CHIRAL PERTURBATION THEORY IN THE MESON SECTOR

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Various ChPT: http://www.thep.lu.se/∼bijnens/chpt.html

Overview

- 50, 40, 35, 30, 25, 20 and 15 years ago
- Chiral Perturbation Theory (ChPT, CHPT, χPT)
- Expand in which quantities
- Two-flavour ChPT at NNLO: one mass
  - Calculations
  - LECs and Quark-mass dependence of m_π^2, F_π
- Three-flavour ChPT at NNLO: 3-5 masses
  - Calculations
  - What about p^6 LECs and can we test ChPT at NNLO
  - Fits to data (some preliminary new ones); some quark mass dependences
  - $\eta \to 3\pi$

Overview

- Even more flavours at NNLO (Partially Quenched)
- Renormalization group
- Hard pion ChPT: some indications it might exist
- A few words about ChPT and the weak interaction

Jubileum Papers: 50 years

The start:

- M. Goldberger and S. Treiman, *Decay of the pi meson*. Phys. Rev. 110:1178-1184,1958. (330 citations)
- Y. Nambu, *Axial Vector Current Conservation in Weak Interactions*, Phys. Rev. Lett. 4 (1960) 380 (530 citations)
- M. Gell-Mann and M. Lévy, *The axial vector current in beta decay*. Nuovo Cim. 16 (1960) 705 (1229 citations)

Jubileum Papers: 40

Tree level:

- S. Weinberg, *Nonlinear realizations of chiral symmetry*, Phys. Rev. 166 (1968) 1568 (736 citations)
- M. Gell-Mann, R.J. Oakes and B. Renner, *Behavior of current divergences under $SU(3) \times SU(3)$*, Phys. Rev. 175 (1968) 2195 (1264 citations)
- S. Coleman, J. Wess and B. Zumino, *Structure of phenomenological Lagrangians. 1.*, Phys. Rev. 177 (1969) 2239 (1091 citations)
- C. Callan, S. Coleman, J. Wess and B. Zumino, *Structure of phenomenological Lagrangians. 2.*, Phys. Rev. 177 (1969) 2247 (932 citations)

Jubileum Papers: 35 years

Tree level:

- CCWZ
- G. Ecker and J. Honerkamp, *Pion Pion Phase Shifts From Covariant Perturbation Theory For A Chiral Invariant Field Theoretic Model*, Nucl. Phys. B 52 (1973) 211
- P. Langacker and H. Pagels, *Applications of Chiral Perturbation Theory: Mass Formulas and the Decay* $\eta \to 3\pi$ Phys.Rev.D10:2904,1974
- Review early work: H. Pagels, *Departures From Chiral Symmetry: A Review*, Phys. Rept. 16 (1975) 219

Jubileum Papers: 30 and 25 years

The restart:

- Steven Weinberg, *Phenomenological Lagrangians*, Physica A96 (1979) 327 (1884 citations)
- J. Gasser and A. Zepeda, *Approaching The Chiral Limit In QCD*, Nucl. Phys. B174 (1980) 445 (preprint in 1979)
- Juerg Gasser and Heiri Leutwyler, *Chiral Perturbation Theory to One Loop*, Annals Phys. 158 (1984) 142 (2407 citations)
- Juerg Gasser and Heiri Leutwyler, *Chiral Perturbation Theory: Expansions in the Mass of the Strange Quark* Nucl. Phys. B250 (1985) 465 (2431 citations)
- J. Bijnens, H. Sonoda and M. Wise, *On the Validity of Chiral Perturbation Theory for K^0-$\overline{K^0}$ Mixing* , Phys. Rev. Lett. 53 (1984) 2367 Here is where I started

Jubileum Papers: 20 years

LECs from elsewhere:

- G. Ecker, J. Gasser, A. Pich and E. de Rafael, *The Role of Resonances in Chiral Perturbation Theory*, Nucl. Phys. B321 (1989) 311 (826 citations)
- G. Ecker,J. Gasser, H. Leutwyler, A. Pich and E. de Rafael, *Chiral Lagrangians for Massive Spin 1 Fields*, Phys. Lett. B223 (1989) 425 (462 citations)
- J. F. Donoghue, C. Ramirez and G. Valencia, *The Spectrum of QCD and Chiral Lagrangians of the Strong and Weak Interactions*, Phys. Rev. D 39 (1989) 1947 (258 citations)

Jubileum Papers: 15 years

First full two-loop:

- S. Bellucci, J. Gasser and M.E. Sainio, *Low-energy photon-photon collisions to two loop order*, Nucl. Phys. B423 (1994) 80
- H. Leutwyler, *On The Foundations Of Chiral Perturbation Theory*, Ann. Phys. 235 (1994) 165

Chiral Perturbation Theory

Exploring the consequences of the chiral symmetry of QCD and its spontaneous breaking using effective field theory techniques

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Derivation from QCD:

H. Leutwyler, *On The Foundations Of Chiral Perturbation Theory*, Ann. Phys. 235 (1994) 165 [hep-ph/9311274]

For lectures, review articles: see
http://www.thep.lu.se/∼bijnens/chpt.html

Chiral Perturbation Theory

Degrees of freedom: Goldstone Bosons from Chiral Symmetry Spontaneous Breakdown

Power counting: Dimensional counting in momenta/masses

Expected breakdown scale: Resonances, so M_ρ or higher depending on the channel

Chiral Symmetry

QCD: 3 light quarks: equal mass: interchange: $SU(3)_V$

But $\mathcal{L}_{QCD} = \sum_{q=u,d,s} \left[i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q \left(\bar{q}_R q_L + \bar{q}_L q_R \right) \right]$

So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$.

Chiral Perturbation Theory

$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$
$SU(3)_L \times SU(3)_R$ broken spontaneously to $SU(3)_V$

8 generators broken $\Longrightarrow$ 8 massless degrees of freedom
and interaction vanishes at zero momentum

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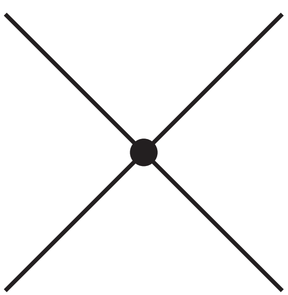
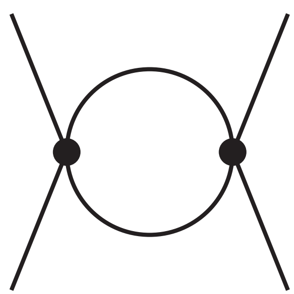
Power counting in momenta: Meson loops



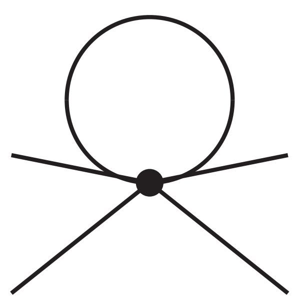
p^2



$(p^2)^2\,(1/p^2)^2\,p^4 = p^4$

$1/p^2$

$\int d^4p$ $\quad p^4$



$(p^2)\,(1/p^2)\,p^4 = p^4$

Chiral Perturbation Theories

- Which chiral symmetry: $SU(N_f)_L \times SU(N_f)_R$, for $N_f = 2, 3, \ldots$ and extensions to (partially) quenched
- Or beyond QCD talk by Neil
- Space-time symmetry: Continuum or broken on the lattice: Wilson, staggered, mixed action
- Volume: Infinite, finite in space, finite T
- Which interactions to include beyond the strong one
- Which particles included as non Goldstone Bosons
- To which order
- What assumptions have been made on the LECs
- Lattice: talks by Hashimoto, Sachrajda, Aoki, Herdoiza, Heller, Juettner, Kaneko, Laiho, Necco

Chiral Perturbation Theories

- Baryons
- Heavy Quarks
- Vector Mesons (and other resonances)
- Structure Functions and Related Quantities
- Light Pseudoscalar Mesons
- . . .
- $\Longrightarrow$ shortage of letters for the constants in the Lagrangians (LECs)

Chiral Perturbation Theories

- Baryons
- Heavy Quarks
- Vector Mesons (and other resonances)
- Structure Functions and Related Quantities
- Light Pseudoscalar Mesons
  - Two or Three (or even more) Flavours
  - Strong interaction and couplings to external currents/densities
  - Including (internal) electromagnetism
  - Including weak nonleptonic interactions
  - Treating kaon as heavy

Lagrangians

$U(\phi) = \exp(i\sqrt{2}\Phi/F_0)$ parametrizes Goldstone Bosons

$$\Phi(x) = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\,\eta_8}{\sqrt{6}} \end{pmatrix} .$$

LO Lagrangian: $\mathcal{L}_2 = \frac{F_0^2}{4}\{\langle D_\mu U^\dagger D^\mu U\rangle + \langle \chi^\dagger U + \chi U^\dagger\rangle\}\,,$

$D_\mu U = \partial_\mu U - i r_\mu U + i U l_\mu\,,$
left and right external currents: $r(l)_\mu = v_\mu + (-)a_\mu$

Scalar and pseudoscalar external densities: $\chi = 2B_0(s + ip)$
quark masses via scalar density: $s = \mathcal{M} + \cdots$

$\langle A\rangle = Tr_F\,(A)$

Lagrangians

$$\mathcal{L}_4 = L_1\langle D_\mu U^\dagger D^\mu U\rangle^2 + L_2\langle D_\mu U^\dagger D_\nu U\rangle\langle D^\mu U^\dagger D^\nu U\rangle$$
$$+L_3\langle D^\mu U^\dagger D_\mu U D^\nu U^\dagger D_\nu U\rangle + L_4\langle D^\mu U^\dagger D_\mu U\rangle\langle \chi^\dagger U + \chi U^\dagger\rangle$$
$$+L_5\langle D^\mu U^\dagger D_\mu U(\chi^\dagger U + U^\dagger\chi)\rangle + L_6\langle \chi^\dagger U + \chi U^\dagger\rangle^2$$
$$+L_7\langle \chi^\dagger U - \chi U^\dagger\rangle^2 + L_8\langle \chi^\dagger U\chi^\dagger U + \chi U^\dagger\chi U^\dagger\rangle$$
$$-iL_9\langle F^R_{\mu\nu} D^\mu U D^\nu U^\dagger + F^L_{\mu\nu} D^\mu U^\dagger D^\nu U\rangle$$
$$+L_{10}\langle U^\dagger F^R_{\mu\nu} U F^{L\mu\nu}\rangle + H_1\langle F^R_{\mu\nu}F^{R\mu\nu} + F^L_{\mu\nu}F^{L\mu\nu}\rangle + H_2\langle\chi^\dagger\chi\rangle$$

L_i: Low-energy-constants (LECs)
H_i: Values depend on definition of currents/densities

These absorb the divergences of loop diagrams: $L_i \to L_i^r$
Renormalization: order by order in the powercounting

Lagrangians

Lagrangian Structure:

	2 flavour		3 flavour		3+3 PQChPT	
p^2	F, B	2	F_0, B_0	2	F_0, B_0	2
p^4	l_i^r, h_i^r	7+3	L_i^r, H_i^r	10+2	$\hat{L}_i^r, \hat{H}_i^r$	11+2
p^6	c_i^r	52+4	C_i^r	90+4	K_i^r	112+3

p^2: Weinberg 1966

p^4: Gasser, Leutwyler 84,85

p^6: JB, Colangelo, Ecker 99,00

- replica method $\Longrightarrow$ PQ obtained from N_F flavour
- All infinities known
- 3 flavour special case of 3+3 PQ: $\hat{L}_i^r, K_i^r \to L_i^r, C_i^r$
- 53 $\to$ 52 arXiv:0705.0576 [hep-ph]

Chiral Logarithms

The main predictions of ChPT:

- Relates processes with different numbers of pseudoscalars
- Chiral logarithms
- includes Isospin and the eightfold way ($SU(3)_V$)

$$m_\pi^2 = 2B\hat{m} + \left(\frac{2B\hat{m}}{F}\right)^2 \left[\frac{1}{32\pi^2}\log\frac{(2B\hat{m})}{\mu^2} + 2l_3^r(\mu)\right] + \cdots$$

$M^2 = 2B\hat{m}$
$B \neq B_0$, $F \neq F_0$ (two versus three-flavour)

LECs and μ

$l_3^r(\mu)$

$$\bar{l}_i = \frac{32\pi^2}{\gamma_i}\, l_i^r(\mu) - \log\frac{M_\pi^2}{\mu^2}\,.$$

Independent of the scale μ.

For 3 and more flavours, some of the $\gamma_i = 0$: $L_i^r(\mu)$

μ :

- m_π, m_K: chiral logs vanish
- pick larger scale
- 1 GeV then $L_5^r(\mu) \approx 0$ large N_c arguments????
- compromise: $\mu = m_\rho = 0.77$ GeV

Expand in what quantities?

