Subtractive renormalization of the chiral potentials up to next-to-next-to-leading order

Chieh Jen Yang (Jerry)
Ohio University

C. J. Yang, C. Elster, and D. R. Phillips

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A simplified overview of nuclear physics

1 TeV

100 GeV Effective fundamental theories (QCD, QED, ect.)

Nuclear physics

Difficulty: Non-perturbative region

By spontaneous symmetry breaking

Lattice QCD

1 GeV

Chiral perturbation theory

100 MeV

NN data

Goal: Effective field theory for low energy NN interaction.

Application: Detail nuclear structure

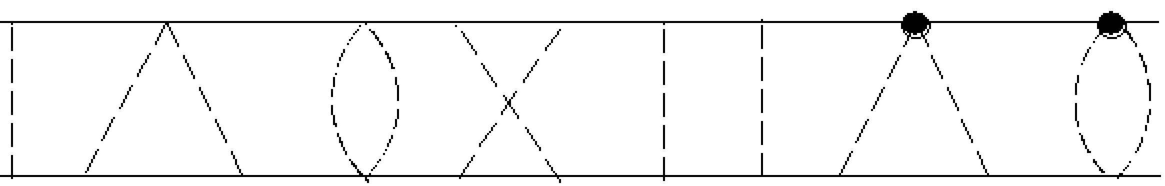
NN interaction and renormalization

Lippmann-Schwinger eq.

(Weinberg, Ordonez, Ray, van Kolck)

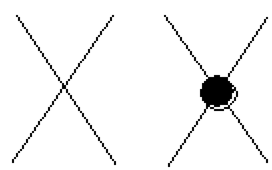
$$T_l(k',k;E) = V_l(k',k;E) + \frac{2}{\pi}\int_0^{\Lambda} \frac{dp\,p^2 V_l(k',p;E)T_l(p,k;E)}{E - E_p + i\varepsilon}$$

iterate...



+ contact terms

Represent short range (high energy) physics.



$\lambda + \gamma \cdot (k-k')^2$

In s-waves, LO: $C_{S(T)}$.

Cutoff in LSE

$C_{S(T)}(\Lambda)$ are determined by fitting to physical observables.

The goal of this work

- We want to evaluate whether (or under what conditions) χET, as an EFT (like QED or QCD), is really improved order by order, after renormalization.
  - * More and more counter terms (with free parameters fitted to data), which somehow make it difficult to see whether the improvement is coming from the inclusion of higher order or the fit.

1. What kind(order) of contact term should one adopt?
2. Cutoff-indep. in phase shift $\overset{?}{\leftrightarrow}$ renormalization point indep.
3. What is the highest Λ can one use?

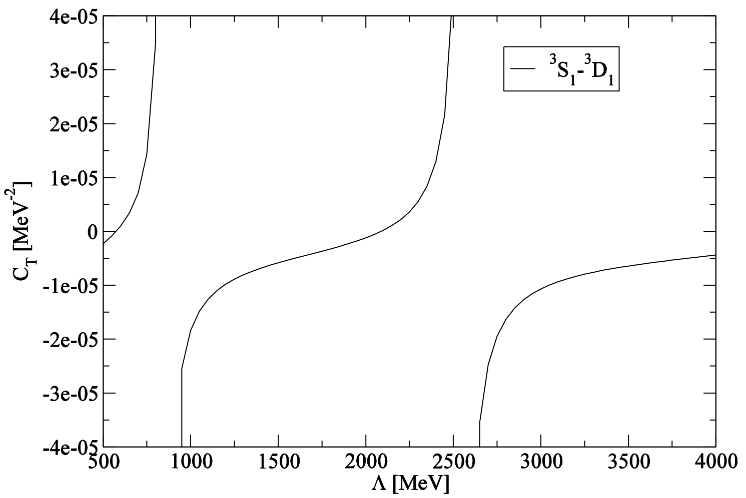
- **We adopt a subtractive renormalization scheme to achieve our goal.**

Determining LEC's in the contact terms

1. Fitting (Ordonez, Epelbaum, Machleidt, Valderrama, etc.)

- Previously, renormalize V by adjusting the unknown constants to fit data, e.s. fit scattering length, effective range or phase shifts.
- Problem: 1. fine-tuning. 2.

Problem of fitting



We develop subtraction scheme to solve the problem.

Method for LO: Three steps

Step 1:

$$T(p,0;0) = V(p,0;0) + C + \frac{2}{\pi}\int_0^{\Lambda} dp' p' [\frac{V(p,p';0)+C}{-p'^2/2\mu}] T(p',0;0) \quad (1) \qquad T(0,0;0) = \frac{f_0}{-2\mu k} = \frac{a}{2\mu}$$

$$T(0,0;0) = V(0,0;0) + C + \frac{2}{\pi}\int_0^{\Lambda} dp' p' [\frac{V(0,p';0)+C}{-p'^2/2\mu}] T(p',0;0) \quad (2)$$

