

UNITARIZED CHIRAL
DYNAMICS IN
FEW-BODY SYSTEMS

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INTRODUCTION

- UNTIL RECENTLY, MESON-MESON AND MESON-BARYON STRUCTURE OF MESON AND BARYON RESONANCES HAS BEEN EXPLORED EXTENSIVELY USING CHIRAL DYNAMICS, E.G. :

$\sigma(600), f_0(980), a_0(980), \kappa, \Lambda(1405), \Lambda(1520), etc.$ ^{1, 2, 3, 4,5}



- SOME STATES IN THE MESON & BARYON SPECTRUM, HOWEVER, COULD VERY WELL POSSESS A MORE COMPLICATED MOLECULAR STRUCTURE



¹ J. A. OLLER, E. OSET, NUCL. PHYS. A 620 (1997) 438.

² J. A. OLLER, ULF-G. MEISSNER, PHYS. LETT. B 500 (2001) 263-272.

³ J. A. OLLER, E. OSET, J. R. PELÁEZ, PHYS. REV. D 59 074001 (199).

⁴ D. JIDO, J. A. OLLER, E. OSET, A. RAMOS, U. G. MEISSNER, NUCL. PHYS. A 725 (2003) 181-200.

⁵ L. ROCA, SOURAV SARKAR, V.K. MAGAS, E. OSET PHYS. REV. C73, 045208 (2006).

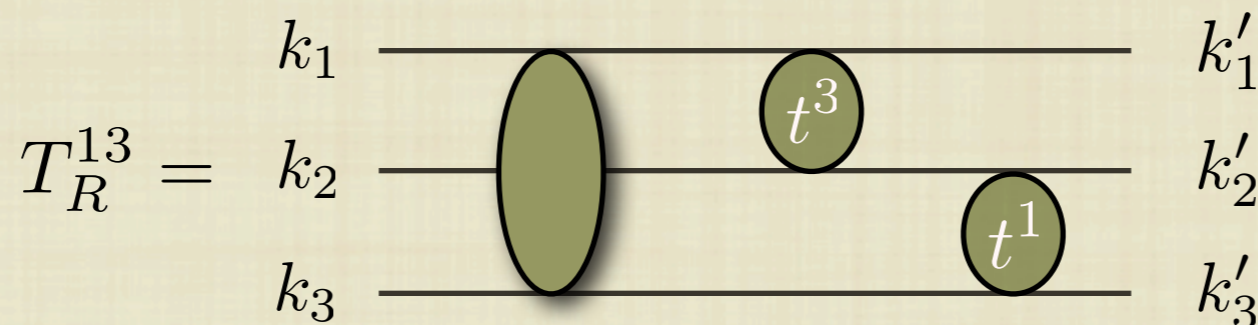
THE MODEL

- WE SOLVE THE FADDEEV EQUATIONS

$$T = T^1 + T^2 + T^3$$

$$T^i = t^i \delta^3(\vec{k}'_i - \vec{k}_i) + T_R^{ij} + T_R^{ik}$$

- THE T_R^{ij} MATRICES CONTAIN ALL THE POSSIBLE DIAGRAMS WHERE THE LAST TWO SUCCESSIVE INTERACTIONS ARE T^i AND T^j



- AND THEY SATISFY THE EQUATIONS:

$$T_R^{12} = t^1 g^{12} t^2 + t^1 \left[G^{121} T_R^{21} + G^{123} T_R^{23} \right]$$

$$T_R^{13} = t^1 g^{13} t^3 + t^1 \left[G^{131} T_R^{31} + G^{132} T_R^{32} \right]$$

$$T_R^{21} = t^2 g^{21} t^1 + t^2 \left[G^{212} T_R^{12} + G^{213} T_R^{13} \right]$$

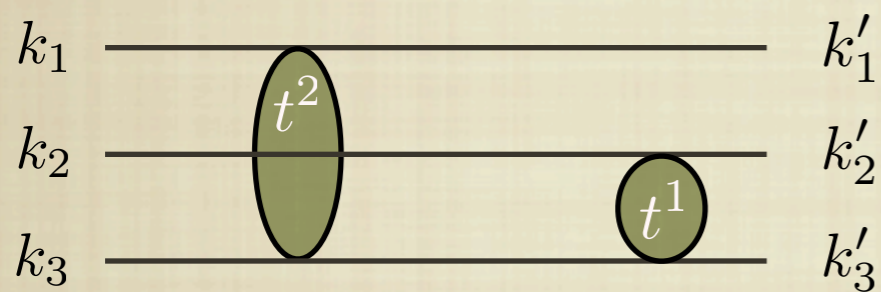
$$T_R^{23} = t^2 g^{23} t^3 + t^2 \left[G^{231} T_R^{31} + G^{232} T_R^{32} \right]$$

$$T_R^{31} = t^3 g^{31} t^1 + t^3 \left[G^{312} T_R^{12} + G^{313} T_R^{13} \right]$$

$$T_R^{32} = t^3 g^{32} t^2 + t^3 \left[G^{321} T_R^{21} + G^{323} T_R^{23} \right]$$

■ t^1 IS THE TWO BODY t -MATRIX $\longrightarrow t = V + V\tilde{g}t$

■ g^{1j} IS THE THREE-BODY GREEN FUNCTION.

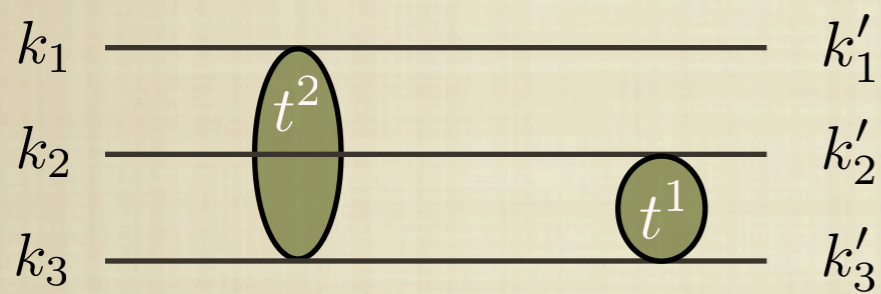


$$g^{ij}(\vec{k}_i', \vec{k}_j) = \left(\prod_{r=1}^D \frac{N_r}{2E_r} \right) \frac{1}{\sqrt{s} - E_i(\vec{k}_i') - E_l(\vec{k}_i' + \vec{k}_j) - E_j(\vec{k}_j)}$$

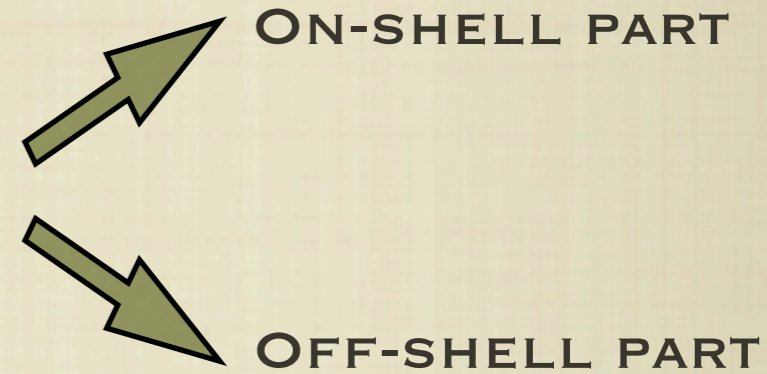
$$N_r = \begin{cases} 1 & \text{meson-meson interaction} \\ 2M_r & \text{meson-baryon interaction} \end{cases}$$

