Nuclear *Forces* on the Lattice

Silas Beane

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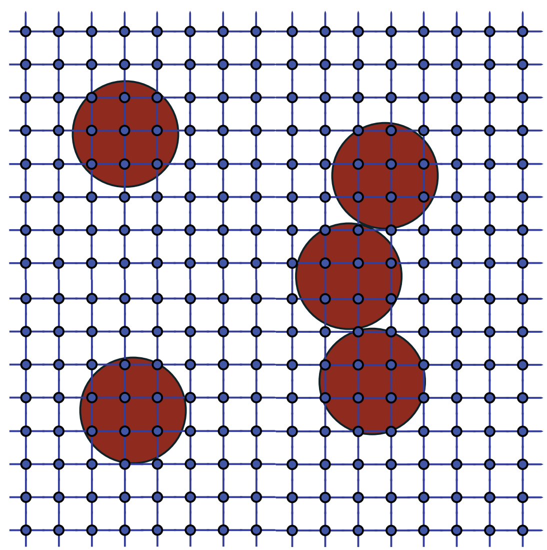

In general, Forces/Potentials are not observables in quantum mechanics!

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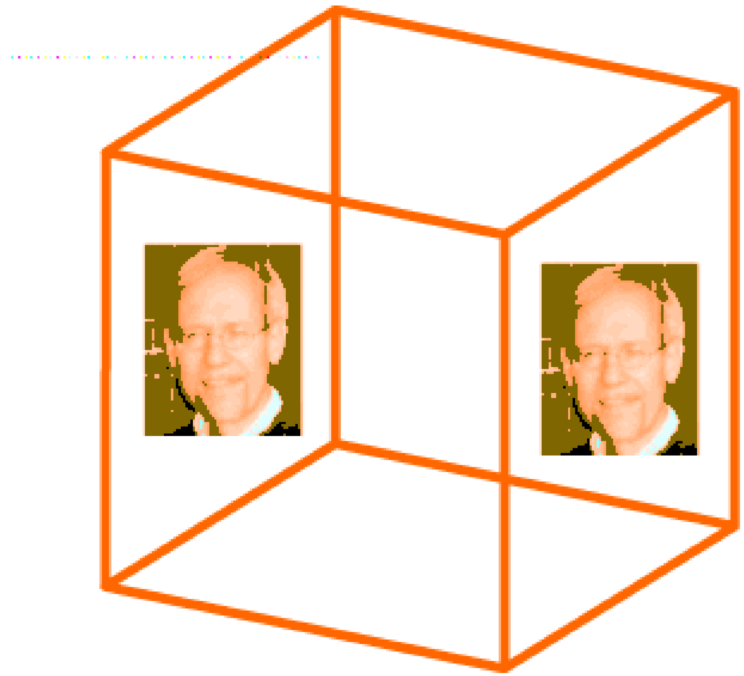
On the lattice we obtain observables from energy levels!



$$\Delta E_0(2,L) \;=\; \frac{4\pi\, a}{m_\pi\, L^3}\left\{1-\left(\frac{a}{\pi\, L}\right)\mathcal{I}+\left(\frac{a}{\pi\, L}\right)^2\left[\mathcal{I}^2-\mathcal{J}\right]+\left(\frac{a}{\pi\, L}\right)^3\left[-\mathcal{I}^3+3\mathcal{I}\mathcal{J}-\mathcal{K}\right]\right\}+\frac{8\pi^2 a^3}{m_\pi\, L^6}\, r+\mathcal{O}\left(L^{-7}\right)$$

$$\mathcal{I} \;=\; \lim_{\Lambda_j\to\infty}\sum_{\mathbf{i}\neq\mathbf{0}}^{|\mathbf{i}|\leq\Lambda_j}\frac{1}{|\mathbf{i}|^2}-4\pi\Lambda_j=-8.91363291781 \quad , \quad \mathcal{J} \;=\; \sum_{\mathbf{i}\neq\mathbf{0}}\frac{1}{|\mathbf{i}|^4}=16.532315959 \quad , \quad \mathcal{K}=\sum_{\mathbf{i}\neq\mathbf{0}}\frac{1}{|\mathbf{i}|^6}=8.401923974433$$

- Will Detmold (Washington)
- Tom Luu (Livermore)
- Kostas Orginos (William and Mary/JLab)
- Assumpta Parreño (Barcelona)
- Martin Savage (Washington)
- Aaron Torok (New Hampshire)
- André Walker-Loud (Maryland)
- Silas Beane (New Hampshire)

Outline

- Introduction
  - Signal/noise estimates
  - The relevance of chiral dynamics
- $\pi\pi$, $K\pi$
- MB
- NN
- Signal/noise revisited: NNN *and beyond..*
- Conclusion

Un-Outline

- $\pi\pi\ldots\pi$ and $KK\ldots K$
- YN Parreño/WG3-Thu
- V_{NN} Hatsuda/WG3-Tue

Signal/Noise Estimates Lepage (1989)

Correlators with pions

$$\langle \theta(t) \rangle \quad = \quad \langle \left(\sum_x \pi^-(\mathbf{x}, t) \right)^n \left(\pi^+(\mathbf{0}, 0) \right)^n \rangle \ \rightarrow \ A_0 \ e^{-n m_\pi t}$$

Signal/Noise Estimates Lepage (1989)

Correlators with pions

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$$\begin{aligned}
N\sigma^2 &\sim \langle\theta(t)^\dagger\theta(t)\rangle - \langle\theta(t)\rangle^2 \\
&= \langle\left(\sum_x \pi^-(\mathbf{x},t)\right)^n \left(\sum_y \pi^+(\mathbf{y},t)\right)^n \left(\pi^+(\mathbf{0},0)\right)^n \left(\pi^-(\mathbf{0},0)\right)^n\rangle - \langle\theta(t)\rangle^2 \\
&\rightarrow \left(A_2 - A_0^2\right)\ e^{-2n m_\pi t}
\end{aligned}$$

Signal/Noise Estimates Lepage (1989)

Correlators with pions

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$$\frac{\text{noise}}{\text{signal}} \quad \sim \quad \frac{\sigma(t)}{\langle\theta(t)\rangle} \sim \frac{\sqrt{\left(A_2 - A_0^2\right)\; e^{-n m_\pi t}}}{\sqrt{N} A_0\; e^{-n m_\pi t}} \sim \frac{1}{\sqrt{N}}$$

Noise independent of t!

Correlators with nucleons

$$\langle \theta^{ii}(t) \rangle \quad = \quad \sum_{\mathbf{x}} \langle p^i(\mathbf{x}, t) \overline{p}^i(\mathbf{0}, 0) \rangle \ \rightarrow \ A^{ii}_{p0} \ e^{-m_p t}$$

Correlators with nucleons

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$$N\sigma^2 \quad \sim \quad \langle \theta^{ii\dagger}(t) \theta(t)^{ii} \rangle - \langle \theta^{ii}(t) \rangle^2 \; = \; \sum_x \langle p^i(\mathbf{x},t) \overline{p}^i(\mathbf{x},t) p^i(\mathbf{0},0) \overline{p}^i(\mathbf{0},0) \rangle \quad - \quad \langle \theta^{ii}(t) \rangle^2$$

$$\rightarrow \; A_{p2} \; e^{-3m_\pi t} - A^2_{p0} \; e^{-2m_p t} \quad \rightarrow \; A_{p2} \; e^{-3m_\pi t}$$

Correlators with nucleons

$$\langle \theta^{ii}(t) \rangle \quad = \quad \sum_{\mathbf{x}} \langle p^i(\mathbf{x},t)\overline{p}^i(\mathbf{0},0)\rangle \;\rightarrow\; A_{p0}^{ii}\; e^{-m_p t}$$

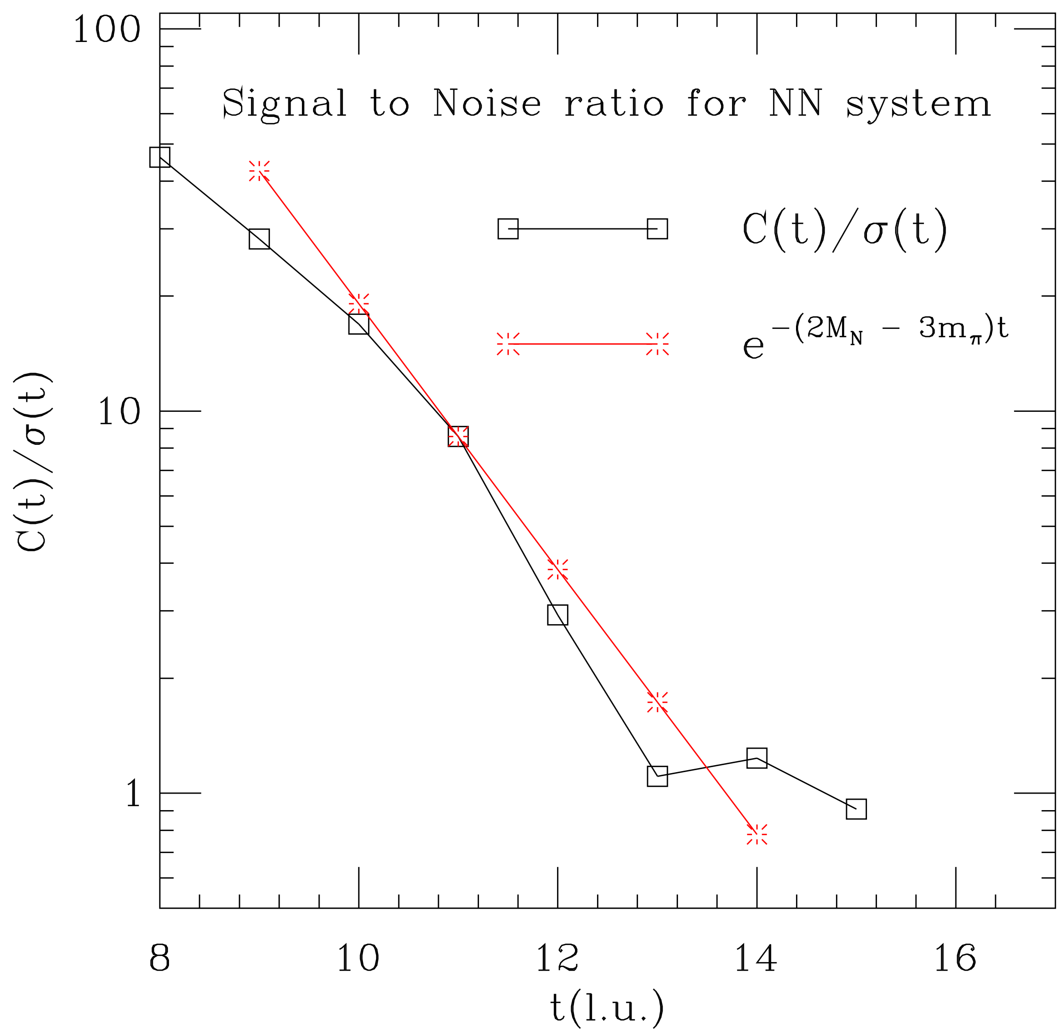
$$N\sigma^2 \quad \sim \quad \langle \theta^{ii\dagger}(t)\theta(t)^{ii}\rangle - \langle \theta^{ii}(t)\rangle^2 \;=\; \sum_{x} \langle p^i(\mathbf{x},t)\overline{p}^i(\mathbf{x},t)p^i(\mathbf{0},0)\overline{p}^i(\mathbf{0},0)\rangle \quad - \quad \langle \theta^{ii}(t)\rangle^2$$

$$\rightarrow\; A_{p2}\; e^{-3m_\pi t} - A_{p0}^2\; e^{-2m_p t} \quad \rightarrow\; A_{p2}\; e^{-3m_\pi t}$$

$$\frac{\text{noise}}{\text{signal}} \quad \sim \quad \frac{\sigma(t)}{\langle \theta(t)\rangle} \sim \frac{1}{\sqrt{N}}\; e^{(m_p - \frac{3}{2}m_\pi)t}$$

Exponential growth of noise!

$np\ (^1S_0)$ NPLQCD MILC/2064f21b676m010m050



(Courtesy of P. Bedaque and A. Walker-Loud)

The Relevance of Chiral Dynamics

With ∞ resources chiral dynamics is irrelevant to lattice QCD!

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However...

$$\mathrm{COST} \sim (L)^4 \, (b)^{-6.5} \, (M_q)^{-2.5}$$

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However...

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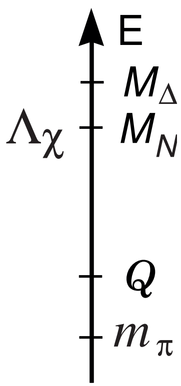
We need chiral dynamics because we are poor..