- Expansion is in momenta and masses
- But is not unique: relations between masses (Gell-Mann–Okubo) exists
- Express orders in terms of physical masses and quantities (F_π, F_K)?
- Express orders in terms of lowest order masses?
- E.g. $s + t + u = 2m_\pi^2 + 2m_K^2$ in πK scattering

I prefer physical masses

- Thresholds correct
- Chiral logs are from physical particles propagating

An example

$$m_\pi = \frac{m_0}{1 + a\frac{m0}{f0}} \qquad f_\pi = \frac{f_0}{1 + b\frac{m0}{f0}}$$

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$$m_\pi = m_0 - a\frac{m_0^2}{f_0} + a^2\frac{m_0^3}{f_0^2} + \cdots$$

$$f_\pi = f_0\left(1 - b\frac{m_0}{f_0} + b^2\frac{m_0^2}{f_0^2} + \cdots\right)$$

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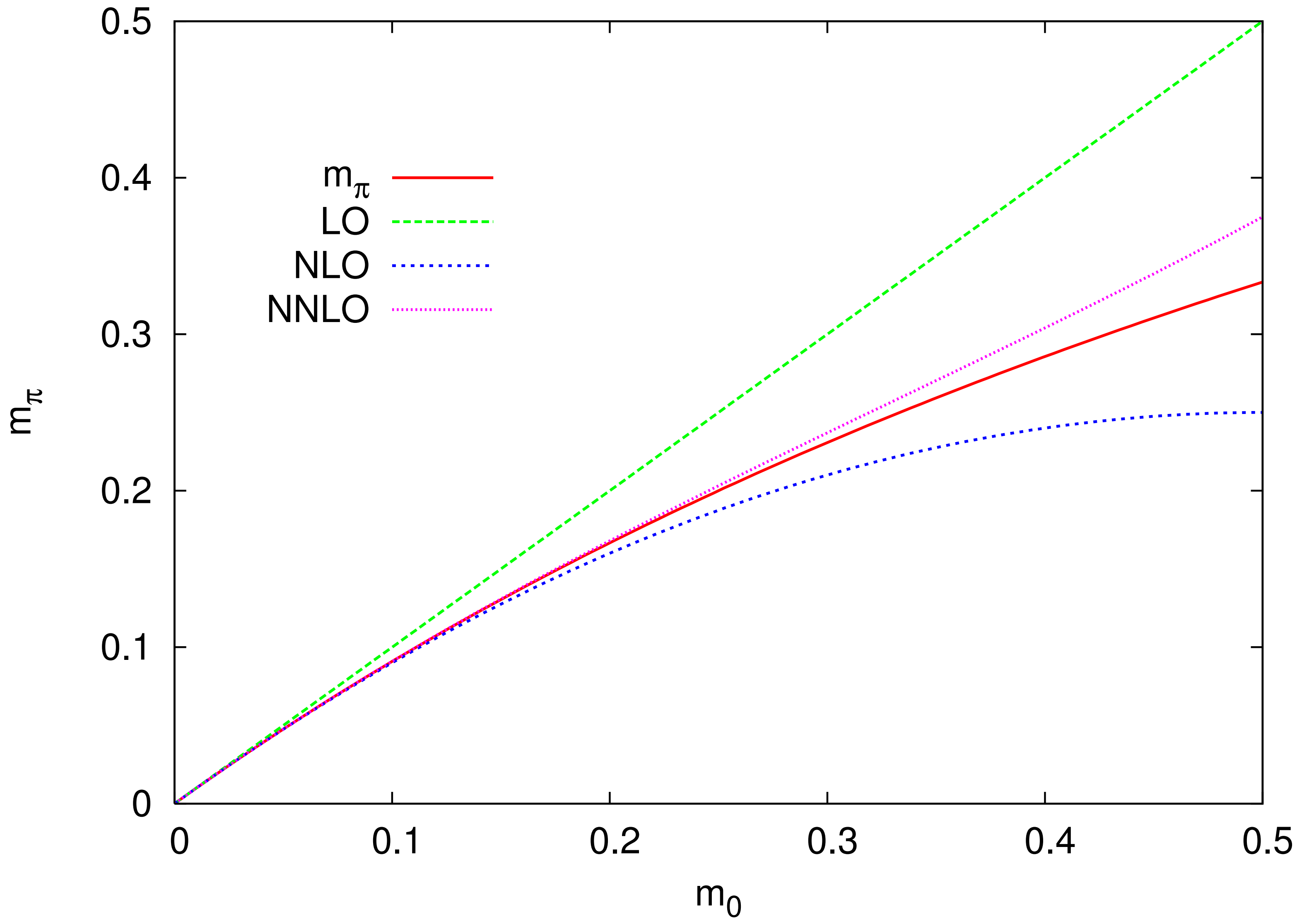
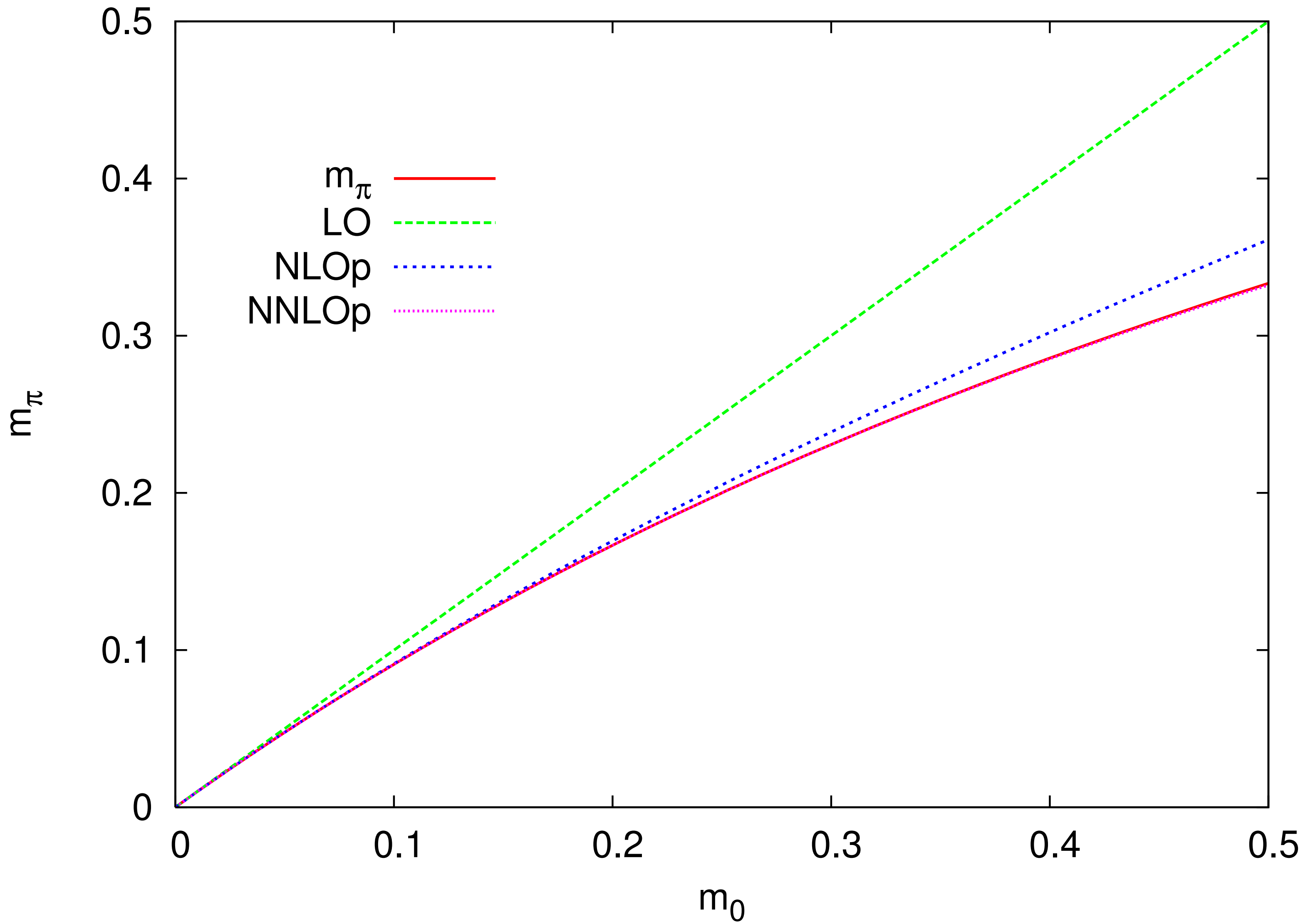
$$f_\pi = f_0\left(1 - b\frac{m_\pi}{f_\pi} + b(2b-a)\frac{m_\pi^2}{f_\pi^2} + \cdots\right)$$

$$a = 1 \quad b = 0.5 \quad f_0 = 1$$

An example: m_0/f_0



An example: m_π / f_π



Two-loop Two-flavour

Review paper on Two-Loops: JB, hep-ph/0604043 Prog. Part. Nucl. Phys. 58 (2007) 521

Dispersive Calculation of the nonpolynomial part in q^2, s, t, u

- Gasser-Meißner: F_V, F_S: 1991 numerical
- Knecht-Moussallam-Stern-Fuchs: $\pi\pi$: 1995 analytical
- Colangelo-Finkemeier-Urech: F_V, F_S: 1996 analytical

Two-Loop Two-flavour

- Bellucci-Gasser-Sainio: $\gamma\gamma \to \pi^0\pi^0$: 1994
- Bürgi: $\gamma\gamma \to \pi^+\pi^-$, F_π, m_π: 1996
- JB-Colangelo-Ecker-Gasser-Sainio: $\pi\pi$, F_π, m_π: 1996-97
- JB-Colangelo-Talavera: $F_{V\pi}(t)$, $F_{S\pi}$: 1998
- JB-Talavera: $\pi \to \ell\nu\gamma$: 1997
- Gasser-Ivanov-Sainio: $\gamma\gamma \to \pi^0\pi^0$, $\gamma\gamma \to \pi^+\pi^-$: 2005-2006
- m_π, F_π, F_V, F_S, $\pi\pi$: simple analytical forms
- Colangelo-(Dürr-)Haefeli: Finite volume F_π, m_π 2005-2006
- Kampf-Moussallam: $\pi^0 \to \gamma\gamma$ 2009 talk by Moussallam

LECs

$\bar{l}_1$ to $\bar{l}_4$: ChPT at order p^6 and the Roy equation analysis in $\pi\pi$ and F_S Colangelo, Gasser and Leutwyler, *Nucl. Phys.* B 603 (2001) 125 [hep-ph/0103088]

$\bar{l}_5$ and $\bar{l}_6$: from F_V and $\pi \to \ell\nu\gamma$ JB,(Colangelo,)Talavera and from $\Pi_V - \Pi_A$ González-Alonso, Pich, Prades, talk by González-Alonso

$$\begin{aligned} \bar{l}_1 &= -0.4 \pm 0.6\,, & \bar{l}_2 = 4.3 \pm 0.1\,,\\ \bar{l}_3 &= 2.9 \pm 2.4\,, & \bar{l}_4 = 4.4 \pm 0.2\,,\\ \bar{l}_5 &= 12.24 \pm 0.21\,, & \bar{l}_6 - \bar{l}_5 = 3.0 \pm 0.3\,,\\ \bar{l}_6 &= 16.0 \pm 0.5 \pm 0.7\,. \end{aligned}$$

$l_7 \sim 5 \cdot 10^{-3}$ from π^0-η mixing Gasser, Leutwyler 1984

LECs

Some combinations of order p^6 LECs are known as well: curvature of the scalar and vector formfactor, two more combinations from $\pi\pi$ scattering (implicit in b_5 and b_6),