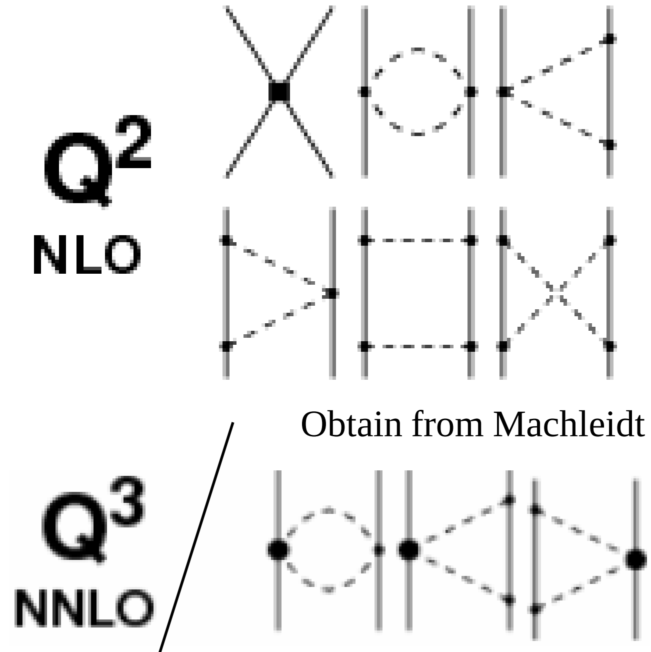
$$(1)-(2): \; T(p,0;0) - \frac{a}{2\mu} = V(p,0;0) + \frac{2}{\pi}\int_0^{\Lambda} dp' p'^2 [\frac{V(p,p';0)-V(0,p';0)}{-p'^2/2\mu}] T(p',0;0)$$

Step 2: Use the same idea to get T(p,k;0).

Step 3: Use property of LSE to relate T(0) to T(E).

$$[1+T(0)(G(0)-G(E))]T(E) = T(0)$$

- In NLO(Q^2) and NNLO(Q^3) we have TPE, which diverges as $Q^{2(3)}$. => Include the O(Q^2) contact term to renormalize it.
- V=OPE+TPE+ $\lambda+\gamma^*$ [O(Q^2)].
- Further develop our subtraction technique to solve it.



Obtain from Machleidt

Use dimensional regularization (DR) or spectral function regularization (SFR) to regularize the diverge loop integral.

Contact terms: from LO to NLO/NNLO

In p-waves: none $\longrightarrow$ Cpp'.

s-waves: $C_{S(T)} \longrightarrow \lambda_{ll'}+\gamma_{ll'}[O(Q^2)]$.

More complicated cases of Subtraction Method

A. In p-waves (Cp'k):

Key point: **divide LSE by p'k, then proceed as before.**

B. Tensor contact interaction:

Divide by $p^l p'^{l'}$ to eliminate $\lambda_t p^2$.

$$\begin{pmatrix} \lambda + C_2(p^2 + p'^2) & \lambda_t p'^2 \\ \lambda_t p^2 & 0 \end{pmatrix}$$

C. Energy-dep. Contact term:

Eliminate γE* by doing subtraction at E*.

D. Momentum-dep. Contact term in s-waves:

Need to replace one subtraction by one fitting, due to the fact that there is no on-shell observable corresponds to it.

Input:

p-waves: Generalized scattering length $\lim_{k\to 0}[\frac{t_{11}(k,k;0)}{kk}] = \frac{\alpha_{11}}{M}$

S-waves: a_0, $\delta(E^*)$ for the singlet; with additional α_{20} for triplet.

P-waves results at Leading order(LO):

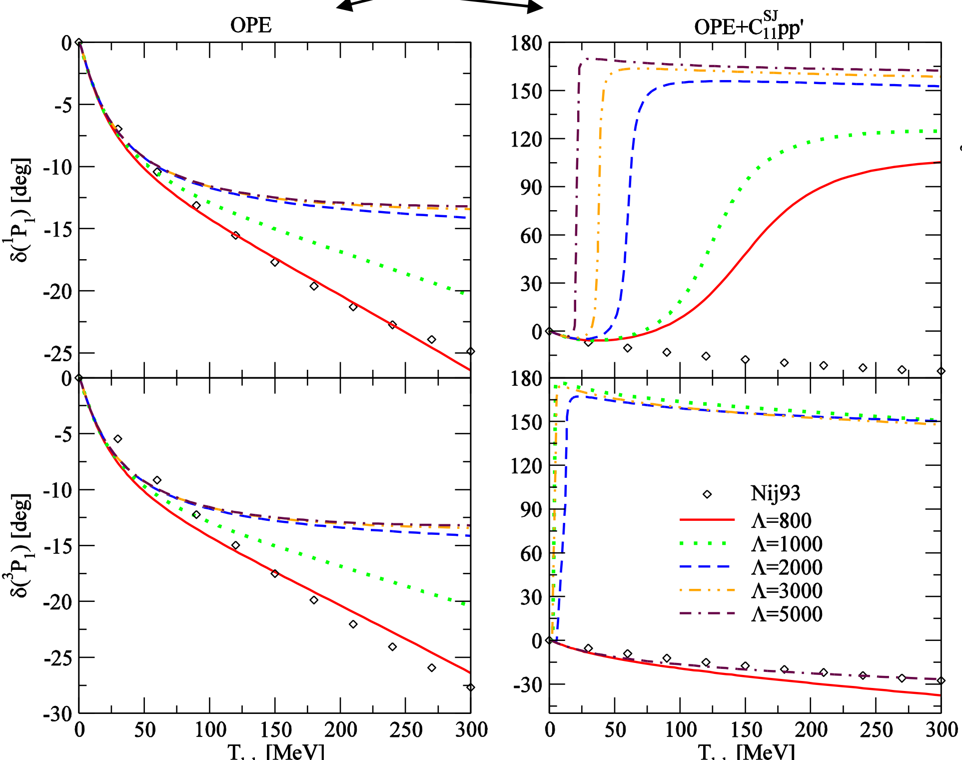
(no counter term in Weinberg counting)

