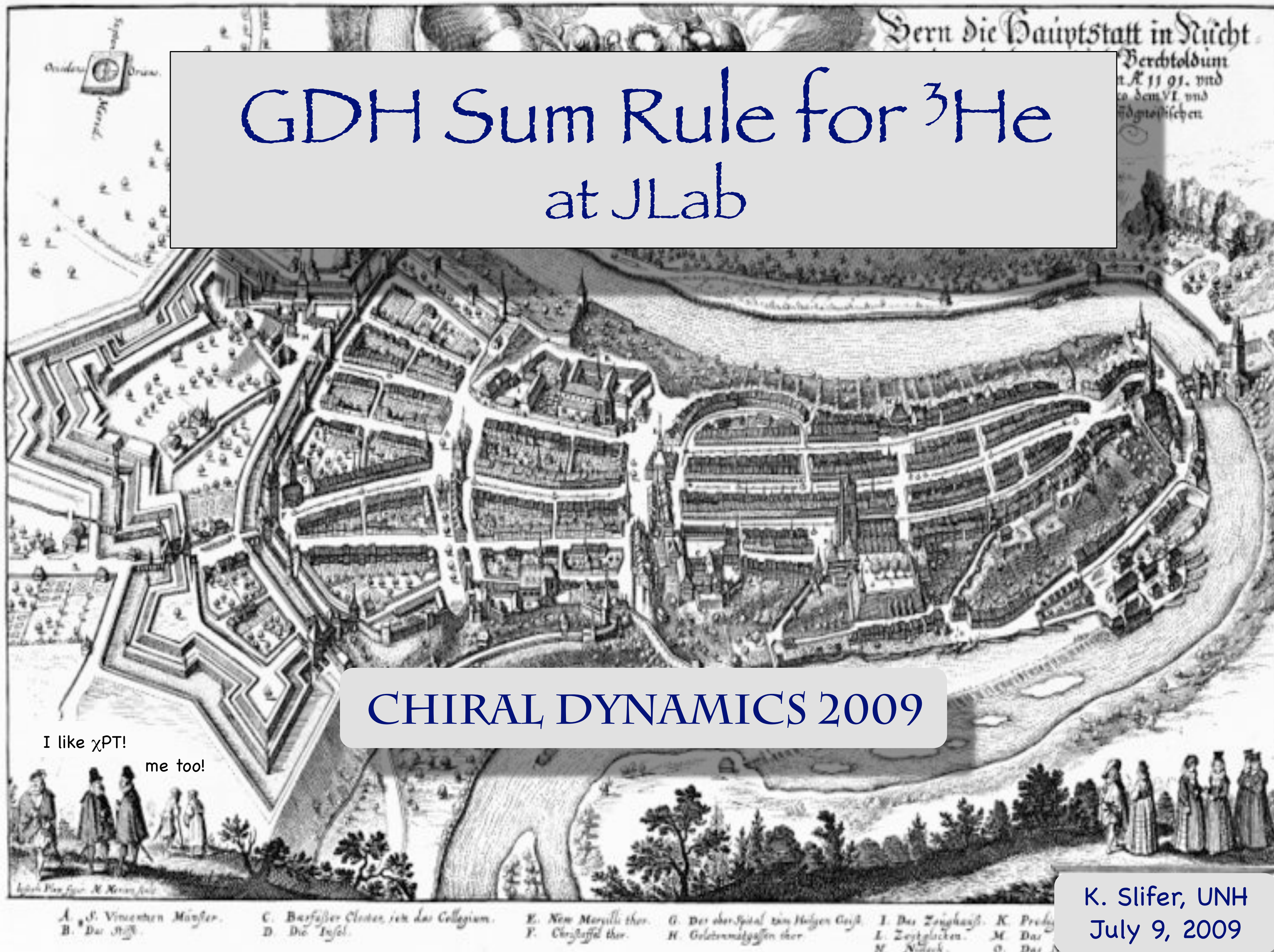
GDH Sum Rule for ^{3}He
at JLab



CHIRAL DYNAMICS 2009

K. Slifer, UNH
July 9, 2009

Overview

Inclusive Electron Scattering

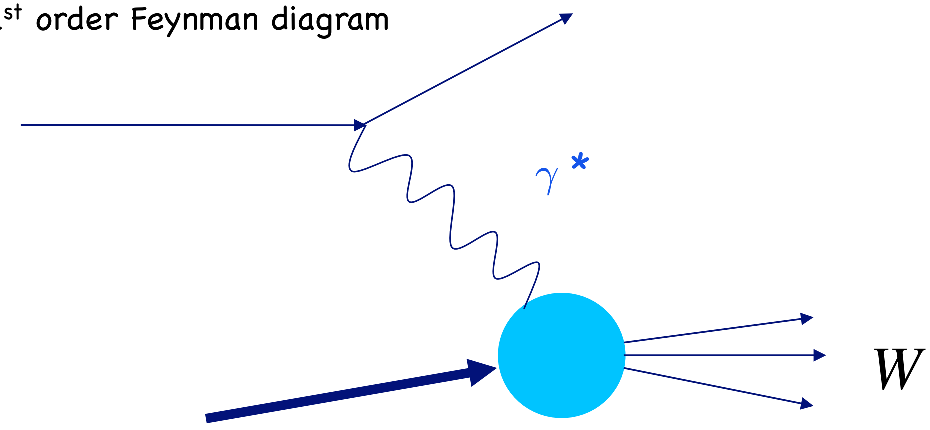
Dispersion Relations & Sum Rules

Published ^{3}He Data

Preliminary ^{3}He Data

Inclusive Scattering

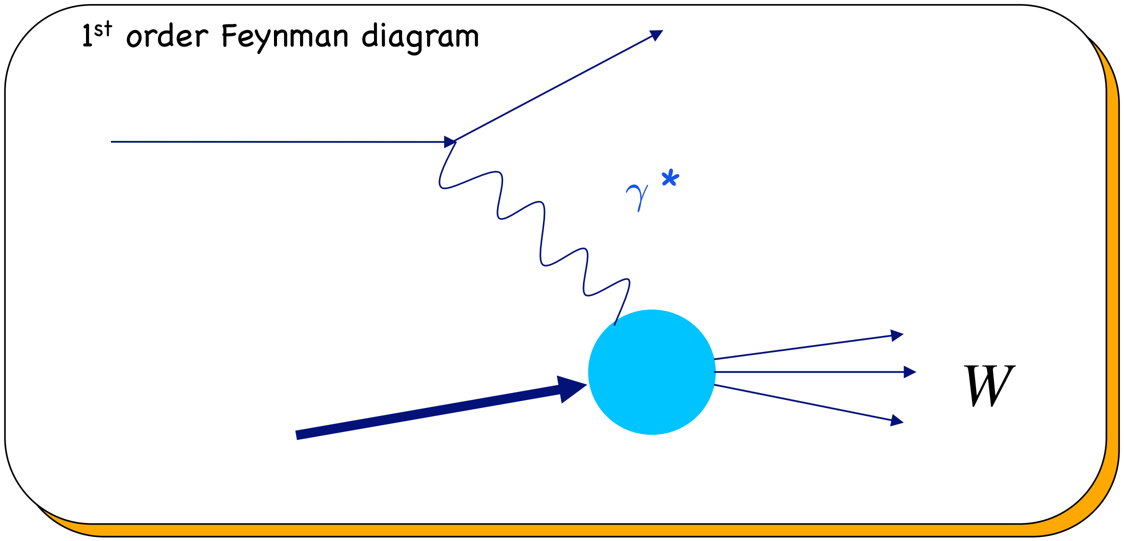
1st order Feynman diagram



Kinematics

Q^2 : 4-momentum transfer
X : Bjorken Scaling var
W : Invariant mass of target

Inclusive Scattering



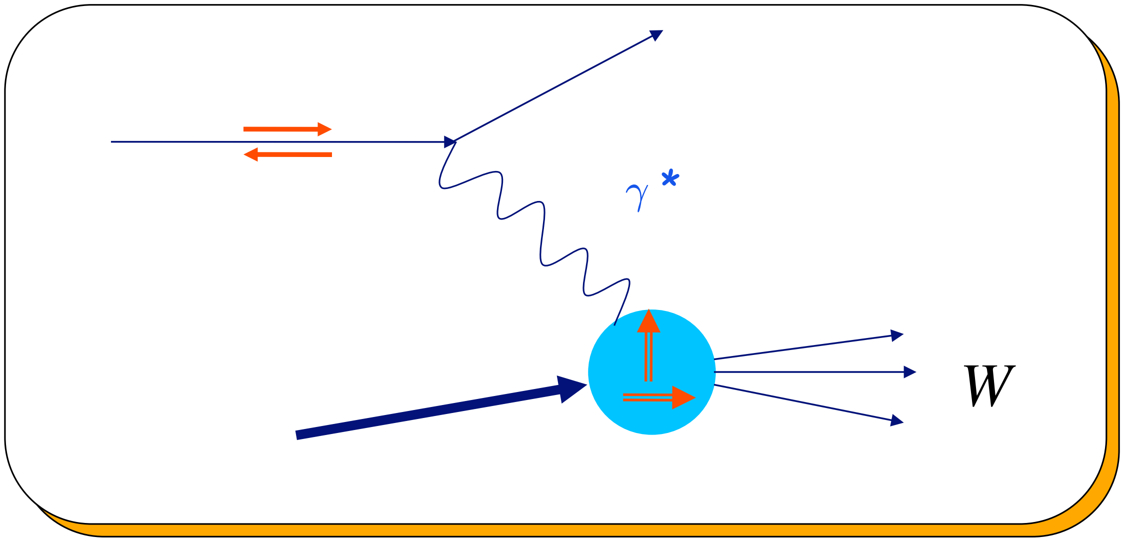
Q^2 : 4-momentum transfer
X : Bjorken Scaling var
W : Invariant mass of target

$$\frac{d^2\sigma}{d\Omega dE'} = \sigma_{Mott}\left[\frac{1}{\nu}F_2(x,Q^2) + \frac{2}{M}F_1(x,Q^2)\tan^2\frac{\theta}{2}\right]$$

Inclusive Cross Section

deviation from point-like behavior
characterized by the Structure Functions

Inclusive Scattering



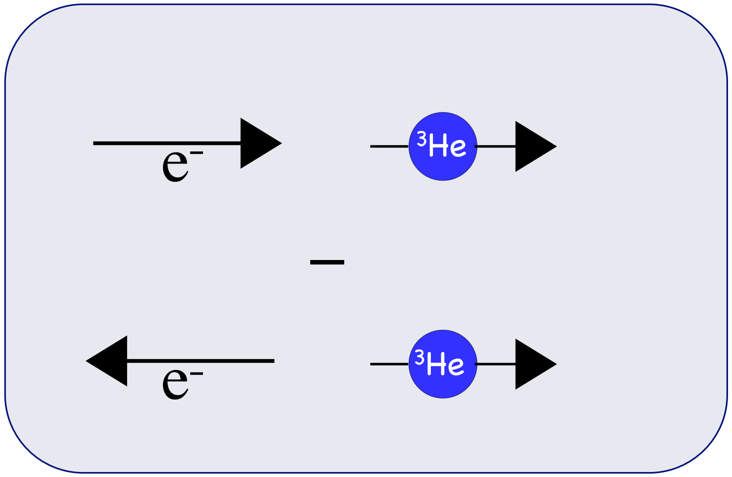
When we add spin degrees of freedom to the target and beam, 2 Addiitonal SF needed.

$$\frac{d^2\sigma}{d\Omega dE'} = \sigma_{Mott}\left[\frac{1}{\nu}F_2(x,Q^2) + \frac{2}{M}F_1(x,Q^2)\tan^2\frac{\theta}{2}\right]$$

$$+\,\gamma\, g_1(x,Q^2) + \delta\, g_2(x,Q^2)$$

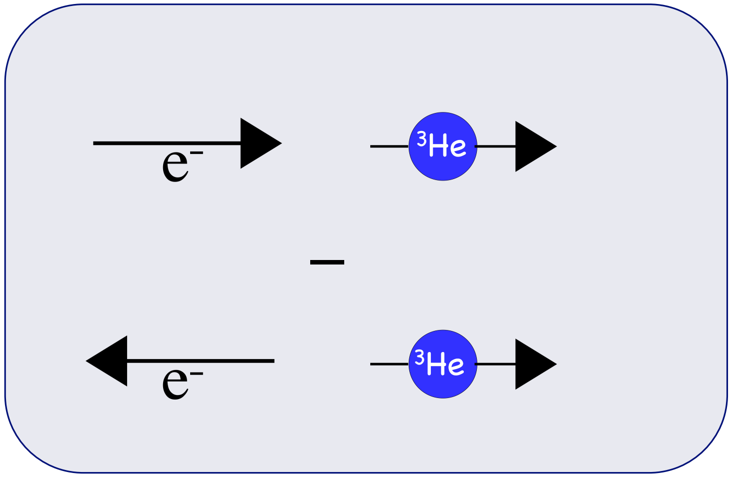
Inclusive **<u>Polarized</u>** Cross Section

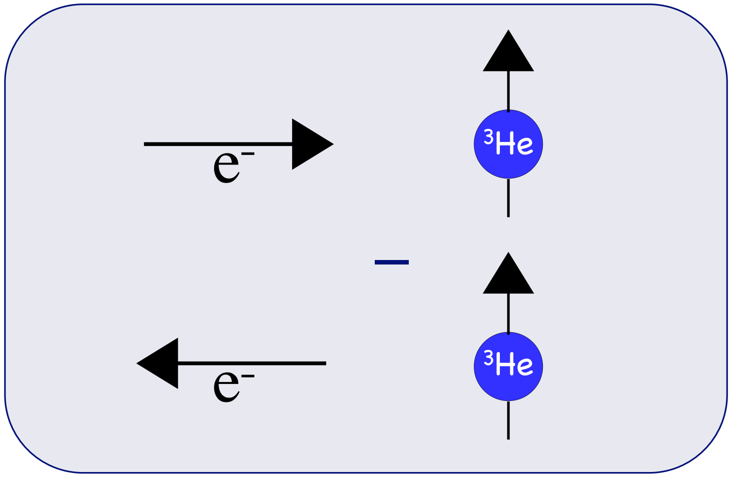
Accessing the polarized SFs



$$\frac{d^2\sigma^{\uparrow\Uparrow}}{d\Omega dE'} - \frac{d^2\sigma^{\downarrow\Uparrow}}{d\Omega dE'} = \frac{4\alpha^2}{\nu Q^2}\frac{E'}{E}\left[(E + E'\cos\theta)\, g_1 - 2Mx g_2\right]$$