χ-PT

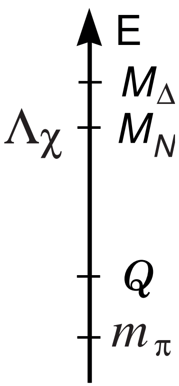
QCD:



$$\frac{Q}{\Lambda_\chi}, \quad \frac{m_\pi}{\Lambda_\chi}, \quad \frac{M_\Delta - M_N}{\Lambda_\chi}, \ \ldots$$

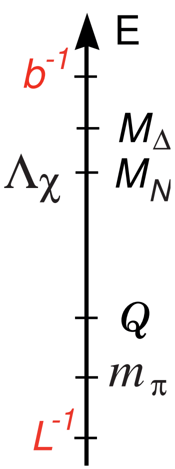
χ-PT

QCD:



$$\frac{Q}{\Lambda_\chi}, \quad \frac{m_\pi}{\Lambda_\chi}, \quad \frac{M_\Delta - M_N}{\Lambda_\chi}, \ \ldots$$

Lattice QCD :



$$b\, m_\pi, \quad e^{-m_\pi L}, \quad m_\pi L, \quad \frac{1}{L\Lambda_\chi}, \ \ldots$$

Hybrid of staggered sea quarks (MILC) and domain-wall valence quarks

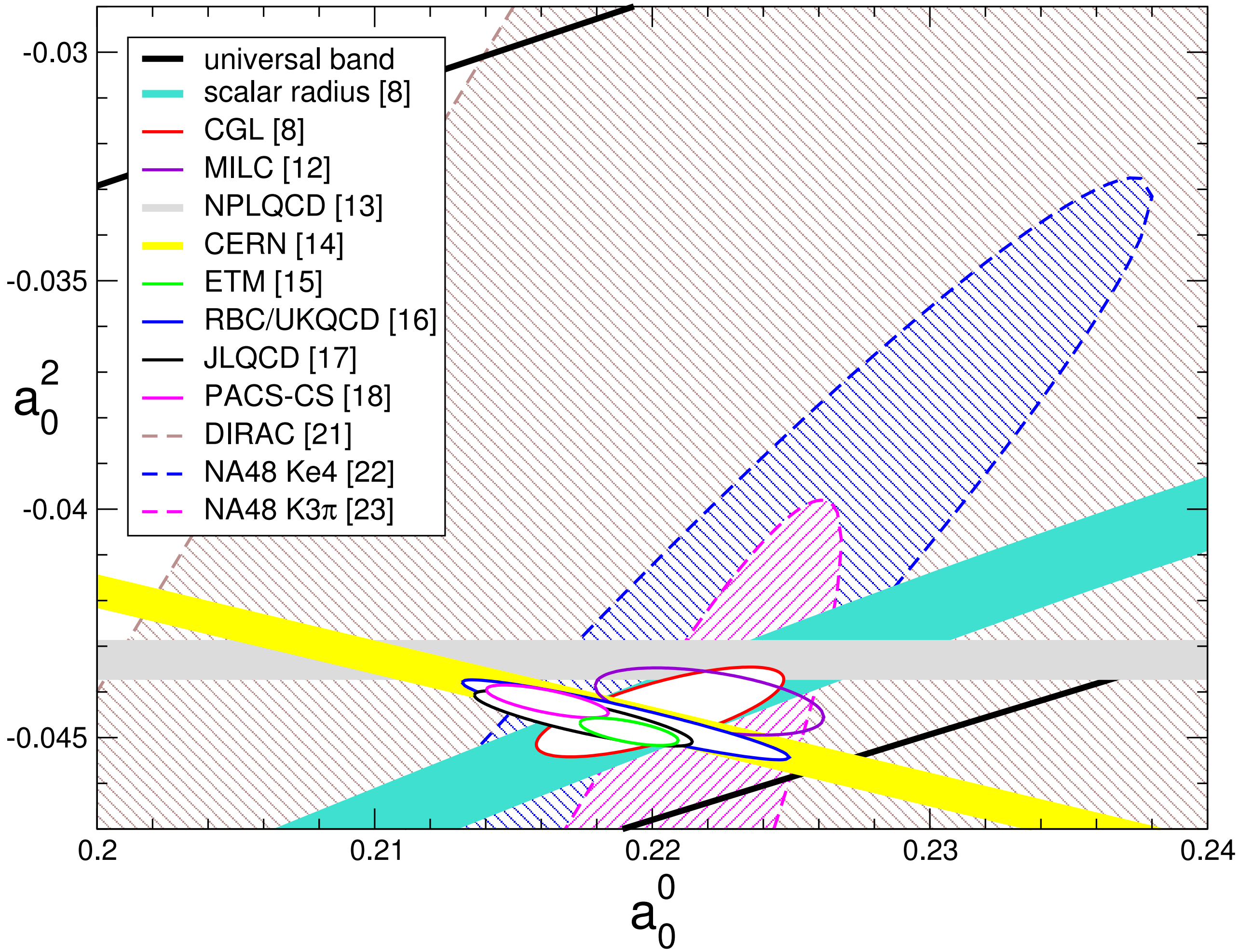
(2+1) dynamical flavors

Config Set	Dimensions	m_π	# configs	# sources
2064f21b676m007m050	$20^3 \times 64$	291 MeV	1039	24
2064f21b676m010m050	$20^3 \times 64$	352 MeV	769	24
2064f21b679m020m050	$20^3 \times 64$	491 MeV	486	24
2064f21b681m030m050	$20^3 \times 64$	591 MeV	564	24

$b_{MILC}^{coarse} \sim 0.125$ fm $\qquad L \sim 2.5$ fm

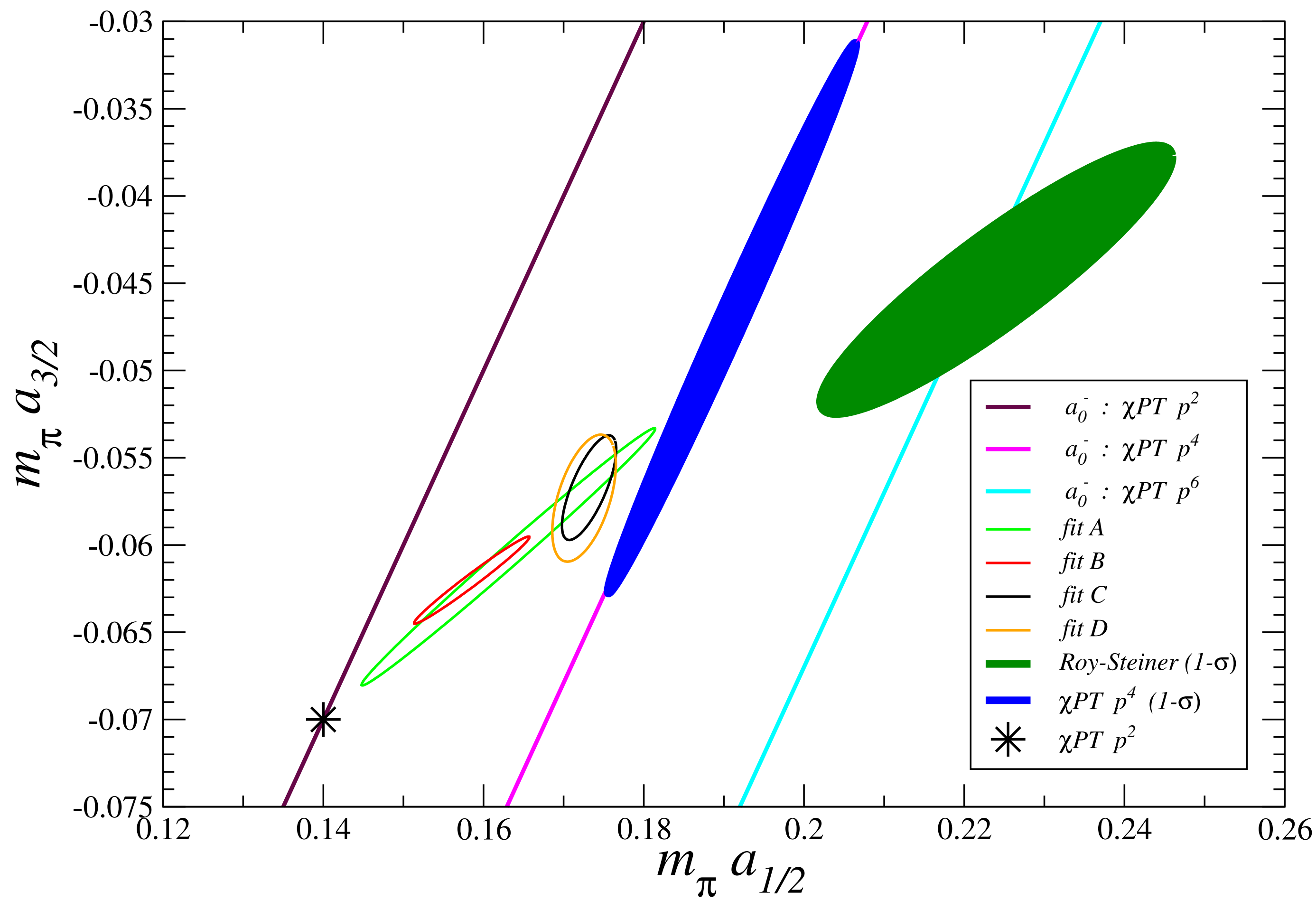
$\pi\pi$

NPLQCD (2007)



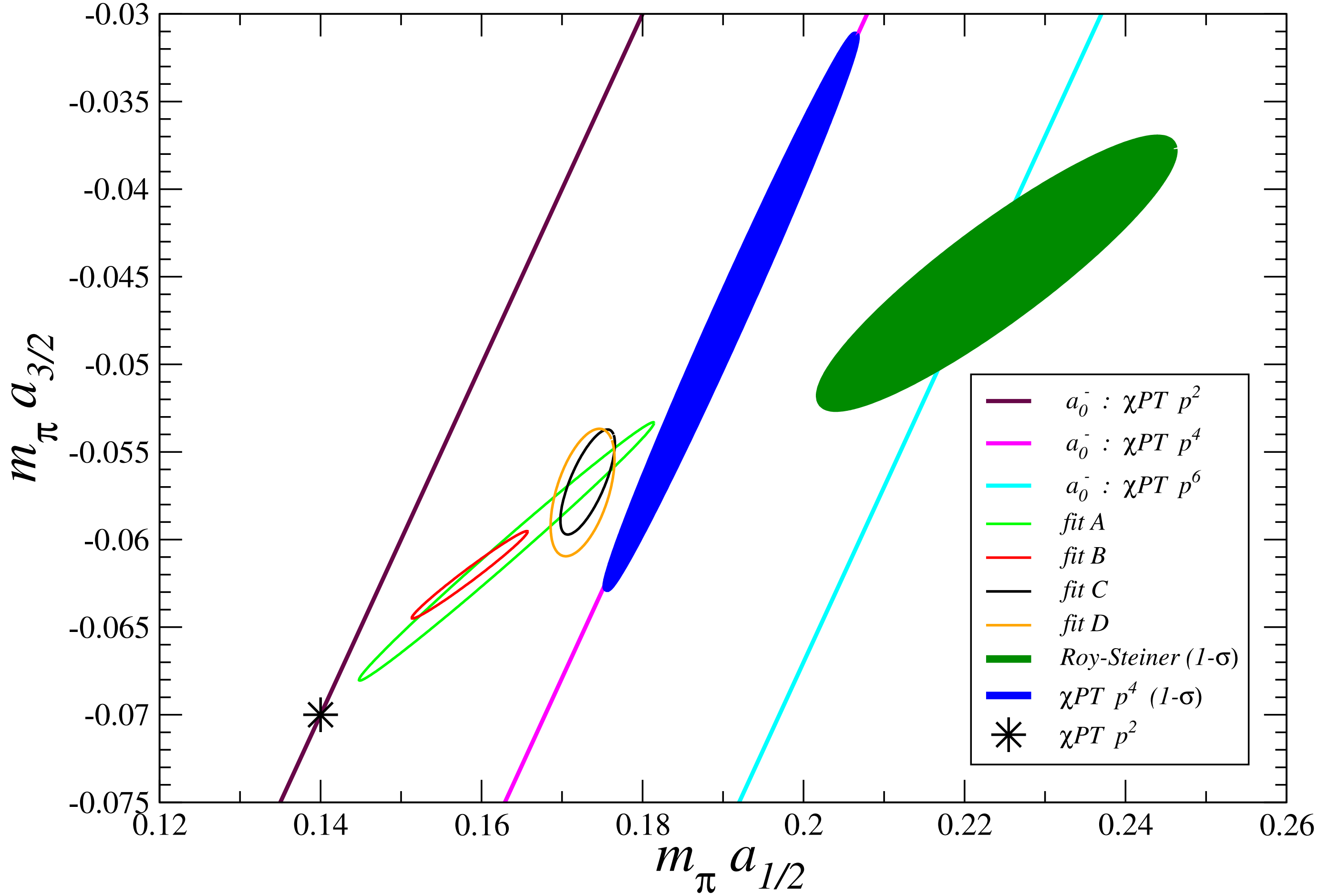
(Courtesy of H. Leutwyler)