Note: c_i^r for m_π, f_π, $\pi\pi$: small effect

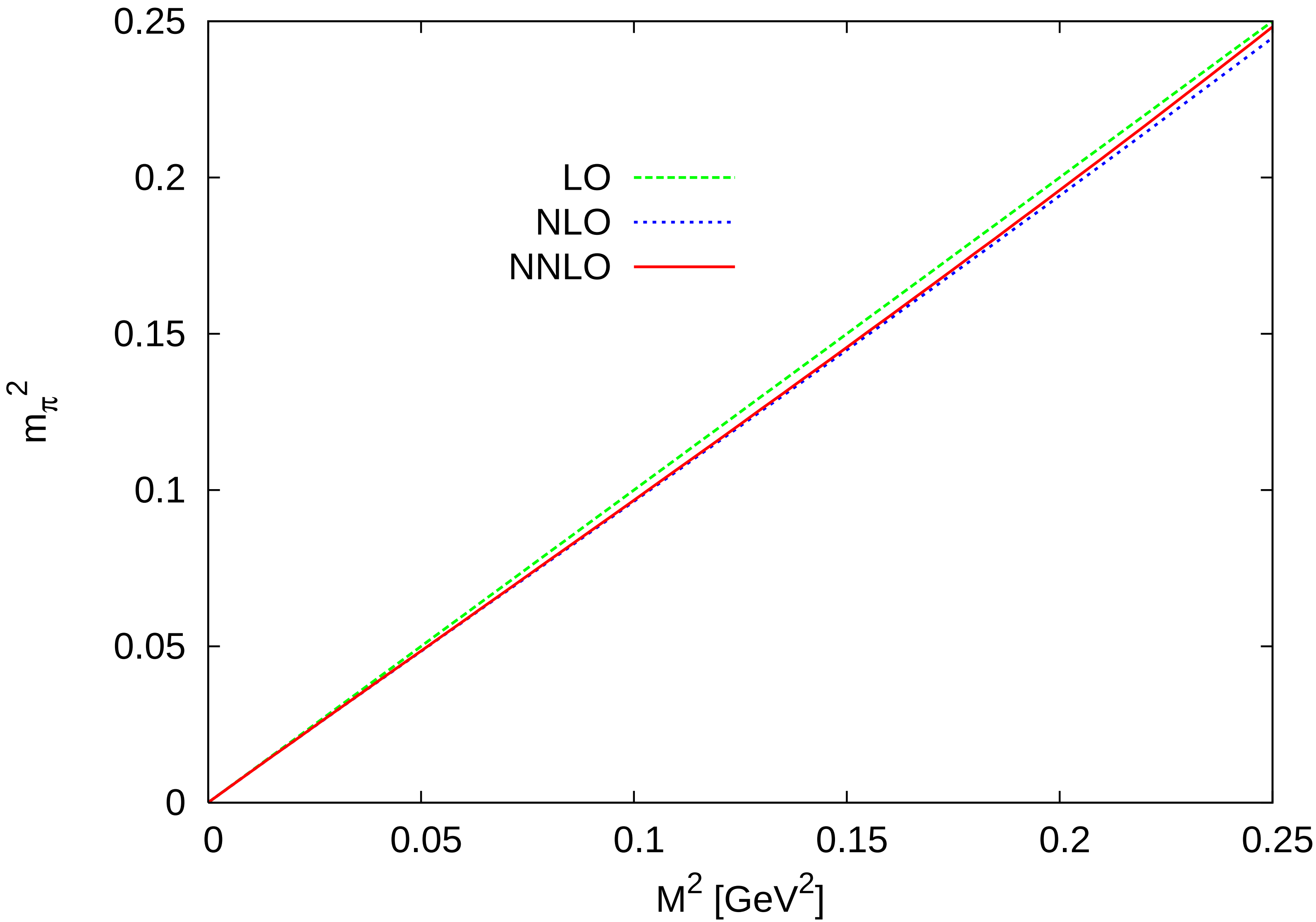
$c_i^r(770 MeV) = 0$ for plots shown

expansion in m_π^2/F_π^2 shown

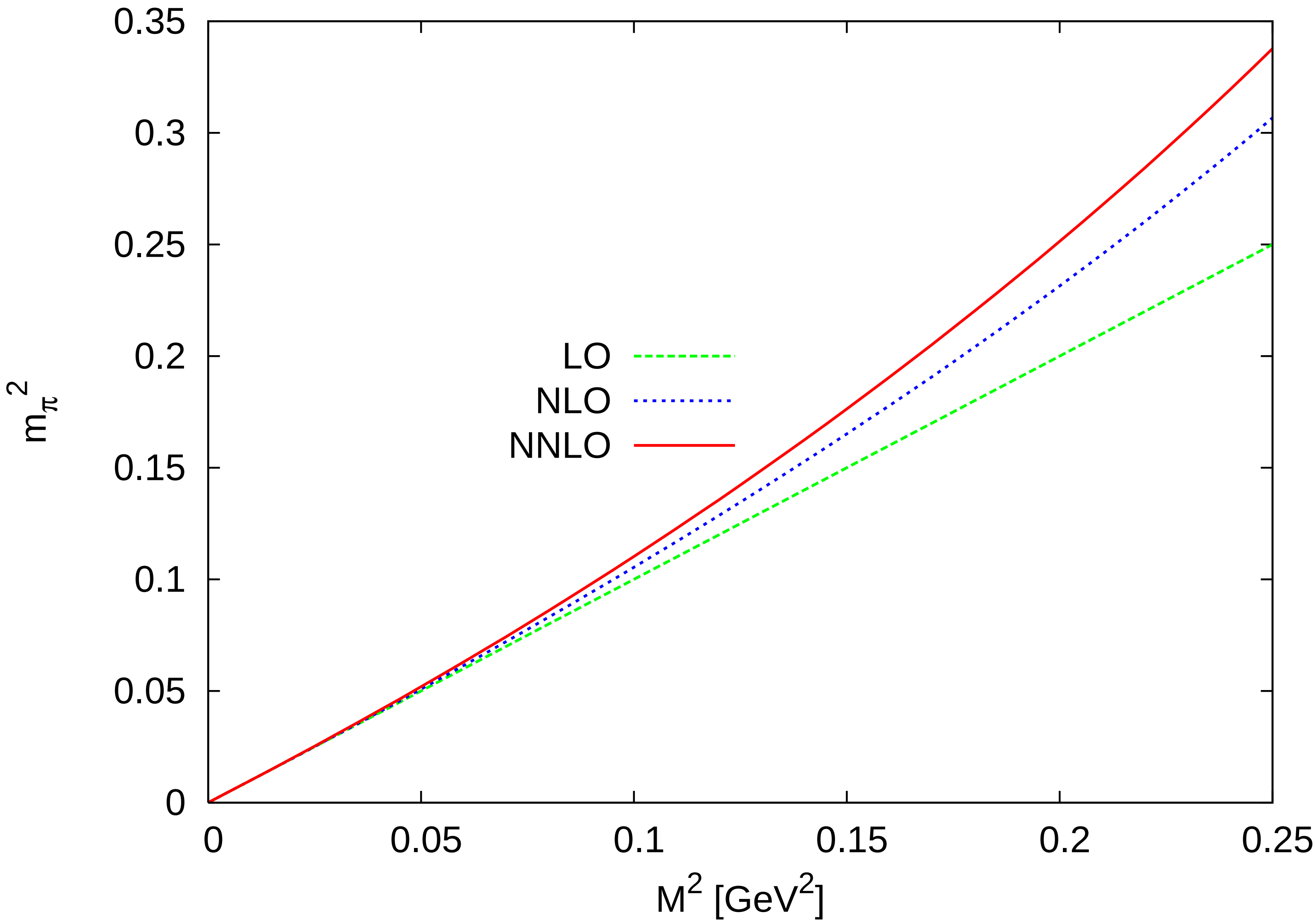
General observation:

- Obtainable from kinematical dependences: known
- Only via quark-mass dependence: poorely known

m_π^2

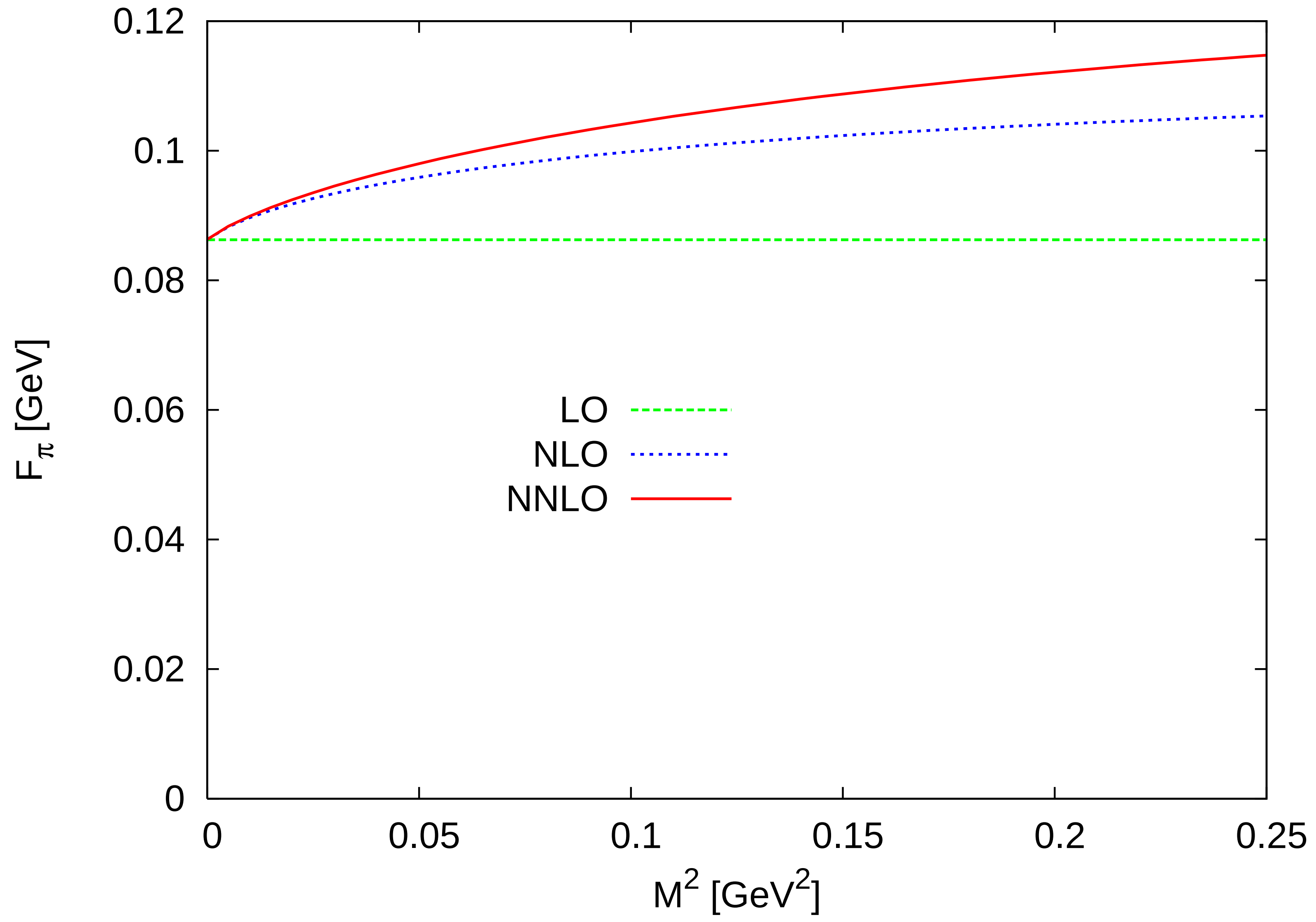


m_π^2 ($\bar{l}_3 = 0$)



0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
0.05
0.1
0.15
0.2
0.25
m_π^2
M^2 [GeV^2]
LO
NLO
NNLO

F_π


0.12
0.1
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0.04
0.02
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0
0.05
0.1
0.15
0.2
0.25
F_π [GeV]
M^2 [GeV^2]
LO
NLO
NNLO

Pion polarizabilities

Pion polarizabilities as calculated/measured/derived:

- ChPT:
  $(\alpha_1 - \beta_1)_{\pi^\pm} = (5.7 \pm 0.1) \cdot 10^{-4}\ \mathrm{fm}^3$ Ivanov-Gasser-Sainio
- Latest experiment Mainz 2005
  $(\alpha_1 - \beta_1)_{\pi^\pm} = (11.6 \pm 1.5_{stat} \pm 3.0_{syst} \pm 0.5_{mod}) \cdot 10^{-4}\ \mathrm{fm}^3$
  Possible problem background direct $\gamma N \to \gamma N \pi$
- $(\alpha_1 - \beta_1)_{\pi^\pm} = (13.6 \pm 2.8_{stat} \pm 2.4_{syst}) \cdot 10^{-4}\ \mathrm{fm}^3$ Serpukhov 1983
- Dispersive analysis from $\gamma\gamma \to \pi\pi$:
  - $(\alpha_1 - \beta_1) = (13.0 + 3.6 - 1.9) \cdot 10^{-4}\mathrm{fm}^3$ Fil'kov-Kashevarov
  - Large model dependence in their extraction, "Our calculations. . . are in reasonable agreement with ChPT for charged pions" Pasquini-Drechsel-Scherer
- Talks by: Fil'kov, Drechsel and Friedrich (Compass)

Two-loop Three-flavour, $\leq$2001

- $\Pi_{VV(\pi,\eta,K)}$ Kambor, Golowich; Kambor, Dürr; Amorós, JB, Talavera $\boxed{L_{10}^r}$
- $\Pi_{VV\rho\omega}$ Maltman
- $\Pi_{AA\pi}$, $\Pi_{AA\eta}$, F_π, F_η, m_π, m_η Kambor, Golowich; Amorós, JB, Talavera
- Π_{SS} Moussallam $\boxed{L_4^r, L_6^r}$
- Π_{VVK}, Π_{AAK}, F_K, m_K Amorós, JB, Talavera
- $K_{\ell 4}$, $\langle \overline{q}q \rangle$ Amorós, JB, Talavera $\boxed{L_1^r, L_2^r, L_3^r}$
- F_M, m_M, $\langle \overline{q}q \rangle$ $(m_u \neq m_d)$ Amorós, JB, Talavera $\boxed{L_{5,7,8}^r, m_u/m_d}$

Two-loop Three-flavour, $\geq$2001

- $F_{V\pi}$, F_{VK^+}, F_{VK^0} — Post, Schilcher; JB, Talavera — L_9^r
- $K_{\ell 3}$ — Post, Schilcher; JB, Talavera — V_{us}
- $F_{S\pi}$, F_{SK} (includes σ-terms) — JB, Dhonte — L_4^r, L_6^r
- $K, \pi \to \ell\nu\gamma$ — Geng, Ho, Wu — L_{10}^r
- $\pi\pi$ — JB,Dhonte,Talavera
- πK — JB,Dhonte,Talavera
- relation l_i^r, c_r^i and L_i^r, C_i^r — Gasser,Haefeli,Ivanov,Schmid talk by Ivanov
- Finite volume $\langle \overline{q}q \rangle$ — JB,Ghorbani
- $\eta \to 3\pi$: — JB,Ghorbani
- $K_{\ell 3}$ isospin breaking — JB,Ghorbani

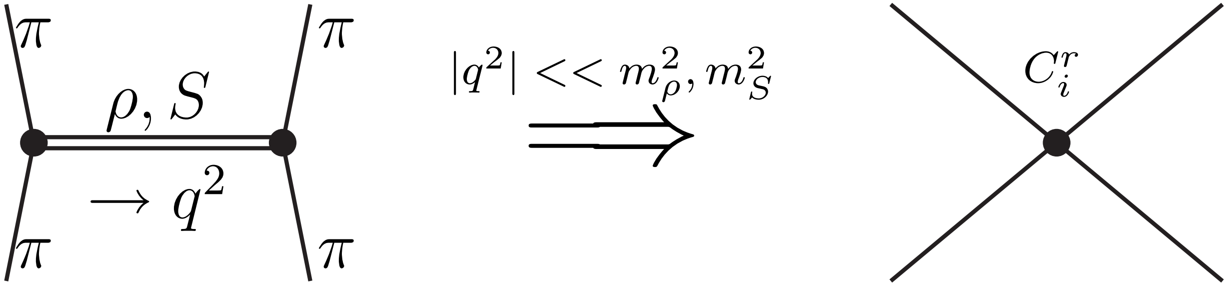
Two-loop Three-flavour

Known to be in progress

- Finite Volume: sunsetintegrals JB,Lähde
- More analytical work on $K_{\ell 3}$ Greynat et al.