■ t^i IS THE TWO BODY t -MATRIX $\longrightarrow t = V + V\tilde{g}t$

■ g^{ij} IS THE THREE-BODY GREEN FUNCTION

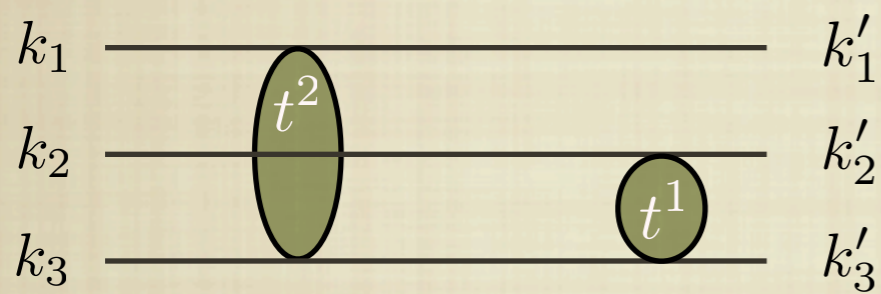


CHIRAL AMPLITUDES

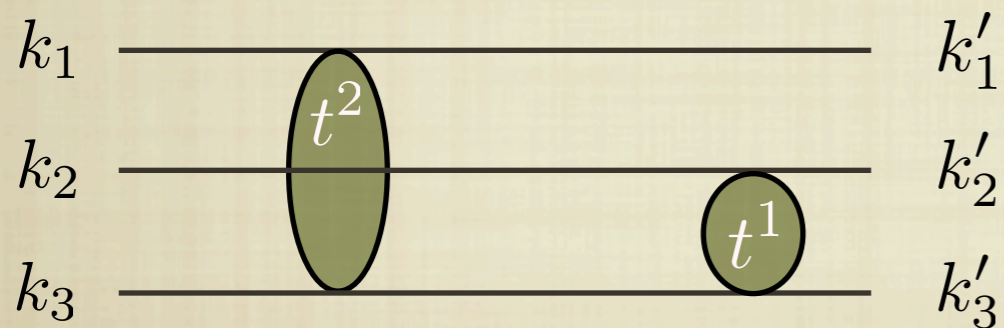


■ t^i IS THE TWO BODY t -MATRIX $\longrightarrow t = V + V\tilde{g}t$

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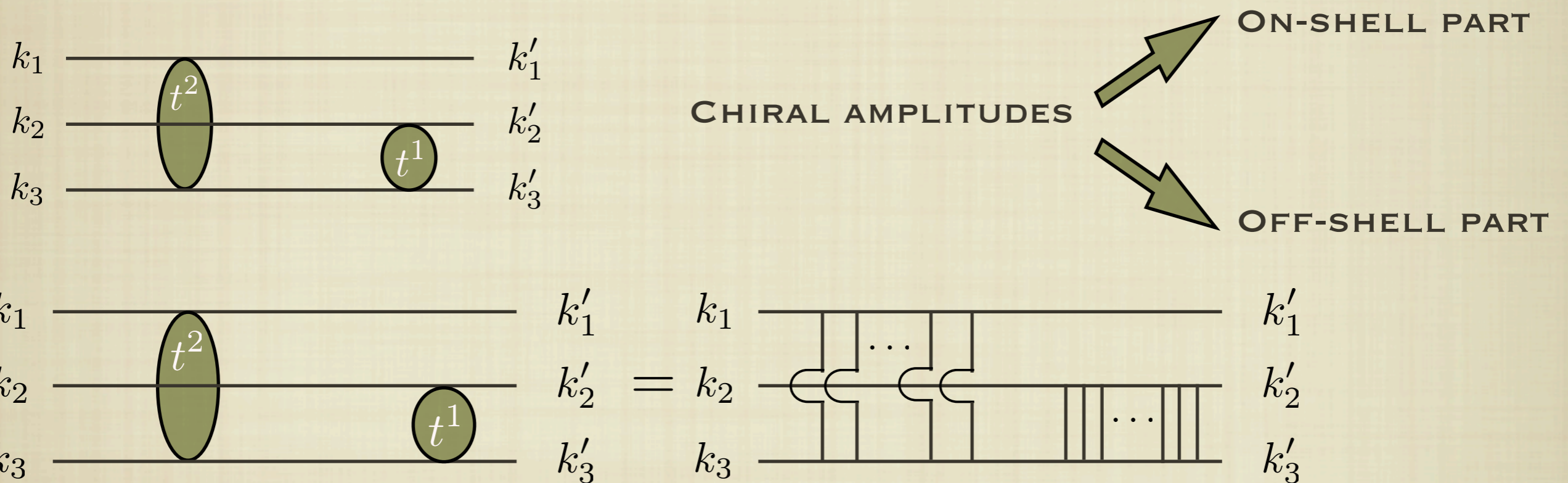