Accessing the polarized SFs

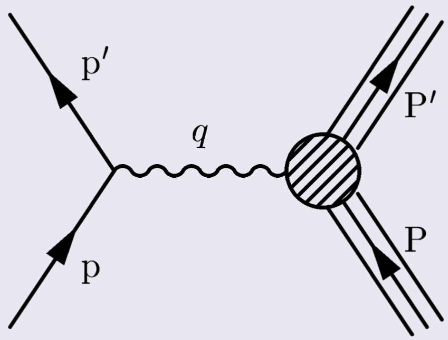




$$\frac{d^2\sigma^{\uparrow\Uparrow}}{d\Omega dE'} - \frac{d^2\sigma^{\downarrow\Uparrow}}{d\Omega dE'} = \frac{4\alpha^2}{\nu Q^2}\frac{E'}{E}\left[(E + E'\cos\theta)\, g_1 - 2Mx g_2\right]$$

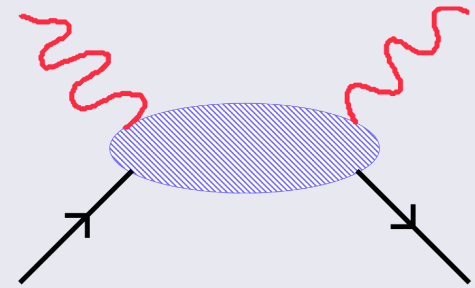
$$\frac{d^2\sigma^{\uparrow\Rightarrow}}{d\Omega dE'} - \frac{d^2\sigma^{\downarrow\Rightarrow}}{d\Omega dE'} = \frac{4\alpha^2}{\nu Q^2}\frac{E'}{E}\sin\theta\left[g_1 + \frac{2ME}{\nu} g_2\right]$$

Inclusive Electron Scattering



$$W_{\mu\nu} = \frac{1}{4\pi}\int d^4 e^{iq\cdot\xi}\langle PS|J(\xi)J(0)|PS\rangle$$

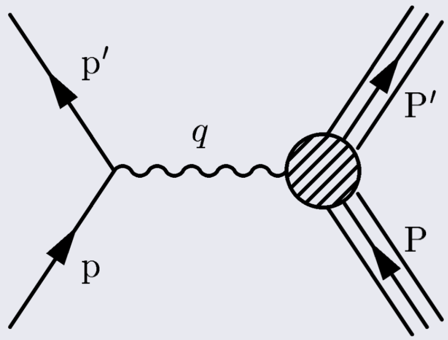
Doubly Virtual Compton Scattering



$$T_{\mu\nu} = i\int d^4 e^{iq\cdot\xi}\langle PS|\mathcal{T}J(\xi)J(0)|PS\rangle$$

Compton Scattering Tensor differs from inclusive scattering Tensor only by the time ordering of the EM currents

Inclusive Electron Scattering



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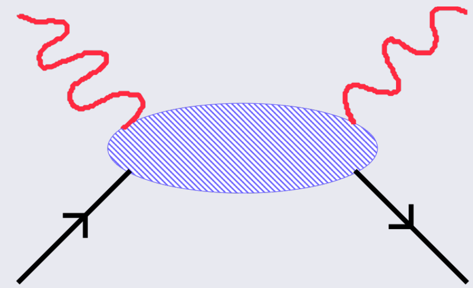
Doubly Virtual Compton Scattering



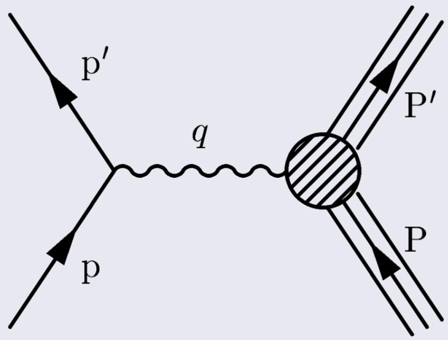
$$T_{\mu\nu} = i\int d^4 e^{iq\cdot\xi}\langle PS|\mathcal{T}J(\xi)J(0)|PS\rangle$$

Compton Scattering Tensor differs from inclusive scattering Tensor only by the time ordering of the EM currents

$$\begin{aligned} W^{\mu\nu} &= i\epsilon^{\mu\nu\alpha\beta}[s_\beta G_1(\nu,Q^2) + (M\nu s_\beta - s\cdot q P_\beta)G_2(\nu,Q^2)] \\ T^{\mu\nu} &= -i\epsilon^{\mu\nu\alpha\beta}q_\alpha[s_\beta S_1(\nu,Q^2) + (M\nu s_\beta - s\cdot q P_\beta)S_2(\nu,Q^2)] \end{aligned}$$

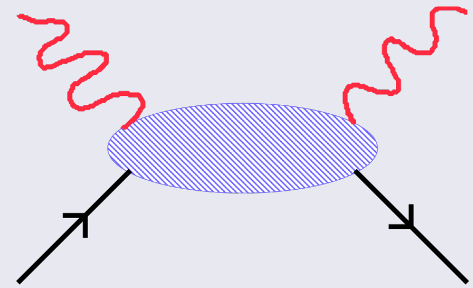
$$\begin{aligned} g_1(x,Q^2) &= M\nu G_1(\nu,Q^2) \\ g_2(x,Q^2) &= \nu^2 G_2(\nu,Q^2) \end{aligned}$$

Inclusive Electron Scattering



$$W_{\mu\nu} = \frac{1}{4\pi}\int d^4 e^{iq\cdot\xi}\langle PS|J(\xi)J(0)|PS\rangle$$

Doubly Virtual Compton Scattering



$$T_{\mu\nu} = i\int d^4 e^{iq\cdot\xi}\langle PS|\mathcal{T}J(\xi)J(0)|PS\rangle$$

$$W_{\mu\nu}(\nu, Q^2) = \frac{1}{2\pi M} Im\ T_{\mu\nu}(\nu, Q^2)$$

Kramers-Kronig type dispersion relation

$$S_1(\nu, Q^2) = 4\int_0^\infty \frac{d\nu'\nu'}{\nu'^2 - \nu^2} G_1(\nu', Q^2)$$

$$S_2(\nu, Q^2) = 4\int_0^\infty \frac{d\nu'\nu}{\nu'^2 - \nu^2} G_2(\nu', Q^2)$$

Generalized Sum Rules

Ji and Osborne, J. Phys. **G27**, 127 (2001)

Unsubtracted Dispersion Relation + Optical Theorem:

$$S_1(\nu, Q^2) = 4\int_0^\infty \frac{d\nu'\nu'}{\nu'^2 - \nu^2} G_1(\nu', Q^2)$$

↓

Extended GDH Sum

$$\Gamma_1 = \int g_1 dx = \frac{Q^2}{8} S_1(0, Q^2)$$

$Q^2 = 0$ ➔ GDH Sum Rule

$Q^2 = \infty$ ➔ Bjorken Sum Rule

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$$\int_{\nu_{\rm th}}^{\infty} \frac{\sigma_{\rm A}(\nu) - \sigma_{\rm P}(\nu)}{\nu} d\nu = -4\pi^2 \mathcal{S}\alpha \left(\frac{\kappa}{M}\right)^2$$

$$= -234\ \mu b\ \ (\text{ Neutron};\ \ \kappa = -1.91\ )$$

$$= -496\ \mu b\ \ (\ {}^3\text{He};\ \ \kappa = -8.366\ )$$

Generalized Sum Rules

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huge impact of QE and threshold e-disintegration

$$\int_{\nu_{\rm th}}^\infty \frac{\sigma_{\rm A}(\nu) - \sigma_{\rm P}(\nu)}{\nu} d\nu = -4\pi^2 \mathcal{S}\alpha \left(\frac{\kappa}{M}\right)^2$$

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$Q^2 = 0$ ➔GDH Sum Rule

$Q^2 = \infty$ ➔Bjorken Sum Rule

$$\Gamma_1^p(Q^2) - \Gamma_1^n(Q^2) = \frac{g_A}{6} \cdot C_{\mathrm{NS}}(\alpha_s)$$

Generalized Sum Rules

Ji and Osborne, J. Phys. **G27**, 127 (2001)

Unsubtracted Dispersion Relation + Optical Theorem:

$$S_1(\nu, Q^2) = 4 \int_0^\infty \frac{d\nu' \nu'}{\nu'^2 - \nu^2} G_1(\nu', Q^2)$$

Extended GDH Sum

$$\Gamma_1 = \int g_1 dx = \frac{Q^2}{8} S_1(0, Q^2)$$

$Q^2 = 0$ ➔ GDH Sum Rule

$Q^2 = \infty$ ➔ Bjorken Sum Rule

$$\frac{\Gamma_1^{(^3\mathrm{H})} - \Gamma_1^{(^3\mathrm{He})} = \frac{g_A^{tri}}{6} \cdot C_{\mathtt{NS}}(\alpha_s)}{\Gamma_1^p(Q^2) - \Gamma_1^n(Q^2) = \frac{g_A}{6} \cdot C_{\mathtt{NS}}(\alpha_s)} = 0.965 \pm 0.004$$

Generalized Sum Rules

Ji and Osborne, J. Phys. **G27**, 127 (2001)

Unsubtracted Dispersion Relation + Optical Theorem:

$$S_1(\nu, Q^2) = 4\int_0^\infty \frac{d\nu'\nu'}{\nu'^2 - \nu^2} G_1(\nu', Q^2)$$

$$S_2(\nu, Q^2) = 4\int_0^\infty \frac{d\nu'\nu}{\nu'^2 - \nu^2} G_2(\nu', Q^2)$$

Extended GDH Sum

$$\Gamma_1 = \int g_1 dx = \frac{Q^2}{8} S_1(0, Q^2)$$

$Q^2 = 0$ ➔GDH Sum Rule

$Q^2 = \infty$ ➔Bjorken Sum Rule

BC Sum Rule

$$\Gamma_2 = \int_0^1 g_2(x, Q^2) dx = 0$$

Superconvergence relation valid at any Q^2

B&C, Annals Phys. **56**, 453 (1970).

Generalized Forward Spin Polarizabilities

Drechsel, Pasquini and Vanderhaehen, Phys. Rep. **378**, 99 (2003).

$$g_{TT}(\nu, Q^2) = \frac{\nu}{2\pi^2}\mathcal{P}\int_{\nu_0}^{\infty} \frac{d\nu' K}{\nu'^2 - \nu^2}\sigma_{TT}(\nu', Q^2) \qquad g_{LT}(\nu, Q^2) = \frac{1}{2\pi^2}\mathcal{P}\int_{\nu_0}^{\infty} \frac{d\nu' \nu' K}{\nu'^2 - \nu^2}\sigma_{LT}(\nu', Q^2)$$

LEX of g_{TT} and g_{LT} lead to the Generalized Forward Spin Polarizabilities

$$\gamma_0(Q^2) = \left(\frac{1}{2\pi^2}\right)\int_{\nu_0}^{\infty} \frac{K(\nu, Q^2)}{\nu}\frac{\sigma_{TT}(\nu, Q^2)}{\nu^3}d\nu$$

$$= \frac{16\alpha M^2}{Q^6}\int_0^{x_0} x^2 \left[g_1(x, Q^2) - \frac{4M^2}{Q^2}x^2 g_2(x, Q^2)\right]$$

x^2 weighting dominated by RR

$$\delta_{LT}(Q^2) = \left(\frac{1}{2\pi^2}\right)\int_{\nu_0}^{\infty} \frac{K(\nu, Q^2)}{\nu}\frac{\sigma_{LT}(\nu, Q^2)}{Q\nu^2}d\nu$$

$$= \frac{16\alpha M^2}{Q^6}\int_0^{x_0} x^2 \left[g_1(x, Q^2) + g_2(x, Q^2)\right]$$