$K\pi$



-0.03
-0.035
-0.04
-0.045
-0.05
-0.055
-0.06
-0.065
-0.07
-0.075
0.12
0.14
0.16
0.18
0.2
0.22
0.24
0.26
$m_\pi a_{3/2}$
$m_\pi a_{1/2}$
a_0^- : χPT p^2
a_0^- : χPT p^4
a_0^- : χPT p^6
fit A
fit B
fit C
fit D
Roy-Steiner (1-σ)
χPT p^4 (1-σ)
χPT p^2

$K\pi$



Caveat: $b = 0.125$ fm , $L = 2.5$ fm

refined analysis in progress NPLQCD (2009)

Meson-Baryon scattering

Motivation

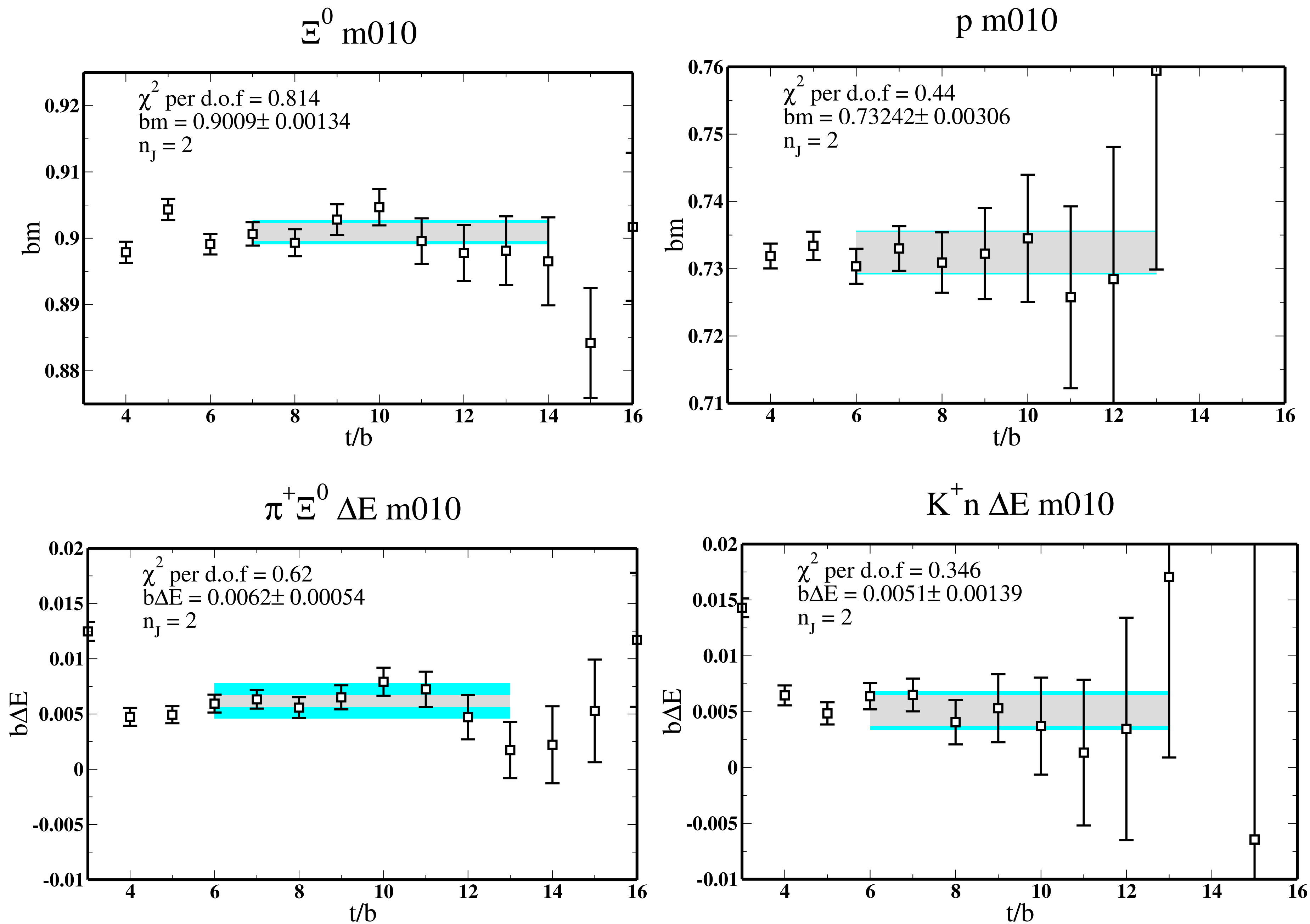
- Interesting for physics of dense matter: e.g. K^-n and kaon condensation
- Interesting χPT phenomenology: e.g. low-energy constants, πN σ-term, *etc.*
- Final state interactions: e.g. weak $\Xi \to \Lambda\pi$
- Test of convergence of $SU(3)$ HBχPT

Five channels with no annihilation

Particles	Isospin	Quark Content
$\pi^+\Sigma^+$	2	$uuu\bar{d}s$
$\pi^+\Xi^0$	3/2	$uu\bar{d}ss$
K^+p	1	$uuud\bar{s}$
K^+n	0 *and* 1	$uudd\bar{s}$
$\overline{K}^0\Sigma^{+\star}$	3/2	$uu\bar{d}ss$
$\overline{K}^0\Xi^0$	1	$u\bar{d}sss$

$\star$ $K\Sigma$ and $\pi\Xi$ carry same quantum numbers: mixed channel analysis required

$\overline{K}^0\Sigma^+$ not included in analysis


Ξ0 m010
χ2 per d.o.f = 0.814
bm = 0.9009± 0.00134
nJ = 2
bm
t/b
p m010
χ2 per d.o.f = 0.44
bm = 0.73242± 0.00306
nJ = 2
bm
t/b
π+Ξ0 ΔE m010
χ2 per d.o.f = 0.62
bΔE = 0.0062± 0.00054
nJ = 2
bΔE
t/b
K+n ΔE m010
χ2 per d.o.f = 0.346
bΔE = 0.0051± 0.00139
nJ = 2
bΔE
t/b

$SU(3)$ HBχPT to NNLO   Kaiser (2001), Liu and Zhu (2007)

$$a_{\pi^+\Sigma^+} = \frac{1}{4\pi}\frac{m_\Sigma}{m_\pi + m_\Sigma}\left[-\frac{m_\pi}{f_\pi^2} + \frac{m_\pi^2}{f_\pi^2}C_1 + \mathcal{Y}_{\pi^+\Sigma^+} + 4\frac{m_\pi^3}{f_\pi^2}h_{123}\right]$$

$$a_{\pi^+\Xi^0} = \frac{1}{4\pi}\frac{m_\Xi}{m_\pi + m_\Xi}\left[-\frac{m_\pi}{2f_\pi^2} + \frac{m_\pi^2}{2f_\pi^2}C_{01} + \mathcal{Y}_{\pi^+\Xi^0} + 4\frac{m_\pi^3}{f_\pi^2}h_1\right]$$

$$a_{K^+p} = \frac{1}{4\pi}\frac{m_N}{m_K + m_N}\left[-\frac{m_K}{f_K^2} + \frac{m_K^2}{f_K^2}C_1 + \mathcal{Y}_{K^+p} + 4\frac{m_K^3}{f_K^2}h_{123}\right]$$

$$a_{K^+n} = \frac{1}{4\pi}\frac{m_N}{m_K + m_N}\left[-\frac{m_K}{2f_K^2} + \frac{m_K^2}{2f_K^2}C_{01} + \mathcal{Y}_{K^+n} + 4\frac{m_K^3}{f_K^2}h_1\right]$$

$$a_{\overline{K}^0\Xi^0} = \frac{1}{4\pi}\frac{m_\Xi}{m_K + m_\Xi}\left[-\frac{m_K}{f_K^2} + \frac{m_K^2}{f_K^2}C_1 + \mathcal{Y}_{\overline{K}^0\Xi^0} + 4\frac{m_K^3}{f_K^2}h_{123}\right]$$

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a_{\pi^+\Xi^0} &= \frac{1}{4\pi}\frac{m_\Xi}{m_\pi+m_\Xi}\left[-\frac{m_\pi}{2f_\pi^2}+\frac{m_\pi^2}{2f_\pi^2}C_{01}+\mathcal{Y}_{\pi^+\Xi^0}+4\frac{m_\pi^3}{f_\pi^2}h_1\right] \\
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\end{aligned}
$$

Overconstrained system!

Many possible fitting strategies

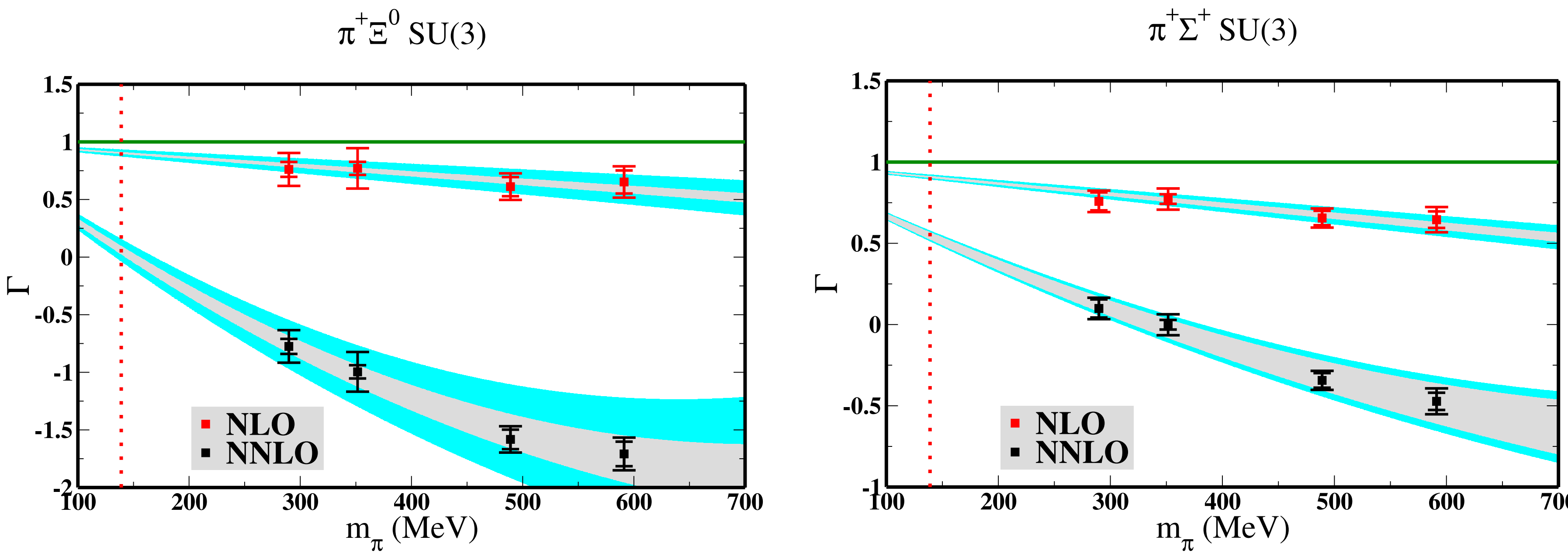
It is convenient to form:

$$\Gamma_{NLO} \equiv -\frac{4\pi a f_\phi^2}{m_\phi}\left(1+\frac{m_\phi}{m_B}\right) = 1 - C_{01} m_\phi$$