CHIRAL AMPLITUDES



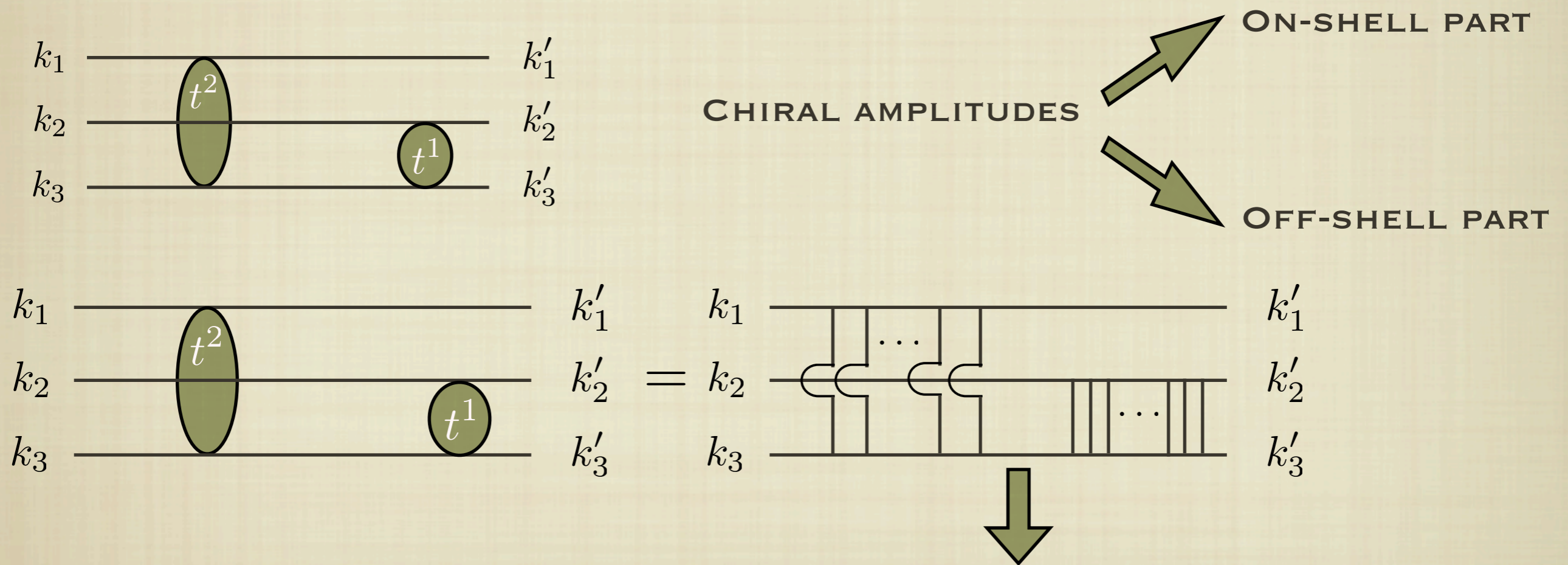
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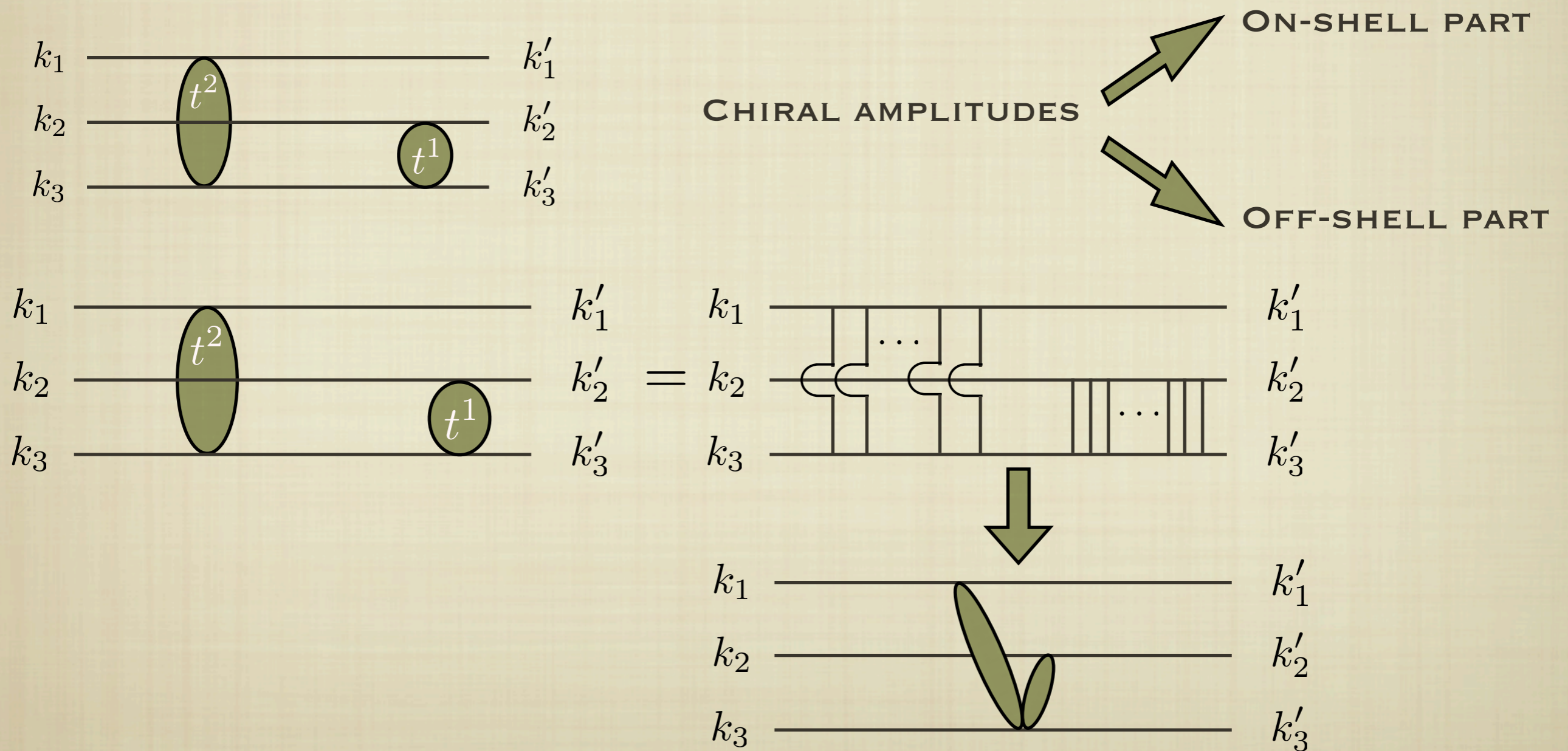
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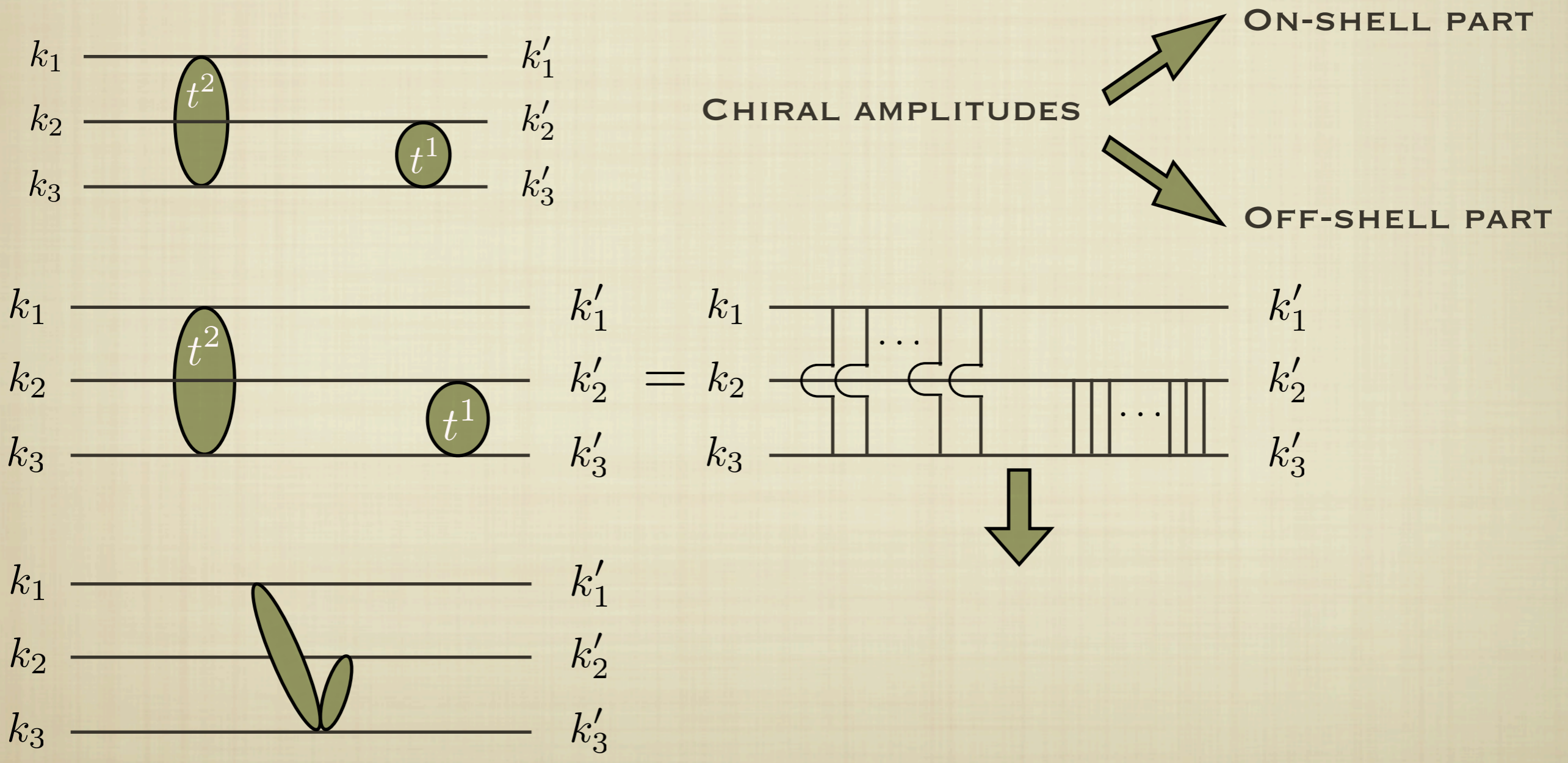
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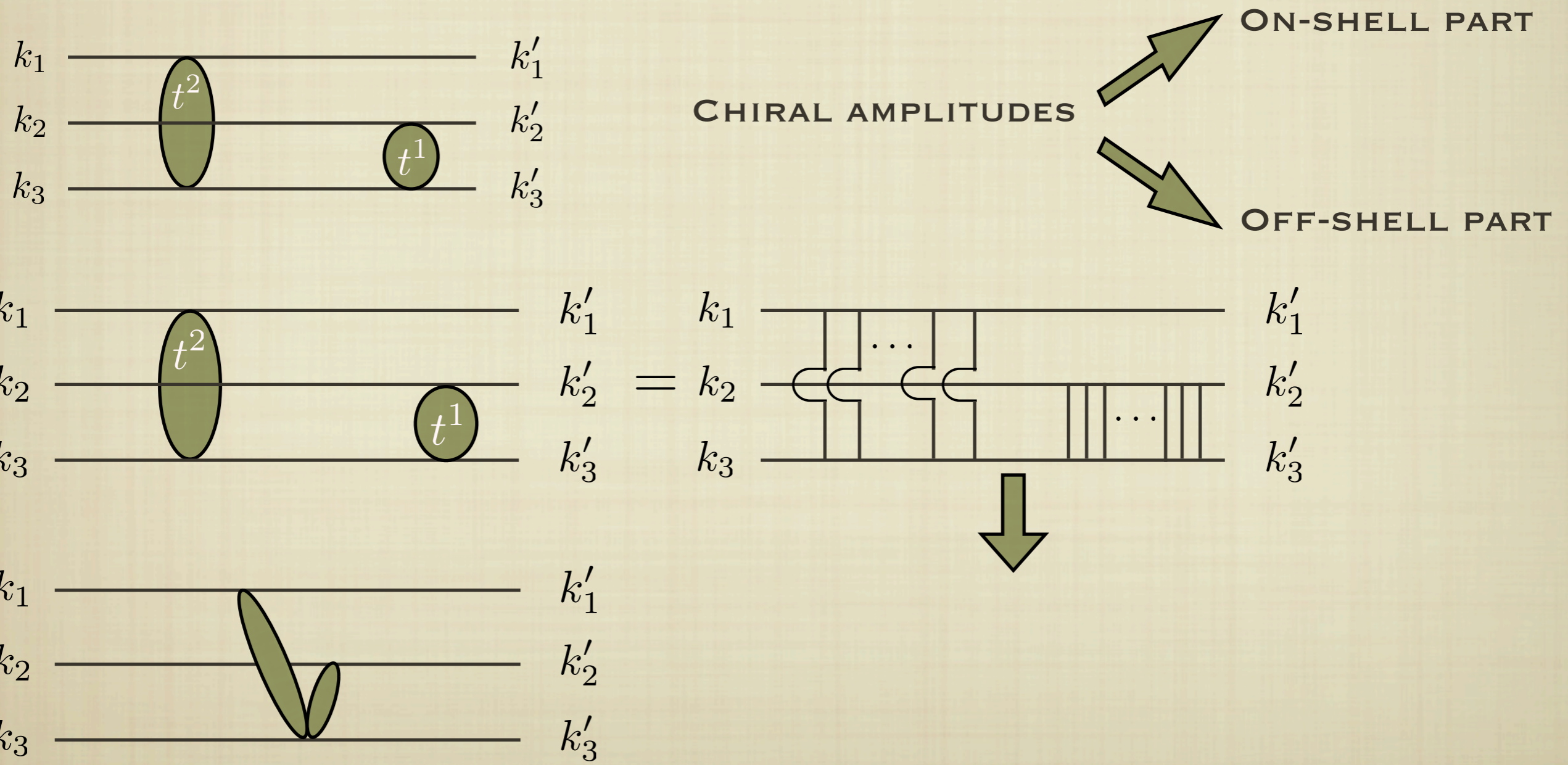
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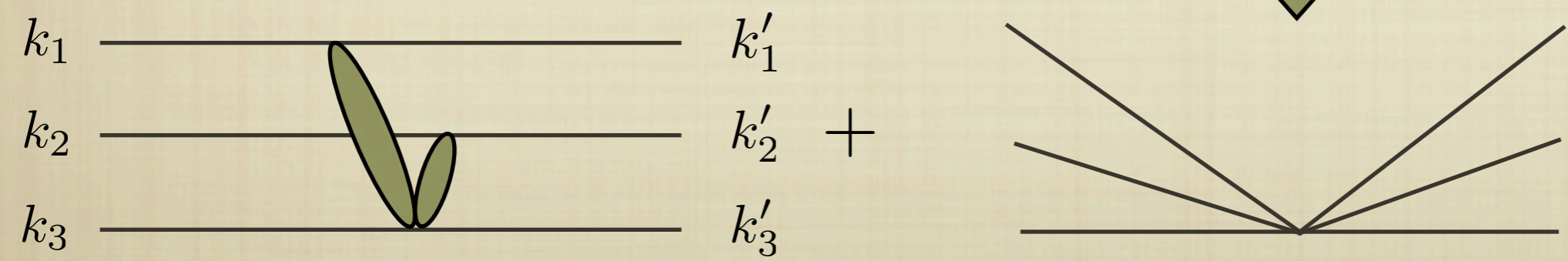
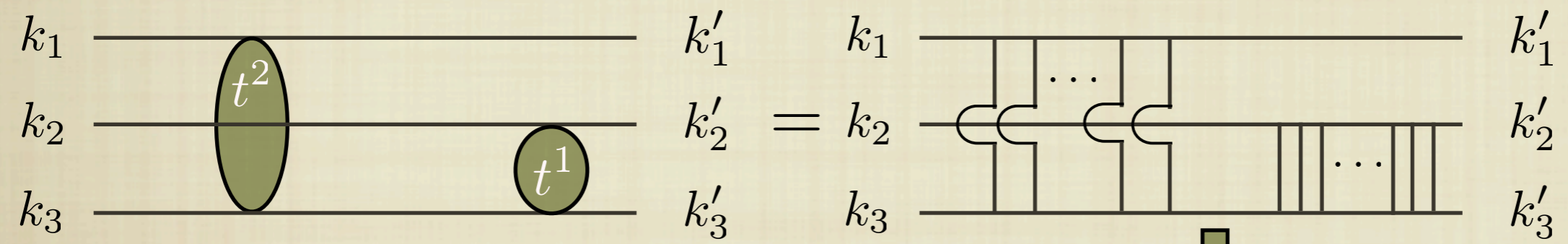
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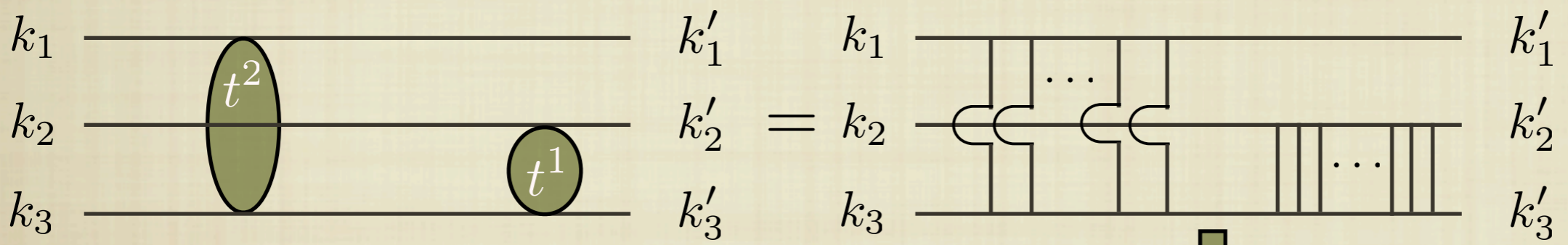
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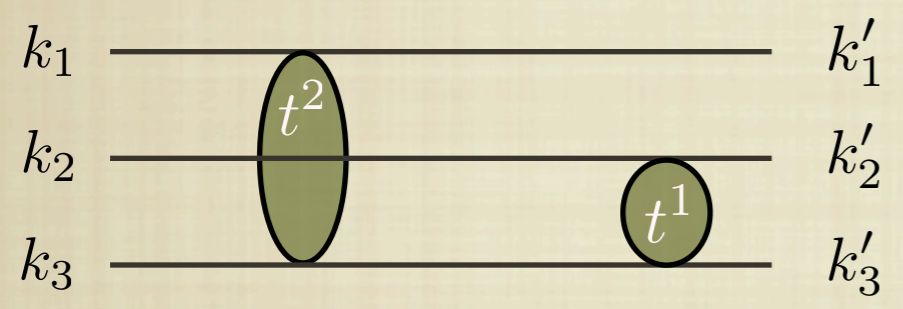
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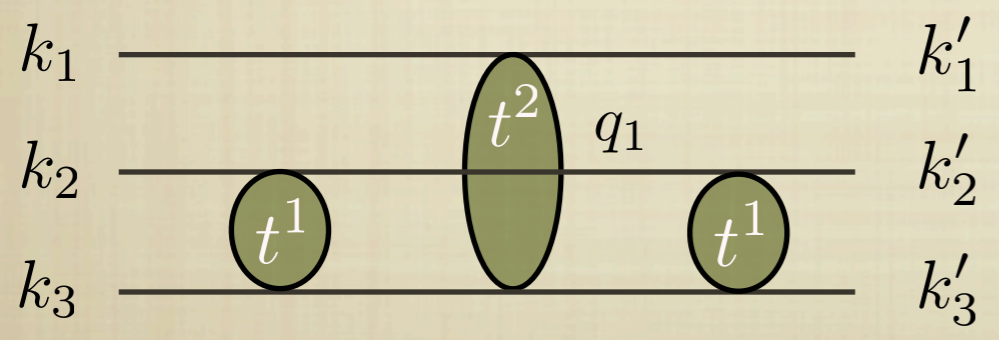
■ g^{ij} IS THE THREE-BODY GREEN