Thomas Jefferson National Accelerator Facility

JLab

Jefferson Lab

CWLinear Accelerator

3 Exp. Halls

0.1 nA to 200 μA

P_b ~ 85%

6 GeV Max Energy

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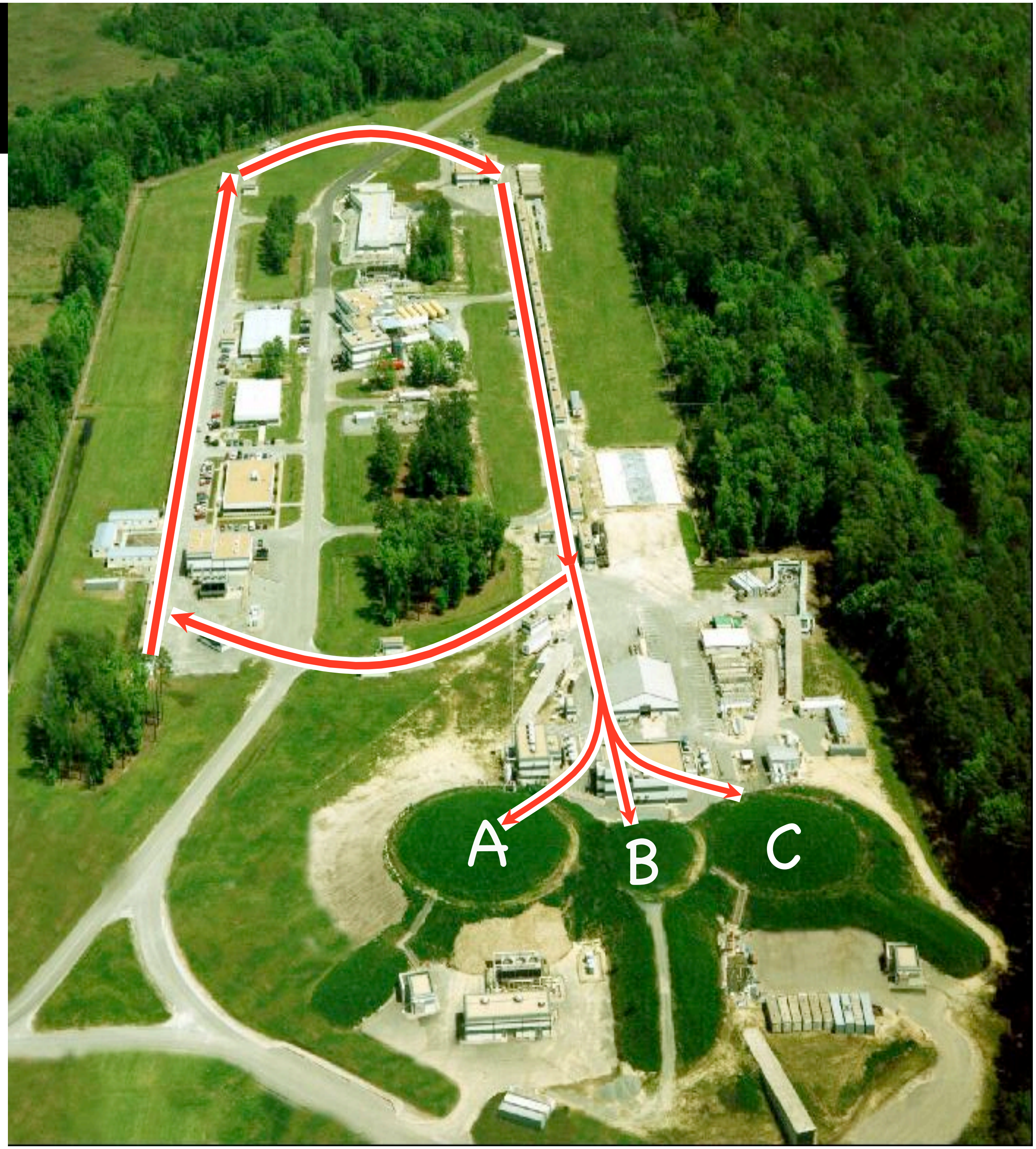
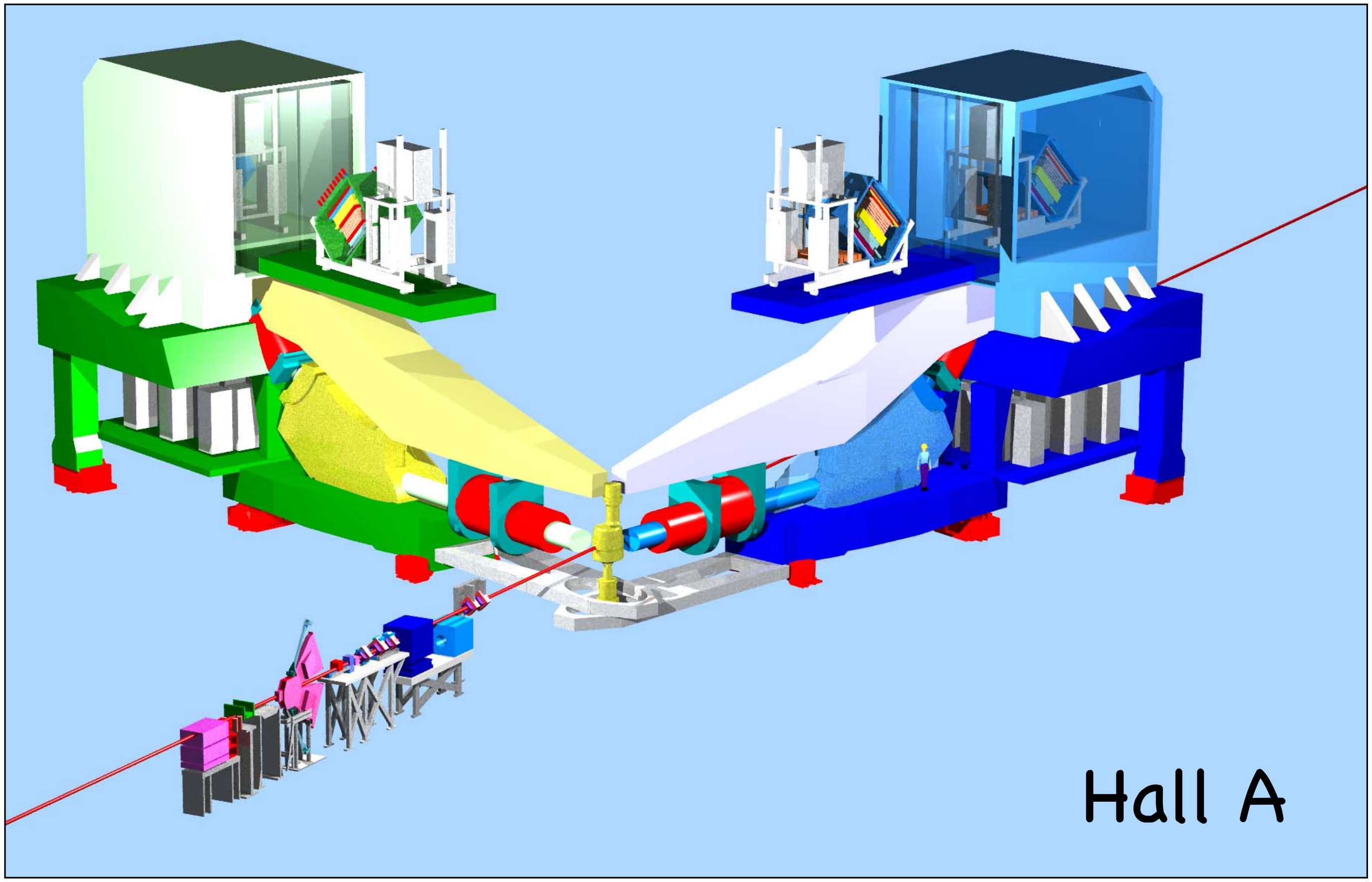


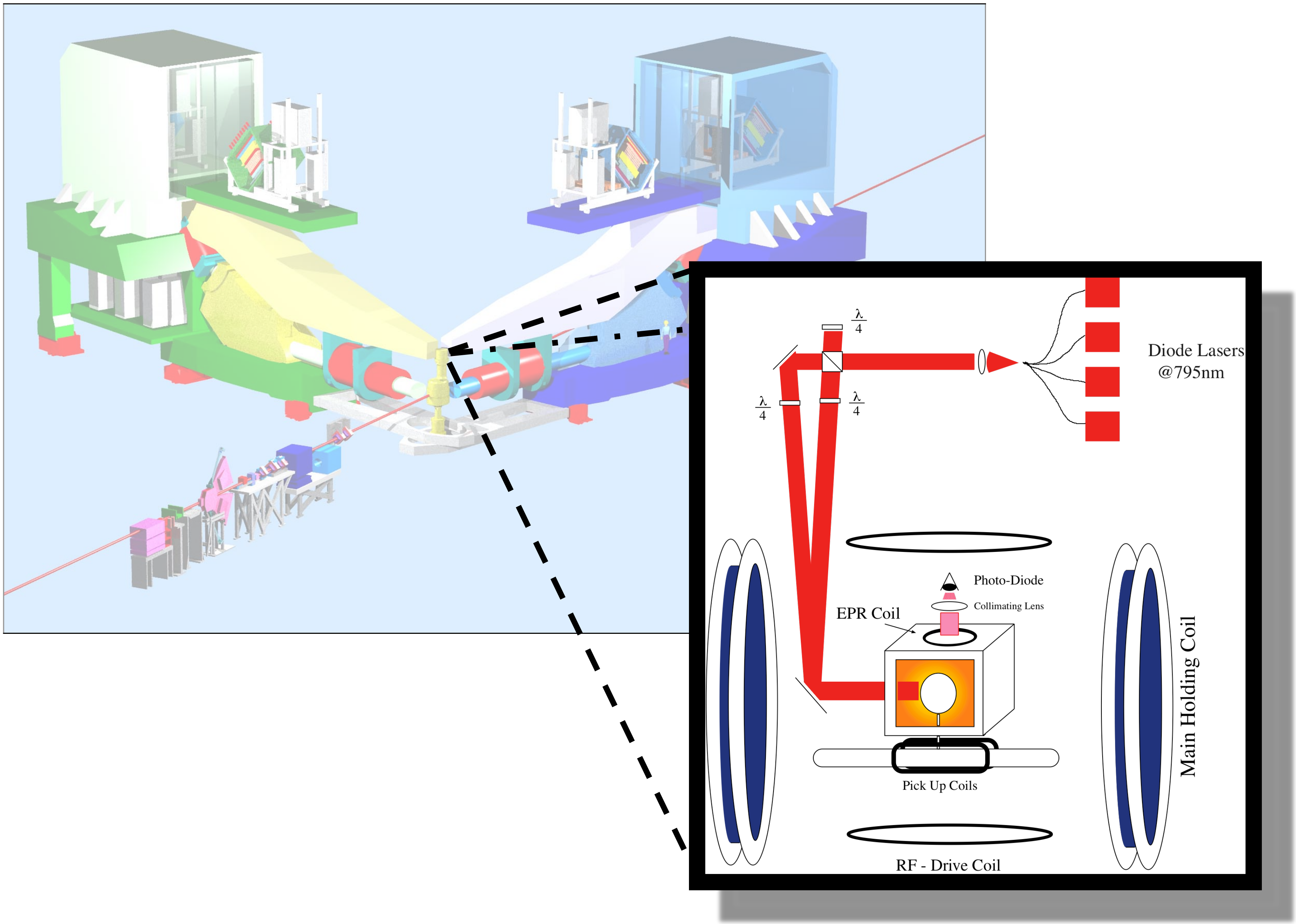
Hall A



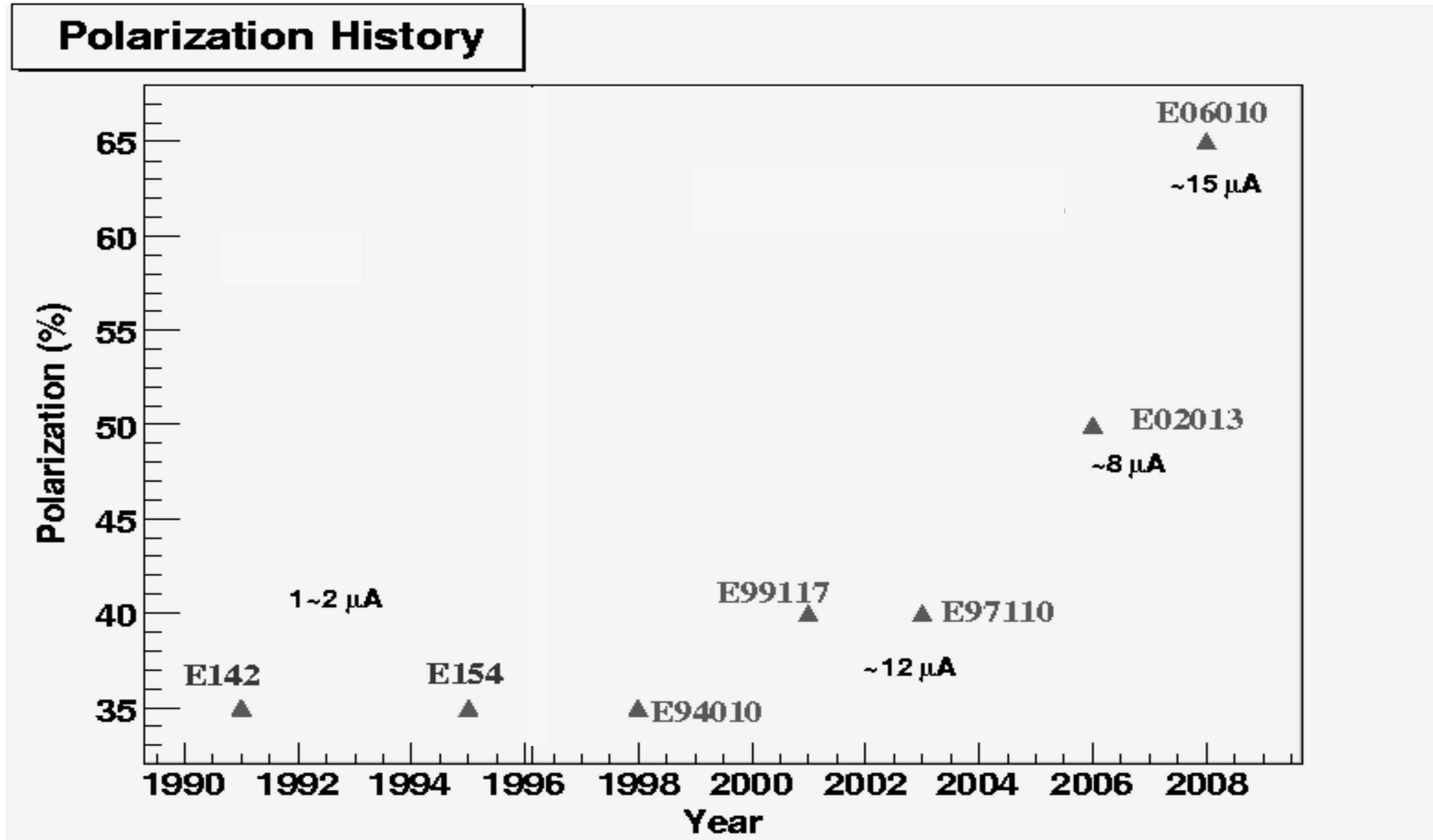
High Resolution Spectrometers (HRS)

- 10^{-4} Resolution
- Momentum : 0.3–4.3 GeV/c
- Max $\mathcal{L} = 10^{38} cm^{-2}s^{-1}$
- Anglular acceptance $\approx$ 4msr

