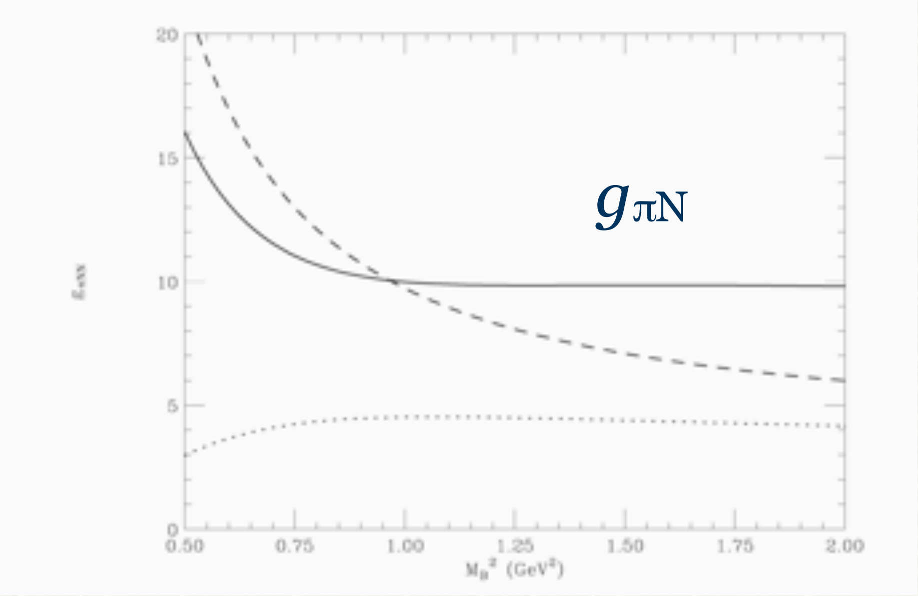
α_F = F/(F+D) = 0.394

= 2/5 for SU(6)

The SU(3) is fairly good for πBB' coupling consistent with the phenomenological models, ex. Nijmegen potential.

Projected correlation function

- **The most reliable estimate of the absolute value of the pi-N-N coupling is by the projected correlated function method: Kondo-Morimatsu Nucl. Phys. A717 (2003)**



$g_{\pi N}$ = 9.6 ± 1.6

v.s.

$g_{\pi N}$ (exp.) ~12.8

slightly underestimated

QCDSR summary

- **Other mesons**
  - **K and η: p^2 expansion is marginally justifiable.**
  - **scalar mesons: σ, a_0, f_0, κ**
    G. Erkol, M. O., T. Rijken, R. Timmermans,
    PRC73 (2006) 044009
    $\alpha_F \sim 0.55$, but observe significant SU(3) breaking
    - The Σ-σ and Ξ-σ couplings are enhanced by 30-50%.
  - **The baryon-scalar-meson coupling strengths can be a good indicator of the 4-quark structure of the scalar mesons.**
- **A challenging subject for LQCD.**

Lattice QCD calculation of the ps-meson-octet-baryon couplings

- **Unquenched** lattice QCD is applied to the pseudoscalar-meson-octet-baryon coupling form factors.

  T. T. Takahashi, G. Erkol, MO PRD 79 (2009) 074509

  - CP-PACS gauge configuration: 2-flavor dynamical quarks on the 16^3x32 lattice
  - RG improved gauge action + the mean-field improved clover quark action
  - $\beta=1.95 \rightarrow a = 0.16$ fm $a^{-1} = 1.267$ GeV
  - The ratio and absolute values of the coupling constants are obtained for several quark masses: $m_q \sim 150, 100, 65, 35$ MeV

Lattice QCD calculation

- **3-point function**

$$\langle G^{\mathcal{B}\mathcal{P}\mathcal{B}'}(t_2,t_1;\mathrm{p}',\mathrm{p};\Gamma)\rangle = -i \sum_{\mathbf{x_2},\mathbf{x_1}} e^{-i\mathbf{p}\cdot\mathbf{x_2}} e^{i\mathbf{q}\cdot\mathbf{x_1}}$$

$$\times\, \Gamma^{\alpha\alpha'} \langle \mathrm{vac}|T[\eta^{\alpha}_{\mathcal{B}}(x_2)P(x_1)\bar{\eta}^{\alpha'}_{\mathcal{B}'}(0)]|\mathrm{vac}\rangle,$$