See also: Nogga, Timmermans and van Kolck (2005), Valderrama (2006).

unrenormalized v.s. renormalized

singular & repulsive

(contact term enforced)

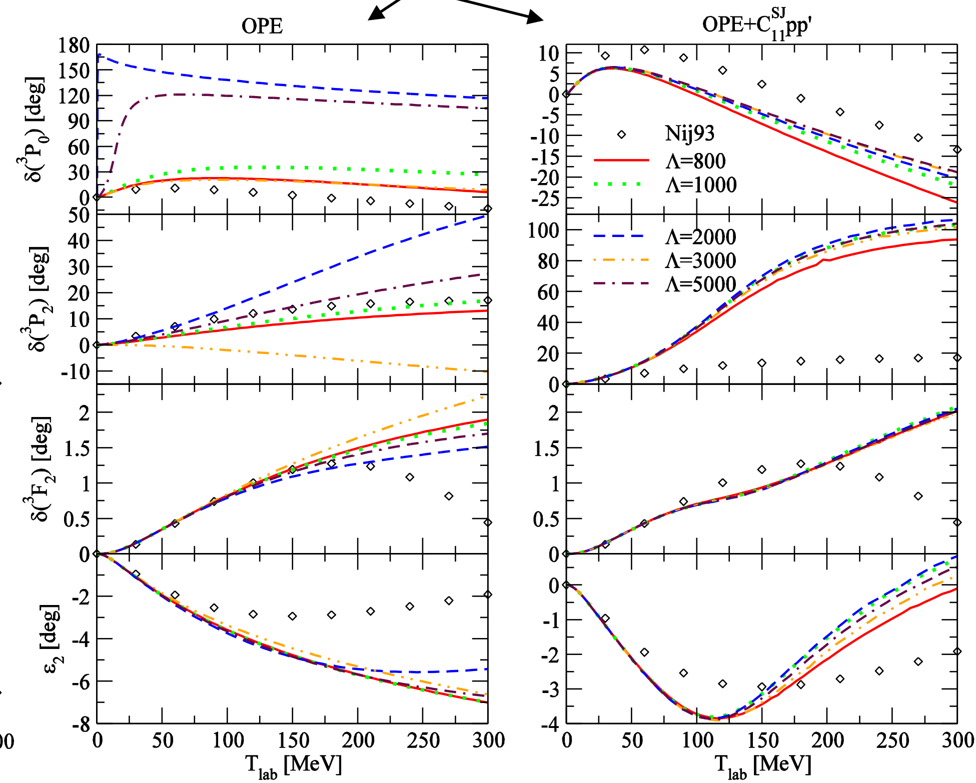


convergent

Divergent w.r.t. Λ

singular & attractive

(contact term enforced)