^{3}He Polarized Target

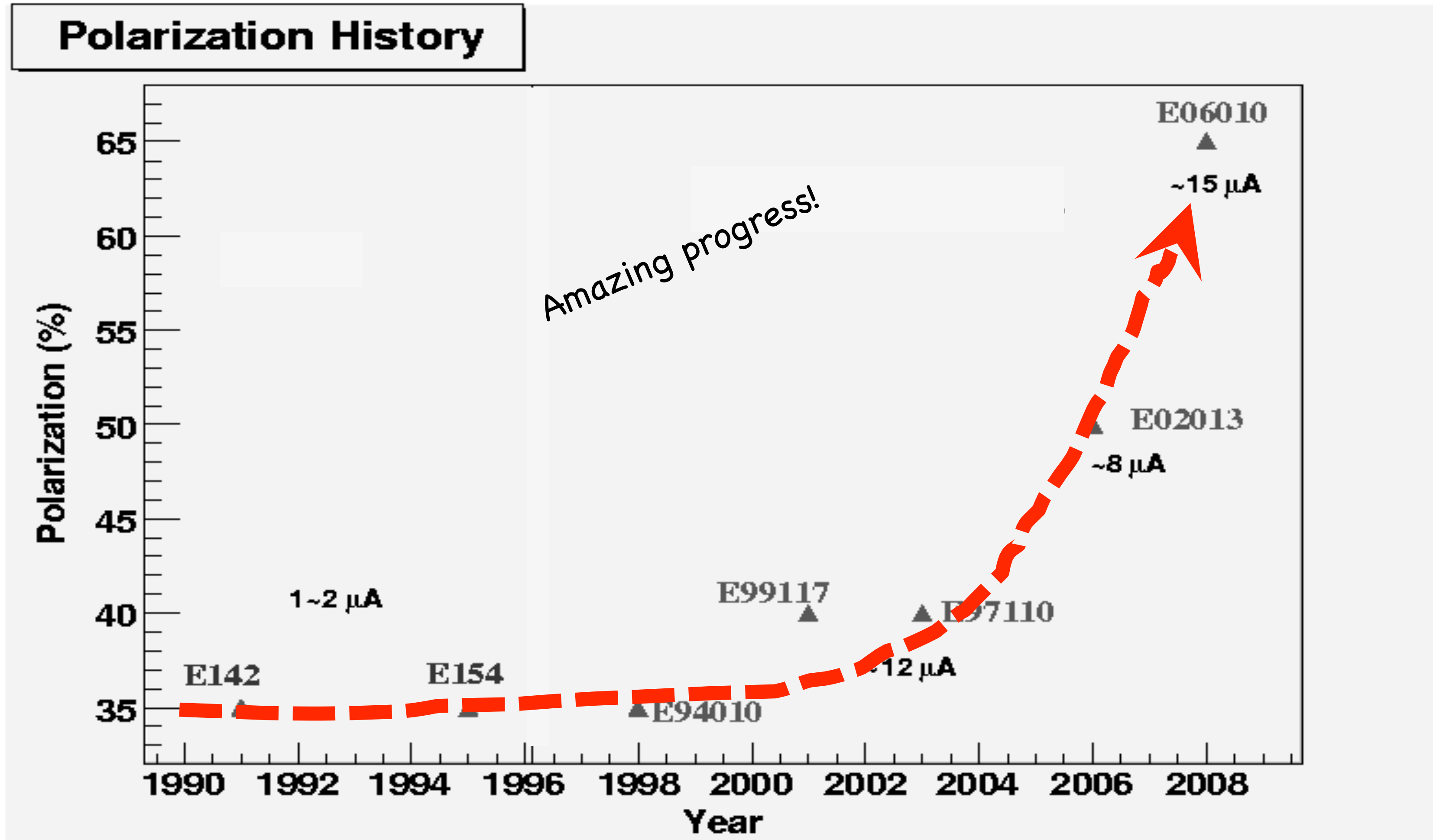


^{3}He Target Polarizations



Courtesy of Chiranjib Dutta

^{3}He Target Polarizations



Several Target Groups: JLab, UVa, W&M, Temple, Kentucky, UNH, Duke ...

^{3}He Data From JLab

Resonance Region Experiments

E01012 Spokesmen: J.P. Chen, S. Choi, and N. Liyanage

E94010 Spokesmen: J.P. Chen, G. Cates, and Z.E. Meziani

E97110 Spokesmen: J.P. Chen, A. Deur, and F. Garibaldi

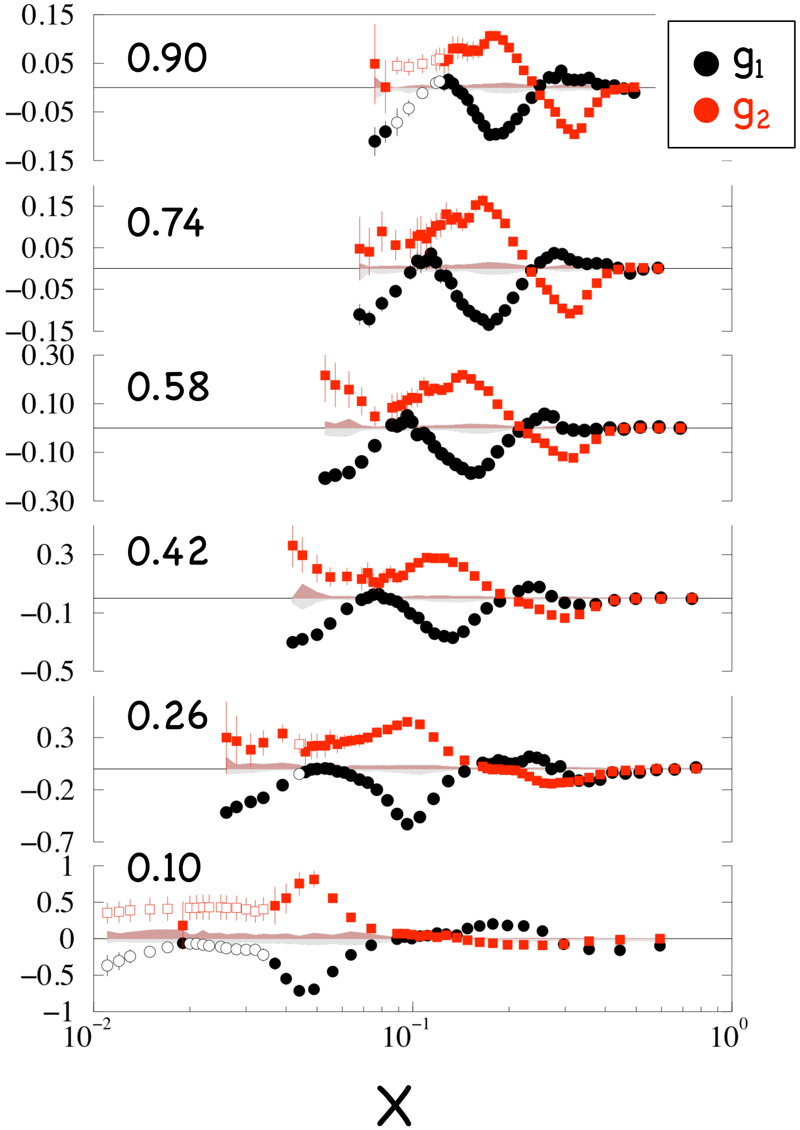
Relevent Publications

KS, PRL 101, 022303 (2008)

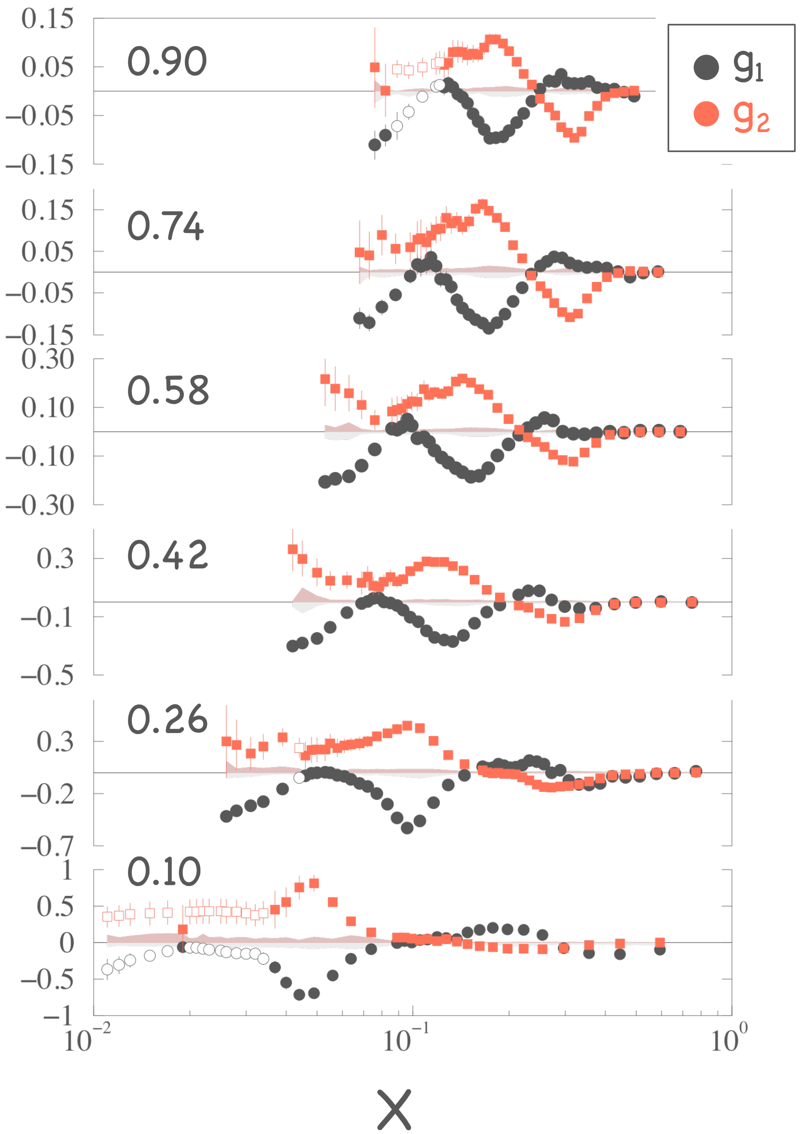
P. Solvignon et al PRL 101, 182502 (2008)

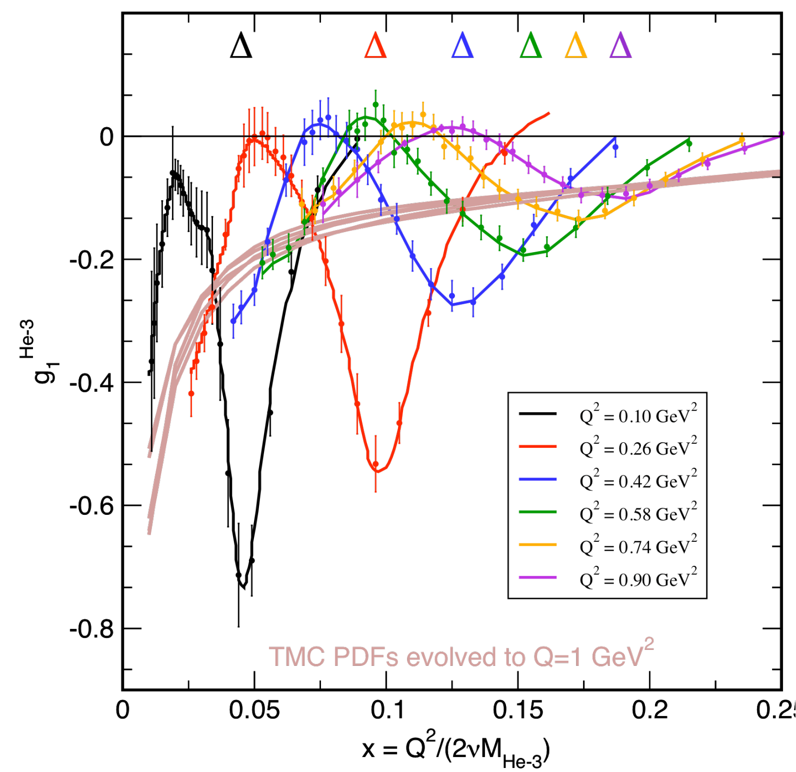
Thanks to Patricia Solvignon and Vince Sulkosky for providing plots

³He Structure Functions



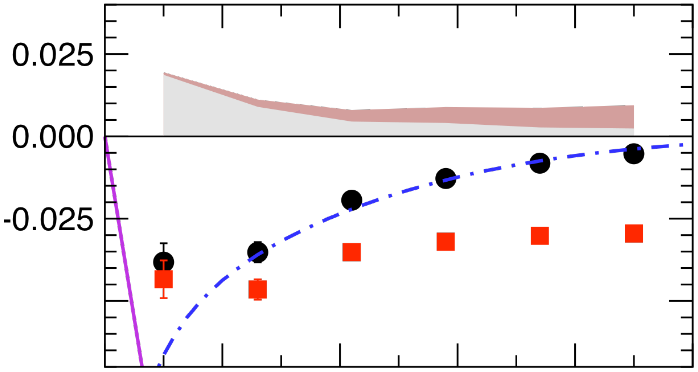
^{3}He Structure Functions





Compared to DIS expectations

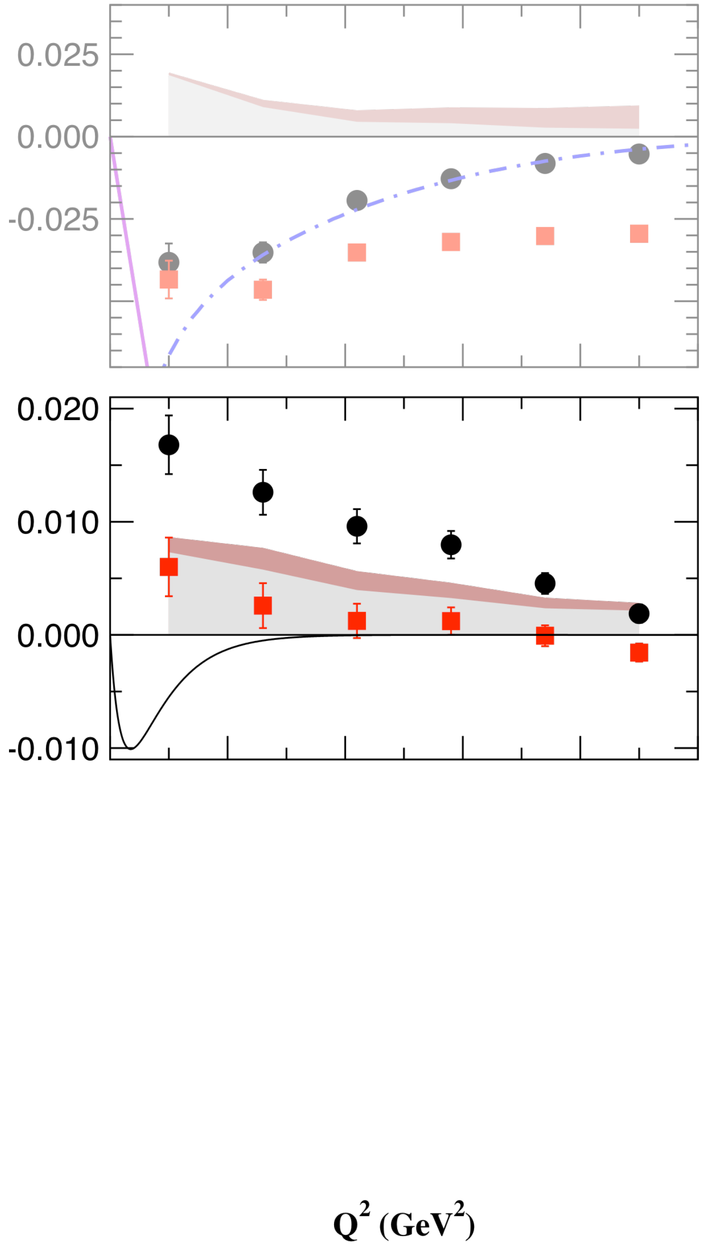
^{3}He Moments



$$\Gamma_1(Q^2) = \int g_1(x, Q^2)dx$$

Q^2 (GeV^2)