$$g^{ij}(\vec{k}_i', \vec{k}_j) = \left(\prod_{r=1}^D \frac{N_r}{2E_r} \right) \frac{1}{\sqrt{s} - E_i(\vec{k}_i') - E_l(\vec{k}_i' + \vec{k}_j) - E_j(\vec{k}_j)}$$

$$N_r = \begin{cases} 1 & \text{meson-meson interaction} \\ 2M_r & \text{meson-baryon interaction} \end{cases}$$

■ G^{ijk} IS THE LOOP FUNCTION FOR DIAGRAMS INVOLVING THREE t MATRICES.



$$\int \frac{d^3 q_1}{(2\pi)^3} t^1(\sqrt{s_{23}}) g^{12} t^2(q_1^2) g^{21} t^1(\sqrt{s_{23}})$$

$$G^{121} = \int \frac{d^3 q_1}{(2\pi)^3} g^{12} t^2(q_1^2) g^{21} [g^{21}(\vec{k}'_2, \vec{k}_1)]^{-1} [t^2(\sqrt{s_{23}})]^{-1}$$

$$t^1(\sqrt{s_{23}}) G^{121} t^2(\sqrt{s_{13}}) g^{21}(\vec{k}'_2, \vec{k}_1) t^1(\sqrt{s_{23}})$$

TWO MESON-ONE BARYON SYSTEMS

- $\pi \bar{K} N, \pi \pi \Sigma, \pi \pi \Lambda, \pi \eta \Sigma, \pi \eta \Lambda, \pi K \Xi$

TWO MESON-ONE BARYON SYSTEMS


■ $\pi \bar{K} N, \pi \pi \Sigma, \pi \pi \Lambda, \pi \eta \Sigma, \pi \eta \Lambda, \pi K \Xi$

S-WAVE



$$J^{\pi} = 1/2^{+}$$


TWO MESON-ONE BARYON SYSTEMS

- $\pi \bar{K} N, \pi \pi \Sigma, \pi \pi \Lambda, \pi \eta \Sigma, \pi \eta \Lambda, \pi K \Xi$  FOUR Σ AND TWO Λ STATES WITH $J^P = 1/2^+$ IN THE ENERGY REGION 1500-1800 MEV.

$$\Sigma(1660) : K^- p \rightarrow \pi \pi \Sigma$$

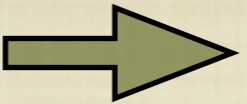
$$\Lambda(1600) : K^- p \rightarrow \pi \pi \Lambda$$

TWO MESON-ONE BARYON SYSTEMS

■ $\pi \bar{K} N, \pi \pi \Sigma, \pi \pi \Lambda, \pi \eta \Sigma, \pi \eta \Lambda, \pi K \Xi$  FOUR Σ AND TWO Λ STATES WITH $J^P = 1/2^+$ IN THE ENERGY REGION 1500-1800 MEV.