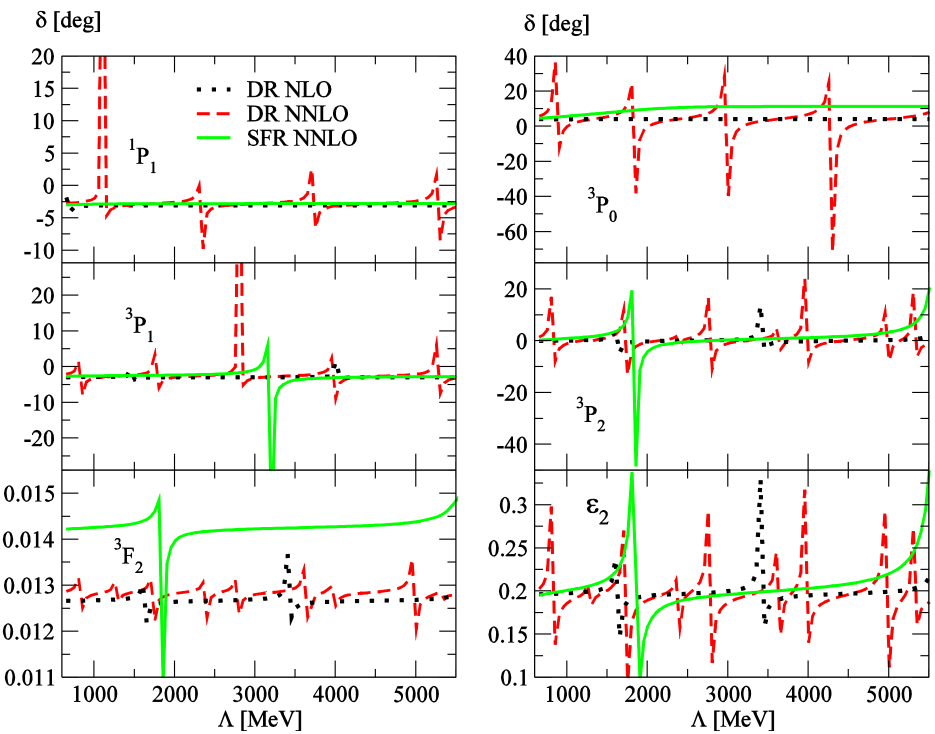
Divergent w.r.t. Λ

convergent

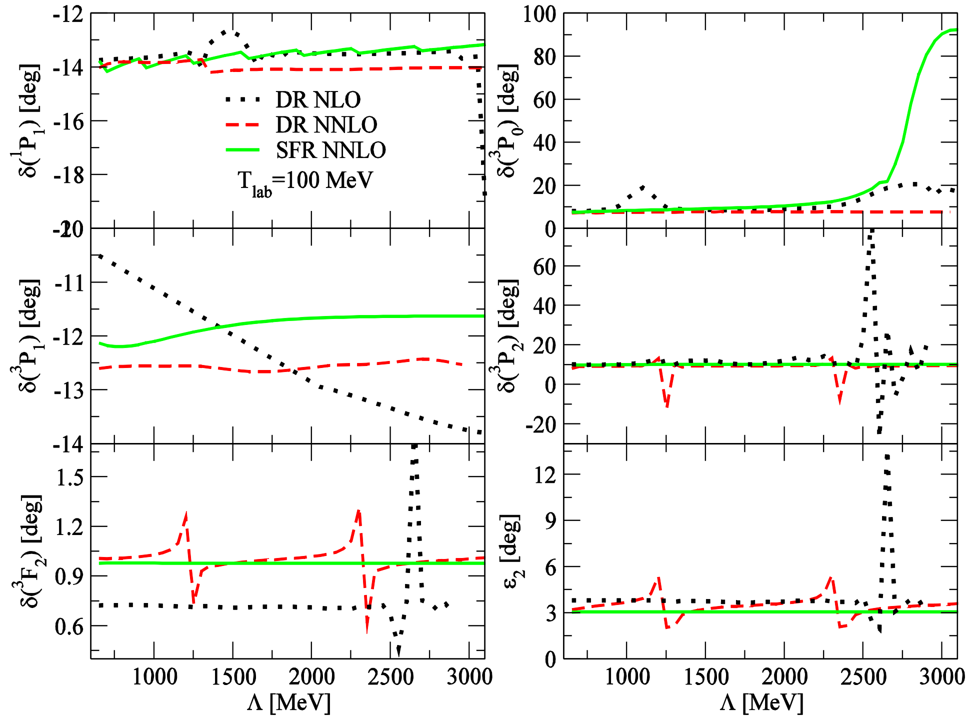
α_{ll} from extracted value of NijmII & Reid93.

P-waves results
r→0 connection for the higher order

Un-renormalized



Renormalized



	$^1P_1$	$^3P_0$	$^3P_1$	$^3P_2$
OPE	U	R	U	R
NLO (DR)	U	U	R	R
NNLO (DR)	R	R	R	R
NLO (DR) + NNLO (SFR)	U	U	R	R
NNLO (SFR)	U	U	R	R

Need counter term to stabilize?

Attractive: Yes(R on the left).

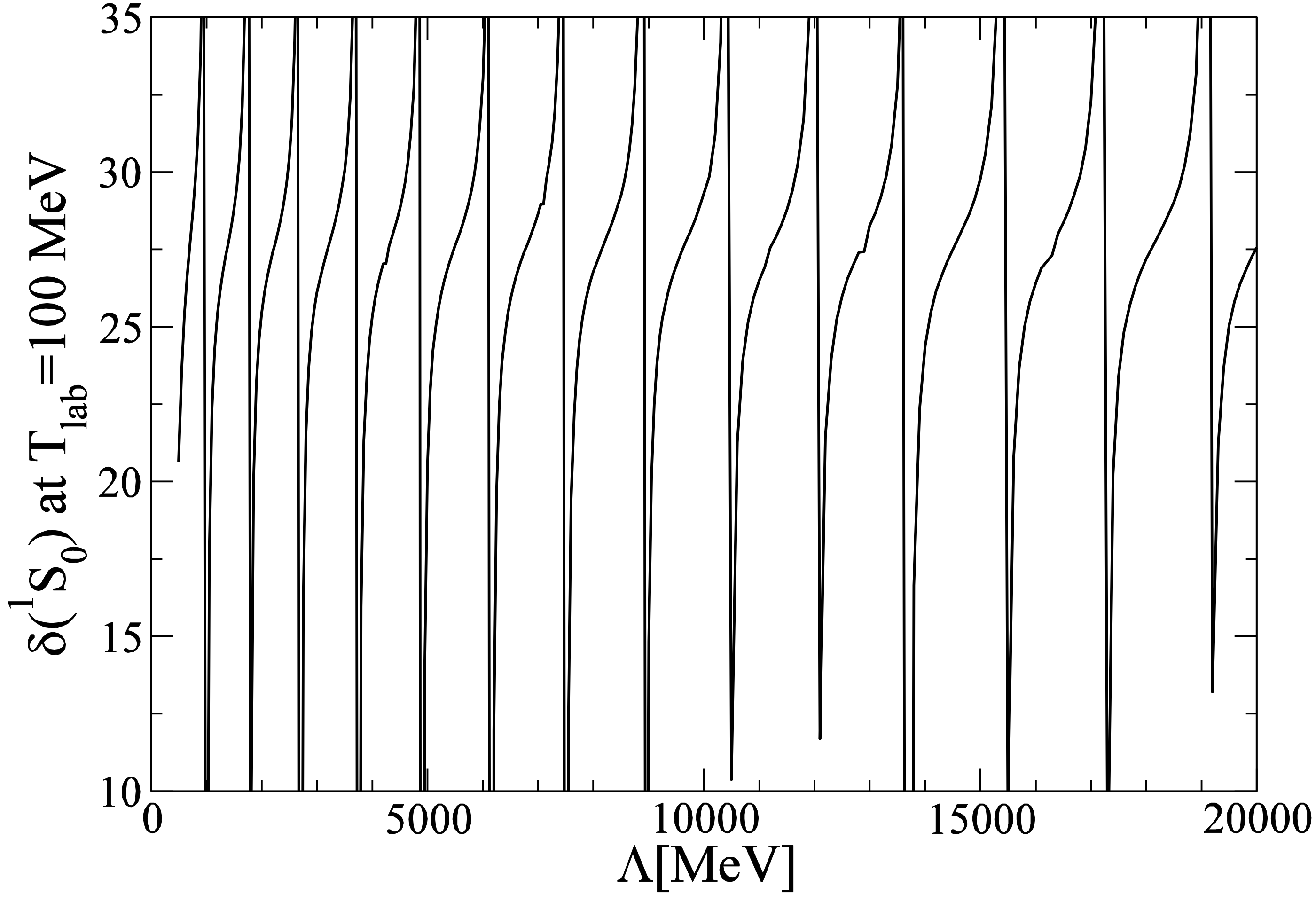
Repulsive: No(U on the left).