^{3}He Moments

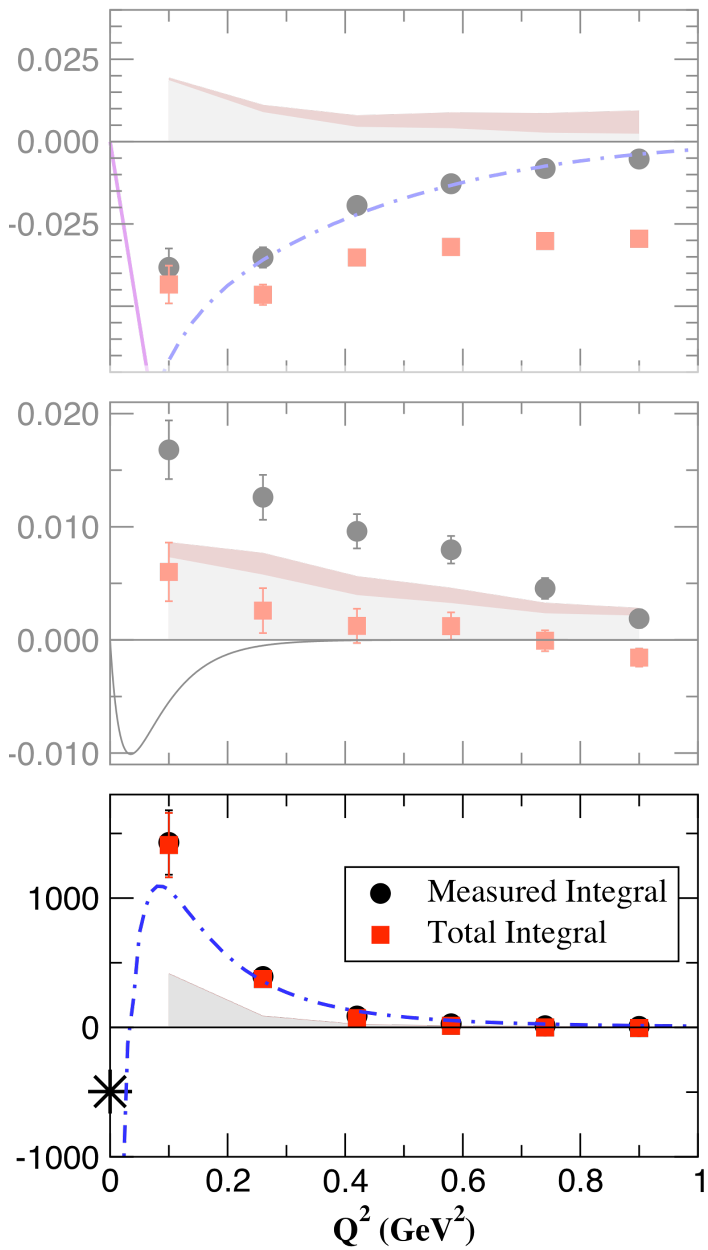


$$\Gamma_1(Q^2) = \int g_1(x, Q^2)dx$$

$$\Gamma_2(Q^2) = \int g_2(x, Q^2)dx$$

more on later slide...

^{3}He Moments



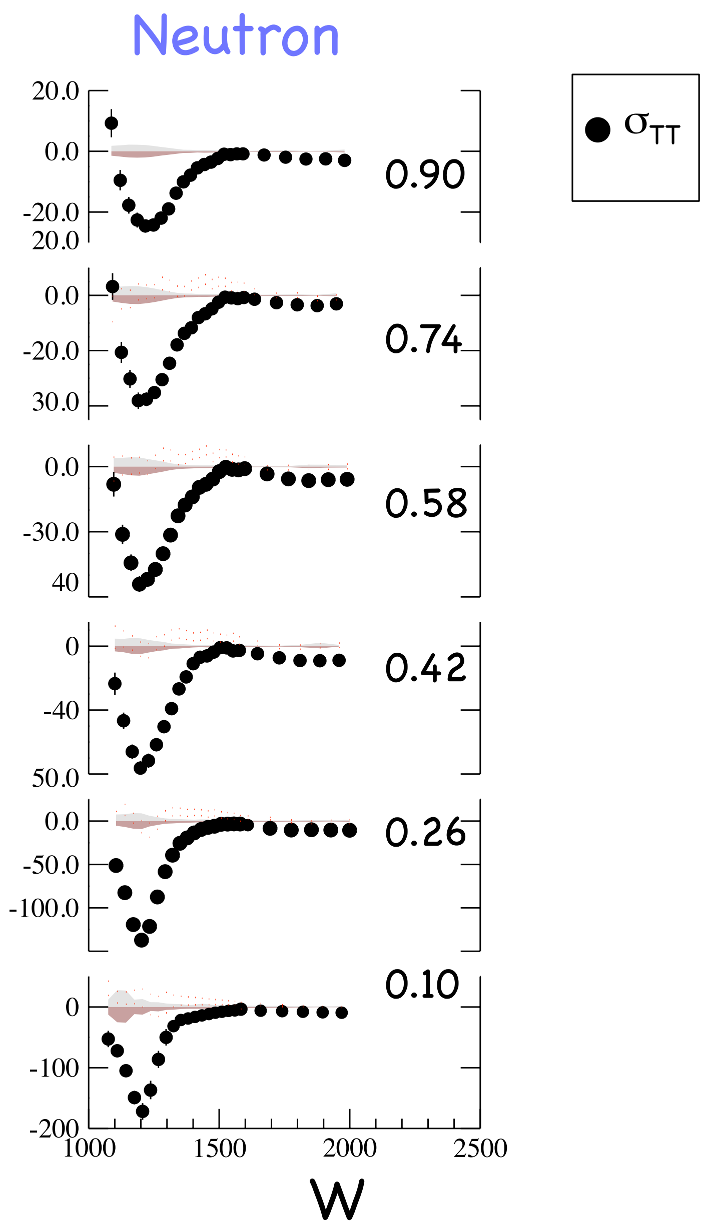
$$\Gamma_1(Q^2) = \int g_1(x, Q^2)dx$$

$$\Gamma_2(Q^2) = \int g_2(x, Q^2)dx$$

$$I_A(Q^2) = \frac{16\pi^2\alpha}{Q^2} \int_0^{x_{th}} \left[g_1(x, Q^2) - \frac{4M^2x^2}{Q^2}\, g_2(x, Q^2) \right] dx$$

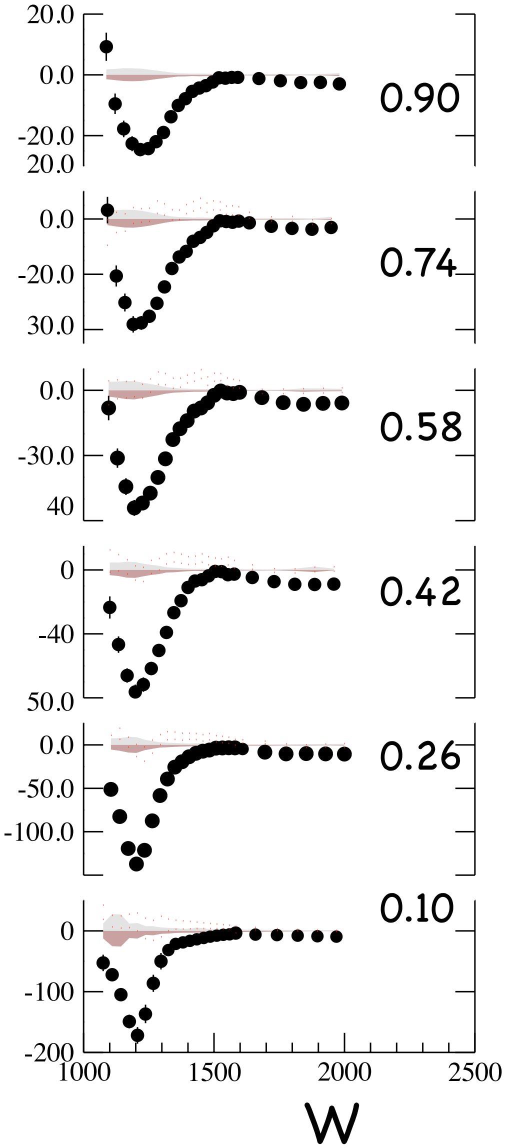
Extended GDH Sum

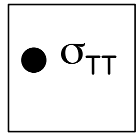
σ_{TT} (GDH Integrand)



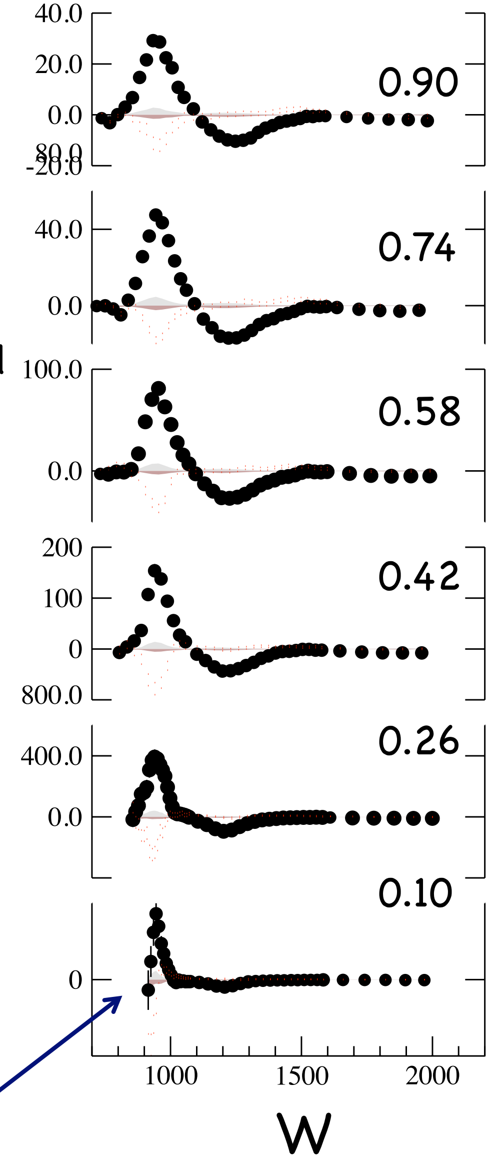
σ_{TT} (GDH Integrand)