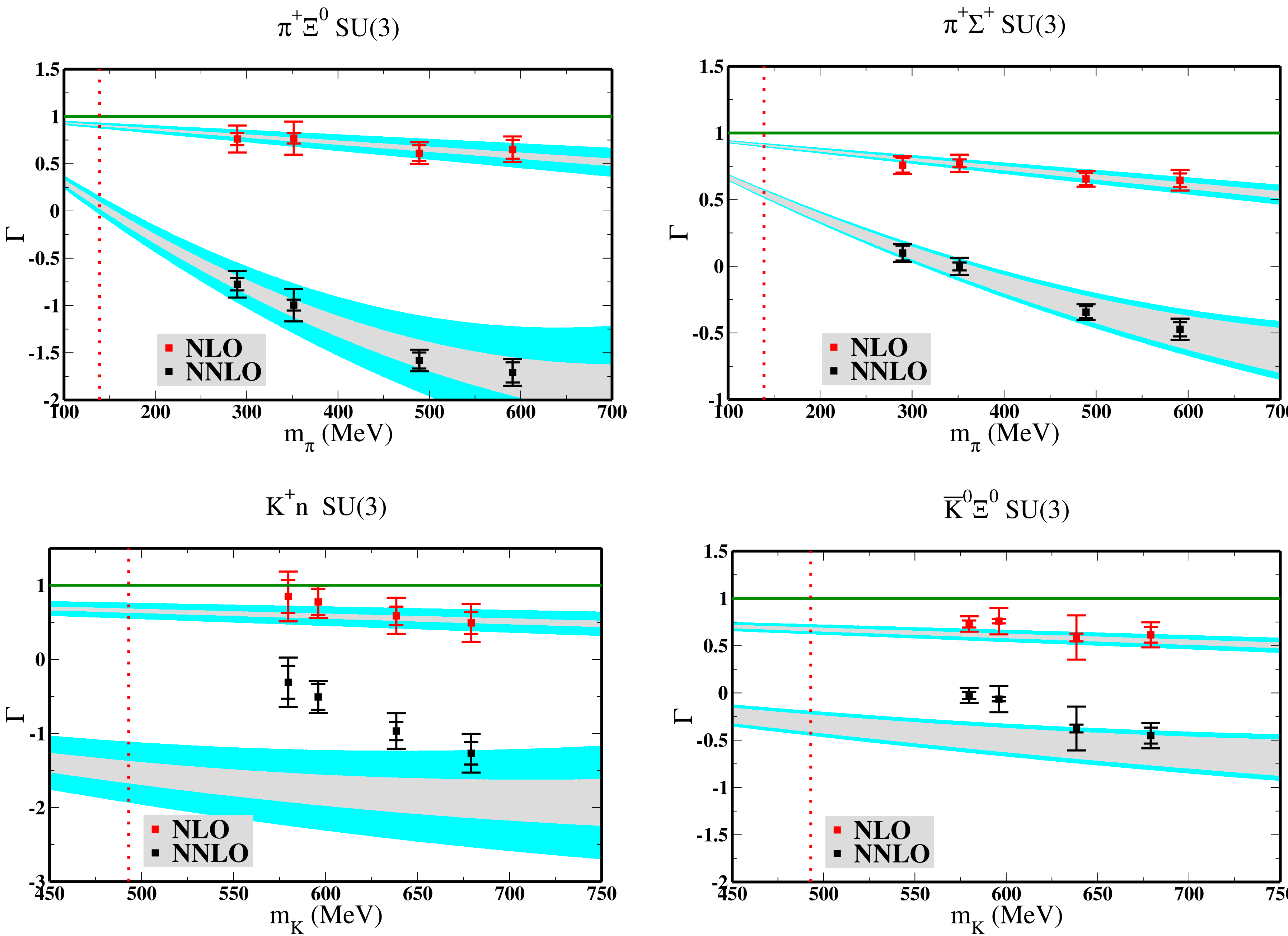
$$\Gamma_{NNLO} \equiv -\frac{4\pi a f_\phi^2}{m_\phi}\left(1+\frac{m_\phi}{m_B}\right) + \frac{f_\phi^2}{m_\phi}\mathcal{Y}_{\phi B}(\Lambda_\chi) = 1 - C_{01} m_\phi - 8 h_1(\Lambda_\chi) m_\phi^2$$

Deviation from unity in Γ is deviation from current algebra

(Note: $\Lambda_\chi = 4\pi f_\pi^{latt}$)



π+Ξ0 SU(3)
π+Σ+ SU(3)
Γ
mπ (MeV)
NLO
NNLO
1.5
1
0.5
0
-0.5
-1
-1.5
-2
100
200
300
400
500
600
700



π+Ξ0 SU(3)
Γ
mπ (MeV)
NLO
NNLO
π+Σ+ SU(3)
K+n SU(3)
mK (MeV)
K̄0Ξ0 SU(3)

$SU(3)$ Results

Quantity	NLO fit each process	NNLO fit $\pi^+\Sigma^+$,$\pi^+\Xi^0$
$C_1(\pi^+\Sigma^+)$	0.66(04)(11) GeV^{-1}	3.51(18)(25) GeV^{-1}
$C_{01}(\pi^+\Xi^0)$	0.69(06)(22) GeV^{-1}	7.44(29)(69) GeV^{-1}
$C_1(K^+p)$	0.44(09)(23) GeV^{-1}	-
$C_{01}(K^+n)$	0.56(11)(27) GeV^{-1}	-
$C_1(\overline{K}^0\Xi^0)$	0.50(06)(14) GeV^{-1}	-
h_1	-	-0.59(08)(14) GeV^{-2}
h_{123}	-	-0.42(10)(10) GeV^{-2}

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Quantity	LO (fm)	NLO fit (fm)	NLO (NNLO fit) (fm)	NNLO (fm)
$a_{\pi\Sigma}$	-0.2294	-0.208(01)(03)	-0.117(06)(08)	-0.197(06)(08)
$a_{\pi\Xi}$	-0.1158	-0.105(01)(04)	0.004(05)(11)	-0.096(05)(12)
a_{Kp}	-0.3971	-0.311(18)(44)	0.292(35)(48)	-0.154(51)(63)
a_{Kn}	-0.1986	-0.143(10)(27)	0.531(28)(68)	0.128(42)(87)
$a_{K\Xi}$	-0.4406	-0.331(12)(31)	0.324(39)(54)	-0.127(57)(70)

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NLO looks perturbative!

NNLO looks perturbative for pion processes only..

$SU(2)$ HBχPT to NNLO Mai *et al* (2009)

$$a_{\pi^+\Sigma^+} = \frac{1}{4\pi}\frac{m_\Sigma}{m_\pi + m_\Sigma}\left[-\frac{2m_\pi}{f_\pi^2} + \frac{2m_\pi^2}{f_\pi^2}\mathrm{C}_{\pi^+\Sigma^+} + \frac{m_\pi^3}{\pi^2 f_\pi^4}\log\frac{m_\pi}{\mu} + \frac{2m_\pi^3}{f_\pi^2}h_{\pi^+\Sigma^+}(\mu)\right]$$

$$a_{\pi^+\Xi^0} = \frac{1}{4\pi}\frac{m_\Xi}{m_\pi + m_\Xi}\left[-\frac{m_\pi}{f_\pi^2} + \frac{m_\pi^2}{f_\pi^2}\mathrm{C}_{\pi^+\Xi^0} + \frac{m_\pi^3}{2\pi^2 f_\pi^4}\log\frac{m_\pi}{\mu} + \frac{m_\pi^3}{f_\pi^2}h_{\pi^+\Xi^0}(\mu)\right]$$

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–alternatively–

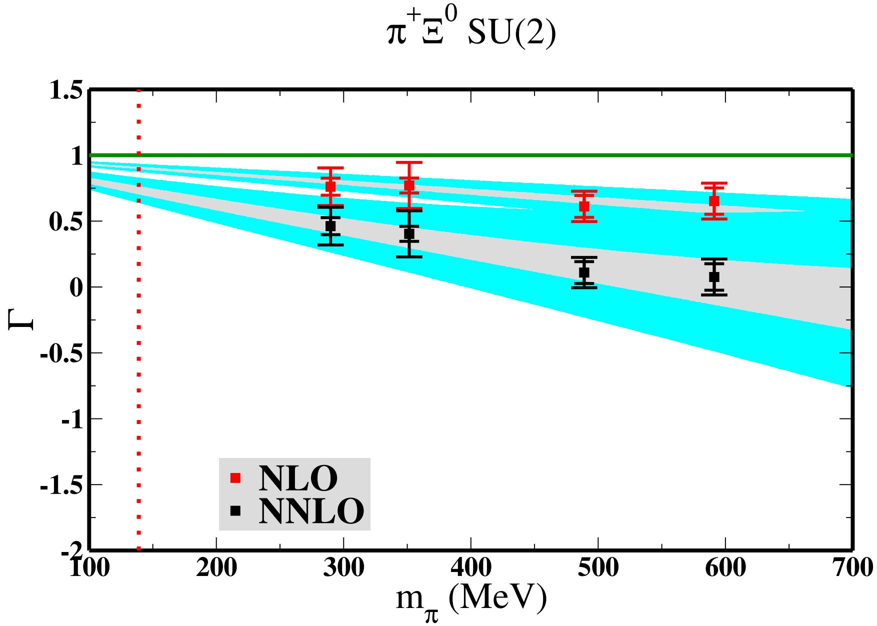
$$a_{\pi^+\Sigma^+} = \frac{1}{2\pi}\frac{m_\Sigma}{m_\pi + m_\Sigma}\left[-\frac{m_\pi}{f^2} + \frac{m_\pi^2}{f^2}\mathrm{C}_{\pi^+\Sigma^+} + \frac{m_\pi^3}{f^2}h'_{\pi^+\Sigma^+}\right]$$

$$a_{\pi^+\Xi^0} = \frac{1}{4\pi}\frac{m_\Xi}{m_\pi + m_\Xi}\left[-\frac{m_\pi}{f^2} + \frac{m_\pi^2}{f^2}\mathrm{C}_{\pi^+\Xi^0} + \frac{m_\pi^3}{f^2}h'_{\pi^+\Xi^0}\right]$$

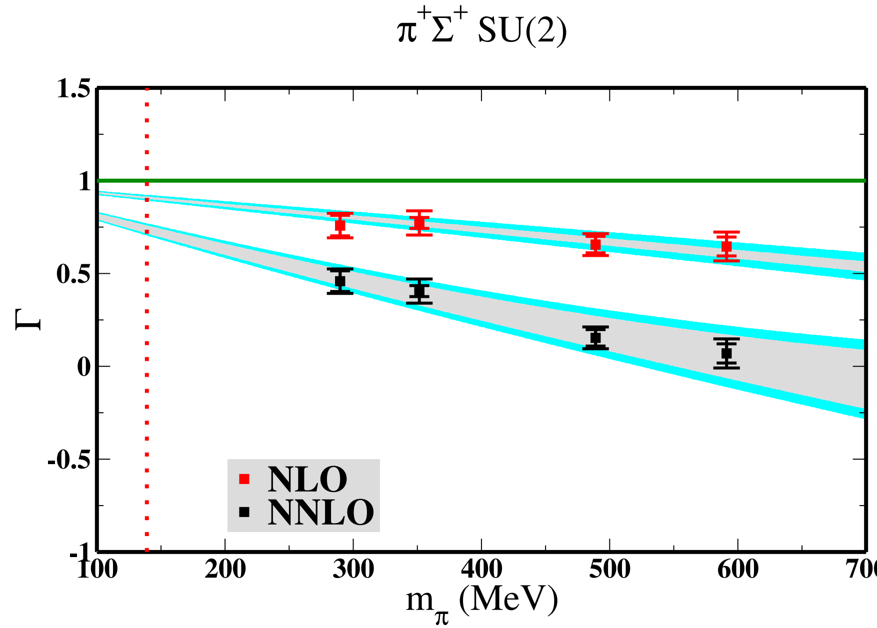
$$h'_{\pi^+\Sigma^+} = \frac{4}{f^2}\ell_4^r + h_{\pi^+\Sigma^+}$$

$$h'_{\pi^+\Xi^0} = \frac{4}{f^2}\ell_4^r + h_{\pi^+\Xi^0}$$

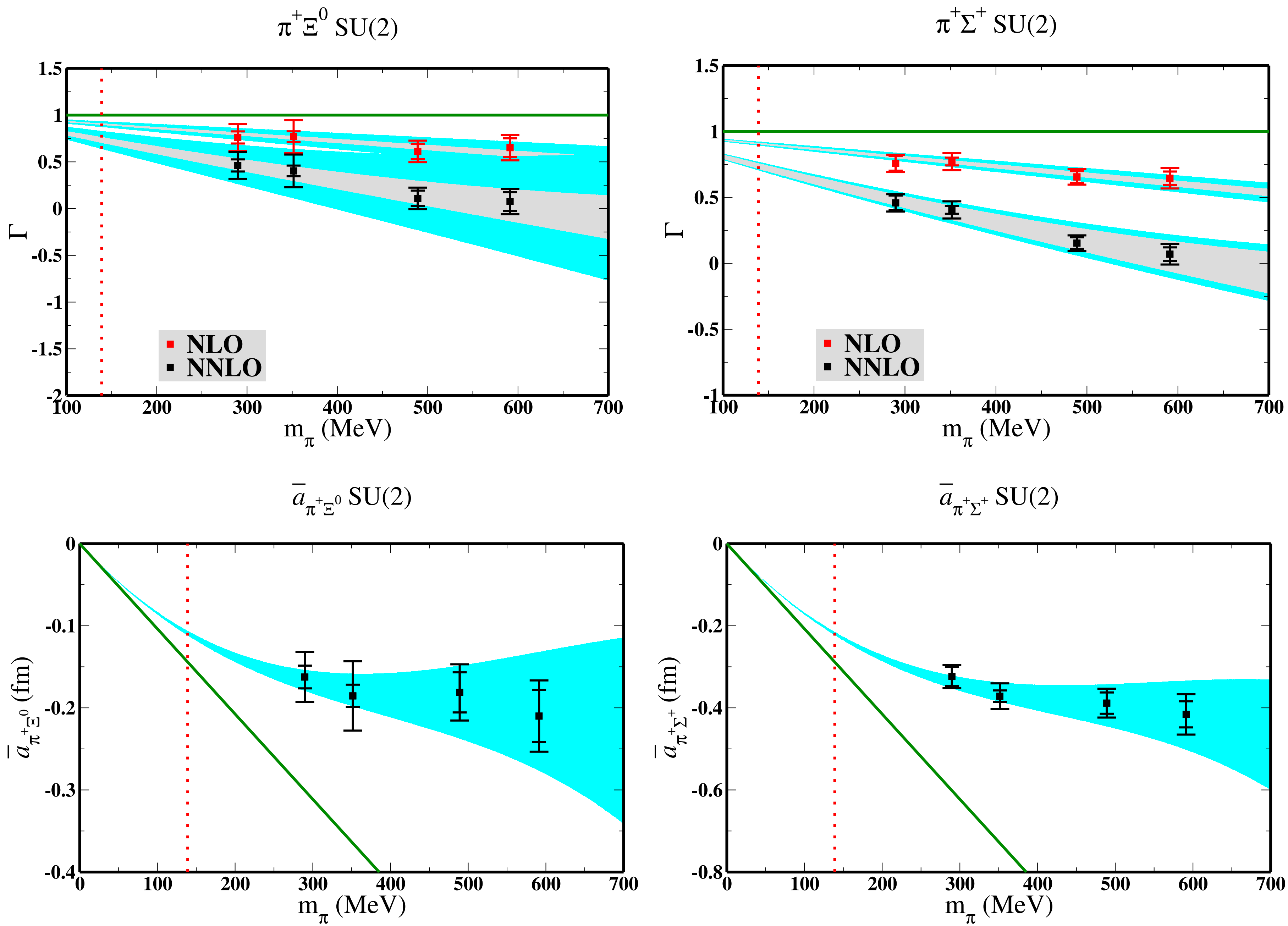
Polynomial extrapolation!