(C.f.: Valderrama, et al (2006))

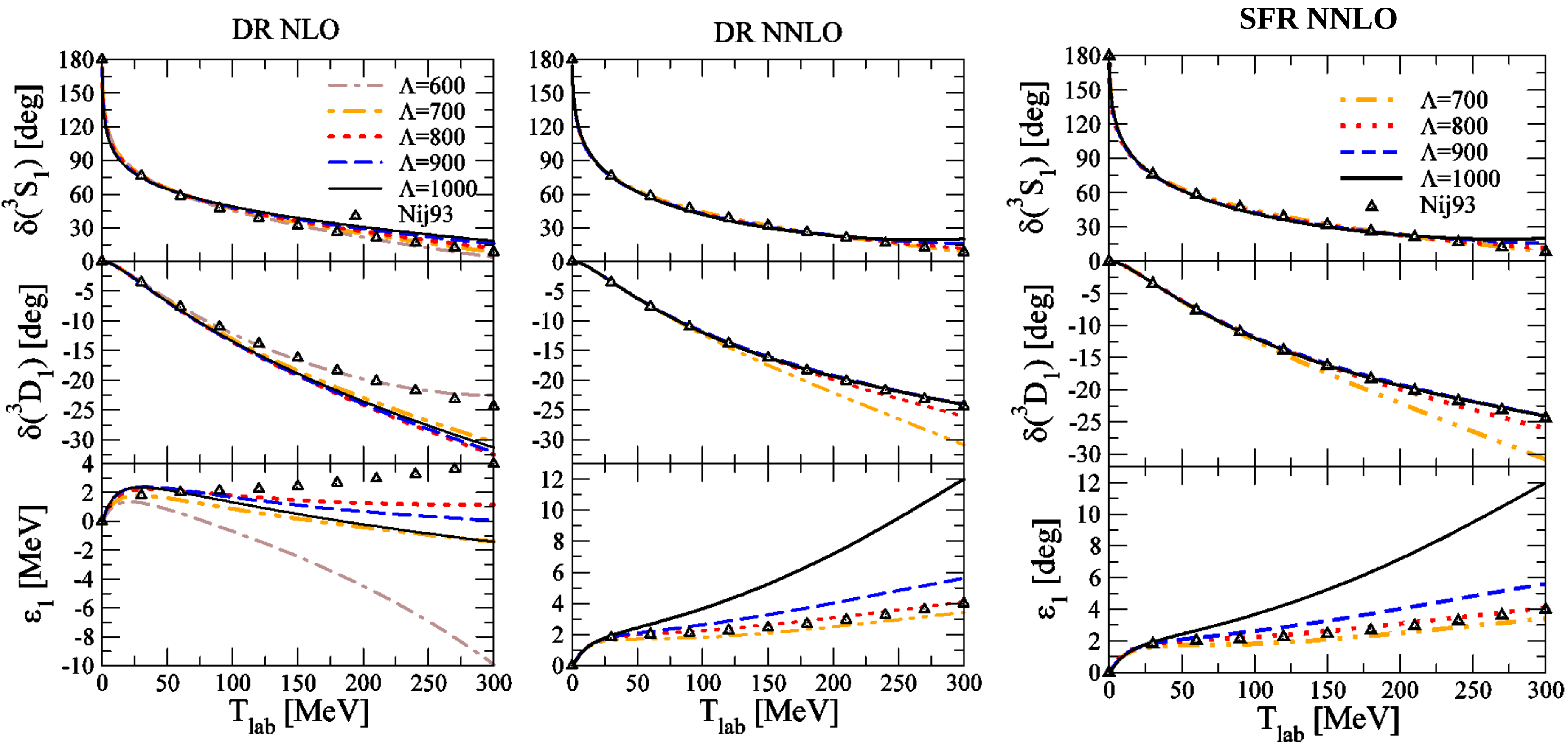
S-waves results: E-dep. counter term

Oscillation with respect to Λ

(in both $^1S_0$ and $^3S_1$-$^3D_1$)

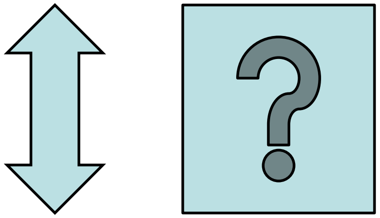


S-waves triplet results
p-dep. counter term



Even if we can fit to the data, does this necessarily mean that the renormalization is done successfully?