Neutron





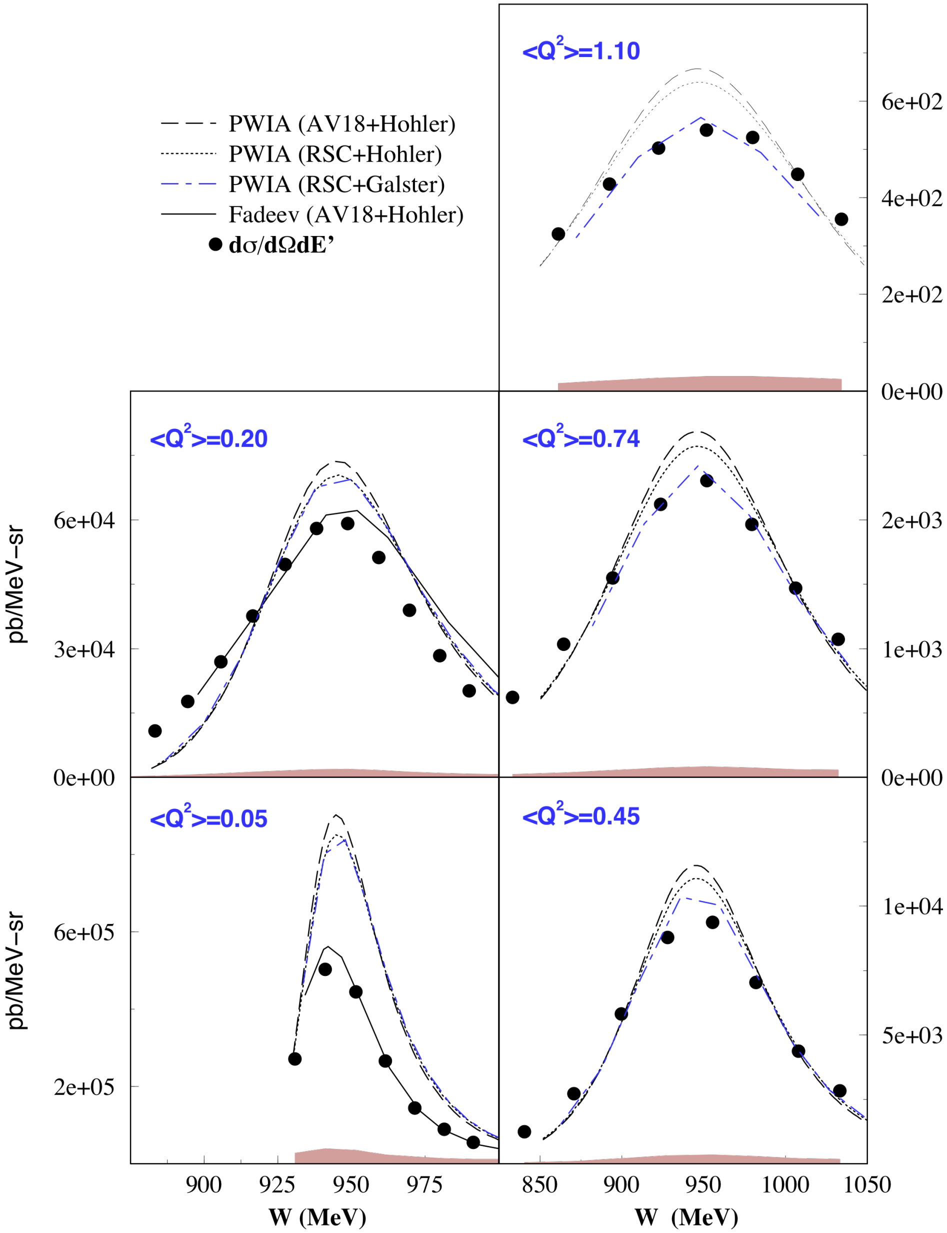
^{3}He



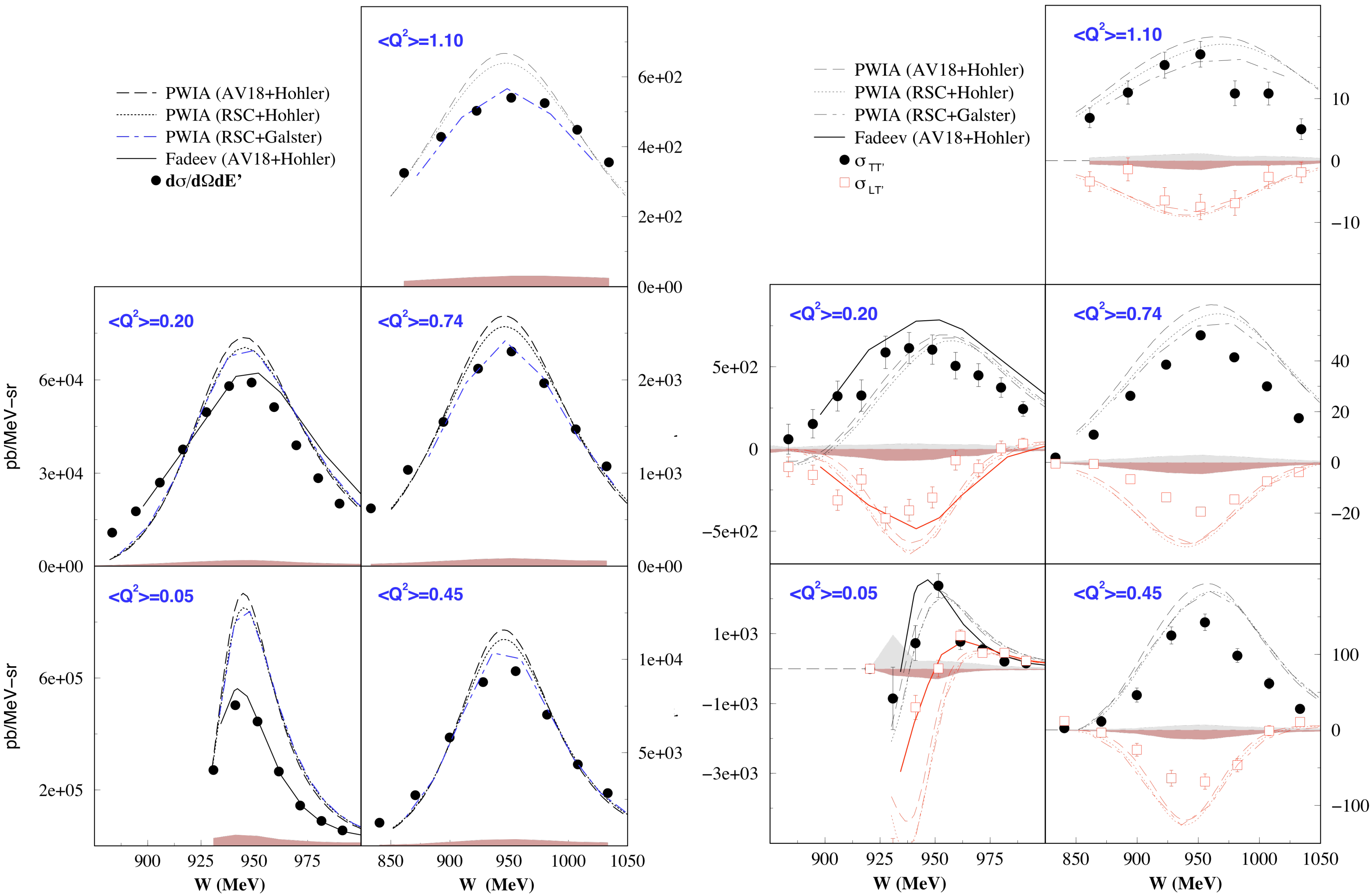
^{3}He GDH dominated by QE region

e-disintegration?

^{3}He Quasi-elastic

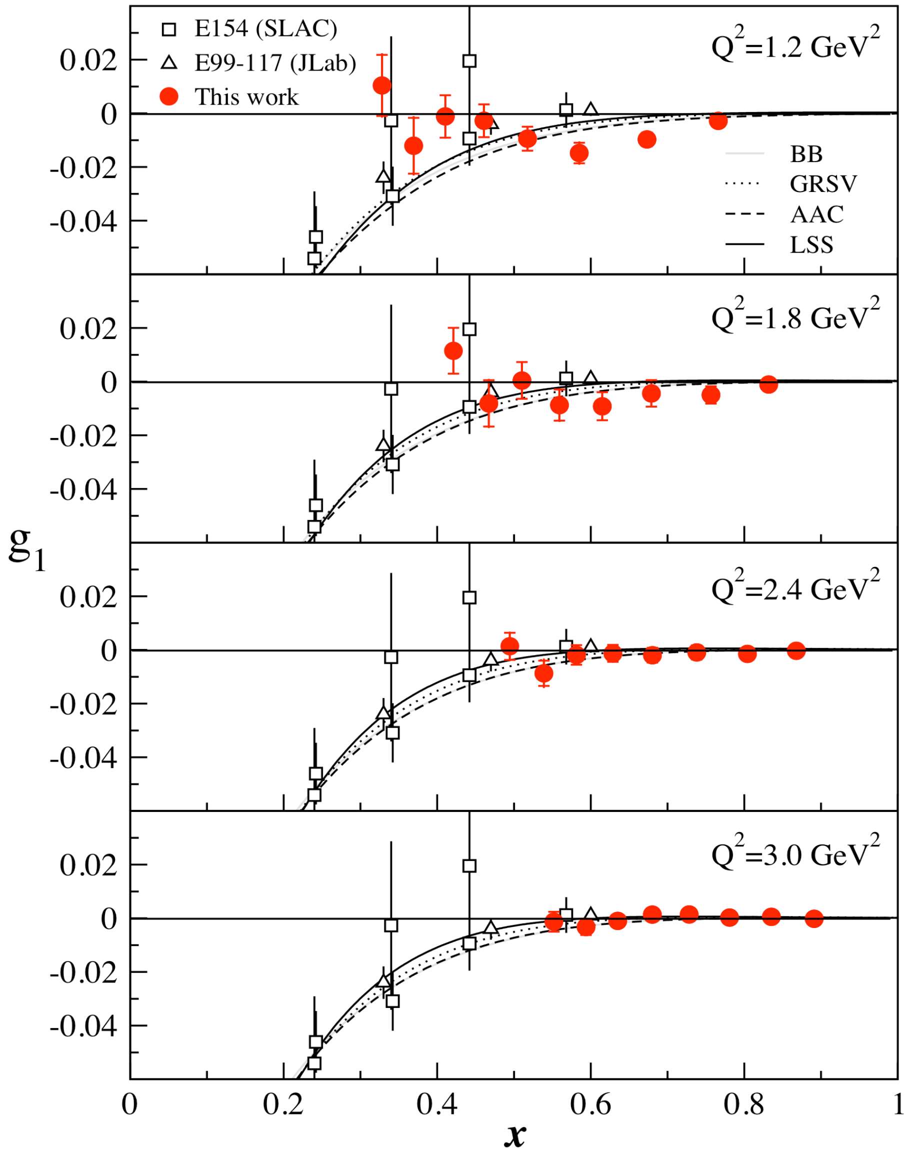


^{3}He Quasi-elastic



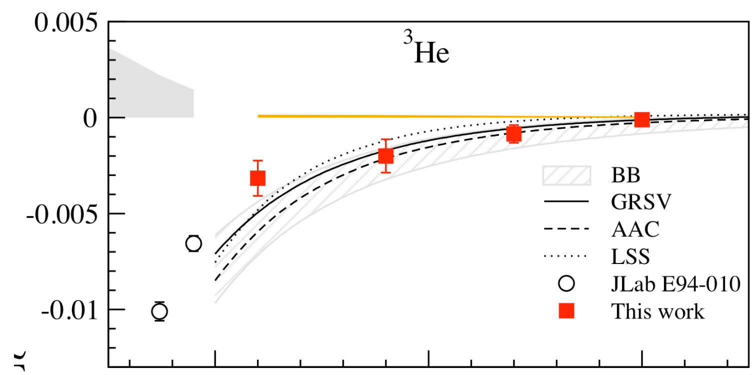
^{3}He Structure Functions

1.2 < Q^2 < 3 GeV^2



E01-012 Collaboration

P. Solvignon et al PRL 101:182502,(2008)



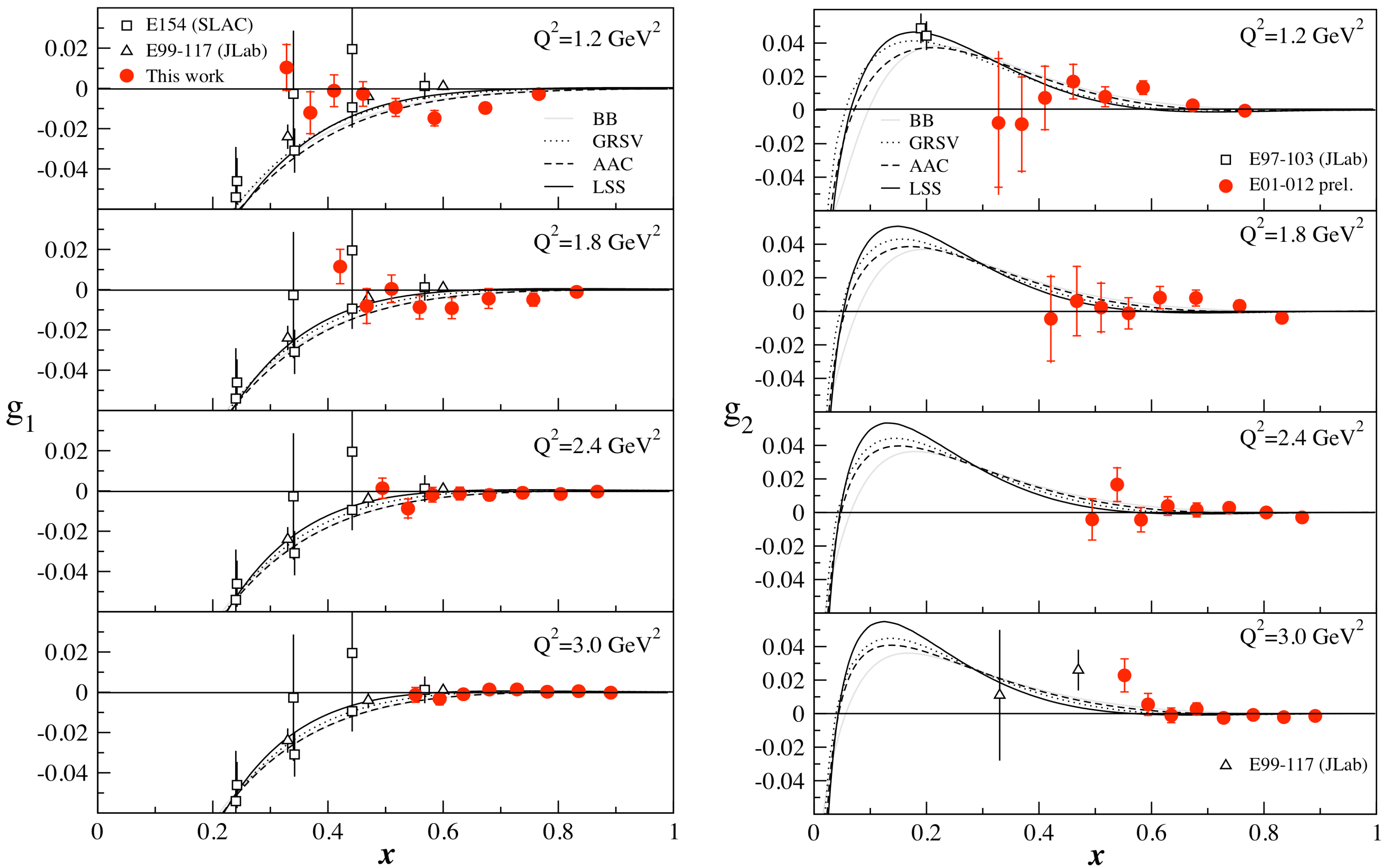
$$\overline{\Gamma}_1(Q^2) = \int g_1(x, Q^2)dx$$