π+Ξ0 SU(2)
1.5
1
0.5
0
-0.5
-1
-1.5
-2
Γ
100
200
300
400
500
600
700
mπ (MeV)
NLO
NNLO



π+Σ+ SU(2)
1.5
1
0.5
0
-0.5
-1
Γ
100
200
300
400
500
600
700
mπ (MeV)
NLO
NNLO


π+Ξ0 SU(2)
π+Σ+ SU(2)
Γ
NLO
NNLO
mπ (MeV)
ā π+Ξ0 SU(2)
ā π+Σ+ SU(2)
ā π+Ξ0 (fm)
ā π+Σ+ (fm)
1.5
1
0.5
0
-0.5
-1
-1.5
-2
100
200
300
400
500
600
700
-0.1
-0.2
-0.3
-0.4
-0.6
-0.8

$SU(2)$ Results

	NLO fit	NNLO fit
$C_{\pi^+\Sigma^+}$	1.28(09)(11) GeV^{-1}	1.90(10)(17) GeV^{-1}
$C_{\pi^+\Xi^0}$	1.84(23)(25) GeV^{-1}	1.93(12)(48) GeV^{-1}
$h'_{\pi^+\Sigma^+}$	-	-1.33(21)(26) GeV^{-2}
$h'_{\pi^+\Xi^0}$	-	-1.36(27)(75) GeV^{-2}

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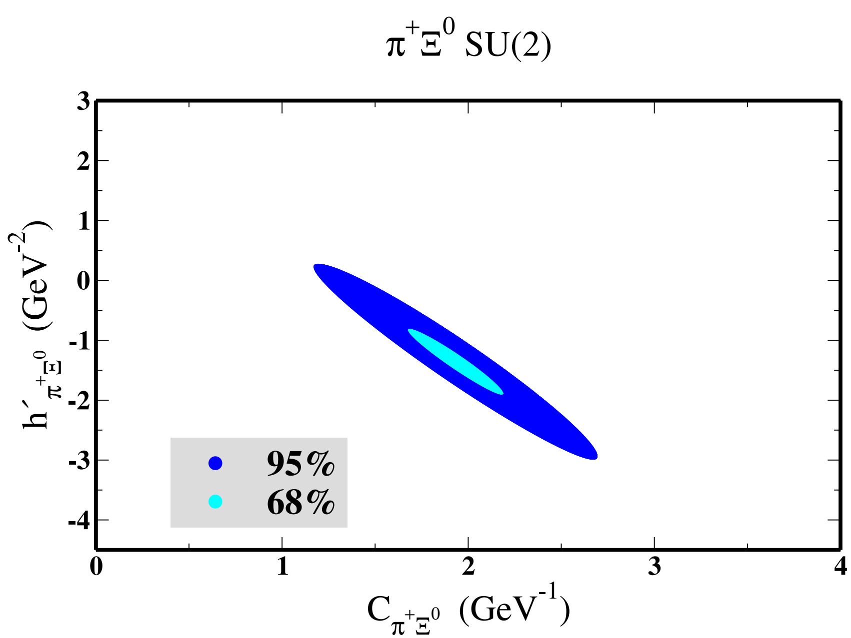
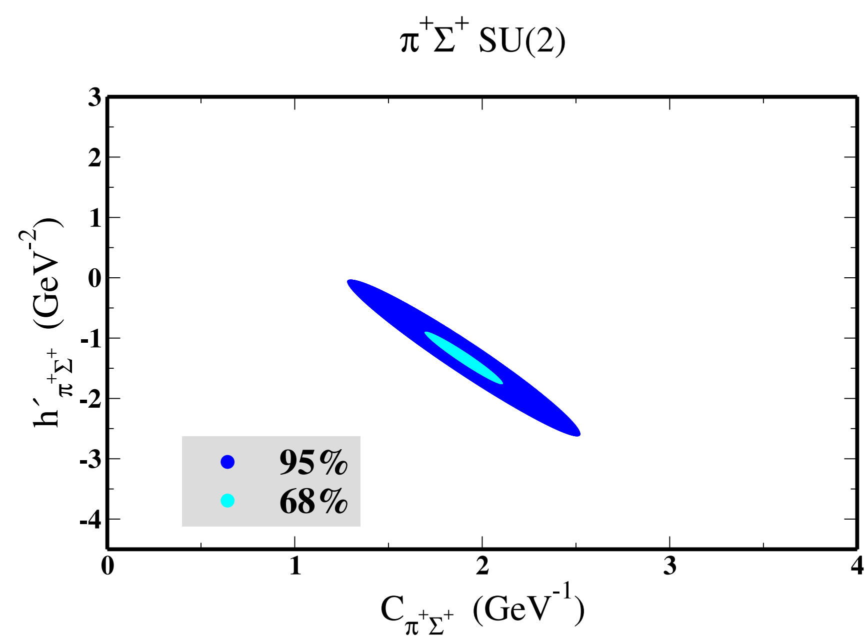
Quantity	LO (fm)	NLO (fm)	NLO (NNLO fit) (fm)	NNLO (fm)
$a_{\pi\Sigma}$	-0.2294	-0.212(03)(04)	-0.190(04)(06)	-0.197(04)(09)
$a_{\pi\Xi}$	-0.1158	-0.106(04)(05)	-0.095(02)(09)	-0.102(02)(09)





Bottom Line:

$$a_{\pi\Xi}^{(3/2)} = -0.098 \pm 0.017 \text{ fm} \qquad NPLQCD$$

$$a_{\pi\Sigma}^{(2)} = -0.197 \pm 0.017 \text{ fm} \qquad NPLQCD$$

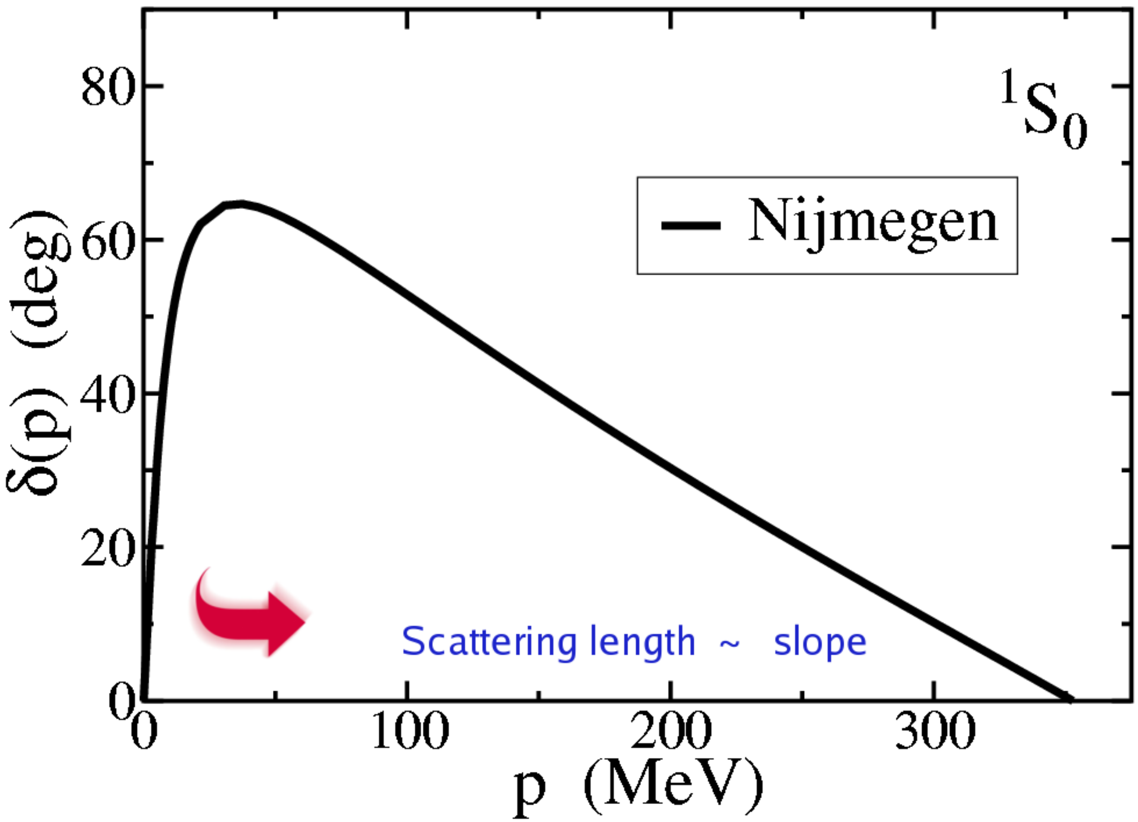
$$a_{\pi\Xi}^{(3/2)} = -0.116 \text{ fm} \qquad Weinberg$$

$$a_{\pi\Sigma}^{(2)} = -0.229 \text{ fm} \qquad Weinberg$$

Enough with toy models..

Now for something hard..

Consider neutron-proton scattering in the $^1S_0$ channel



$$a_s^{^1S_0} \simeq -23 \text{ fm} \simeq \frac{1}{8 \text{ MeV}}$$

Phase shift varies over $\Delta p \sim 8$ MeV: NO Taylor expansion in $\frac{p}{m_\pi}$!

EFT is nonperturbative

Dynamically generated length scale much longer than scale of underlying physics

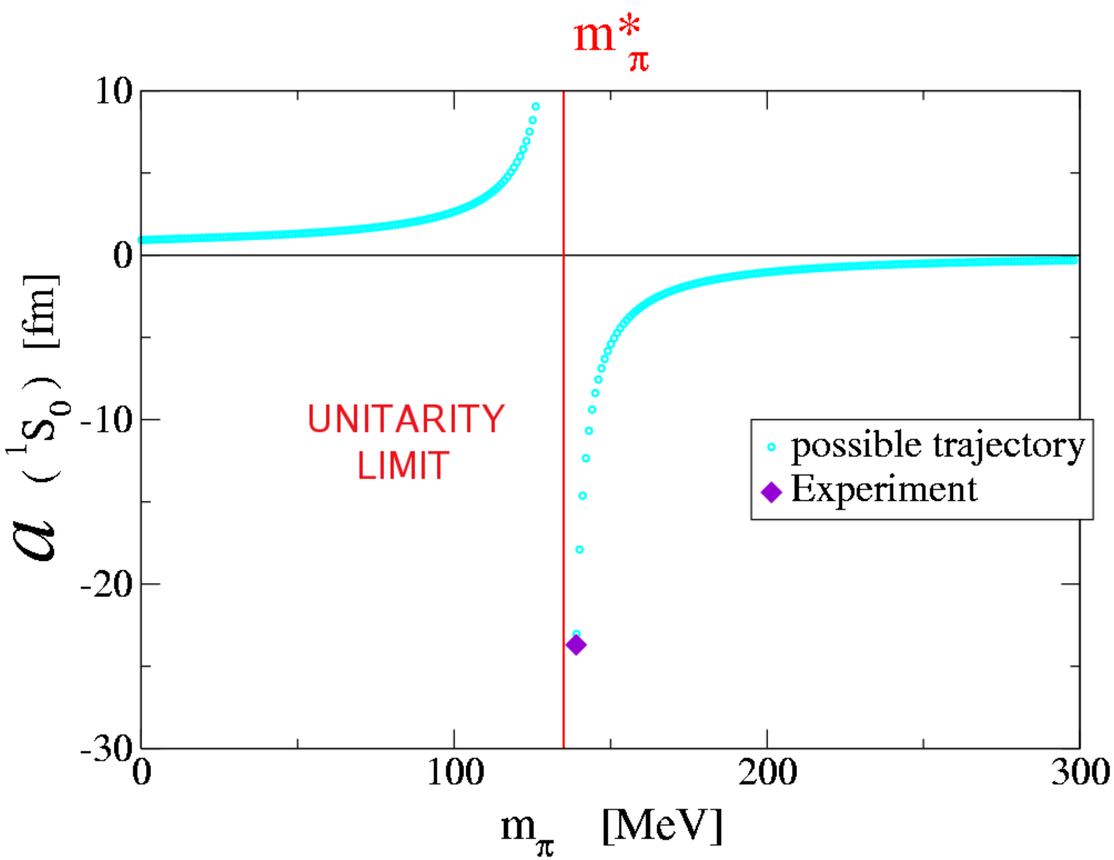
$$a \gg \Lambda_{QCD}^{-1} \;!!$$

EFT is nonperturbative

Dynamically generated length scale much longer than scale of underlying physics

$$a \gg \Lambda_{QCD}^{-1} \;!!$$

Non-relativistic QFT at a non-trivial fixed point!