Cutoff-indep. in phase shift



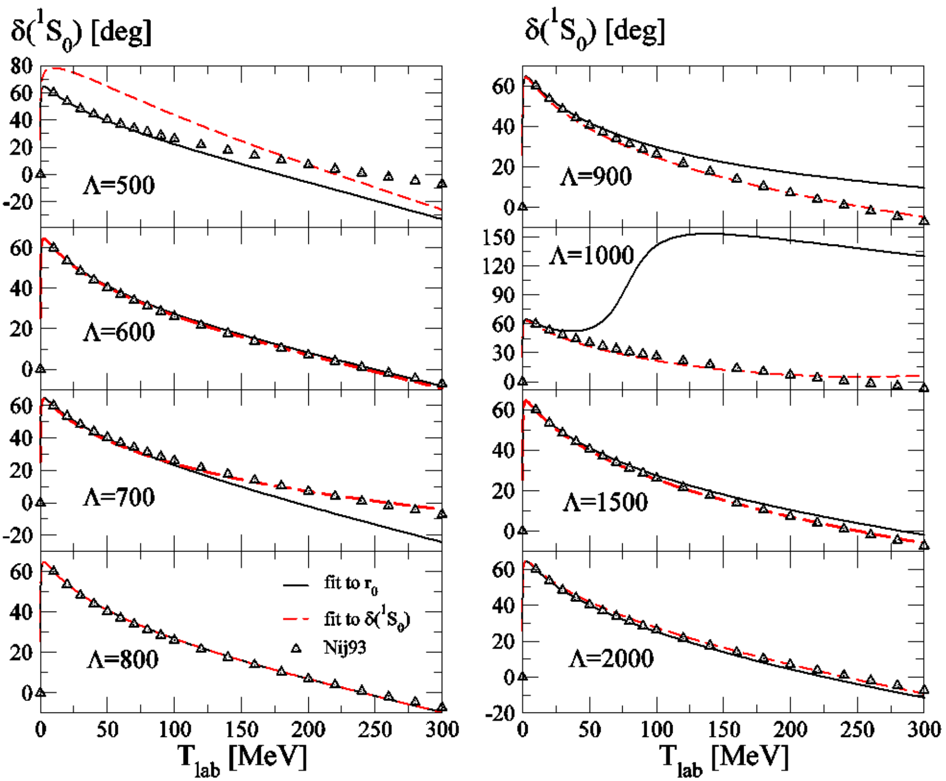
Renormalization point indep.

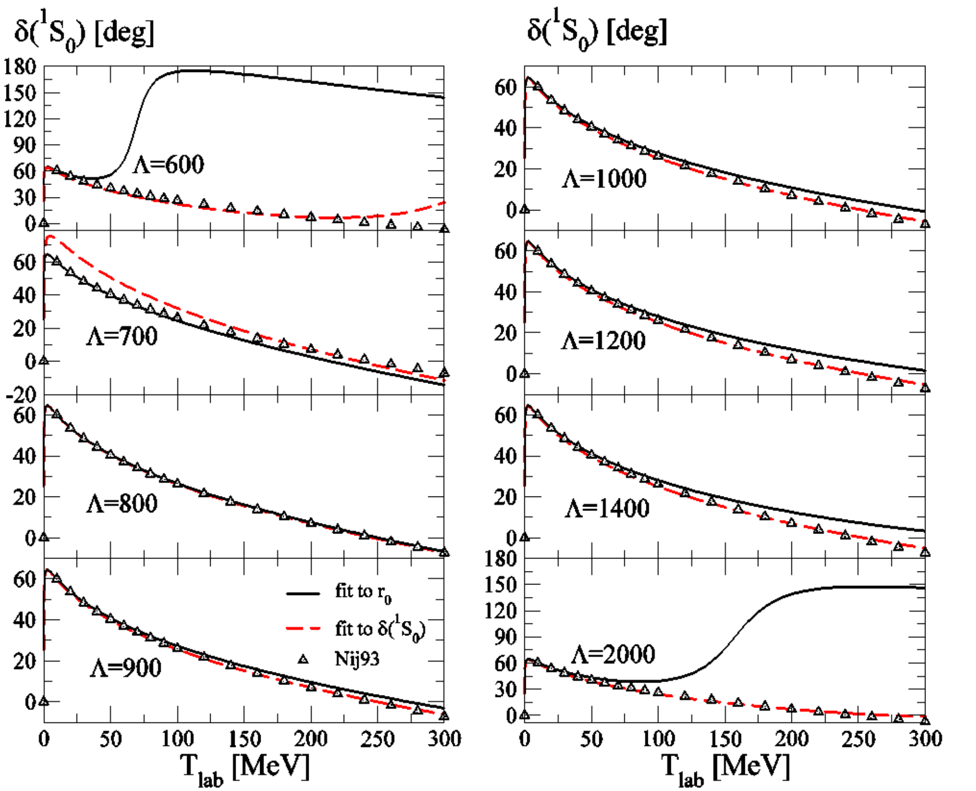
Momentum-dep. Results: Singlet

At higher Λ, has renormalization point dependence.

DR NNLO(fails at 1GeV)

Full SFR NNLO(fails at 2 GeV)





Λ cannot be too low (≤500 MeV) also.

P-waves Summary

Whether need contact term to reach Λ independence?

=> exactly depends on the singularity structure of $V(r\rightarrow 0)$.

Attractive: Yes.

Repulsive: No.

Renormalization point dependence

1. There is a **critical cutoff Λ_C~1 GeV** for DR TPE up to NNLO, after that the contact term in LSE dominate the result.
2. Replacing the whole DR NNLO TPE by **SFR brings Λ_C up to 2.5 GeV.**
3. Λ_C is in the same order of Λ_χ (~1 GeV).