C_i^r

Most analysis use:
C_i^r from (single) resonance approximation



Motivated by large N_c: large effort goes in this

Ananthanarayan, JB, Cirigliano, Donoghue, Ecker, Gamiz, Golterman, Kaiser, Kampf, Knecht, Moussallam, Peris, Pich, Prades, Portoles, de Rafael,. . .

Beyond tree level: $R\chi T$ Cata, Peris, Pich, Portoles, Rosell, . . .

C_i^r

$$\mathcal{L}_V = -\frac{1}{4}\langle V_{\mu\nu}V^{\mu\nu}\rangle + \frac{1}{2}m_V^2\langle V_\mu V^\mu\rangle - \frac{f_V}{2\sqrt{2}}\langle V_{\mu\nu}f_+^{\mu\nu}\rangle - \frac{ig_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu,u^\nu]\rangle + f_\chi\langle V_\mu[u^\mu,\chi_-]\rangle$$

$$\mathcal{L}_A = -\frac{1}{4}\langle A_{\mu\nu}A^{\mu\nu}\rangle + \frac{1}{2}m_A^2\langle A_\mu A^\mu\rangle - \frac{f_A}{2\sqrt{2}}\langle A_{\mu\nu}f_-^{\mu\nu}\rangle$$

$$\mathcal{L}_S = \frac{1}{2}\langle\nabla^\mu S\nabla_\mu S - M_S^2S^2\rangle + c_d\langle Su^\mu u_\mu\rangle + c_m\langle S\chi_+\rangle$$

$$\mathcal{L}_{\eta'} = \frac{1}{2}\partial_\mu P_1\partial^\mu P_1 - \frac{1}{2}M_{\eta'}^2P_1^2 + i\tilde{d}_m P_1\langle\chi_-\rangle\,.$$

$f_V = 0.20$, $f_\chi = -0.025$, $g_V = 0.09$, $c_m = 42$ MeV, $c_d = 32$ MeV, $\tilde{d}_m = 20$ MeV,

$m_V = m_\rho = 0.77$ GeV, $m_A = m_{a_1} = 1.23$ GeV, $m_S = 0.98$ GeV, $m_{P_1} = 0.958$ GeV

f_V, g_V, f_χ, f_A: experiment

c_m and c_d from resonance saturation at $\mathcal{O}(p^4)$

C_i^r

Problems:

- Weakest point in the numerics
- However not all results presented depend on this
- Unknown so far: C_i^r in the masses/decay constants and how these effects correlate into the rest
- No μ dependence: obviously only estimate

C_i^r

Problems:

- Weakest point in the numerics
- However not all results presented depend on this
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- No μ dependence: obviously only estimate

What we did about it:

- Vary resonance estimate by factor of two
- Vary the scale μ at which it applies: 600-900 MeV
- Check the estimates for the measured ones
- Again: kinematic can be had, quark-mass dependence difficult

Comparisons of C_i^r

Kampf-Moussallam 2006 using $\pi\pi$ and πK results of

JB,Dhonte,Talavera

input	$C_1^r + 4C_3^r$	C_2^r	$C_4^r + 3C_3^r$	$C_1^r + 4C_3^r + 2C_2^r$
$\pi K : C_{30}^+, C_{11}^+, C_{20}^-$	20.7 ± 4.9	-9.2 ± 4.9	9.9 ± 2.5	2.3 ± 10.8
$\pi K : C_{30}^+, C_{11}^+, C_{01}^-$	28.1 ± 4.9	-7.4 ± 4.9	21.0 ± 2.5	13.4 ± 10.8
$\pi\pi$			23.5 ± 2.3	18.8 ± 7.2
Resonance model	7.2	-0.5	10.0	6.2

Can this be generalized to test ChPT at NNLO without assumptions on the C_i^r?

Relations at NNLO

Yes: JB, Jemos, talk by Jemos Systematic search for relations between observables that do not depend on the C_i^r.
Included:

- m_M^2 and F_M for π, K, η.
- 11 $\pi\pi$ threshold parameters
- 14 πK threshold parameters
- 6 $\eta \to 3\pi$ decay parameters,
- 10 observables in $K_{\ell 4}$
- 18 in the scalar formfactors
- 11 in the vectorformfactors
- Total: 76

We found 35 relations

Relations at NNLO: πK

$a_\ell^- = a_\ell^{1/2} - a_\ell^{3/2}$, $a_\ell^+ = a_\ell^{1/2} + 2a_\ell^{3/2}$, $\rho = m_K/m_\pi$

$$\left(\rho^4+3\rho^3+3\rho+1\right)\left[a_1^-\right]_{C_i} = 2\rho^2\left(\rho+1\right)^2\left[b_1^-\right]_{C_i} - \frac{2}{3}\rho\left(\rho^2+1\right)\left[b_0^-\right]_{C_i} + \frac{1}{2\rho}\left(\rho^2+\frac{4}{3}\rho+1\right)\left(\rho^2+1\right)\left[a_0^-\right]_{C_i} \qquad (I)$$

$$5\left(\rho^2+1\right)\left[a_2^-\right]_{C_i} = \left[a_1^-\right]_{C_i} + 2\rho\left[b_1^-\right]_{C_i} \qquad (II)$$

$$5\left(\rho+1\right)^2\left[b_2^-\right]_{C_i} = \frac{(\rho-1)^2}{\rho^2}\left[a_1^-\right]_{C_i} - \frac{\rho^4+\frac{2}{3}\rho^2+1}{4\rho^4}\left[a_0^-\right]_{C_i} + \frac{\rho^2-\frac{2}{3}\rho+1}{2\rho^2}\left[b_0^-\right]_{C_i} \qquad (III)$$

$$7\left(\rho^2+1\right)\left[a_3^-\right]_{C_i} = \left[a_2^-\right]_{C_i} + 2\rho\left[b_2^-\right]_{C_i} \qquad (IV)$$

$$7\left[a_3^+\right]_{C_i} = \frac{1}{2\rho}\left[a_2^+\right]_{C_i} - \left[b_2^+\right]_{C_i} + \frac{1}{5\rho}\left[b_1^+\right]_{C_i} - \frac{1}{60\rho^3}\left[a_0^+\right]_{C_i} - \frac{1}{30\rho^2}\left[b_0^+\right]_{C_i} \qquad (V)$$

Relations at NNLO: πK

	Roy-Steiner	NLO 1-loop	NLO LECs	NNLO 2-loop	NNLO 1-loop	remainder
LHS (I)	5.4 ± 0.3	0.16	0.97	0.77	-0.11	0.6 ± 0.3
RHS (I)	6.9 ± 0.6	0.42	0.97	0.77	-0.03	1.8 ± 0.6
10 LHS (II)	0.32 ± 0.01	0.03	0.12	0.11	0.00	0.07 ± 0.01
10 RHS (II)	0.37 ± 0.01	0.02	0.12	0.10	-0.01	0.14 ± 0.01
100 LHS (III)	-0.49 ± 0.02	0.08	-0.25	-0.17	0.05	-0.21 ± 0.02
100 RHS (III)	-0.85 ± 0.60	0.03	-0.25	0.11	-0.03	-0.71 ± 0.60
100 LHS (IV)	0.13 ± 0.01	0.04	0.00	0.01	0.03	0.05 ± 0.01
100 RHS (IV)	0.01 ± 0.01	0.01	0.00	0.00	0.00	-0.01 ± 0.01
10^3 LHS (V)	0.29 ± 0.05	0.09	0.00	0.06	0.01	0.13 ± 0.03
10^3 RHS (V)	0.31 ± 0.07	0.03	0.00	0.06	0.05	0.17 ± 0.07

πK-scattering. The tree level for LHS and RHS of (I) is 3.01 and vanishes for the others.

Problem with (II) but large NNLO corrections

Problem with (IV): a_3^-

Relations at NNLO: summary

- We did numerics for $\pi\pi$, πK and $K_{\ell 4}$: 13 relations
- The two involving a_3^- significantly did not work well
- The relation with $K_{\ell 4}$ also did not work:

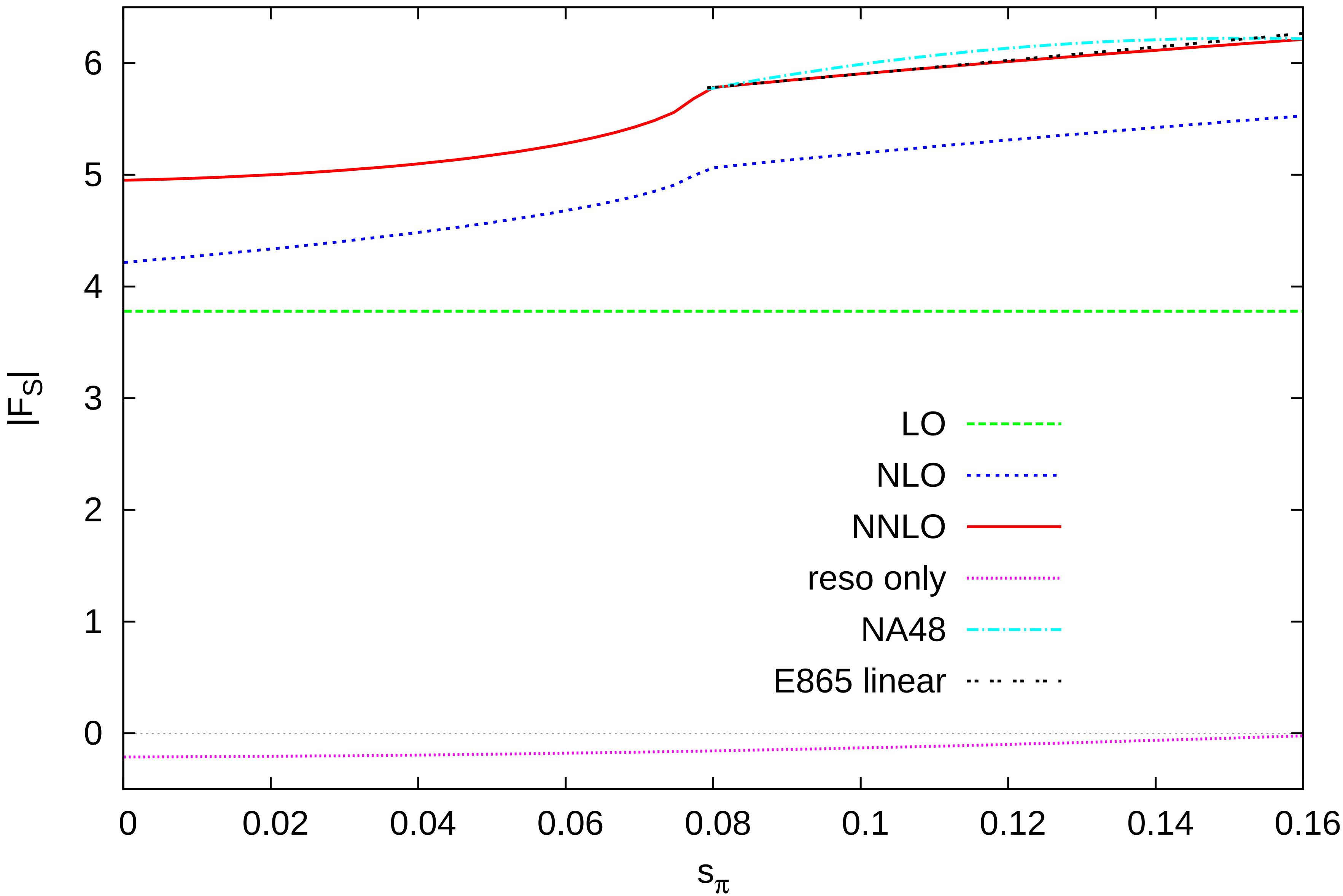
$$\sqrt{2}\,[f_s'']_{C_i} = \frac{32\pi\rho F_\pi}{1+\rho}\left[\frac{35}{6}\left(2+\rho+2\rho^2\right)\left[a_3^+\right]_{C_i} - \frac{5}{4}\left[a_2^+ + 2\rho b_2^+\right]_{C_i}\right]$$