	Γ (PDG) (MeV)	PEAK POSITION (THIS WORK) (MEV)	Γ (THIS WORK) (MEV)
ISOSPIN = 1			
$\Sigma(1560)$	10-100	1590	70
$\Sigma(1620)$	10-100	1630	39
$\Sigma(1660)$	40-200	1656	30
$\Sigma(1770)$	60-100	1790	24
ISOSPIN = 0			
$\Lambda(1600)$	50-250	1568, 1700	60, 136
$\Lambda(1810)$	50-250	1740	20

TWO MESON-ONE BARYON SYSTEMS

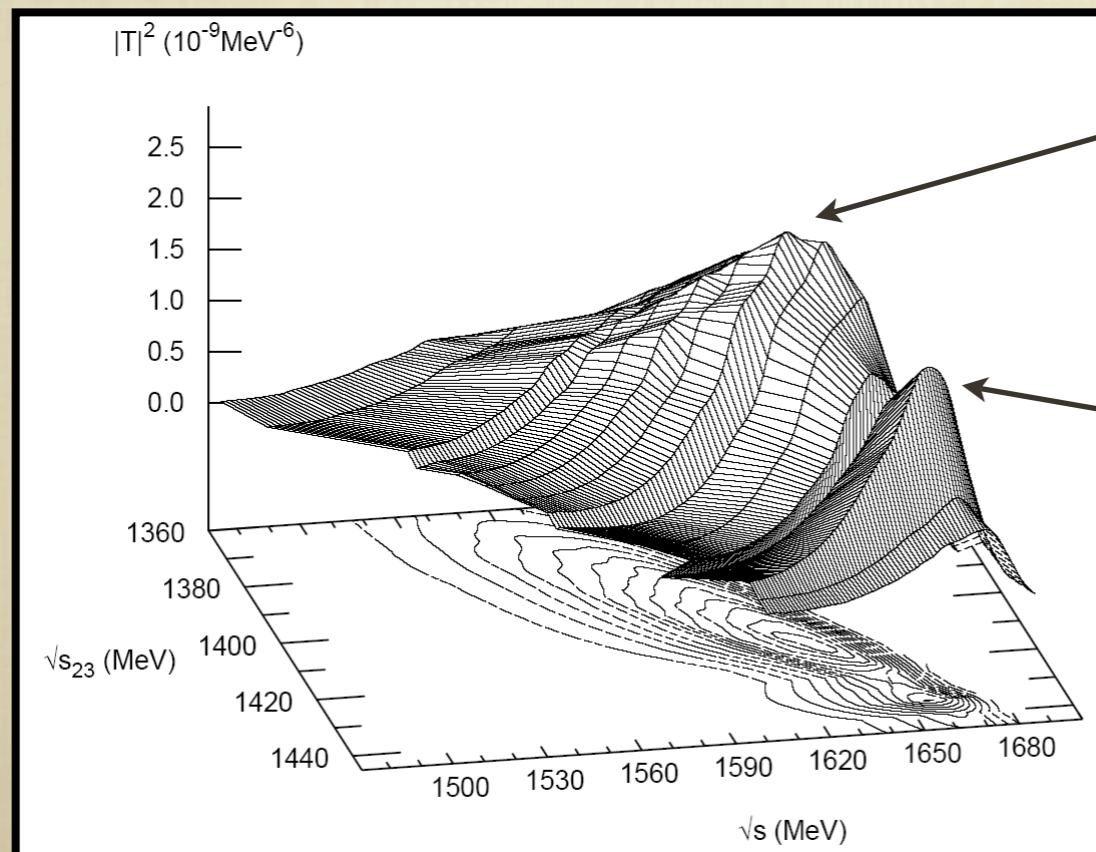
- $\pi \bar{K} N, \pi \pi \Sigma, \pi \pi \Lambda, \pi \eta \Sigma, \pi \eta \Lambda, \pi K \Xi$

 FOUR Σ AND TWO Λ STATES WITH $J^P=1/2^+$ IN THE ENERGY REGION 1500-1800 MEV.

$\Sigma(1660)$

$1656 - i30/2 \text{ MeV}$

$\Sigma(1620)$

$1630 - i39/2 \text{ MeV}$



R. ARMENTEROS ET AL. NUCL. PHYS. B 8, 183 (1968).
 B. R. MARTIN ET AL, NUCL. PHYS. B 127, 349 (1977).

TWO MESON-ONE BARYON SYSTEMS

- $\pi \bar{K} N, \pi \pi \Sigma, \pi \pi \Lambda, \pi \eta \Sigma, \pi \eta \Lambda, \pi K \Xi$ \longrightarrow FOUR Σ AND TWO Λ STATES WITH $J^P = 1/2^+$ IN THE ENERGY REGION 1500-1800 MEV.
- $\pi \pi N, \pi K \Sigma, \pi K \Lambda, \pi \eta N, K \bar{K} N$ \longrightarrow $N^*(1710)$ WITH 40-90% BRANCHING RATIO TO $\pi \pi N$

TWO MESON-ONE BARYON SYSTEMS

- $\pi \bar{K} N, \pi \pi \Sigma, \pi \pi \Lambda, \pi \eta \Sigma, \pi \eta \Lambda, \pi K \Xi$ \Rightarrow FOUR Σ AND TWO Λ STATES WITH $J^P = 1/2^+$ IN THE ENERGY REGION 1500-1800 MEV.
- $\pi \pi N, \pi K \Sigma, \pi K \Lambda, \pi \eta N, K \bar{K} N$ \Rightarrow $N^*(1710)$ WITH 40-90% BRANCHING RATIO TO $\pi \pi N$

	Γ (PDG) (MeV)	PEAK POSITION (THIS WORK) (MEV)	Γ (THIS WORK) (MEV)
$N^*(1710)$	50-250	1704	375

TWO MESON-ONE BARYON SYSTEMS

- $\pi \bar{K} N, \pi \pi \Sigma, \pi \pi \Lambda, \pi \eta \Sigma, \pi \eta \Lambda, \pi K \Xi$ \longrightarrow FOUR Σ AND TWO Λ STATES WITH $J^P = 1/2^+$ IN THE ENERGY REGION 1500-1800 MEV.
- $\pi \pi N, \pi K \Sigma, \pi K \Lambda, \pi \eta N, K \bar{K} N$ \longrightarrow $N^*(1710)$ WITH 40-90% BRANCHING RATIO TO $\pi \pi N$

	Γ (PDG) (MeV)
$N^*(1710)$	50-250
$N^*(2100)$	50-360
$\Delta(1750)$	50-300
$\Delta(1910)$	190-270

TWO MESON-ONE BARYON SYSTEMS

- $\pi \bar{K} N, \pi \pi \Sigma, \pi \pi \Lambda, \pi \eta \Sigma, \pi \eta \Lambda, \pi K \Xi$ \longrightarrow FOUR Σ AND TWO Λ STATES WITH $J^P = 1/2^+$ IN THE ENERGY REGION 1500-1800 MEV.
- $\pi \pi N, \pi K \Sigma, \pi K \Lambda, \pi \eta N, K \bar{K} N$ \longrightarrow $N^*(1710)$ WITH 40-90% BRANCHING RATIO TO $\pi \pi N$

	Γ (PDG) (MeV)
$N^*(1710)$	50-250
$N^*(2100)$	50-360
$\Delta(1750)$	50-300
$\Delta(1910)$	190-270



EXPERIMENTAL
AMPLITUDES

TWO MESON-ONE BARYON SYSTEMS

■ $\pi \bar{K} N, \pi \pi \Sigma, \pi \pi \Lambda, \pi \eta \Sigma, \pi \eta \Lambda, \pi K \Xi$ \longrightarrow FOUR Σ AND TWO Λ STATES WITH $J^P = 1/2^+$ IN THE ENERGY REGION 1500-1800 MEV.