S-waves Summary

- **E-dep. contact term**, there is oscillatory behavior.

1. Singlet channel: The first diverged phase shift appears at **Λ~1000 MeV** for **DR NNLO** **Λ~2000 MeV** for **Full SFR NNLO**.
2. .. Triplet channel: The first diverged phase shift appears at **Λ~1200 MeV for DR NNLO** and **Λ~2300 MeV for Full SFR NNLO**.

- **P-dep. contact term**:

For $^1S_0$ **the indep. of renormalization point breaks once Λ>1~1.2 GeV for DR NNLO**, and **Λ~2000 MeV for SFR NNLO.**

For $^3S_1$-$^3D_1$, **fit breaks down at about Λ~1.2 GeV** in general.

Conclusion

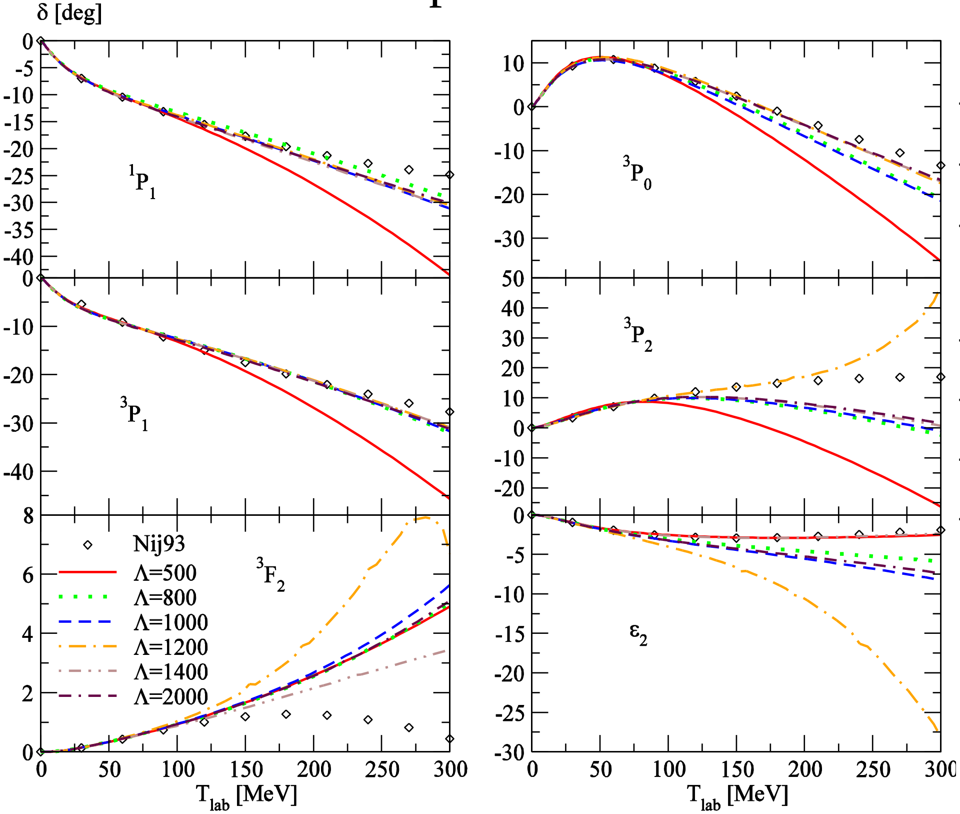
- Subtraction method provides:
  1. An easy way to go to high cutoff in LSE.
  2. A clean information of the dependence of results on the low energy observable (α^{SJ}).
- We found:
  1. **There is a critical cutoff Λ_C~1 GeV for DR TPE up to NNLO, after that the contact term in LSE dominate the result**.
  2. **Replacing the whole DR NNLO TPE by SFR brings Λ_C up to 2 GeV.**

THANK YOU!
VIELEN DANK !
MERCI BEAUCOUP !
GRAZIE MILLE !