	Roy-Steiner NA48	NLO 1-loop	NLO LECs	NNLO 2-loop	NNLO 1-loop	remainder
LHS	-0.73 ± 0.10	-0.23	0.00	-0.15	-0.05	-0.29 ± 0.10
RHS	0.50 ± 0.07	0.19	0.00	0.10	0.03	0.18 ± 0.07

πK-scattering lengths and curvature in F in $K_{\ell 4}$

Resonance p^6 contribution both sides +0.05

Relations at NNLO: summary



Fit: Inputs

Fit: Amoros, JB Talavera 2001

$K_{\ell 4}$: $F(0)$, $G(0)$, λ_F, λ_G E865 BNL $\Longrightarrow$ NA48 talk by Bloch-Devaux

$m^2_{\pi^0}$, m^2_η, $m^2_{K^+}$, $m^2_{K^0}$ em with Dashen violation

F_{π^+} $92.4 \Longrightarrow 92.2 \pm 0.05$ MeV

F_{K^+}/F_{π^+} $1.22 \pm 0.01 \Longrightarrow 1.193 \pm 0.002 \pm 0.006 \pm 0.001$

$m_s/\hat{m}$ 24 (26) (28.8 PACS-CS) talk by Leutwyler

L^r_4, L^r_6

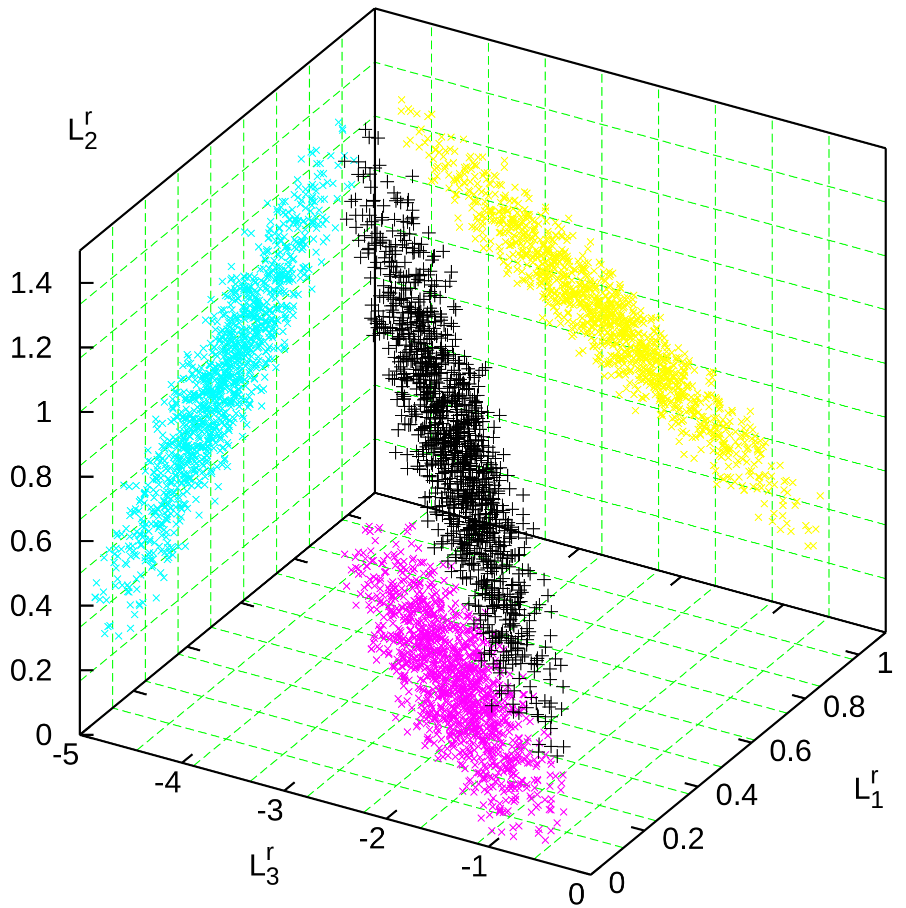
Many more calculations done: include those as well; Comprehensive new fit in progress: preliminary results, see below and talk by Jemos

Fit Outputs: I

	fit 10	same p^4	fit B	fit D	fit 10 iso
$10^3 L_1^r$	0.43 ± 0.12	0.38	0.44	0.44	0.40
$10^3 L_2^r$	0.73 ± 0.12	1.59	0.60	0.69	0.76
$10^3 L_3^r$	-2.35 ± 0.37	-2.91	-2.31	-2.33	-2.40
$10^3 L_4^r$	$\equiv 0$	$\equiv 0$	$\equiv 0.5$	$\equiv 0.2$	$\equiv 0$
$10^3 L_5^r$	0.97 ± 0.11	1.46	0.82	0.88	0.97
$10^3 L_6^r$	$\equiv 0$	$\equiv 0$	$\equiv 0.1$	$\equiv 0$	$\equiv 0$
$10^3 L_7^r$	-0.31 ± 0.14	-0.49	-0.26	-0.28	-0.30
$10^3 L_8^r$	0.60 ± 0.18	1.00	0.50	0.54	0.61

- errors are very correlated
- $\mu = 770$ MeV; 550 or 1000 within errors
- varying C_i^r factor 2 about errors
- $L_4^r, L_6^r \approx -0.3, \ldots, 0.6\ 10^{-3}$ OK
- fit B: small corrections to pion “sigma” term, fit scalar radius JB, Dhonte
- fit D: fit $\pi\pi$ and πK thresholds JB, Dhonte, Talavera

Correlations



(older fit)

$$10^3\, L_1^r = 0.52 \pm 0.23$$

$$10^3\, L_2^r = 0.72 \pm 0.24$$

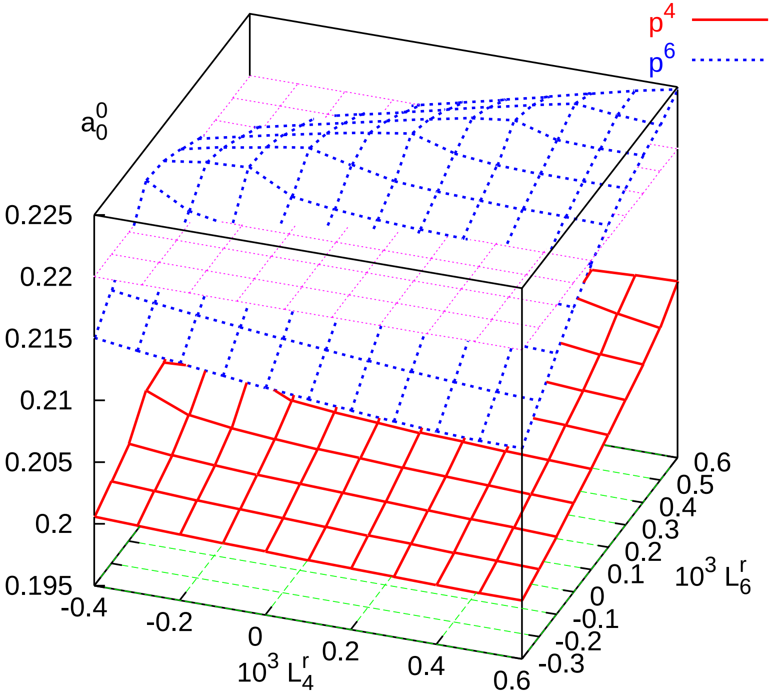
$$10^3\, L_3^r = -2.70 \pm 0.99$$

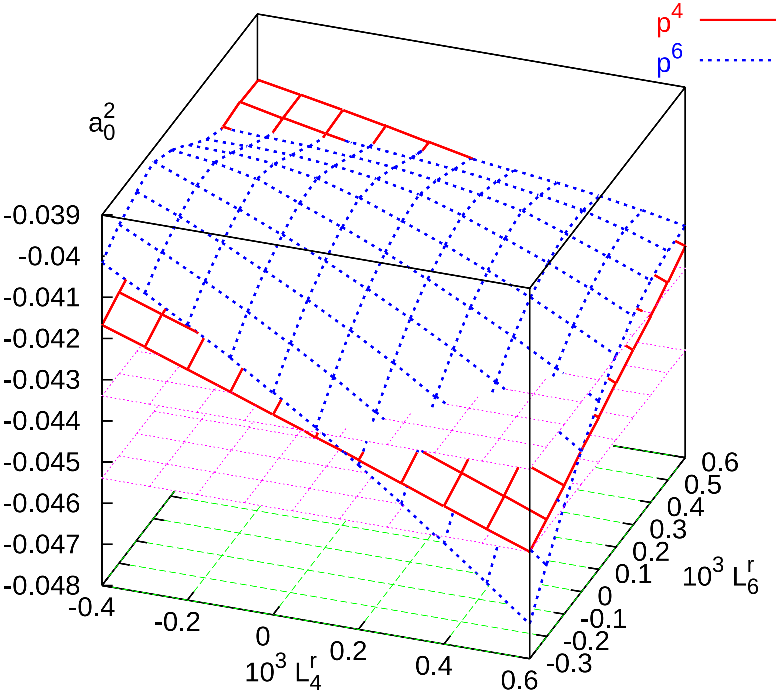
Outputs: II

	fit 10	same p^4	fit B	fit D
$2B_0\hat{m}/m_\pi^2$	0.736	0.991	1.129	0.958
m_π^2: p^4, p^6	0.006,0.258	0.009,$\equiv 0$	$-$0.138,0.009	$-$0.091,0.133
m_K^2: p^4, p^6	0.007,0.306	0.075,$\equiv 0$	$-$0.149,0.094	$-$0.096,0.201
m_η^2: p^4, p^6	$-$0.052,0.318	0.013,$\equiv 0$	$-$0.197,0.073	$-$0.151,0.197
m_u/m_d	0.45$\pm$0.05	0.52	0.52	0.50
F_0 [MeV]	87.7	81.1	70.4	80.4
$\frac{F_K}{F_\pi}$: p^4, p^6	0.169,0.051	0.22,$\equiv 0$	0.153,0.067	0.159,0.061

- $m_u = 0$ always very far from the fits
- F_0: pion decay constant in the chiral limit

$\pi\pi$



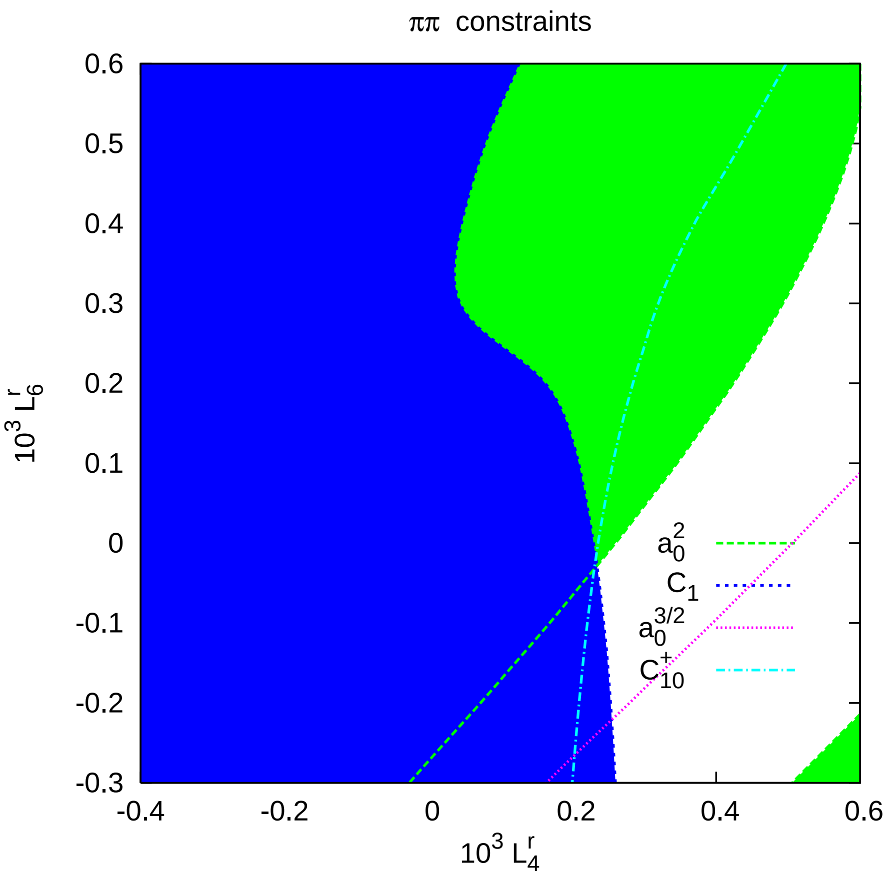


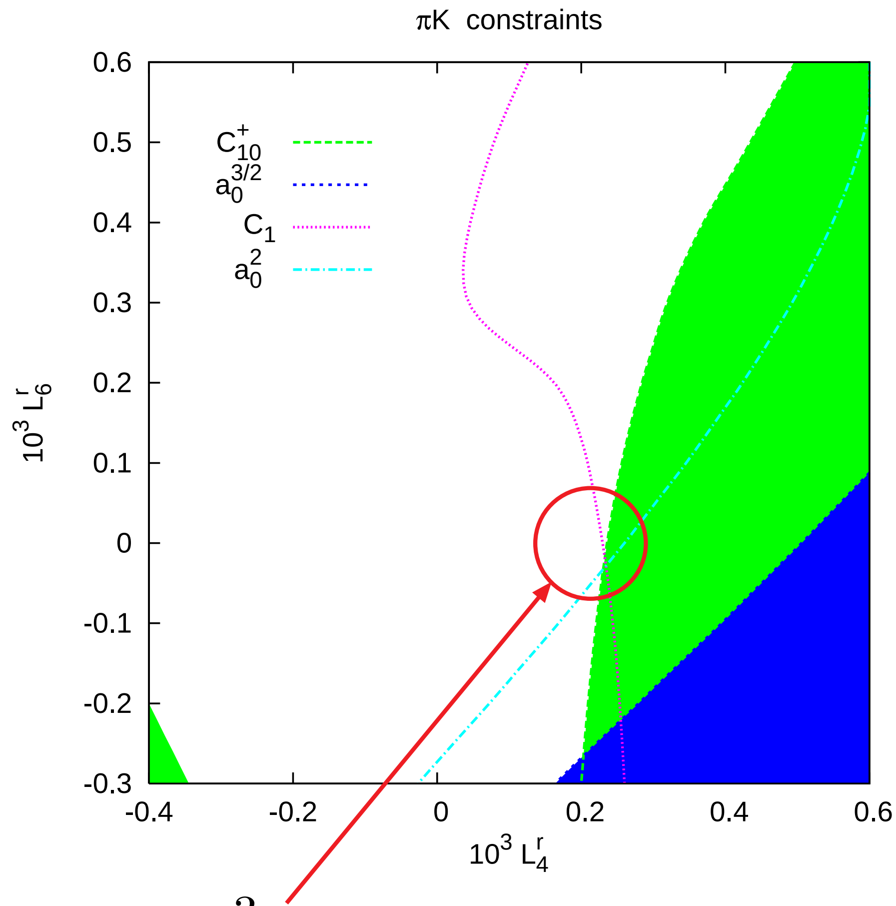
$a_0^0 = 0.220 \pm 0.005$, $a_0^2 = -0.0444 \pm 0.0010$