■ $\pi \pi N, \pi K \Sigma, \pi K \Lambda, \pi \eta N, K \bar{K} N$ \longrightarrow $N^*(1710)$ WITH 40-90% BRANCHING RATIO TO $\pi \pi N$

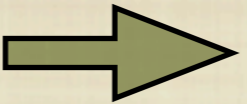
	Γ (PDG) (MeV)	PEAK POSITION (THIS WORK) (MEV)	Γ (THIS WORK) (MEV)
$N^*(1710)$	50-250	1704	375
$N^*(2100)$	50-360	2080	54
$N^*(1920)$?	1924	20
$\Delta(1910)$	190-270	2126	42

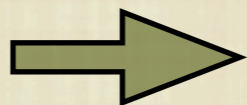
PREDICTED IN
PHYS.REV.C78,035203
(2008)

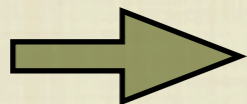
TWO MESON-ONE BARYON SYSTEMS

- $\pi\bar{K}N, \pi\pi\Sigma, \pi\pi\Lambda, \pi\eta\Sigma, \pi\eta\Lambda, \pi K\Xi$ \longrightarrow FOUR Σ AND TWO Λ STATES WITH $J^P=1/2^+$ IN THE ENERGY REGION 1500-1800 MEV.
- $\pi\pi N, \pi K\Sigma, \pi K\Lambda, \pi\eta N, K\bar{K}N$ \longrightarrow $N^*(1710), N^*(2100), N^*(1920), \Delta(1910)$. NO SIGNAL FOR ROPER AND $\Delta(1750)$.
- πKN \longrightarrow STUDY OF THE POSSIBILITY THAT THE Θ^+ COULD BE A πKN BOUND STATE.

TWO MESON-ONE BARYON SYSTEMS

■ $\pi\bar{K}N, \pi\pi\Sigma, \pi\pi\Lambda, \pi\eta\Sigma, \pi\eta\Lambda, \pi K\Xi$  FOUR Σ AND TWO Λ STATES WITH $J^P=1/2^+$ IN THE ENERGY REGION 1500-1800 MEV.

■ $\pi\pi N, \pi K\Sigma, \pi K\Lambda, \pi\eta N, K\bar{K}N$  $N^*(1710), N^*(2100), N^*(1920), \Delta(1910)$.
NO SIGNAL FOR ROPER AND $\Delta(1750)$.

■ πKN  STUDY OF THE POSSIBILITY THAT THE Θ^+ COULD BE A πKN BOUND STATE.



WE DO NOT FIND ANY SIGNAL AROUND 1520 MEV BUT WE OBTAIN A PEAK AROUND 1700 MEV WITH 200 MEV OF WIDTH.