$$\eta_N(x) = \epsilon^{abc}[u^{Ta}(x)C\gamma_5 d^b(x)]u^c(x),$$

$$\eta_\Sigma(x) = \epsilon^{abc}[s^{Ta}(x)C\gamma_5 u^b(x)]u^c(x),$$

$$\eta_\Lambda(x) = \frac{1}{\sqrt{6}}\epsilon^{abc}\{[u^{Ta}(x)C\gamma_5 s^b(x)]d^c(x) - [d^{Ta}(x)C$$

$$\times\, \gamma_5 s^b(x)]u^c(x) + 2[u^{Ta}(x)C\gamma_5 d^b(x)]s^c(x)\},$$

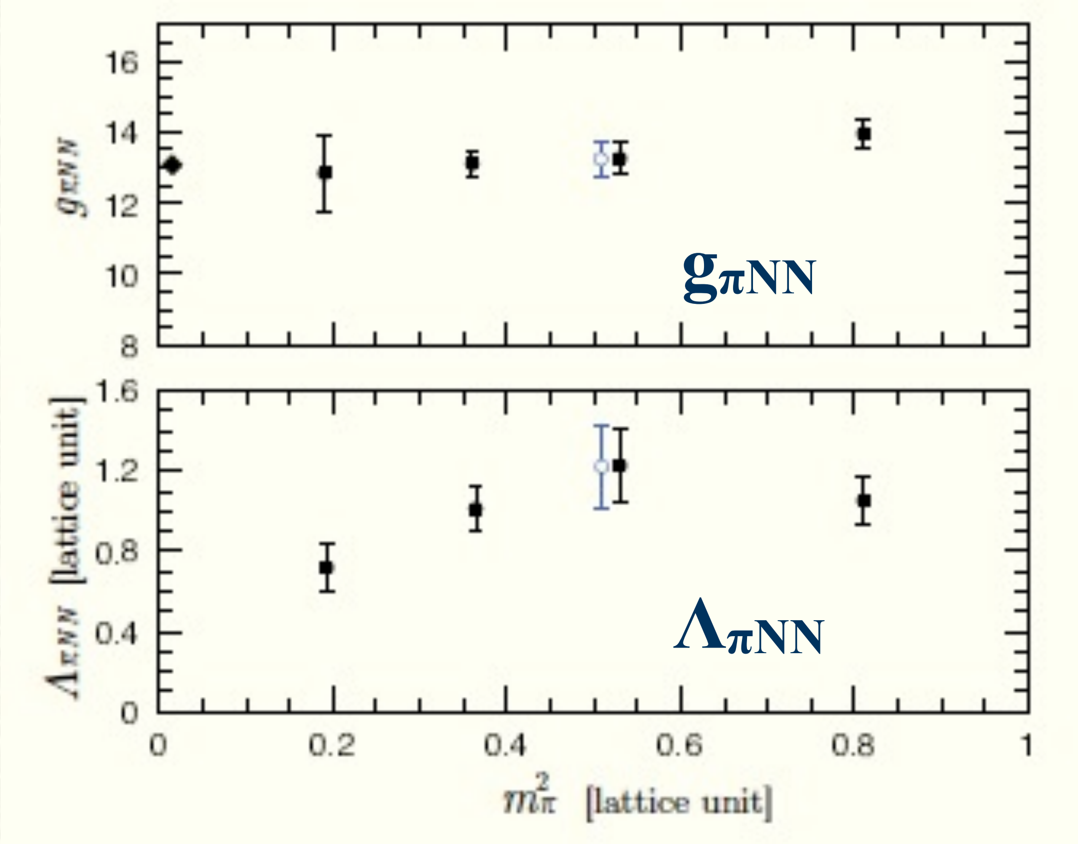
Lattice QCD calculation

- **PS-meson-baryon coupling form factors**

$$R(t_2,t_1;\mathbf{p}',\mathbf{p};\Gamma;\mu) = \frac{\langle G^{\mathcal{B}P\mathcal{B}'}(t_2,t_1;\mathbf{p}',\mathbf{p};\Gamma)\rangle}{\langle G^{\mathcal{B}'}(t_2;\mathbf{p}';\Gamma_4)\rangle}\left[\frac{\langle G^{\mathcal{B}}(t_2-t_1;\mathbf{p};\Gamma_4)\rangle}{\langle G^{\mathcal{B}'}(t_2-t_1;\mathbf{p}';\Gamma_4)\rangle} \times \frac{\langle G^{\mathcal{B}'}(t_1;\mathbf{p}';\Gamma_4)\rangle\langle G^{\mathcal{B}'}(t_2;\mathbf{p}';\Gamma_4)\rangle}{\langle G^{\mathcal{B}}(t_1;\mathbf{p};\Gamma_4)\rangle\langle G^{\mathcal{B}}(t_2;\mathbf{p};\Gamma_4)\rangle}\right]^{1/2},$$