Colangelo, Gasser, Leutwyler

$a_0^0 = 0.159$ $a_0^2 = -0.0454$ at order p^2

$\pi\pi$ and πK





preferred region: fit D: $10^3 L_4^r \approx 0.2$, $10^3 L_6^r \approx 0.0$

General fitting: in progress

New fitting results

	fit 10 iso	NA48	F_K/F_π	Scatt	All	All ($C_i^r = 0$)
$10^3 L_1^r$	0.40 ± 0.12	0.98	0.97	0.97	0.98 ± 0.11	0.75
$10^3 L_2^r$	0.76 ± 0.12	0.78	0.79	0.79	0.59 ± 0.21	0.09
$10^3 L_3^r$	-2.40 ± 0.37	-3.14	-3.12	-3.14	-3.08 ± 0.46	-1.49
$10^3 L_4^r$	$\equiv 0$	$\equiv 0$	$\equiv 0$	$\equiv 0$	0.71 ± 0.67	0.78
$10^3 L_5^r$	0.97 ± 0.11	0.93	0.72	0.56	0.56 ± 0.11	0.67
$10^3 L_6^r$	$\equiv 0$	$\equiv 0$	$\equiv 0$	$\equiv 0$	0.15 ± 0.71	0.18
$10^3 L_7^r$	-0.30 ± 0.15	-0.30	-0.26	-0.23	-0.22 ± 0.15	-0.24
$10^3 L_8^r$	0.61 ± 0.20	0.59	0.48	0.44	0.38 ± 0.18	0.39
χ^2 (dof)	0.25 (1)	0.17 (1)	0.19 (1)	5.38 (5)	1.44 (4)	1.51 (4)

- NA48: use NA48 formfactors but E865 normalization
- F_K/F_π also change this to 1.193
- Scatt: add a_0^0, a_0^2, $a_0^{1/2}$ and $a_0^{3/2}$, $\chi^2 = 5.04$ from a_0^2
- All: add pion scalar radius 0.61 ± 0.04: $\chi^2 = 61$!! for $L_4^r = L_6^r = 0$
- All results preliminary
- In progress: adding more threshold parameters, more knowledge about C_i^r, ...

Quark mass dependences

Updates of plots in

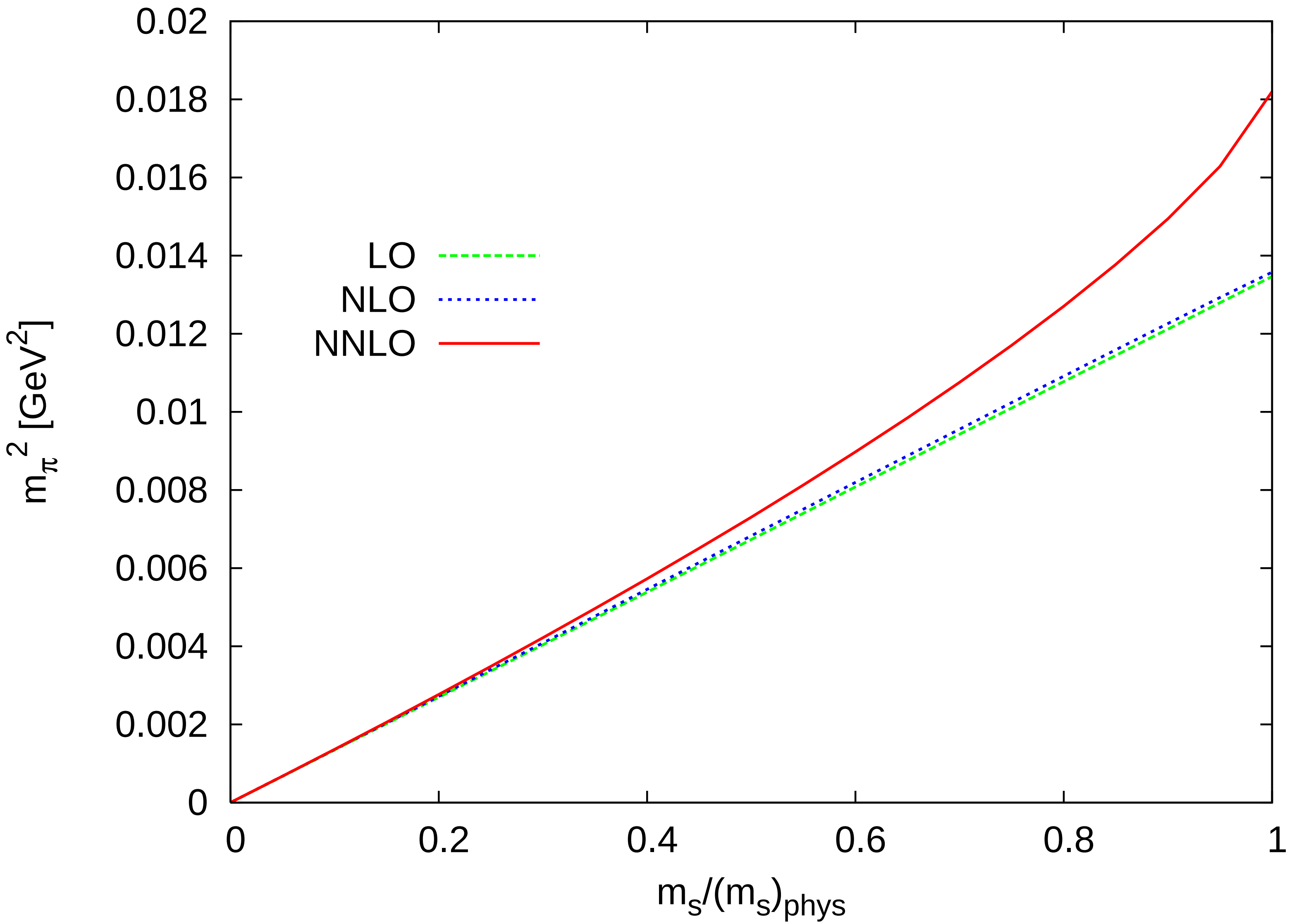
Amorós, JB and Talavera, hep-ph/0003258, Nucl. Phys. B585 (2000) 293

Some new ones

Procedure: calculate a consistent set of $m_\pi, m_K, m_\eta, f_\pi$ with the given input values (done iteratively)

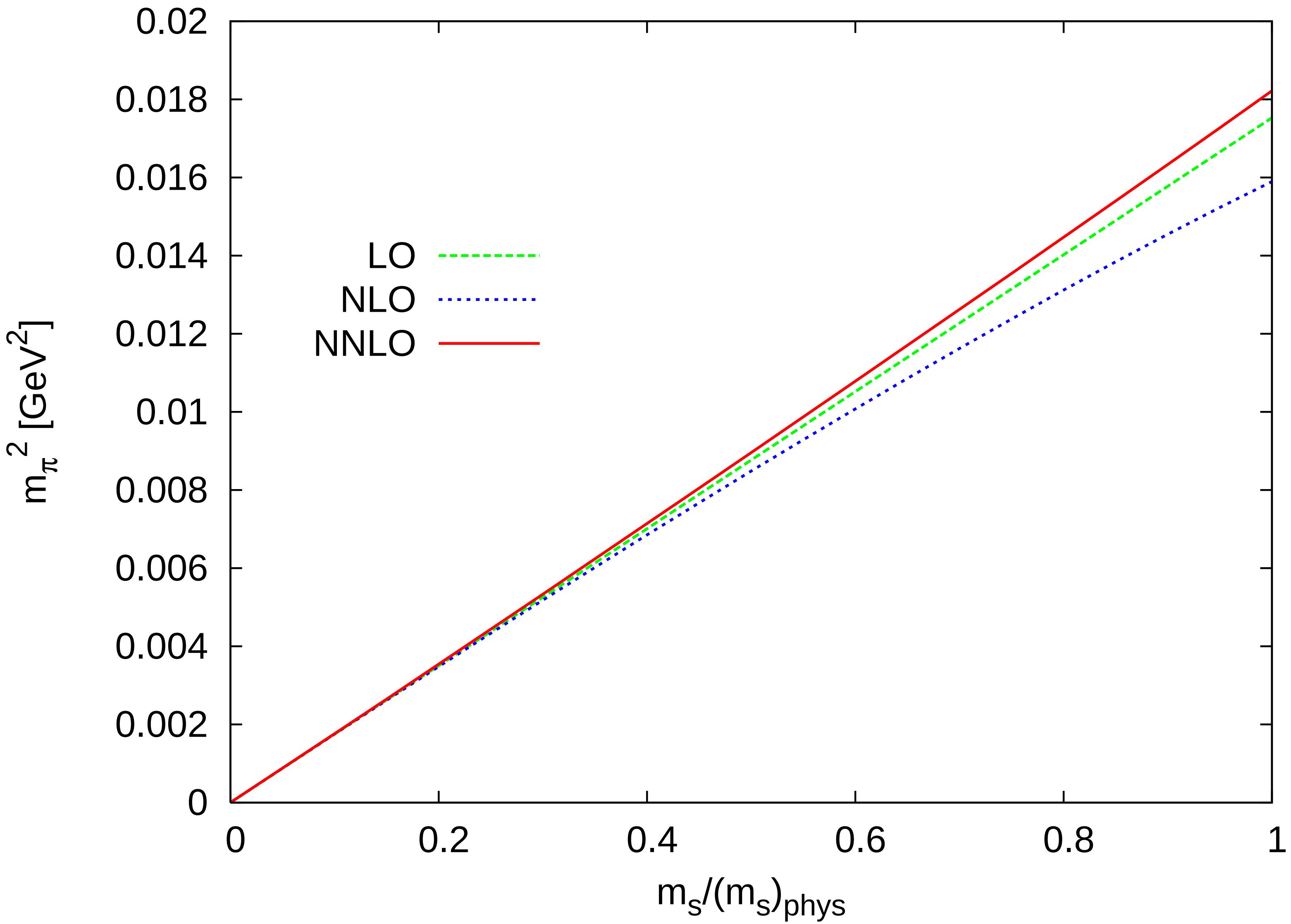
- vary $m_s/(m_s)_{phys}$, keep $m_s/\hat{m} = 24$
  m_π^2, F_K/F_π