THREE-MESON SYSTEMS

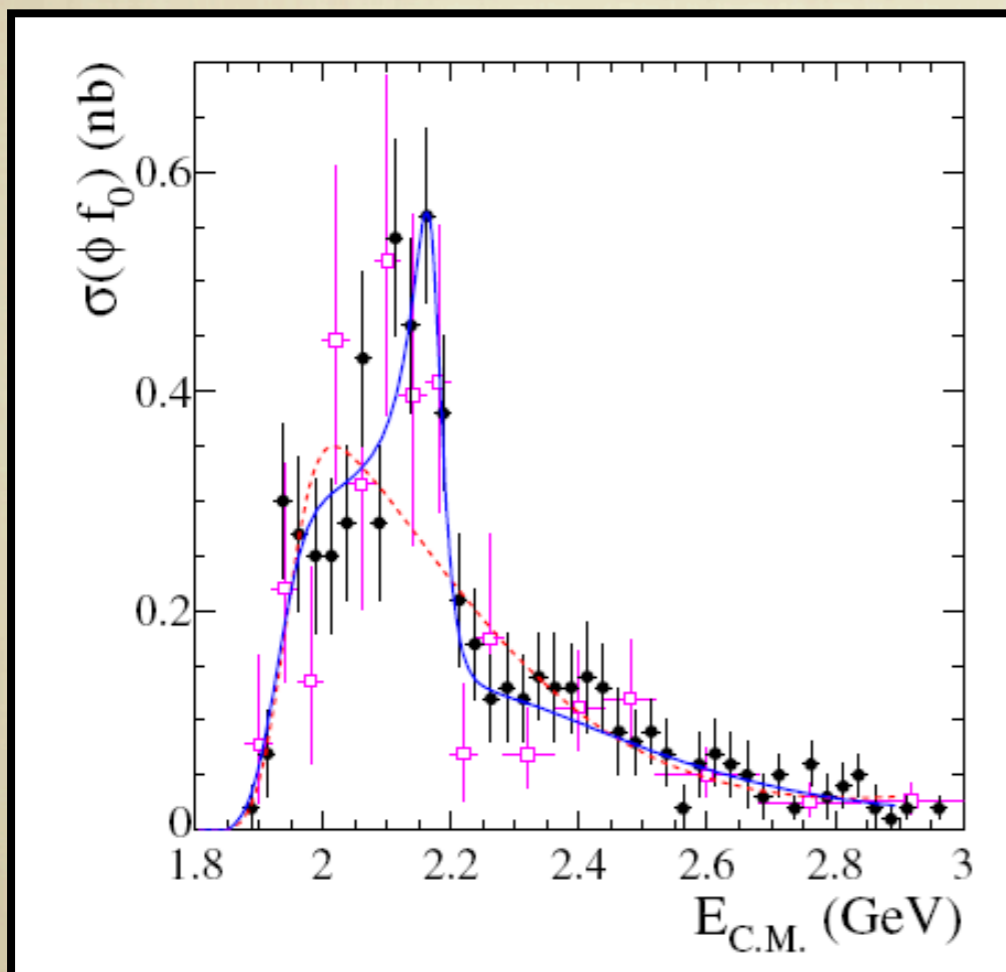
- $\phi K \bar{K}, \phi \pi \pi$

THREE-MESON SYSTEMS

- $\phi K \bar{K}, \phi \pi \pi$

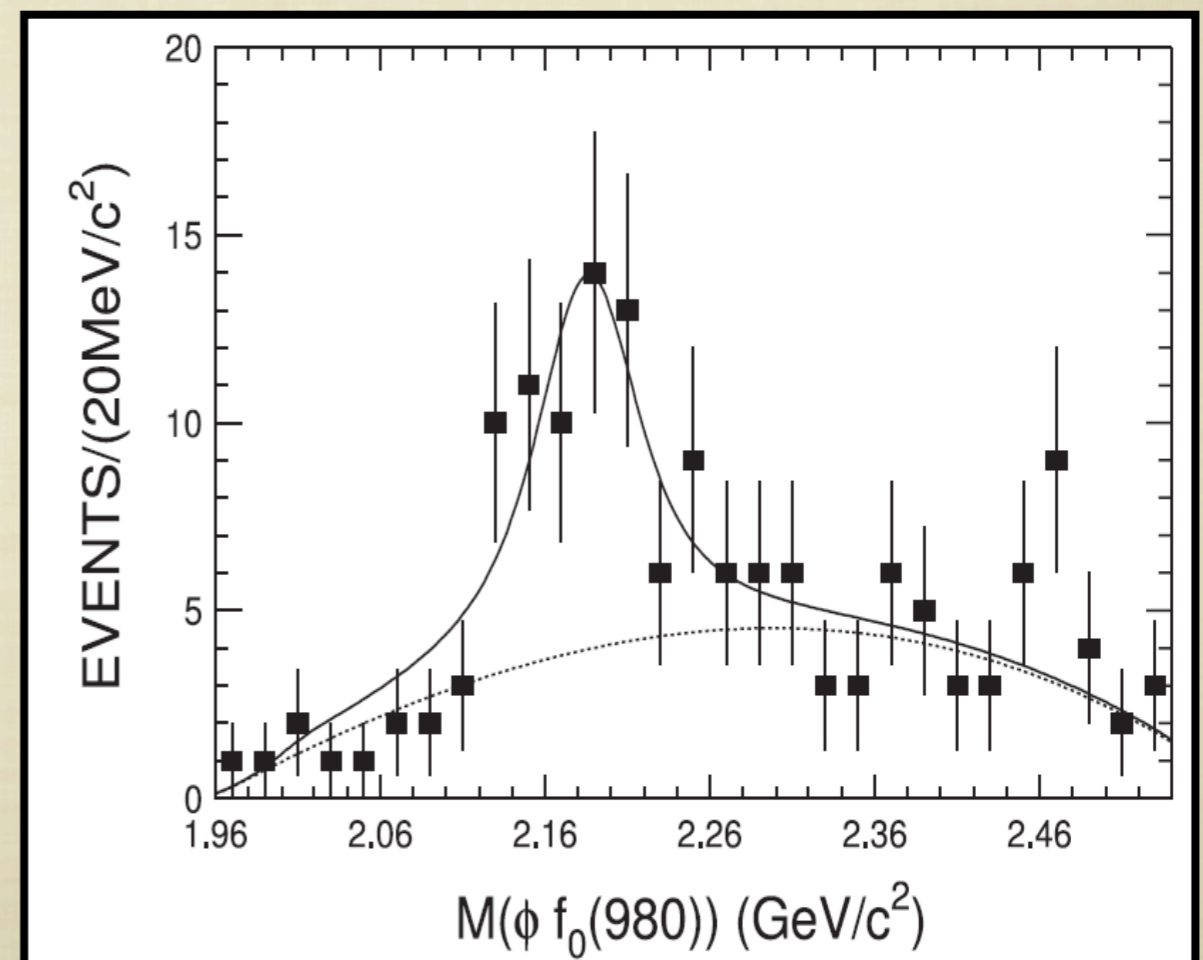
BABAR

$$e^+e^- \rightarrow \phi f_0(980)$$



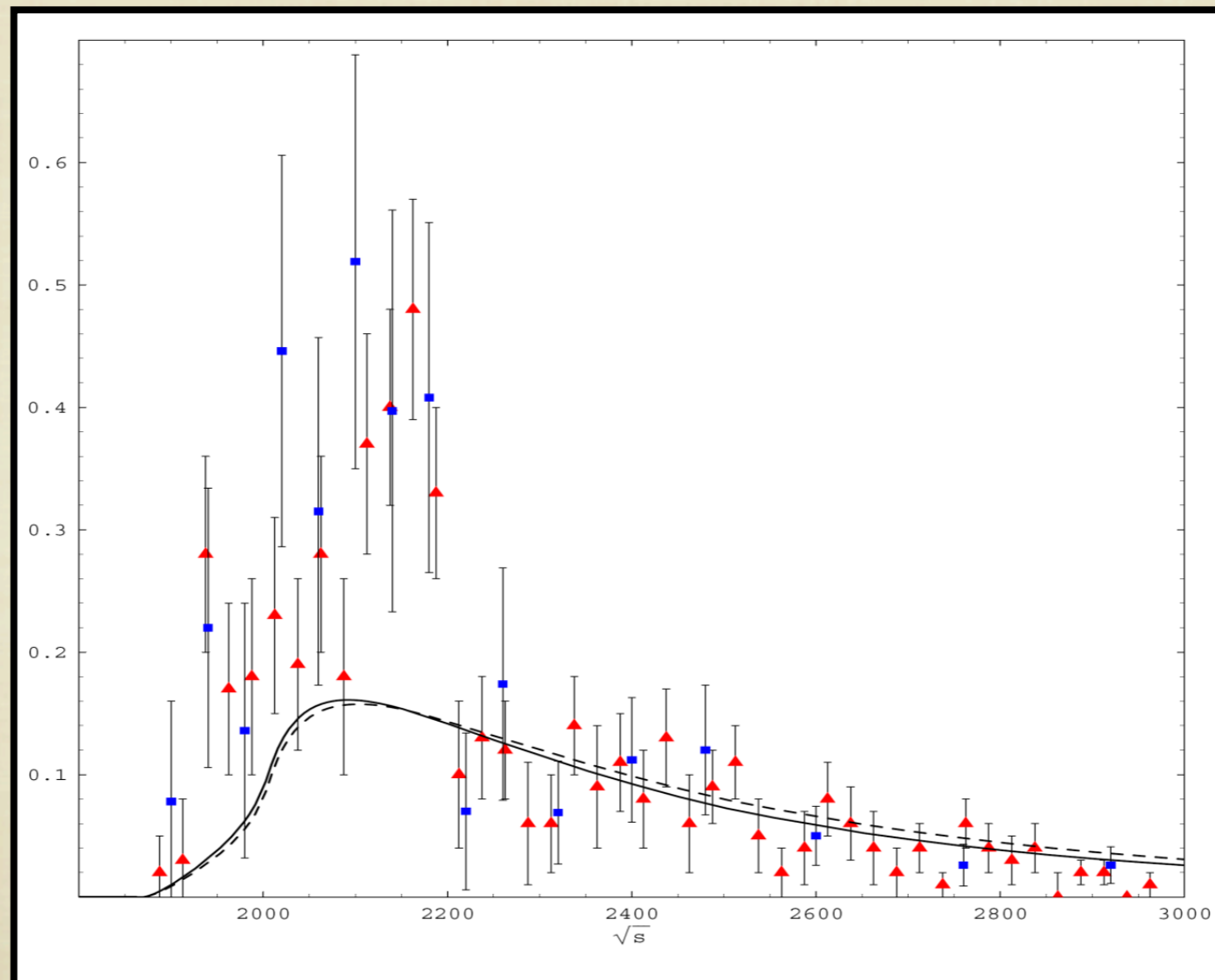
BES

$$J/\Psi \rightarrow \eta \phi f_0(980)$$



THREE-MESON SYSTEMS

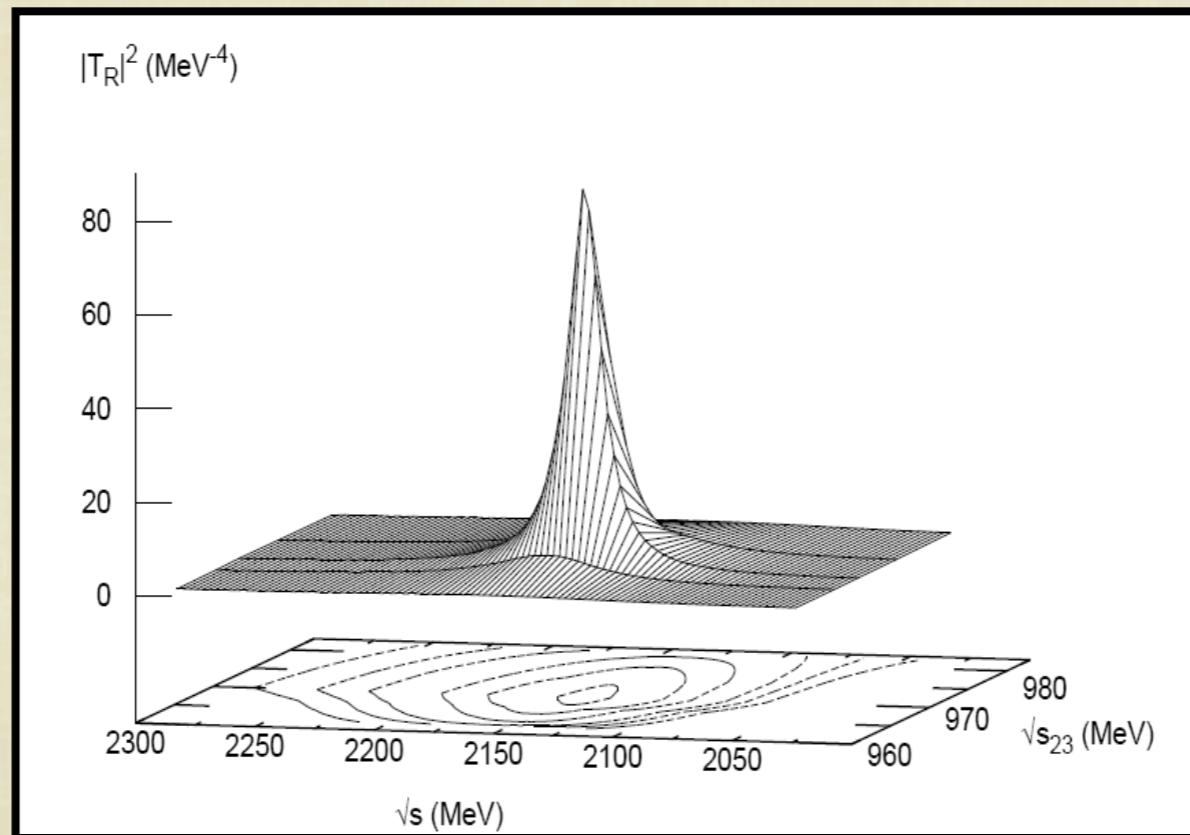
■ $\phi K \bar{K}, \phi \pi \pi$



*see a review on different studies (for example) : "New hadron states", Shi-Lin Zhu , *Int.J.Mod.Phys.E17:283-322,2008*; e-Print: [hep-ph/0703225](https://arxiv.org/abs/hep-ph/0703225)