$$R(t_2,t_1;\mathbf{0},\mathbf{p};\Gamma;\mu) \xrightarrow[t_2-t_1\gg a]{t_1\gg a} \frac{g_P^L(q^2)}{[2E(E+m)]^{1/2}}\, q_3,$$

pi-N-N coupling/form factor



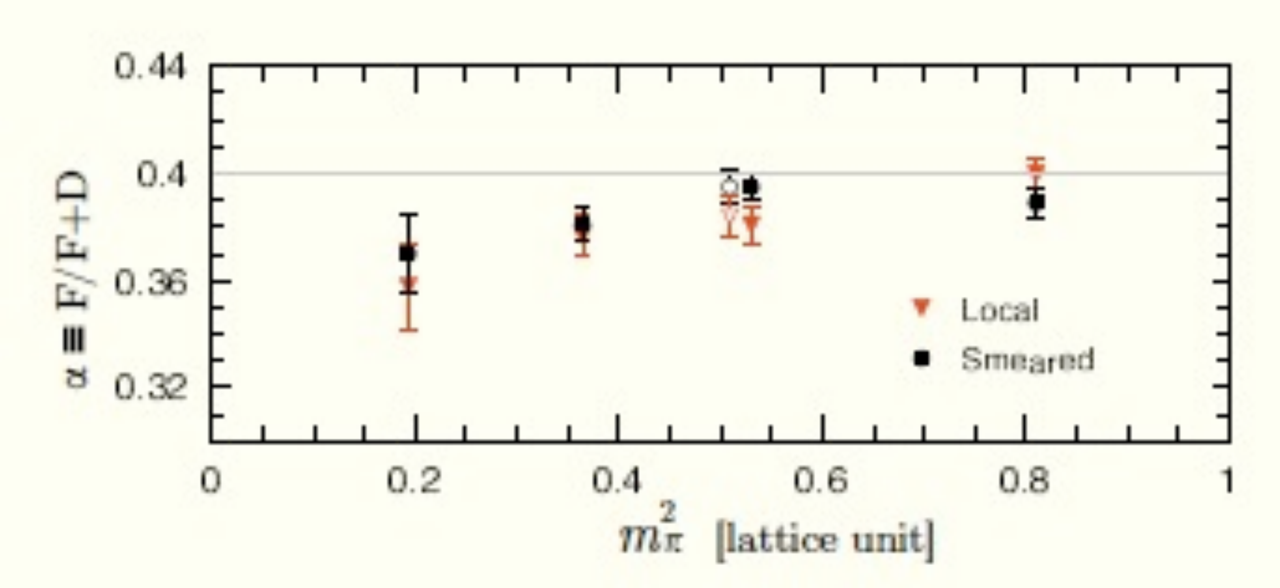
- $g_{\pi NN}$ is consistent with $g_{\pi NN}$ (pheno.) ~ 12.8

- The monopole form factor is softer than the one used in the meson exchange models.

$\Lambda_{\pi NN} \sim 0.6\ a^{-1}$

~ 0.75 GeV

SU(3) couplings



$$\alpha = \frac{F}{F+D} = 0.395 \qquad v.s.\ \mathrm{SU}(6)\ \text{value} \quad \alpha = 0.4$$

SU(3) breaking effect is very small. The deviation δ < 5%

Lattice QCD Summary

- The two-flavor full-QCD lattice calculation was performed for the ps meson-baryon coupling constants and form factors.
- $g_{\pi NN}$ is consistent with $g_{\pi NN}$ (pheno.) ~ 12.8.
- The monopole form factor is softer than the one used in the meson exchange models. $\Lambda_{\pi NN}$ ~ 0.75 GeV.
- The SU(3) symmetry for the ps meson-baryon couplings happens to be “very” good.
- F/D ratio α_F=0395 is close to the SU(6) value.

Conclusion and Outlook

- Serious QCD calculations are inevitable for revealing new (exotic) multi-quark world of hadrons.
- Lattice QCD has reached the position in which the real quark-mass full QCD with chiral symmetry is accessible. (but not yet applied to pentaquarks)
- QCD sum rules also give reliable results with careful evaluations of its validity.
- Strange systems supply many ideal subjects for checking physical ideas of hadron dynamics.
- JPARC is a dream machine for strangeness hadron physics.