m_π^2 fit 10



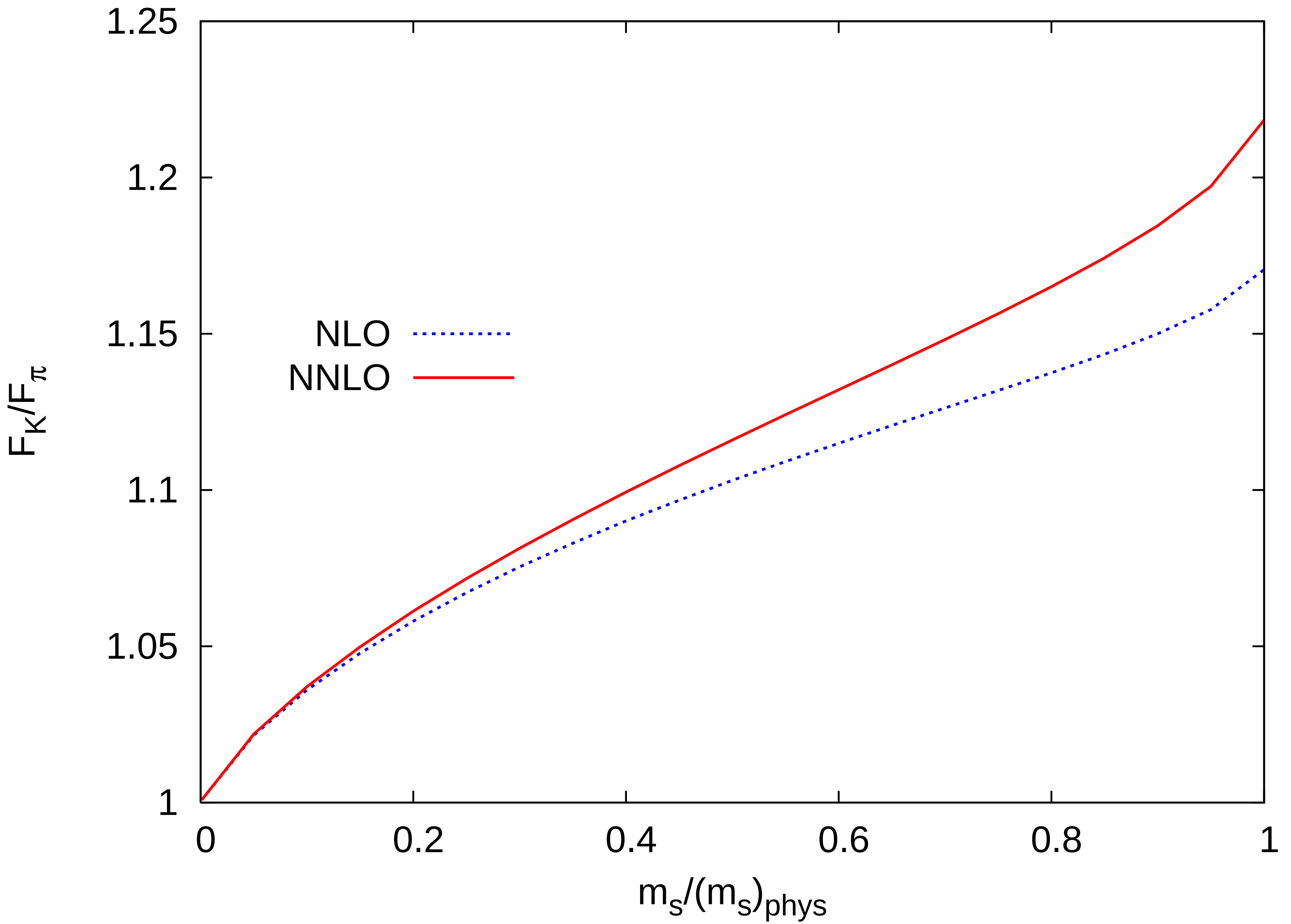
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
0
0.2
0.4
0.6
0.8
1
LO
NLO
NNLO
m_π^2 [GeV^2]
$m_s/(m_s)_{phys}$

m_π^2 fit D



LO
NLO
NNLO
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
0
0.2
0.4
0.6
0.8
1
m_π^2 [GeV^2]
$m_s/(m_s)_{phys}$

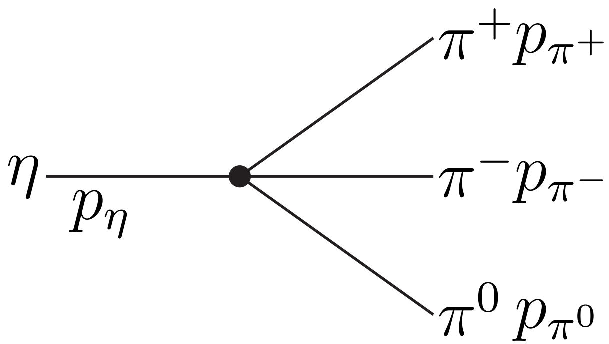
F_K/F_π fit 10



$\eta \to 3\pi$

Reviews: JB, Gasser, Phys.Scripta T99(2002)34 [hep-ph/0202242]

JB, Acta Phys. Slov. 56(2005)305 [hep-ph/0511076]



$$\begin{aligned} s &= (p_{\pi^+} + p_{\pi^-})^2 = (p_\eta - p_{\pi^0})^2 \\ t &= (p_{\pi^-} + p_{\pi^0})^2 = (p_\eta - p_{\pi^+})^2 \\ u &= (p_{\pi^+} + p_{\pi^0})^2 = (p_\eta - p_{\pi^-})^2 \end{aligned}$$

$$s + t + u = m_\eta^2 + 2m_{\pi^+}^2 + m_{\pi^0}^2 \equiv 3s_0\,.$$

$$\langle \pi^0\pi^+\pi^- \text{out}|\eta\rangle = i\,(2\pi)^4\;\delta^4\left(p_\eta - p_{\pi^+} - p_{\pi^-} - p_{\pi^0}\right)\;A(s,t,u)\,.$$

$$\langle \pi^0\pi^0\pi^0 \text{out}|\eta\rangle = i\,(2\pi)^4\;\delta^4\left(p_\eta - p_1 - p_2 - p_3\right)\;\overline{A}(s_1,s_2,s_3)$$

$$\overline{A}(s_1,s_2,s_3) = A(s_1,s_2,s_3) + A(s_2,s_3,s_1) + A(s_3,s_1,s_2)\,,$$

$\eta \to 3\pi$

- Experiment: talks by Prakhov (CB@MAMI), Jacewicz (KLOE) and Kupsc (WASA)
- Theory: talks by Ditsche (electromagnetic effects), Lanz (dispersive) and Gan (new physics in rare decays)

$\eta \to 3\pi$: Lowest order (LO)

Pions are in $I = 1$ state $\Longrightarrow A \sim (m_u - m_d)$ or α_{em}

- α_{em} effect is small (but large via $m_{\pi^+} - m_{\pi^0}$)
- $\eta \to \pi^+\pi^-\pi^0\gamma$ needs to be included directly

$\eta \to 3\pi$: Lowest order (LO)

Pions are in $I = 1$ state $\Longrightarrow$ $A \sim (m_u - m_d)$ or α_{em}

ChPT:Cronin 67: $A(s,t,u) = \frac{B_0(m_u - m_d)}{3\sqrt{3}F_\pi^2}\left\{1 + \frac{3(s - s_0)}{m_\eta^2 - m_\pi^2}\right\}$

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with $Q^2 \equiv \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}$ or $R \equiv \frac{m_s - \hat{m}}{m_d - m_u}$ $\qquad \hat{m} = \frac{1}{2}(m_u + m_d)$

$$A(s,t,u) = \frac{1}{Q^2}\frac{m_K^2}{m_\pi^2}(m_\pi^2 - m_K^2)\frac{\mathcal{M}(s,t,u)}{3\sqrt{3}F_\pi^2},$$

$$A(s,t,u) = \frac{\sqrt{3}}{4R}M(s,t,u)$$

LO: $\mathcal{M}(s,t,u) = \frac{3s - 4m_\pi^2}{m_\eta^2 - m_\pi^2}$ $\qquad M(s,t,u) = \frac{1}{F_\pi^2}\left(\frac{4}{3}m_\pi^2 - s\right)$

$\eta \longrightarrow 3\pi$ beyond p^4: p^2 and p^4

$\Gamma\left(\eta \rightarrow 3\pi\right) \propto |A|^2 \propto Q^{-4}$ allows a PRECISE measurement

$\boxed{Q \approx 24}$ gives lowest order $\Gamma(\eta \rightarrow \pi^+\pi^-\pi^0) \approx 66$ eV .

Other Source from $m^2_{K^+} - m^2_{K^0} \sim Q^{-2} \Longrightarrow \boxed{Q = 20.0 \pm 1.5}$

Lowest order prediction $\Gamma(\eta \rightarrow \pi^+\pi^-\pi^0) \approx 140$ eV .

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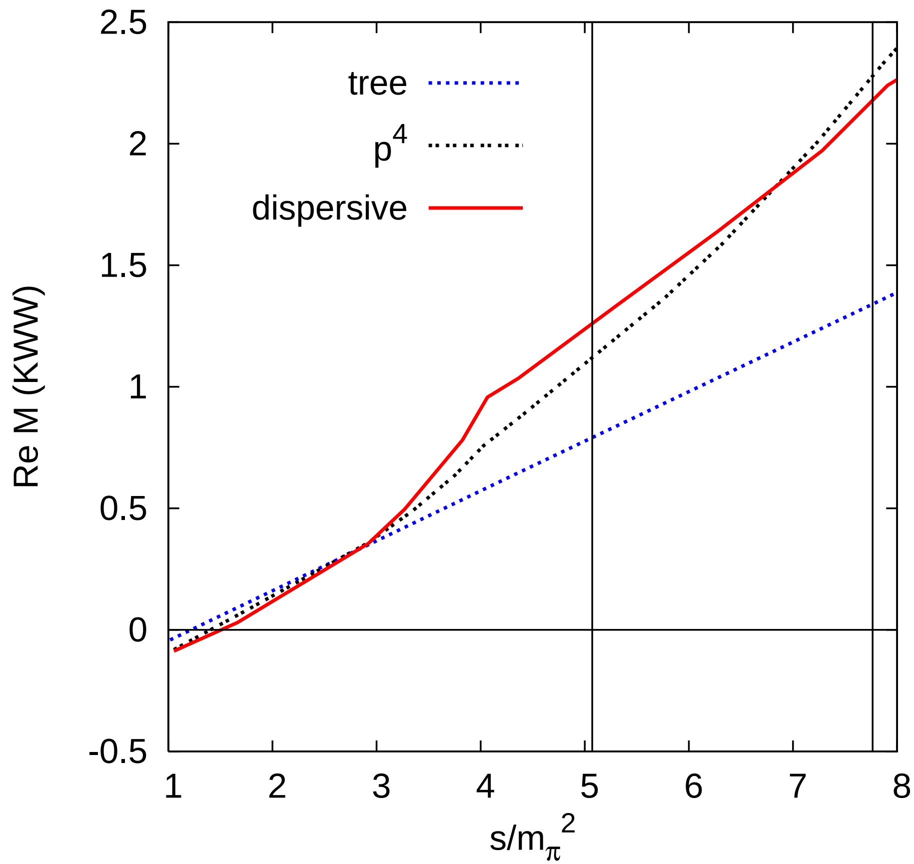
At order p^4 Gasser-Leutwyler 1985: $$\frac{\int dLIPS|A_2 + A_4|^2}{\int dLIPS|A_2|^2} = 2.4\,,$$

($LIPS$=Lorentz invariant phase-space)

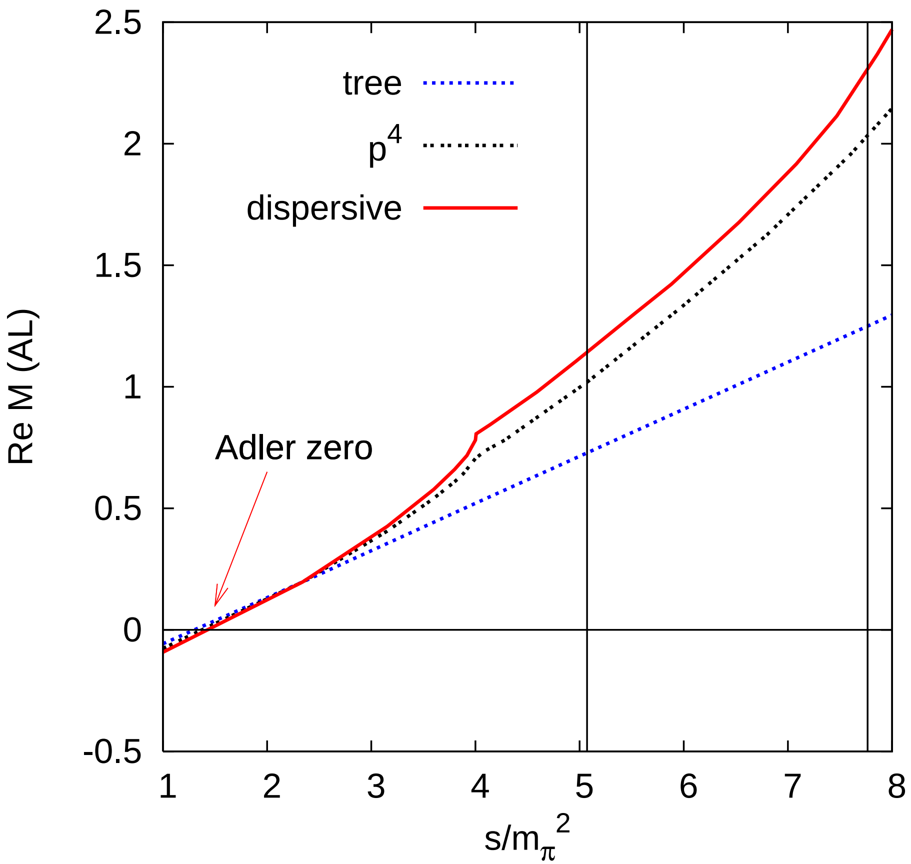
Major source: large S-wave final state rescattering

Experiment: 295 ± 17 eV (PDG 2006)