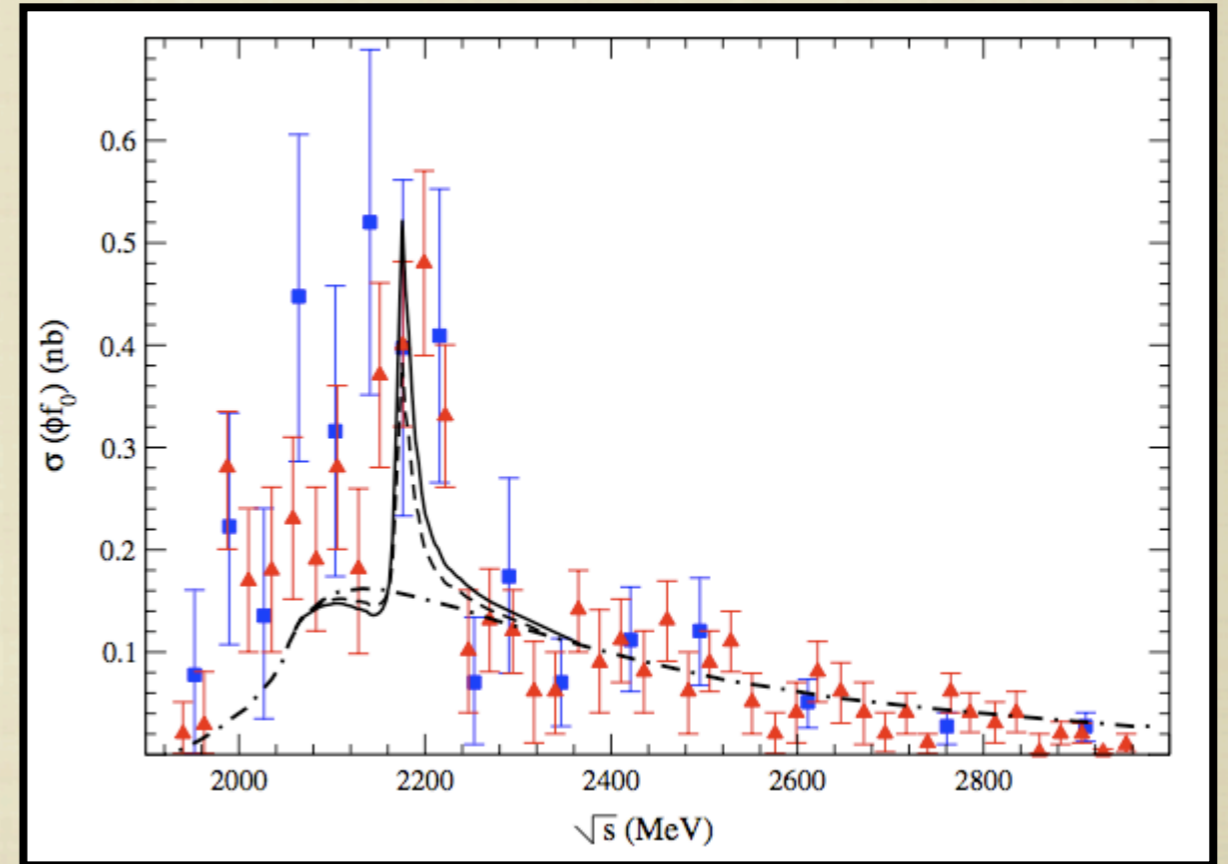
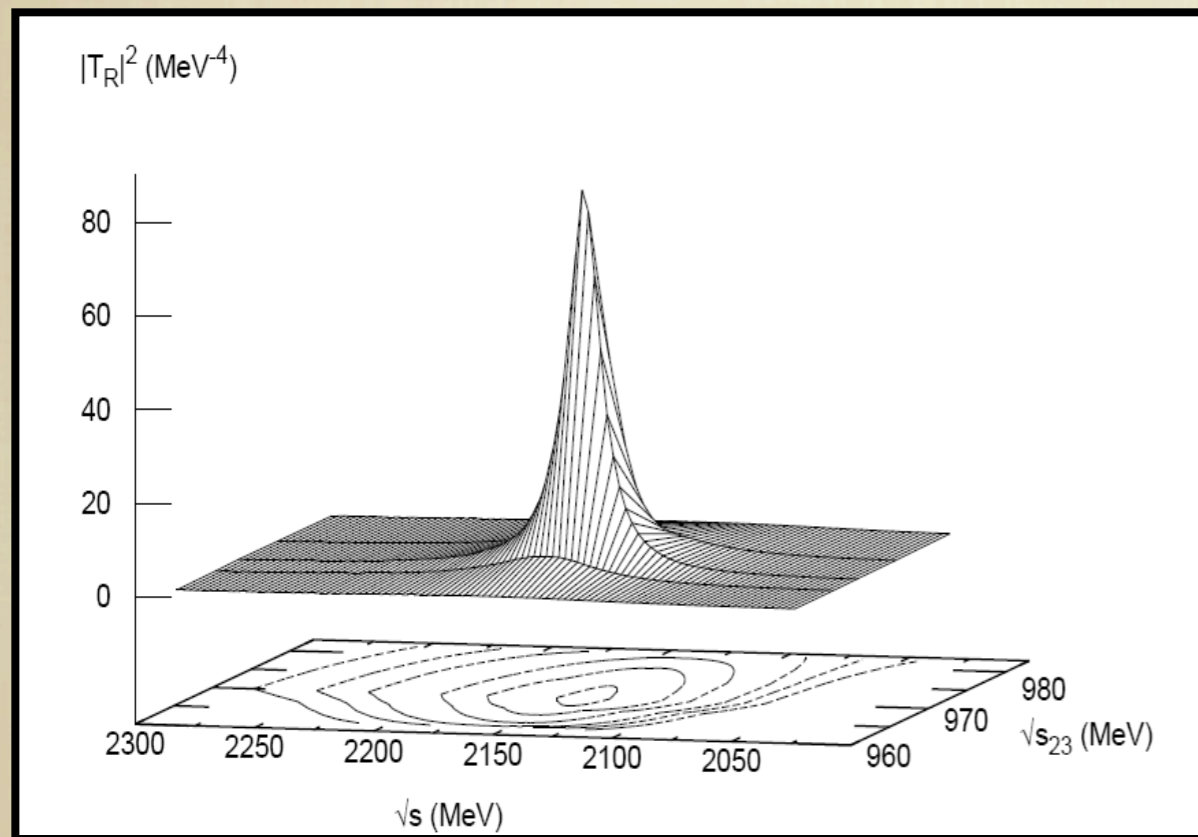
THREE-MESON SYSTEMS

- $\phi K \bar{K}, \phi \pi \pi$



THREE-MESON SYSTEMS

■ $\phi K \bar{K}, \phi \pi \pi$



$$T_{pw}^{\phi f_0} [1 + G_{\phi f_0} T_{\phi f_0}]$$

SUMMARY AND FUTURE PLANS

- WE HAVE OBTAINED FOUR Σ 'S AND TWO Λ 'S RESONANCES IN THE $\pi\bar{K}N$, WHICH CORRESPOND TO ALL THE $1/2^+$ Σ AND Λ STATES IN THE ENERGY REGION 1500-1870.
- WE OBSERVED THE $N^*(1710)$, $N^*(2100)$, $\Delta(1910)$ IN THE $\pi\pi N$ SYSTEM AND COUPLED CHANNELS AND A POSSIBLE $N^*(1910)$ WITH $J^P=1/2^+$ IN THE $K\bar{K}N$ SYSTEM .
- WE HAVE STUDIED THE THREE-MESON SYSTEMS, $\phi K\bar{K}$, $\phi\pi\pi$, WHERE WE GOT THE RESONANCE X(2175) .
- A BROAD BUMP IS OBTAINED IN THE STUDY OF THE πKN SYSTEM AROUND 1700 MEV.
- IN THE $J/\psi K\bar{K}$, $J/\psi\pi\pi$ SYSTEMS WE OBTAIN THE Y(4260).
- STUDY OF THE SYSTEMS $\omega\pi\pi$, $\rho\pi\pi$, $K^*\pi K$, *etc.* , TO GET THE LOW-LYING VECTOR RESONANCES AS W(1420), W(1650), ETC.

AND MANY MORE!!