$$a_s^{-1} \sim \frac{m_\pi - m_\pi^*}{m_\pi} \Lambda_{QCD}$$

Quark mass dependence of NN

Savage and SB (2002), Epelbaum *et al* (2002)

Quark mass dependence of NN

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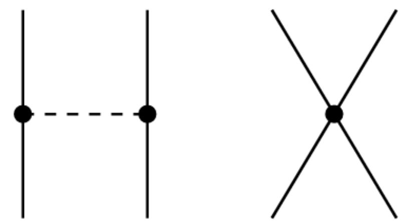
LO :

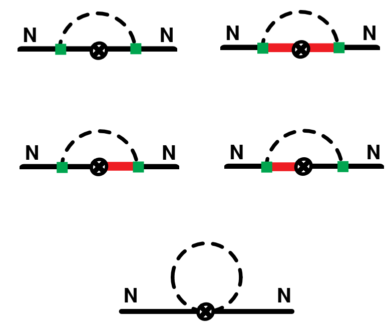


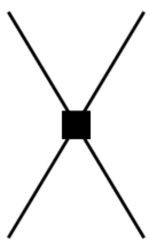
$$V_C^{(QCD)(\pi)}(r;m_\pi) = -\alpha_\pi \, m_\pi^2 \, \frac{e^{-m_\pi r}}{r}$$

$$V_T^{(QCD)(\pi)}(r;m_\pi) = -\alpha_\pi \, \frac{e^{-m_\pi r}}{r} \left(\frac{3}{r^2} + \frac{3m_\pi}{r} + m_\pi^2 \right)$$

$$\alpha_\pi \equiv g_A^2 (1 - 2m_\pi^2 \overline{d}_{18}/g_A)^2/(8\pi f^2)$$

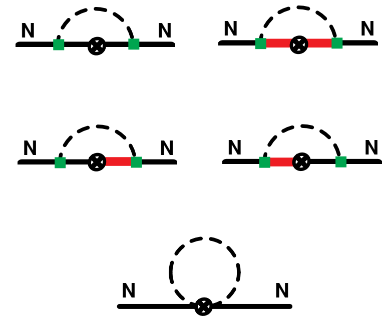
$$g_A = g_A^{(0)} \left[1 - \frac{2(g_A^{(0)})^2 + 1}{4\pi^2 (f^{(0)})^2} m_\pi^2 \log\left(\frac{m_\pi}{\mu}\right) - \frac{(g_A^{(0)})^2 m_\pi^2}{8\pi^2 (f^{(0)})^2} + \frac{4 m_\pi^2}{g_A^{(0)}} \bar{d}_{16} + \ldots \right]$$

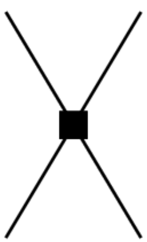




$$\sim \quad D_2 \ \mathrm{Tr}\left(M_q \Sigma\right) \left(N^\dagger N\right)^2$$

$$g_A = g_A^{(0)}\left[1 - \frac{2(g_A^{(0)})^2+1}{4\pi^2 (f^{(0)})^2} m_\pi^2 \log\left(\frac{m_\pi}{\mu}\right) - \frac{(g_A^{(0)})^2 m_\pi^2}{8\pi^2 (f^{(0)})^2} + \frac{4m_\pi^2}{g_A^{(0)}}\bar{d}_{16} + \ldots\right]$$

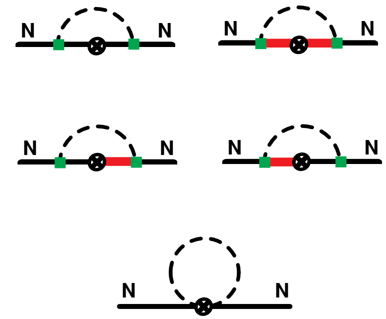


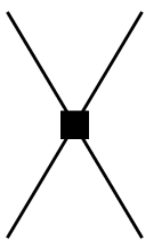


$$\sim \qquad D_2 \, \mathrm{Tr}\,(M_q \Sigma) \left(N^\dagger N\right)^2$$

No easy intuition about quark-mass dependence of nuclear forces!

$$g_A \quad = \quad g_A^{(0)} \left[1 - \frac{2(g_A^{(0)})^2 + 1}{4\pi^2 (f^{(0)})^2} m_\pi^2 \log\left(\frac{m_\pi}{\mu}\right) - \frac{(g_A^{(0)})^2 m_\pi^2}{8\pi^2 (f^{(0)})^2} + \frac{4 m_\pi^2}{g_A^{(0)}} \overline{d}_{16} + \ldots \right]$$



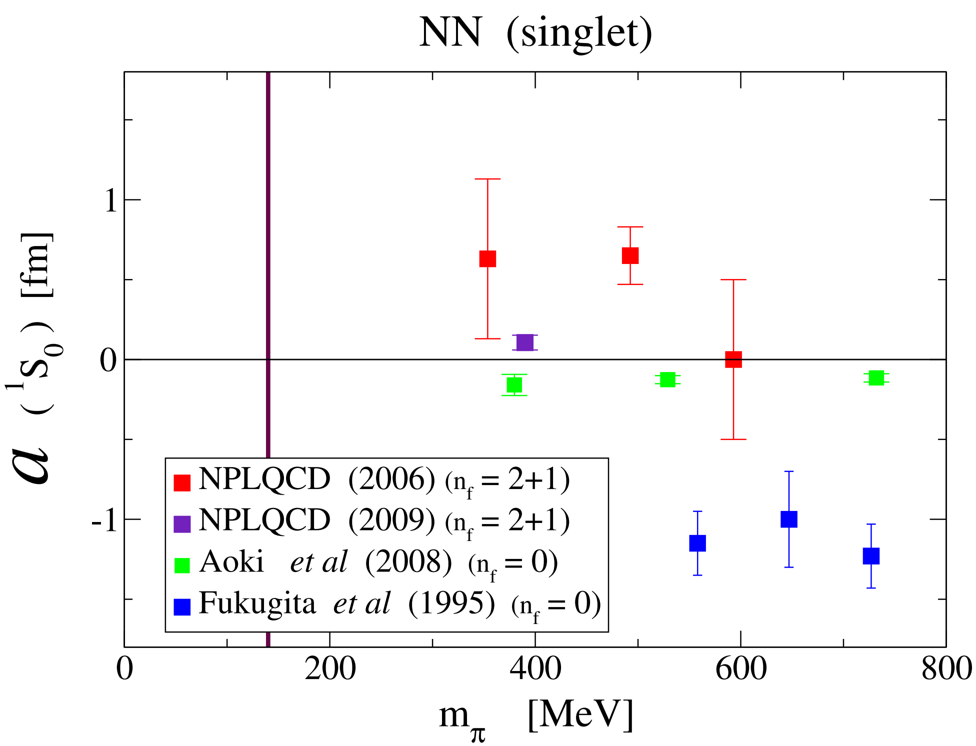


$$\sim \qquad D_2 \; \mathrm{Tr}\left(M_q \Sigma\right) \left(N^\dagger N\right)^2$$

No easy intuition about quark-mass dependence of nuclear forces!

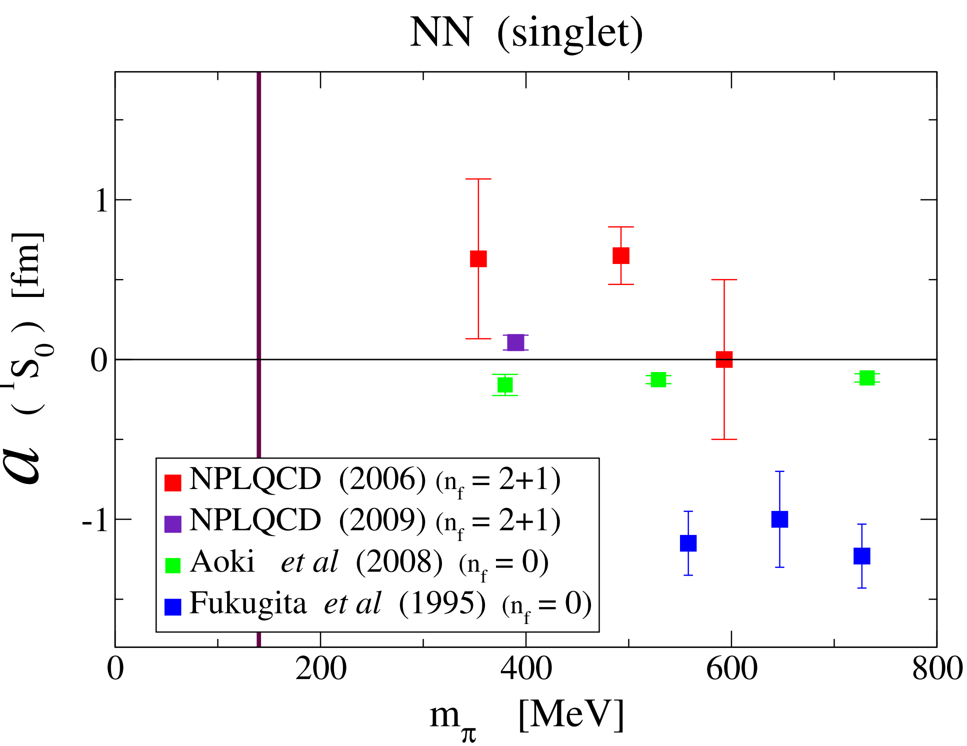
(i.e. please don't guess!!!)

Results: NN



NN (singlet)
a ($^1S_0$) [fm]
1
0
-1
0
200
400
600
800
m_π [MeV]
NPLQCD (2006) (n_f = 2+1)
NPLQCD (2009) (n_f = 2+1)
Aoki *et al* (2008) (n_f = 0)
Fukugita *et al* (1995) (n_f = 0)