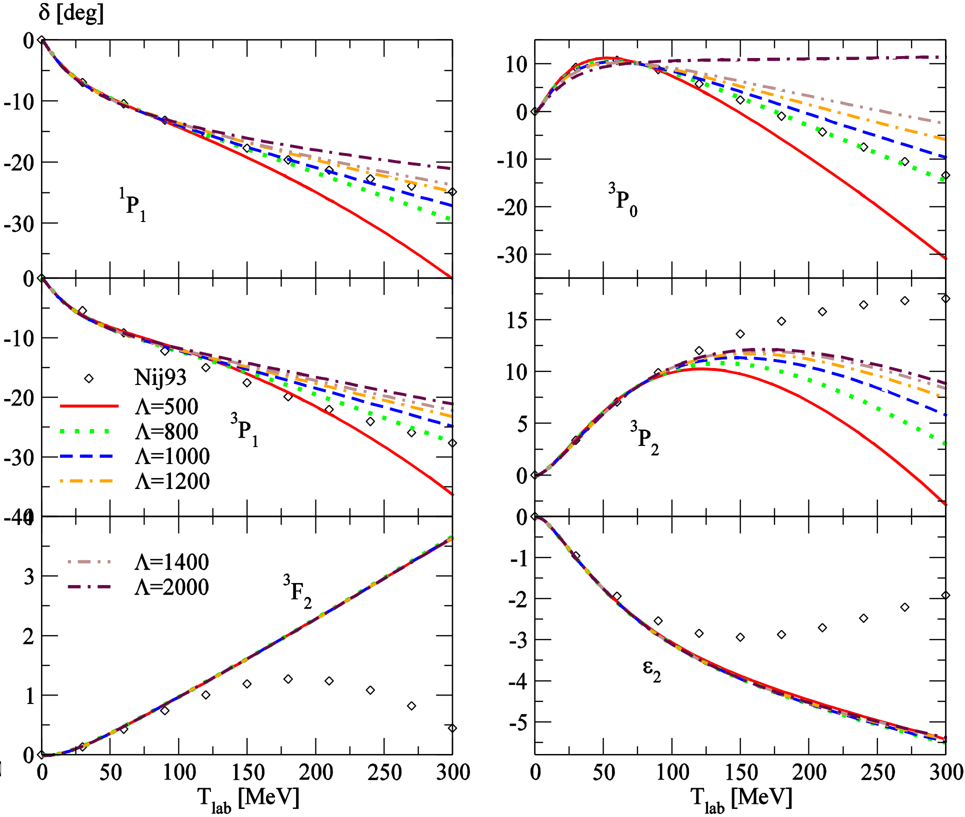
Adjust α^{SJ} for best fit

For DR TPE, need to vary α^{SJ} away from Nijm value up to 30% in some channels.

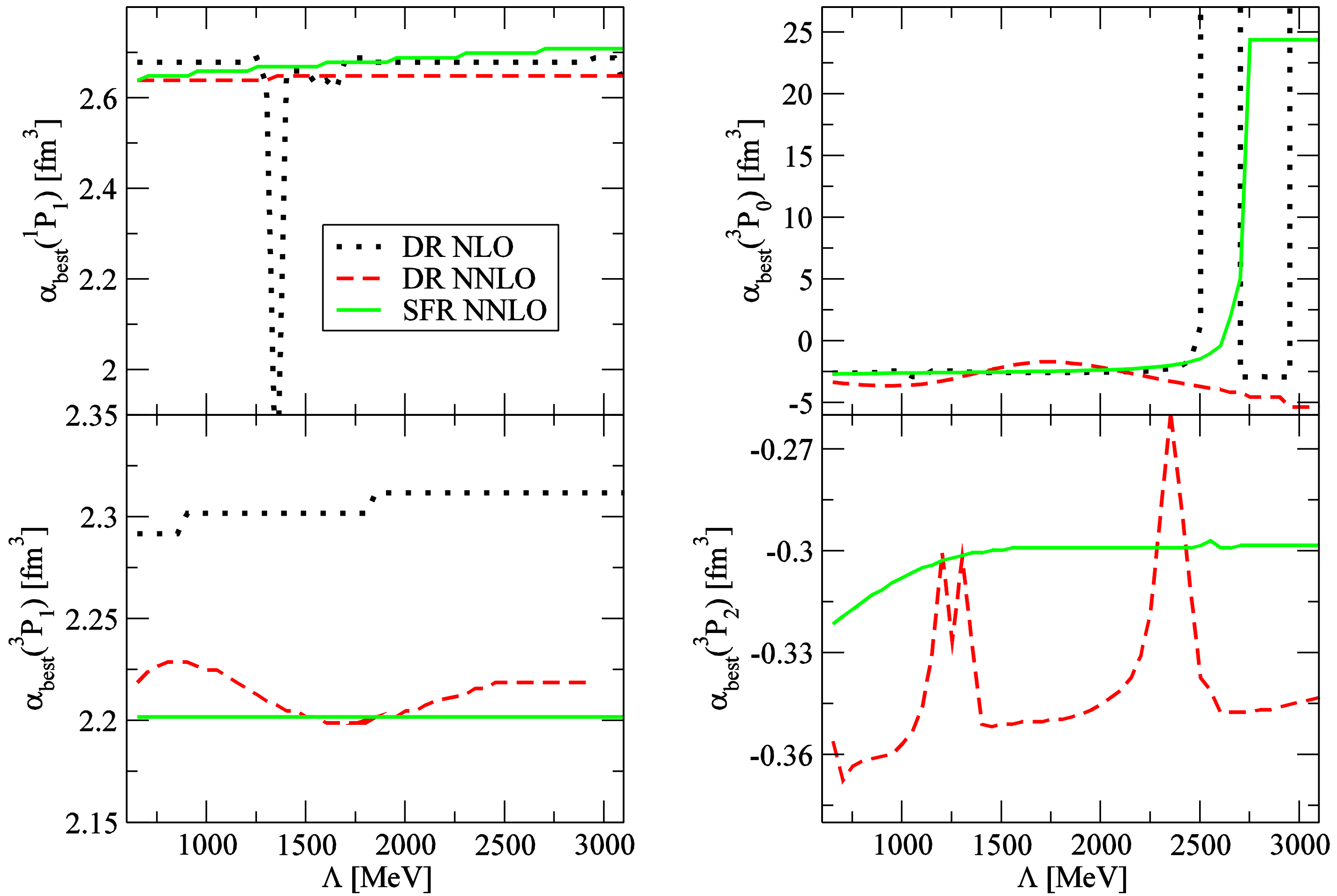
Dimensional regularization (DR)
Up to NNLO



Spectral function regularization (SFR)
Up to NNLO



α_{best} v.s. Λ (DR v.s. SFR)



For SFR NNLO in p-waves, Λ_C~ 2.5GeV.

α_{best} v.s. Λ (DR NLO)