continous with E94010

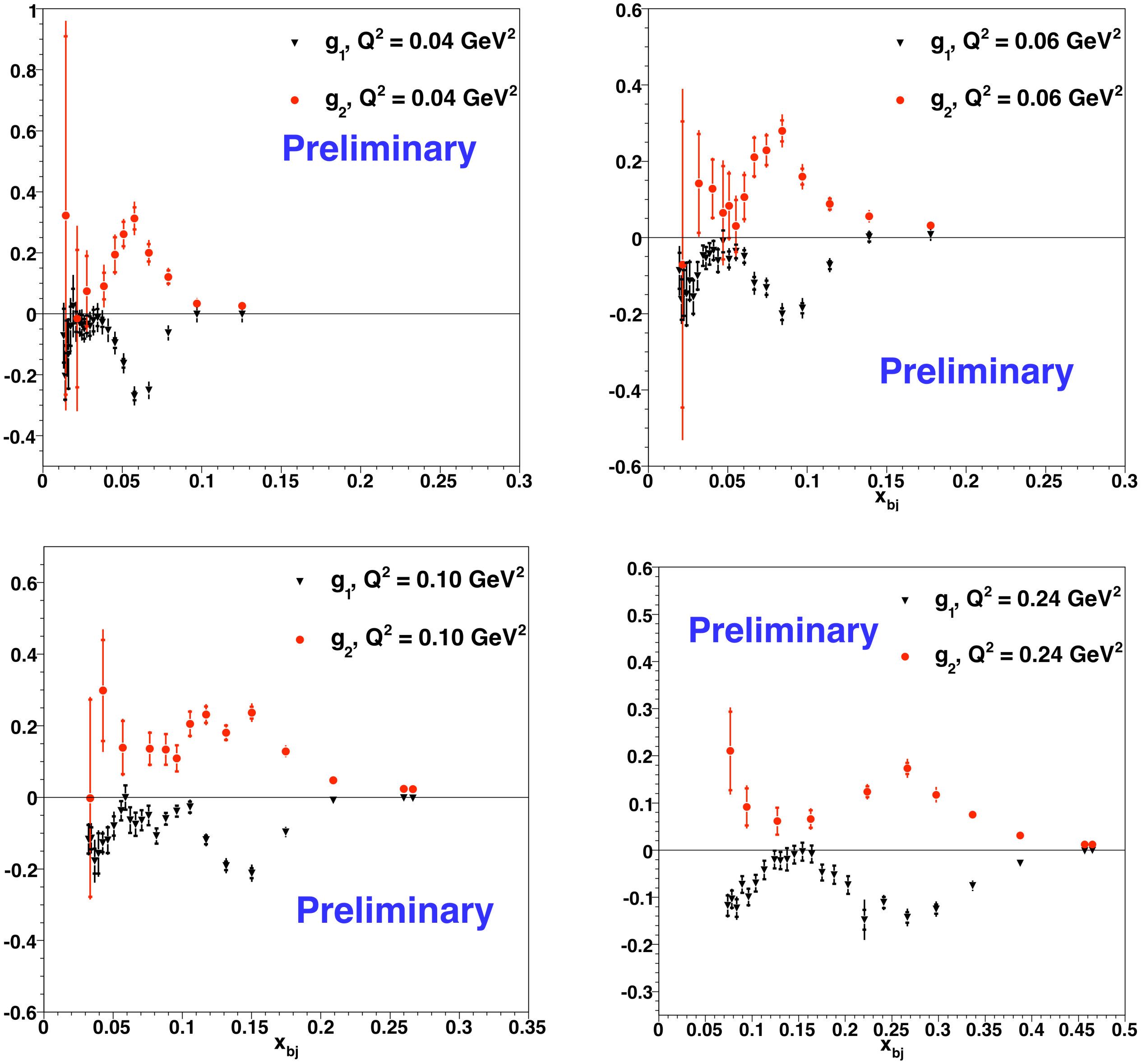
figs courtesy of
P. Solvignon

³He Structure Functions

$1.2 < Q^2 < 3$ GeV²



^{3}He Structure Functions

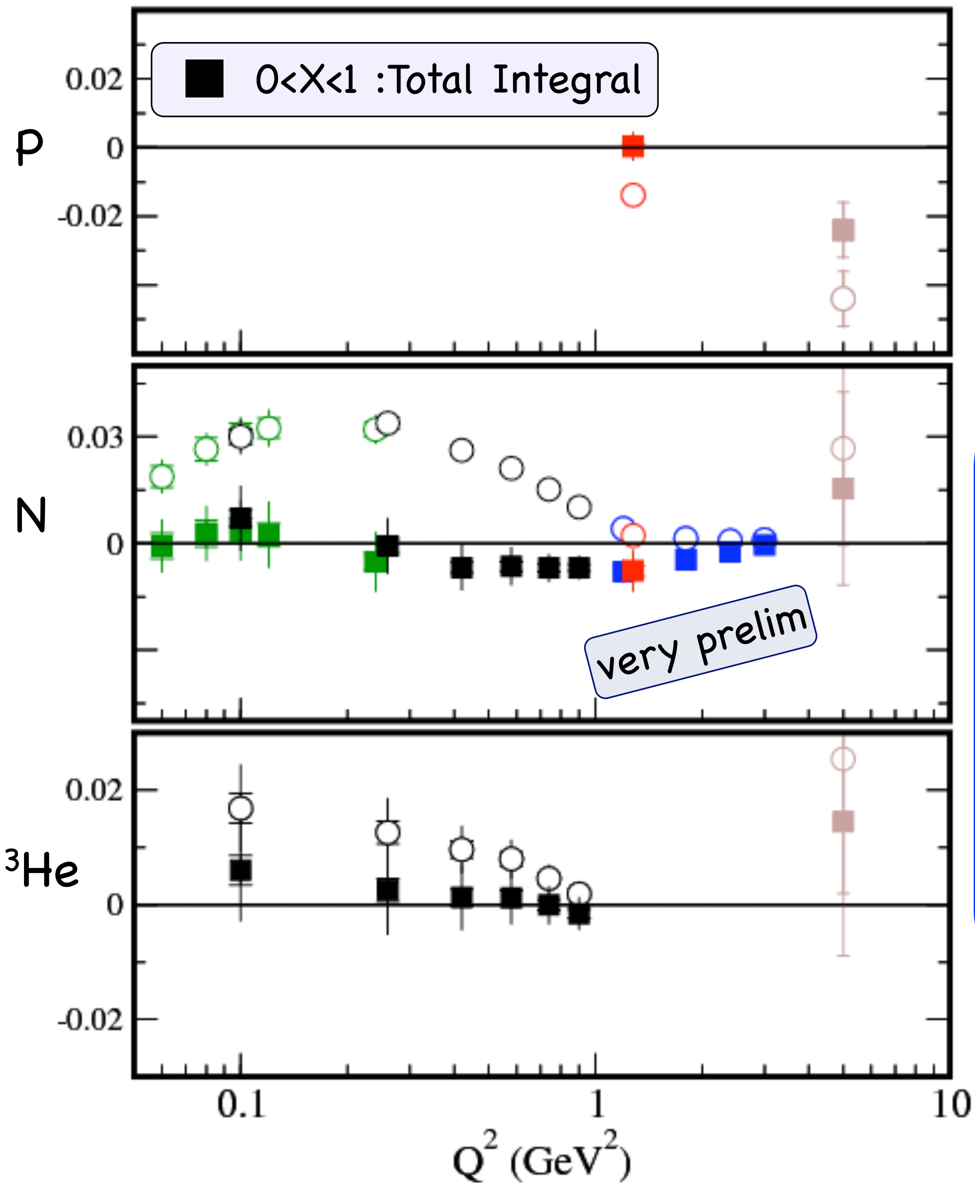


E97110 Preliminary

$0.04 < Q^2 < 0.24$

figs courtesy of
V. Sulkosky

BC Sum Rule



$$\int_0^1 g_2(x, Q^2)dx = 0$$

<u>BC = RES+DIS+ELASTIC</u>

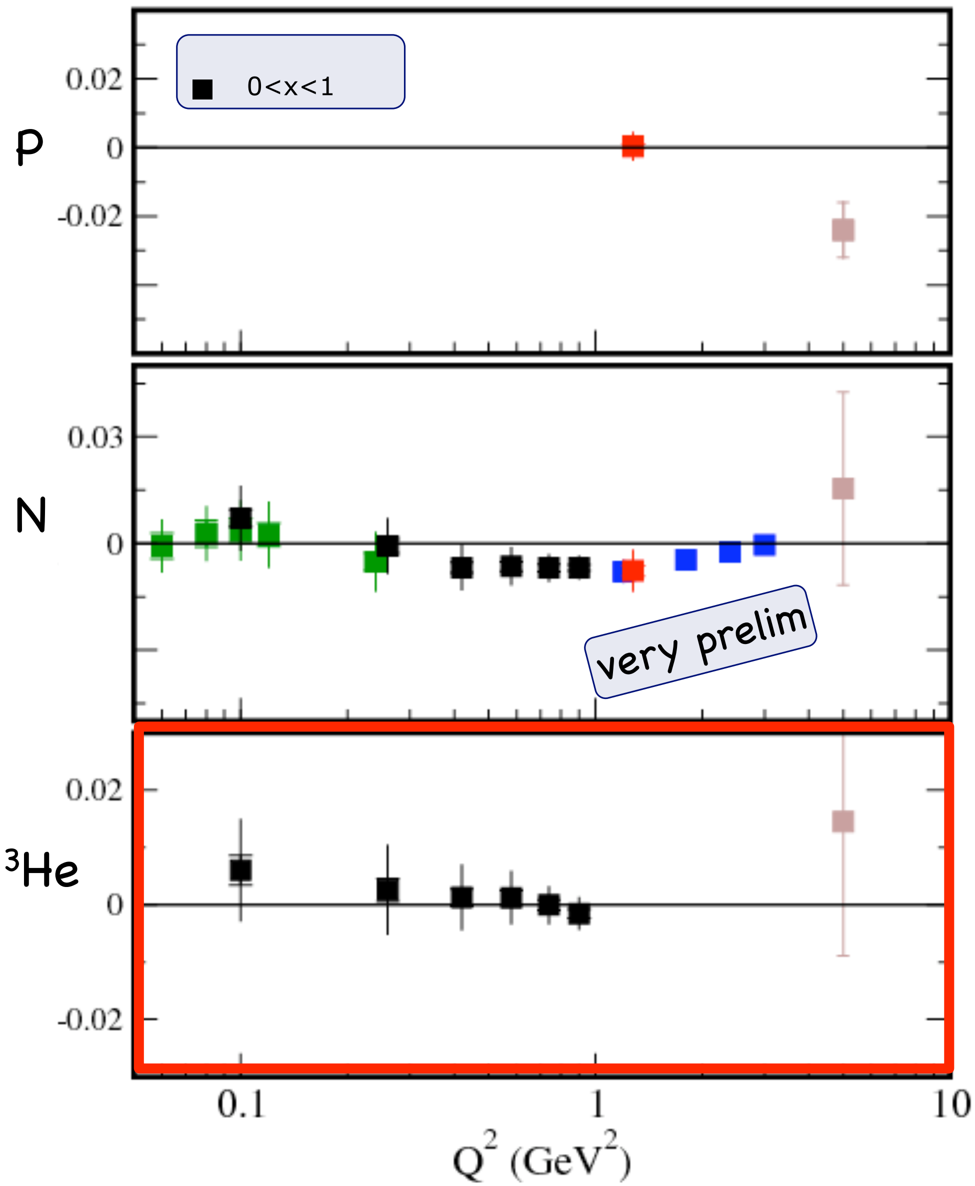
"RES": Here refers to measured x-range

"DIS": refers to unmeasured low x part of the integral. Not strictly Deep Inelastic Scattering due to low Q^2

Assume Leading Twist Behaviour

Elastic: From well know FFs (<5%)

BC Sum Rule

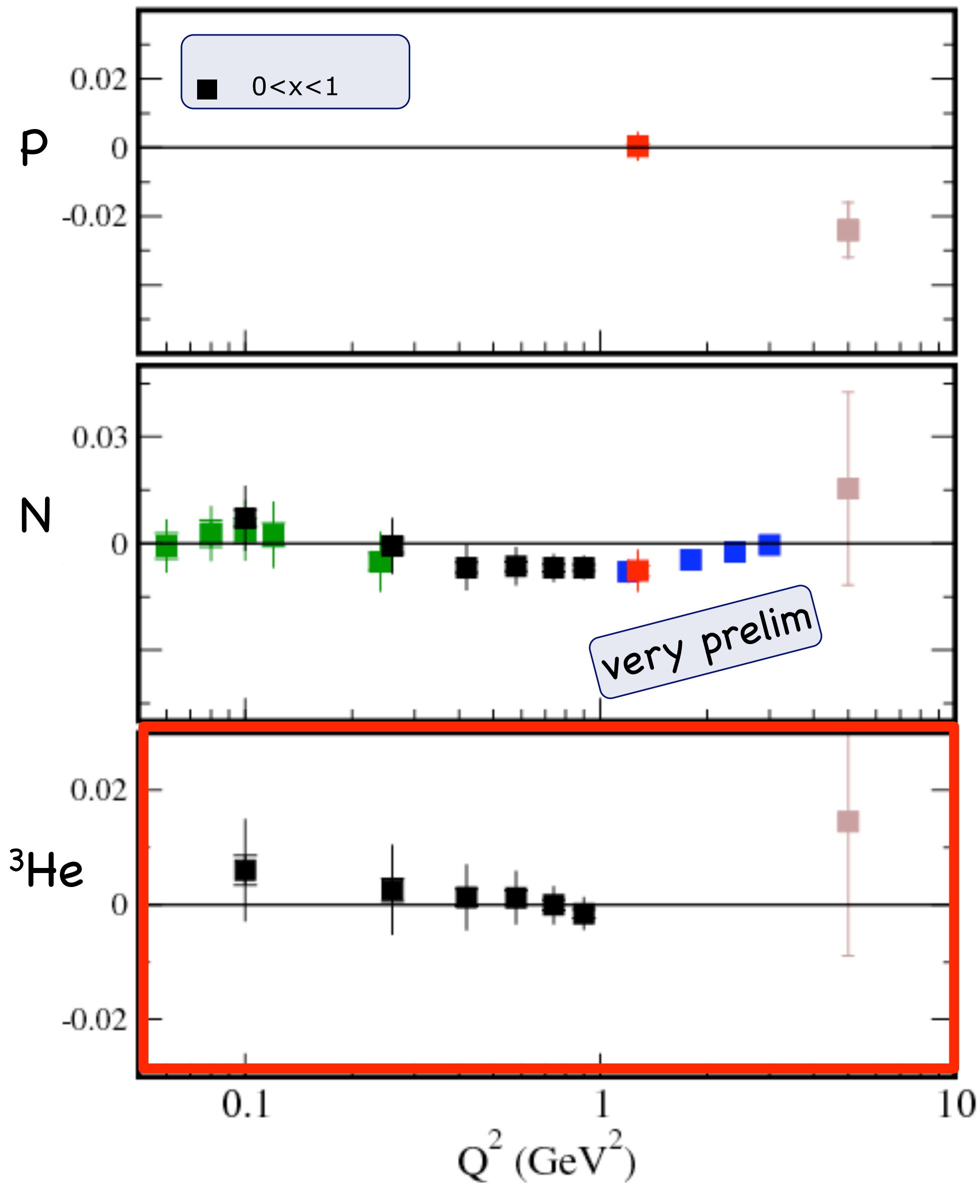


BC satisfied w/in errors for JLab Proton
2.8σ violation seen in SLAC data

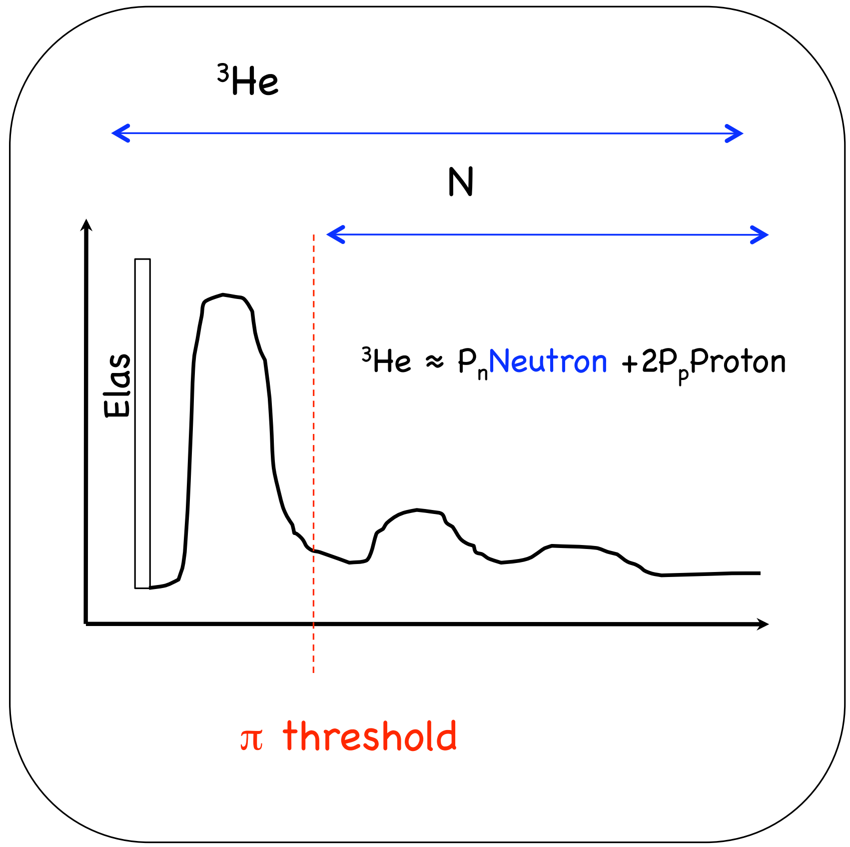
BC satisfied w/in errors for Neutron
(But just barely in vicinity of Q^2=1!)