Results: NN



NN (singlet)
a ($^1S_0$) [fm]
m_π [MeV]
NPLQCD (2006) (n_f = 2+1)
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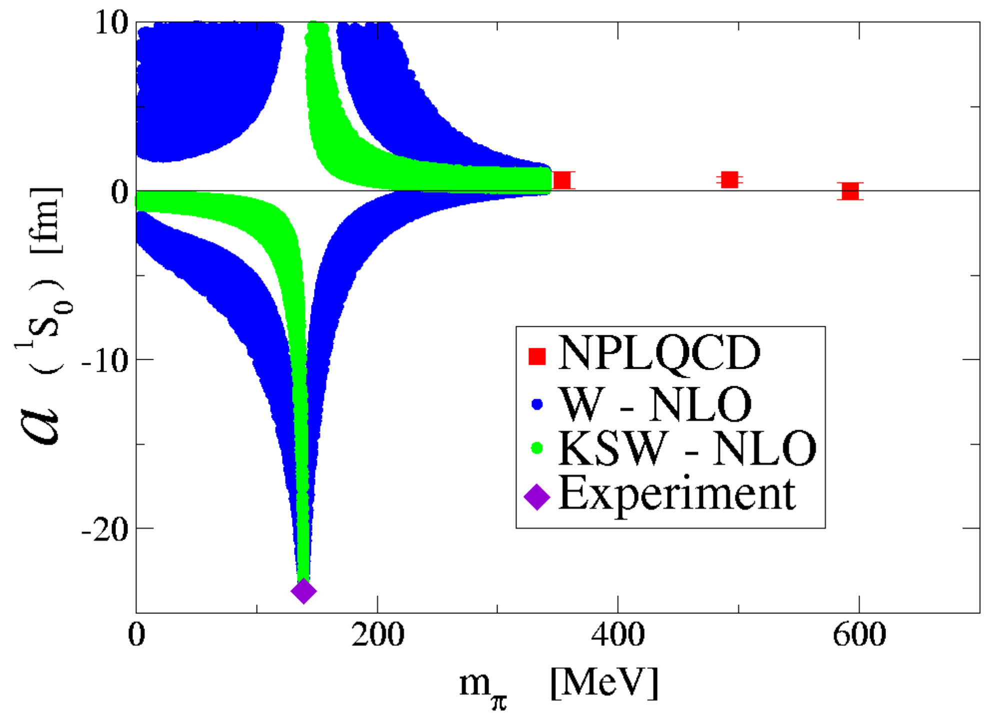
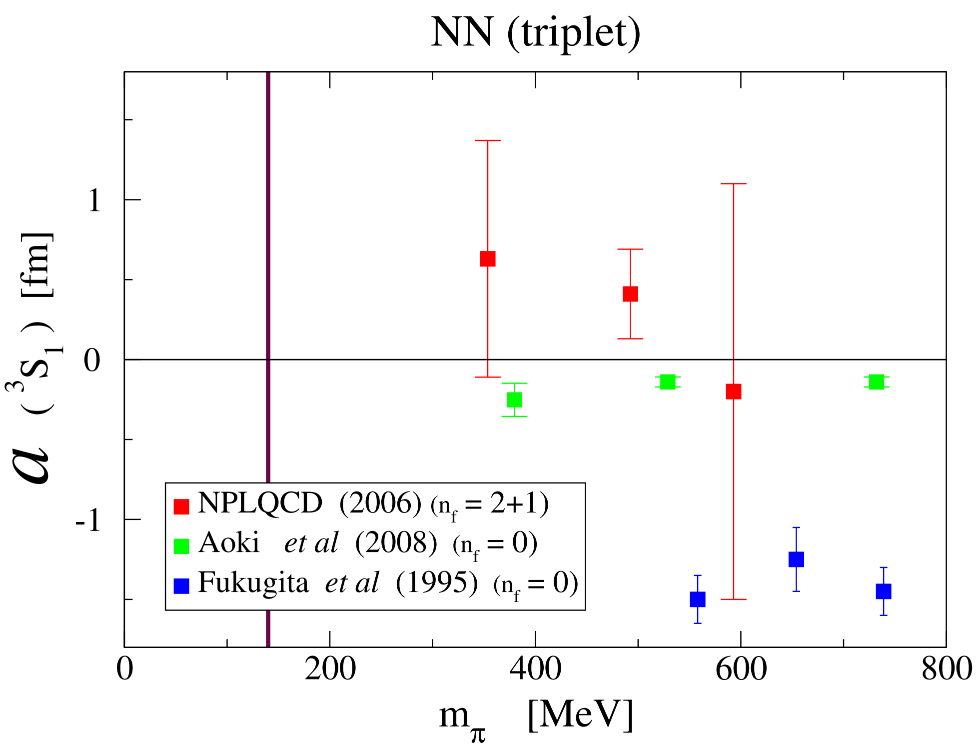
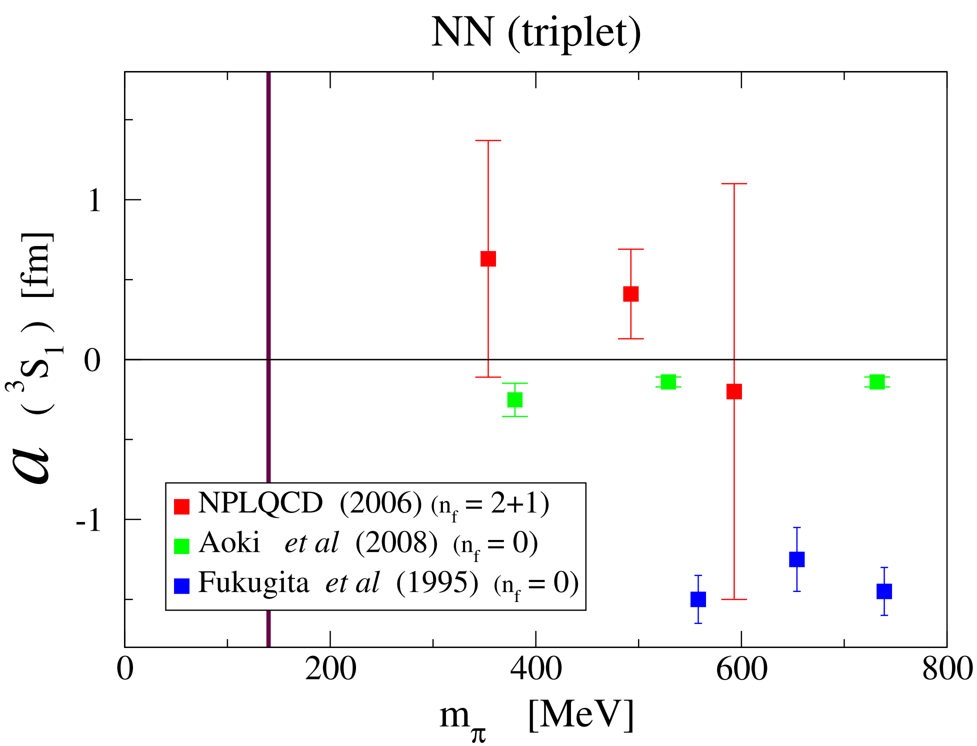


a ($^1S_0$) [fm]
m_π [MeV]
NPLQCD
W - NLO
KSW - NLO
Experiment


NN (triplet)
1
0
-1
0
200
400
600
800
NPLQCD (2006) (n_f = 2+1)
Aoki *et al* (2008) (n_f = 0)
Fukugita *et al* (1995) (n_f = 0)
a ($^3S_1$) [fm]
m_π [MeV]


NN (triplet)
1
0
-1
0
200
400
600
800
a ($^3S_1$) [fm]
m_π [MeV]
NPLQCD (2006) (n_f = 2+1)
Aoki *et al* (2008) (n_f = 0)
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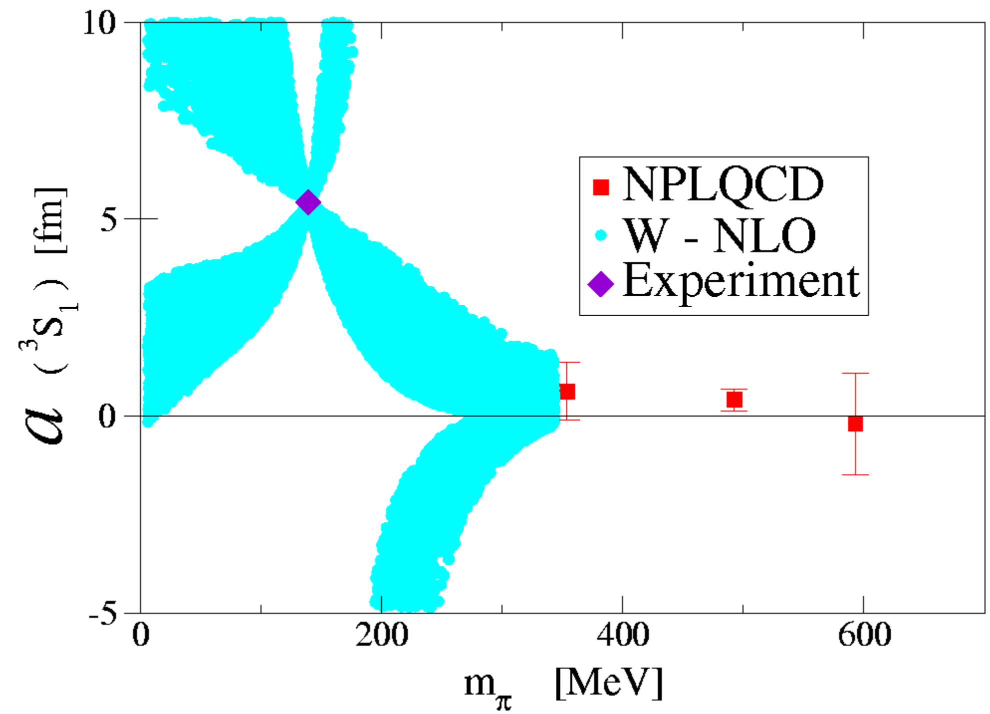
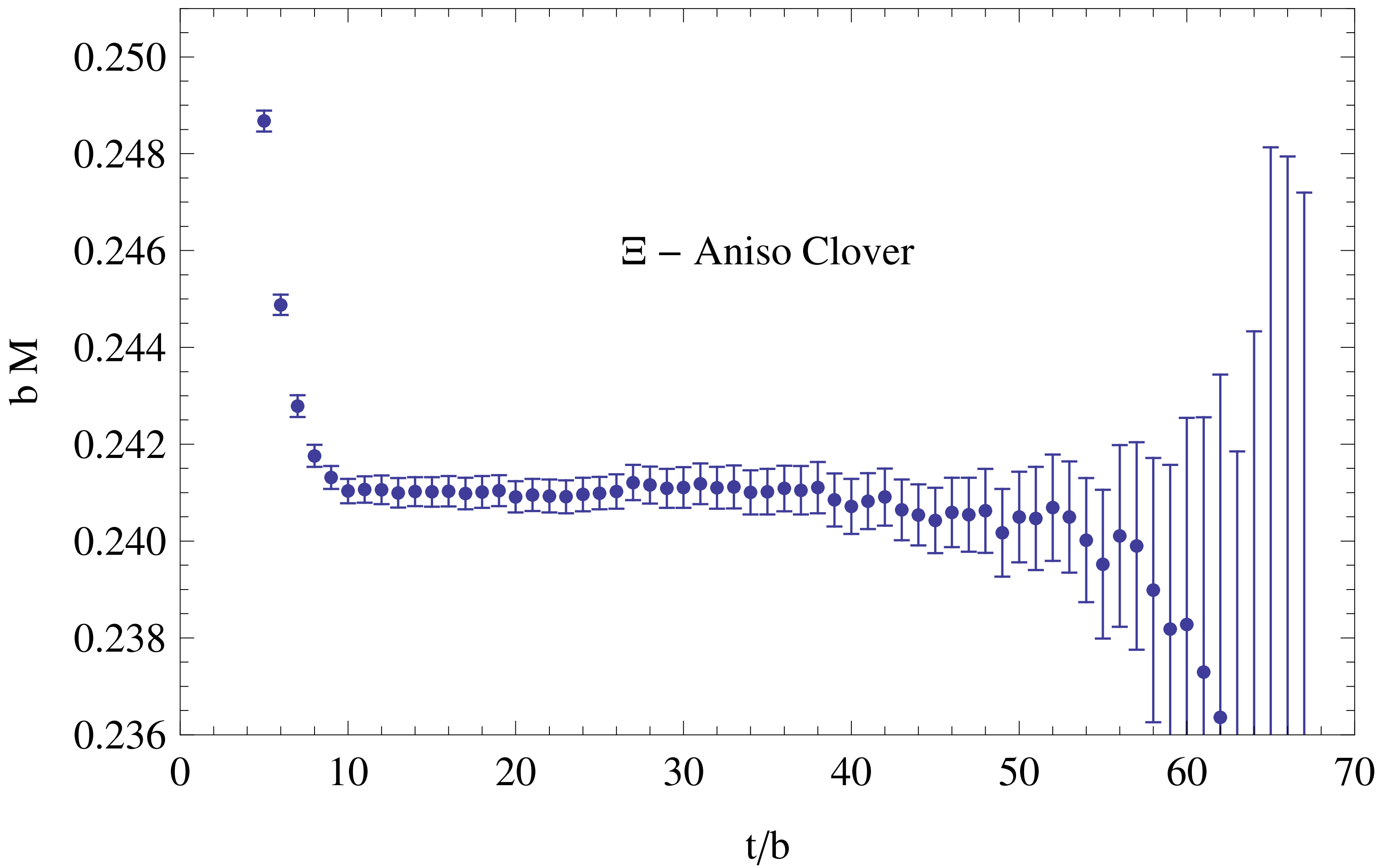
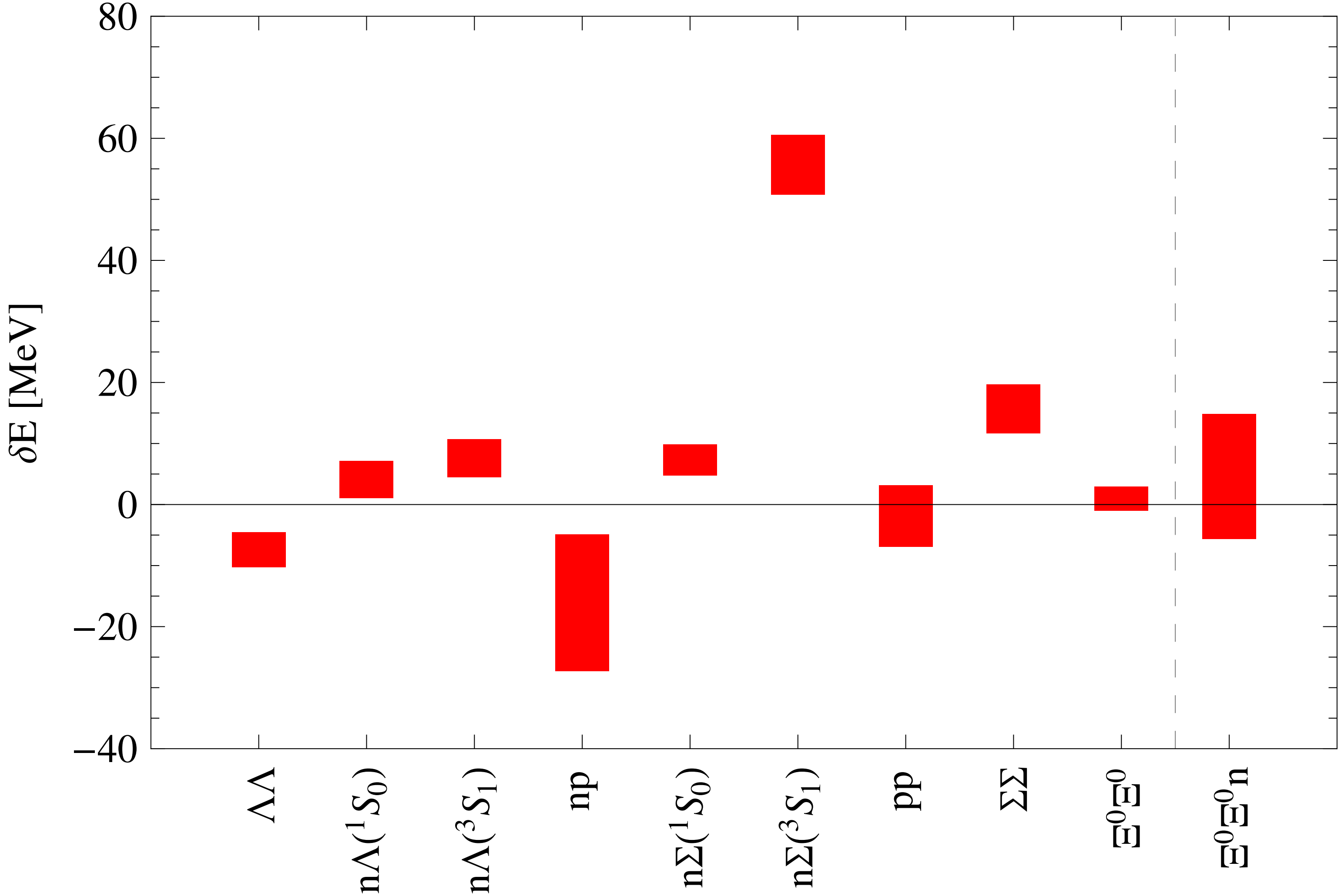

10
5
0
-5
0
200
400
600
a ($^3S_1$) [fm]
m_π [MeV]
NPLQCD
W - NLO
Experiment

New results: NN

Anisotropic clover lattices *with high statistics* NPLQCD (2009)



$$bM_{\Xi} = 0.24112 \pm 0.00021 \pm 0.00006$$

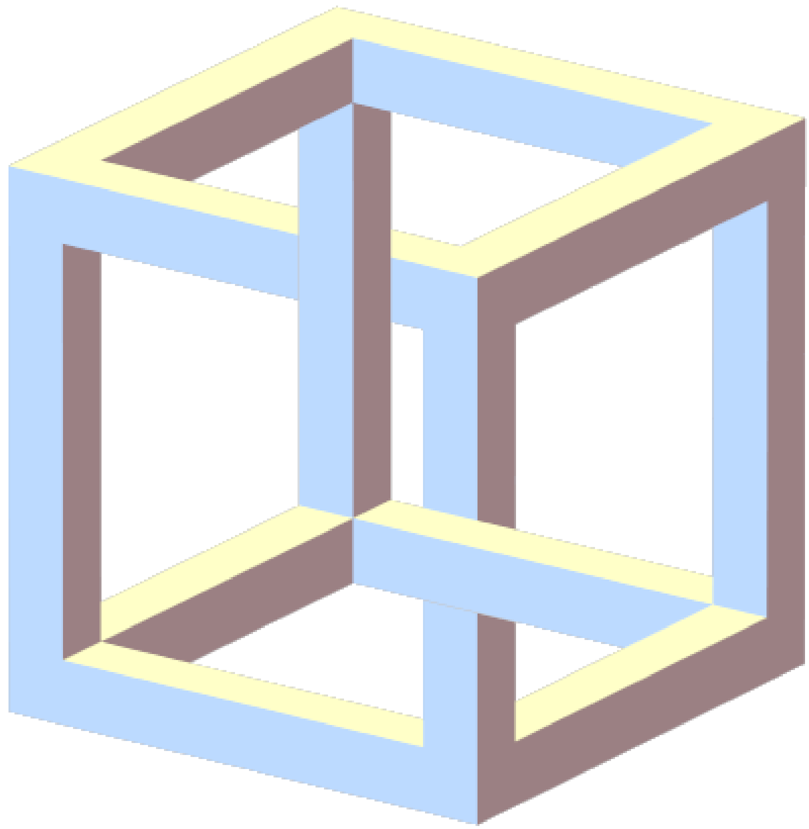


80
60
40
20
0
−20
−40
δE [MeV]
ΛΛ
nΛ(1 S0)
nΛ(3 S1)
np
nΣ(1 S0)
nΣ(3 S1)
pp
ΣΣ
Ξ0Ξ0
Ξ0Ξ0n

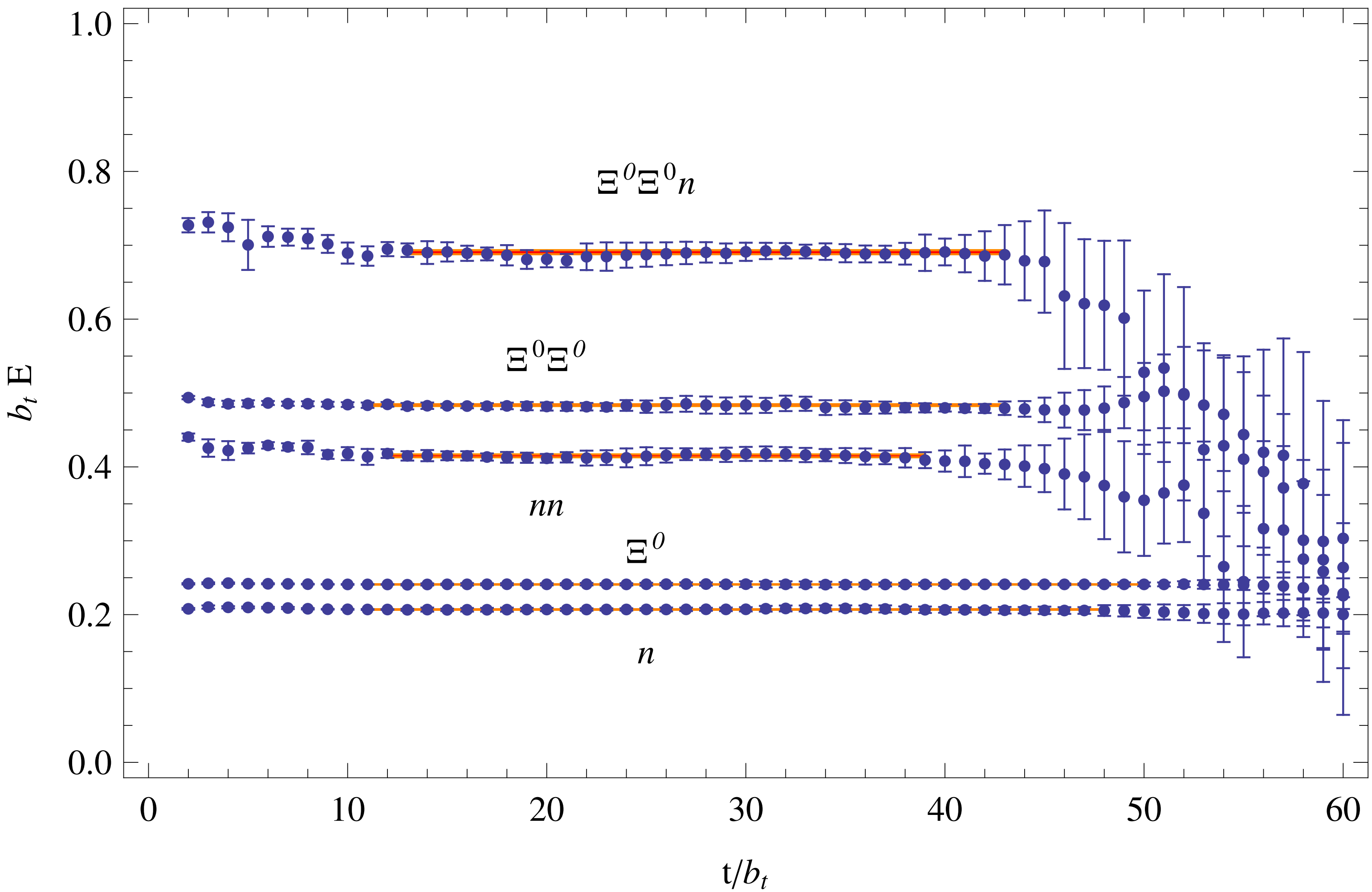
What about 3, 4, *etc* bodies?

What about 3, 4, *etc* bodies?

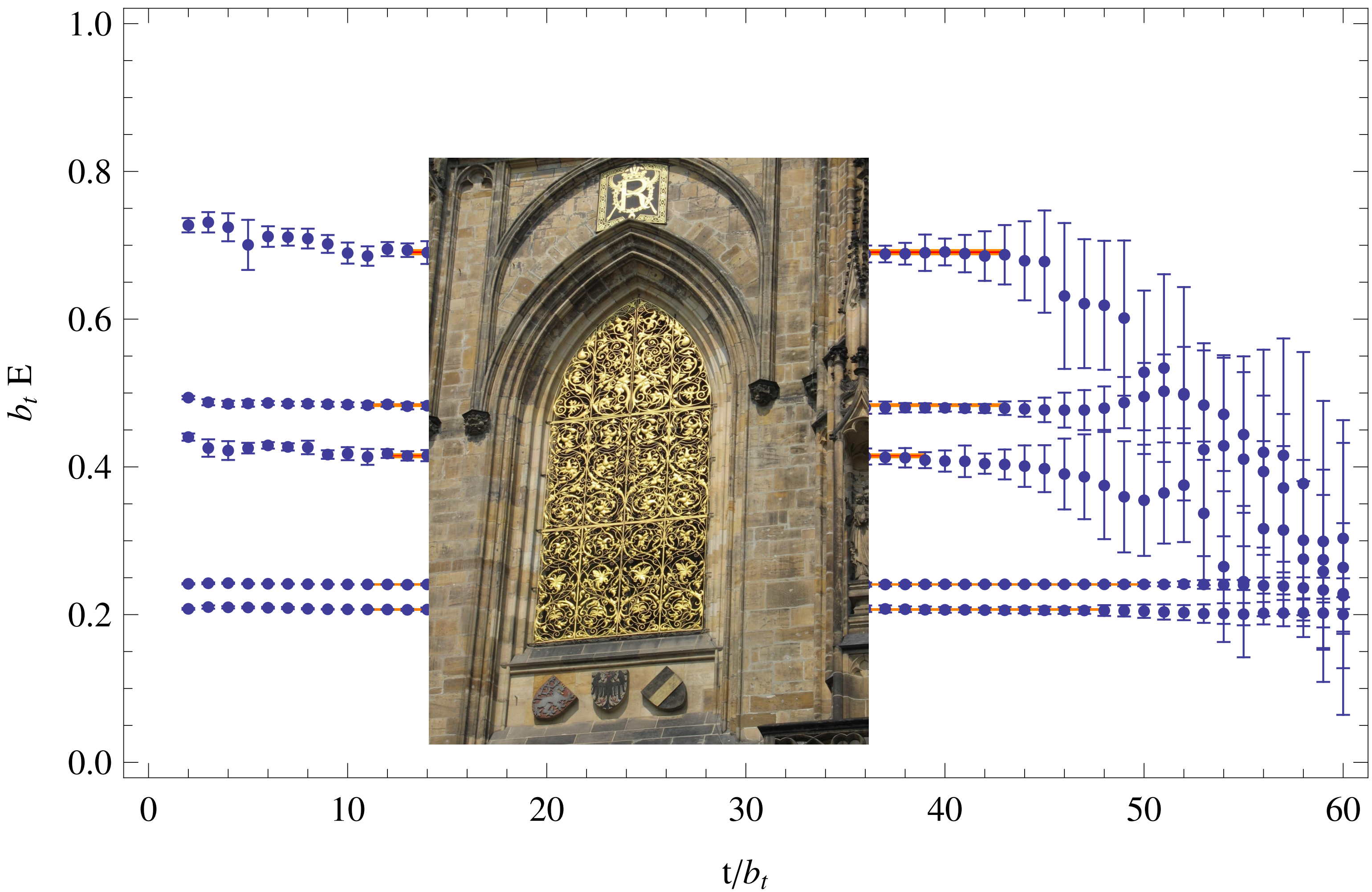
Are we killed by signal/noise?



Anisotropic clover lattices *with high statistics*



Anisotropic clover lattices *with high statistics*



Golden window with no signal/noise problem!

Conclusion

- Meson-meson scattering from lattice QCD is a precision science.
- Kaon-baryon interactions suffer from convergence problems in the $SU(3)$ sector. π-baryon scattering lengths reliably determined in the two-flavor chiral expansion.
- Interactions involving more than one baryon require high statistics. Must do Lattice simulations at other volumes; critical for identification of bound states; e.g. deuterium.
- Signal/noise is not as limiting as thought previously. There is a "golden window" at relatively short times which does not exhibit a signal/noise problem! Opens the door to few-body calculations!