BC satisfied w/in errors for ^{3}He

BC Sum Rule



Difference between N and ^{3}He Sum rules

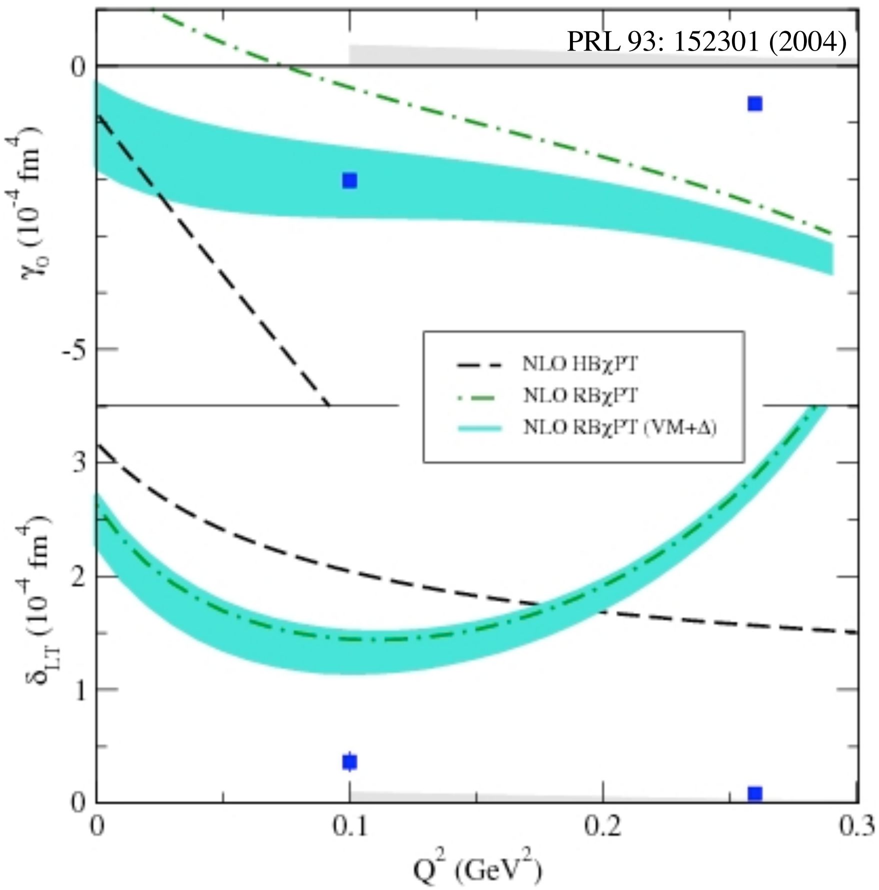


Note: ^{3}He requires use of nuclear elastic

Spin Polarizabilities

Forward Spin Polarizabilities

Neutron



$$\gamma_0 = \frac{16\alpha M^2}{Q^6}\int_0^{x_0} x^2\left[g_1 - \frac{4M^2}{Q^2}x^2 g_2\right]$$

Add Δ by hand:
major effect for γ_0 but not for δ_{LT}

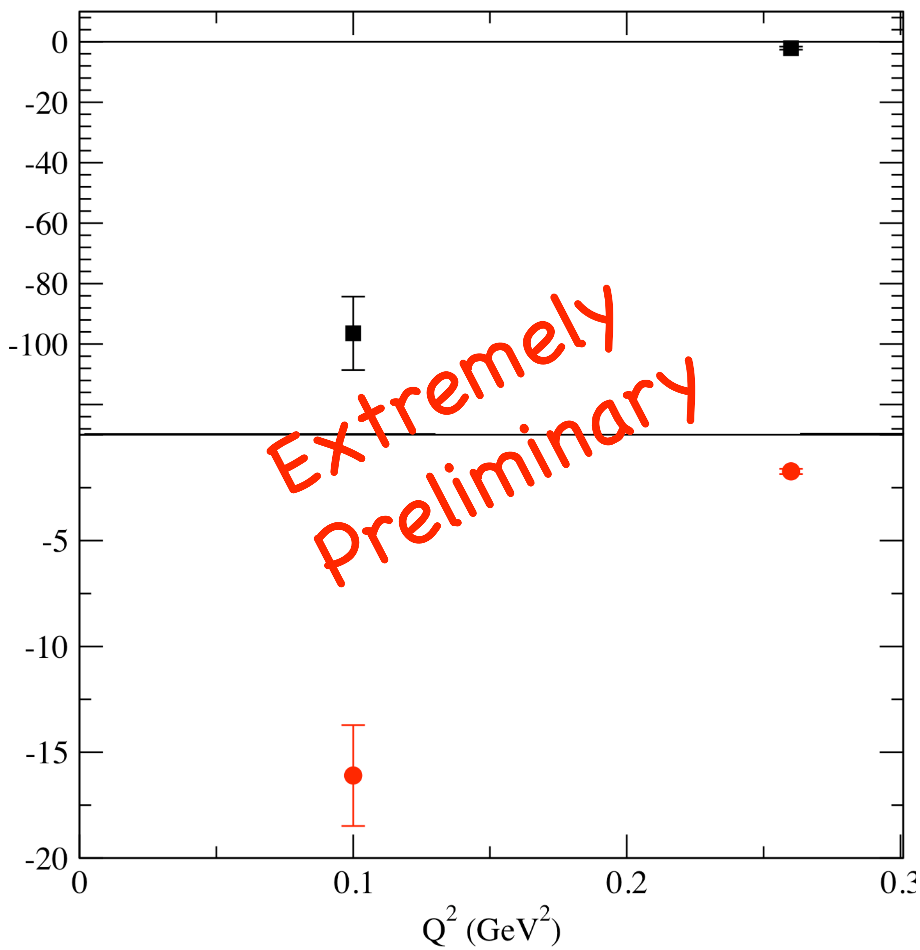
$$\delta_{LT} = \frac{16\alpha M^2}{Q^6}\int_0^{x_0} x^2\left[g_1 + g_2\right]$$

Heavy Baryon χPT Calculation
Kao, Spitzenberg, Vanderhaeghen
PRD 67:016001(2003)

Relativistic Baryon χPT
Bernard, Hemmert, Meissner
PRD 67:076008(2003)

Forward Spin Polarizabilities

^{3}He higher moments



$$\int_{x_0}^{0} dx\ x^2\ g_1^{^3He}(x, Q^2)$$

$$\int_{x_0}^{0} dx\ x^2\ g_2^{^3He}(x, Q^2)$$

Free from Nuclear Corrections

Would love to get theory curves on these plots.

Summary

Dispersion Relations & Sum Rules

JLab Hall A ^{3}He Resonance data

E94-010, E01-012, E97-110

Existing ^{3}He GDH Data

trending positive at Q^2=0.1 GeV^2
while we expect -496 at Q^2=0. E97110 may resolve

Nuclear BC sum rule data

Satisfaction for ^{3}He despite large elastic and QE contributions

Preliminary ^{3}He polarizabilities

Free from nuclear corrections
theory calculations/input needed

Backups

g_2 Structure Function

Wandzura-Wilczek relation PLB **72** (1977) 195

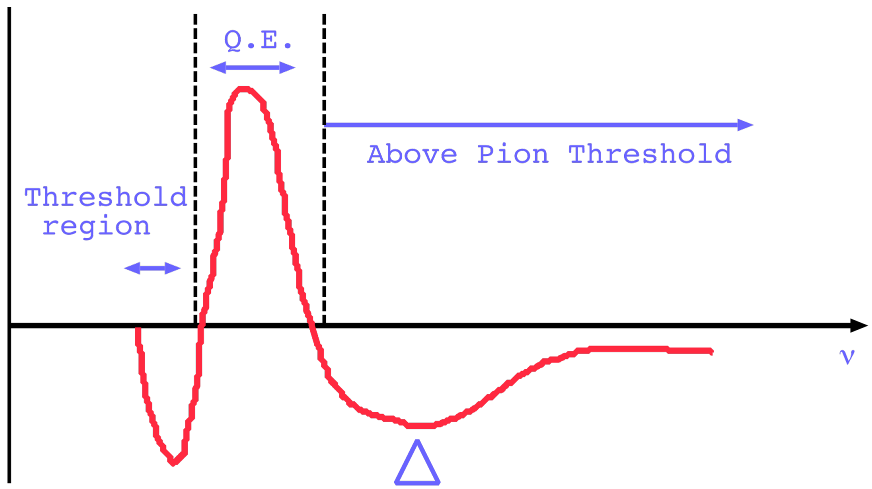
$$g_2^{WW}(x,Q^2) = -g_1(x,Q^2) + \int_x^1 \frac{g_1(y,Q^2)}{y} dy$$

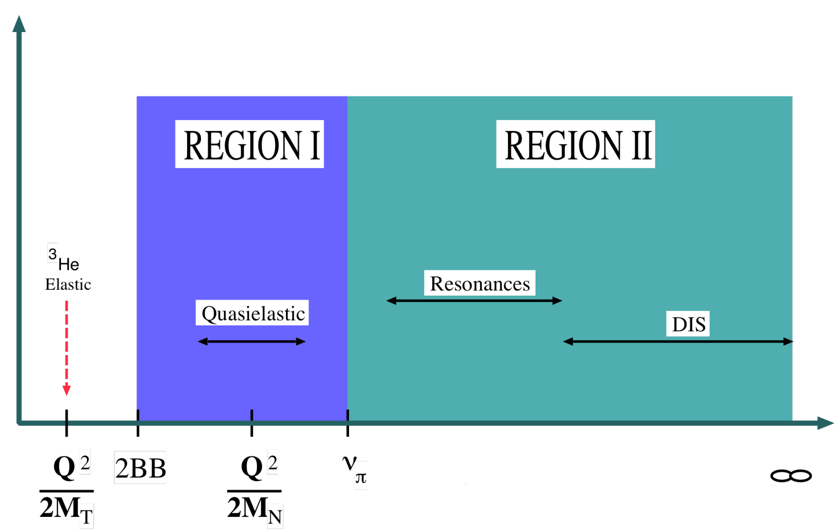
Leading twist determined entirely by g_1

$$g_2 = g_2^{WW} + \overline{g}_2$$

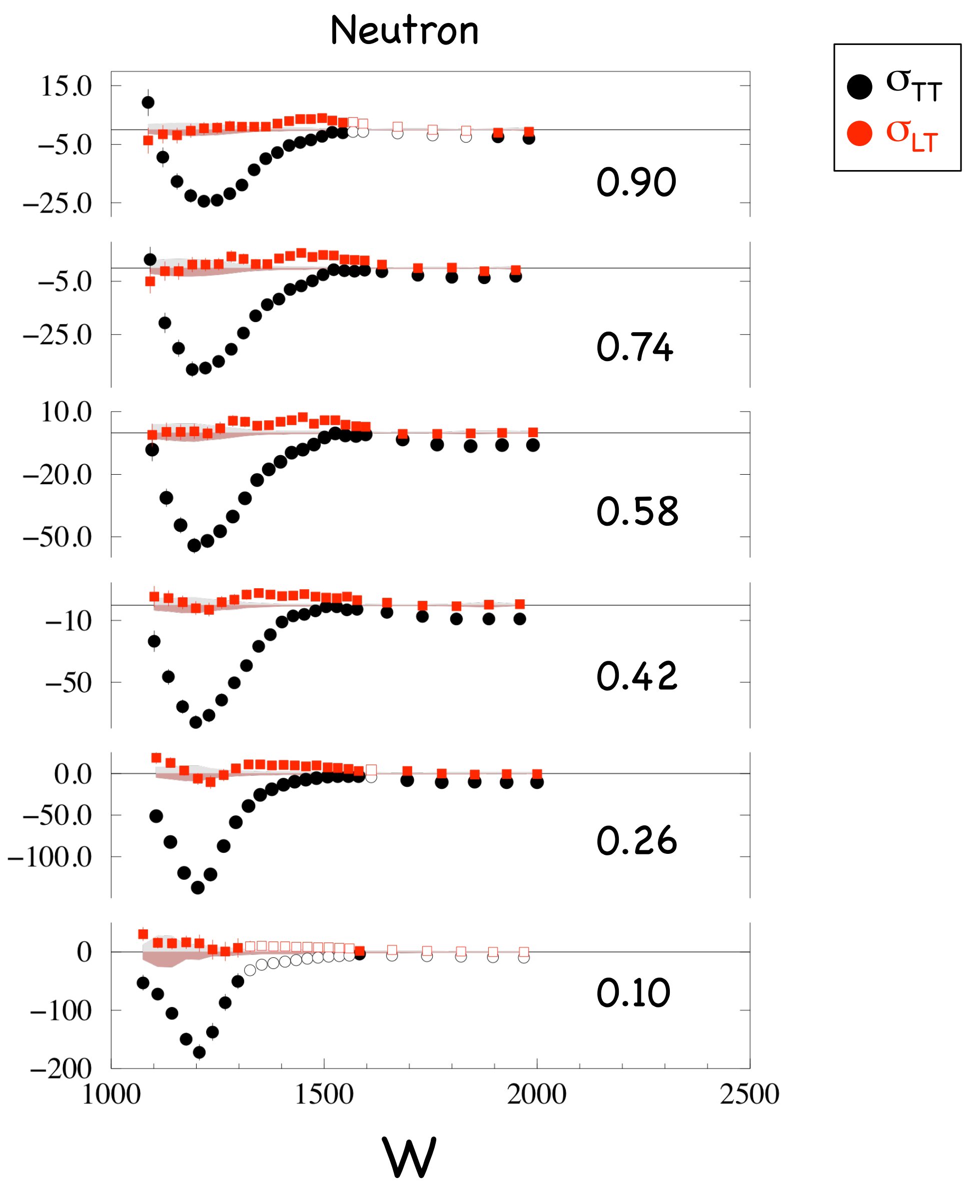
Higher twist

g_2 doesn't exist in Parton Model.
Good quantity to study higher twist


Q.E.
Above Pion Threshold
Threshold region
ν
Δ


REGION I
REGION II
3He Elastic
Quasielastic
Resonances
DIS
Q2/2MT
2BB
Q2/2MN
νπ
∞

σ_{TT} and σ_{LT}



σ_{TT} and σ_{LT}