$\eta \to 3\pi$ beyond p^4: dispersive



Along $s = u$ KWW

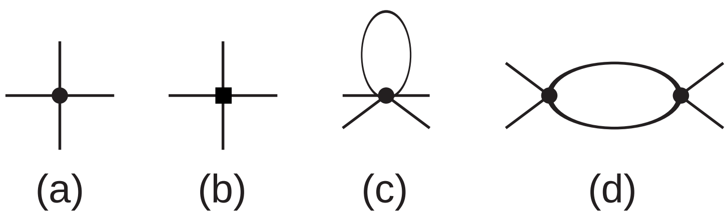


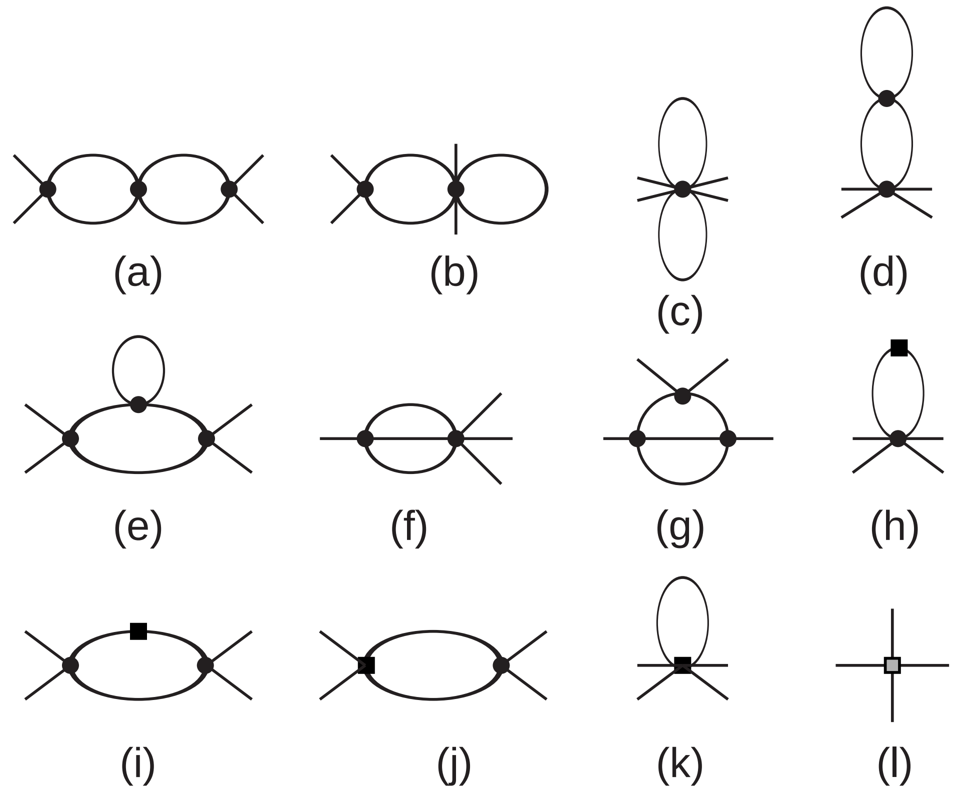
Along $s = u$ AL

Two Loop Calculation: why

- In $K_{\ell 4}$ dispersive gave about half of p^6 in amplitude
- Same order in ChPT as masses for consistency check on m_u/m_d
- Check size of 3 pion dispersive part
- At order p^4 unitarity about half of correction
- Technology exists:
  - Two-loops: Amorós,JB,Dhonte,Talavera,. . .
  - Dealing with the mixing π^0-η: Amorós,JB,Dhonte,Talavera 01
- JB, Ghorbani, arXiv:0709.0230 [hep-ph]
  - Dealing with the mixing π^0-η: extended to $\eta \to 3\pi$

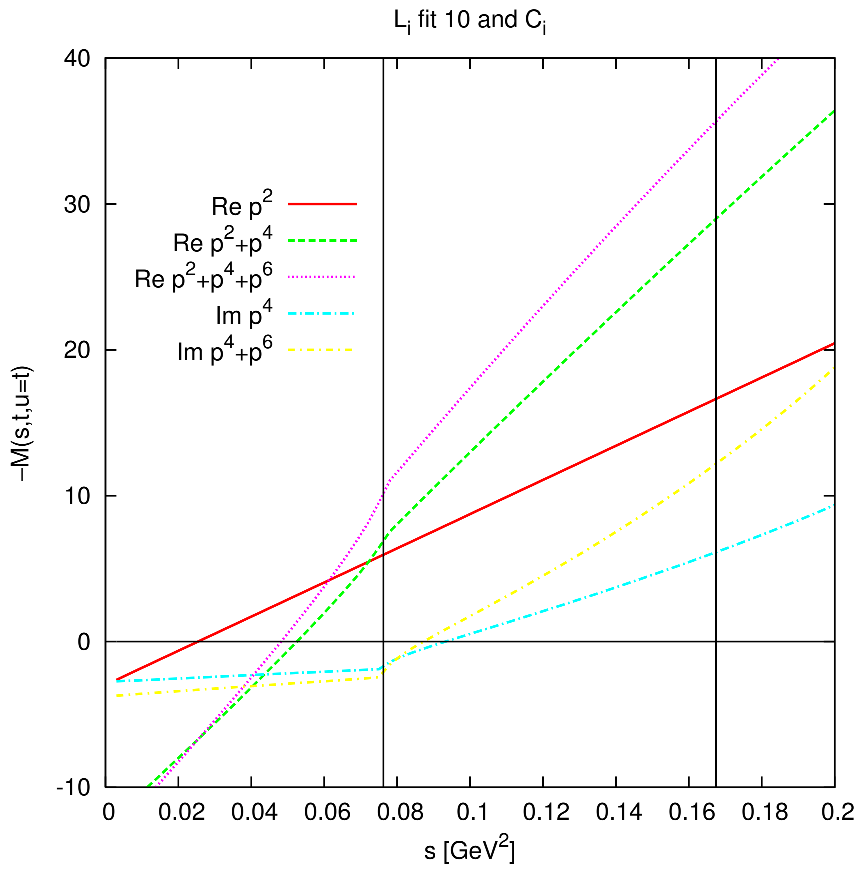
Diagrams



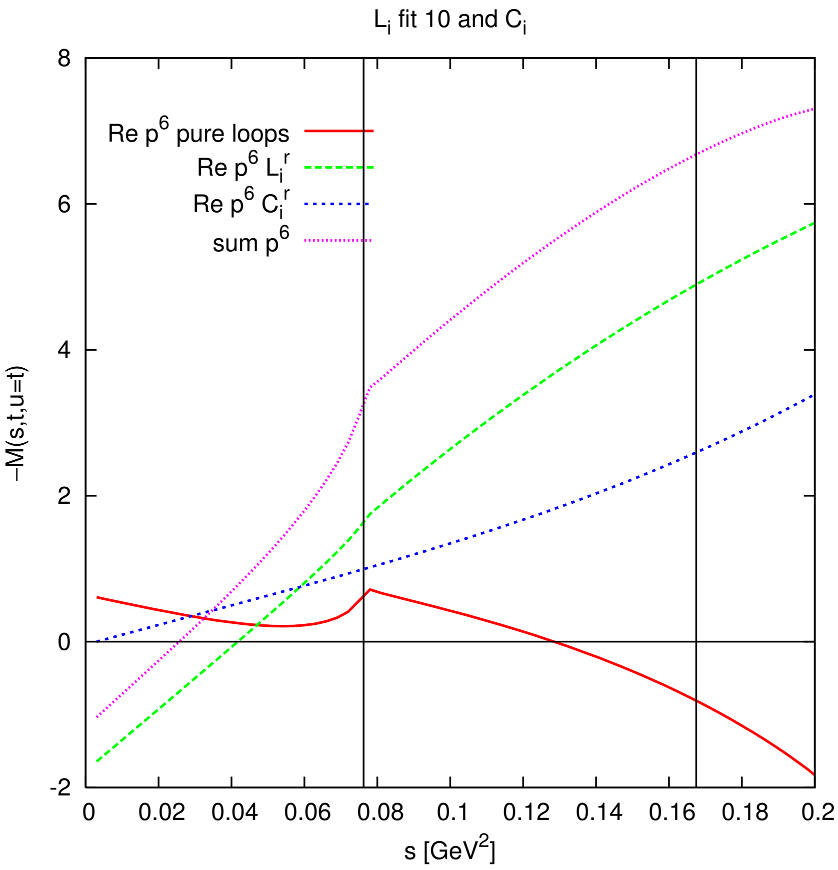


- Include mixing, renormalize, pull out factor $\frac{\sqrt{3}}{4R}, \ldots$
- Two independent calculations (comparison major amount of work)

$\eta \to 3\pi$: $M(s, t = u)$

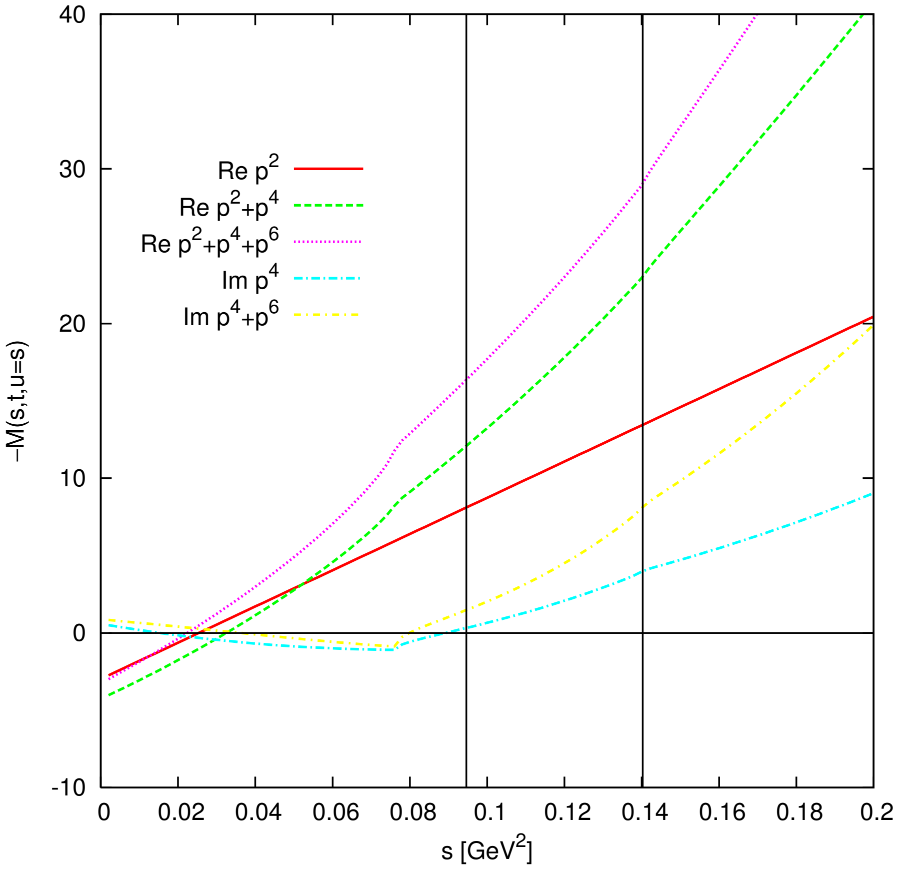


Along $t = u$



Along $t = u$ parts

$\eta \to 3\pi$: $M(s = u, t)$



Along $s = u$

Shape agrees with AL

Correction larger:
20-30% in amplitude

Dalitz plot

$$
\begin{aligned}
x &= \sqrt{3}\frac{T_+ - T_-}{Q_\eta} = \frac{\sqrt{3}}{2m_\eta Q_\eta}(u-t) \\
y &= \frac{3T_0}{Q_\eta} - 1 = \frac{3\left((m_\eta - m_{\pi^o})^2 - s\right)}{2m_\eta Q_\eta} - 1 \stackrel{\text{iso}}{=} \frac{3}{2m_\eta Q_\eta}(s_0 - s) \\
Q_\eta &= m_\eta - 2m_{\pi^+} - m_{\pi^0}
\end{aligned}
$$

T^i is the kinetic energy of pion π^i

$$
z = \frac{2}{3}\sum_{i=1,3}\left(\frac{3E_i - m_\eta}{m_\eta - 3m_\pi^0}\right)^2
$$

E_i is the energy of pion π^i

$$
\begin{aligned}
|M|^2 &= A_0^2\left(1 + ay + by^2 + dx^2 + fy^3 + gx^2y + \cdots\right) \\
|\overline{M}|^2 &= \overline{A}_0^2\left(1 + 2\alpha z + \cdots\right)
\end{aligned}
$$

Experiment: charged

Exp.	a	b	d
KLOE	$-1.090 \pm 0.005^{+0.008}_{-0.019}$	$0.124 \pm 0.006 \pm 0.010$	$0.057 \pm 0.006^{+0.007}_{-0.016}$
Crystal Barrel	-1.22 ± 0.07	0.22 ± 0.11	0.06 ± 0.04 (input)
Layter et al.	-1.08 ± 0.014	0.034 ± 0.027	0.046 ± 0.031
Gormley et al.	-1.17 ± 0.02	0.21 ± 0.03	0.06 ± 0.04

KLOE has: $f = 0.14 \pm 0.01 \pm 0.02$.

Crystal Barrel: d input, but a and b insensitive to d

Theory: charged

	A_0^2	a	b	d	f
LO	120	-1.039	0.270	0.000	0.000
NLO	314	-1.371	0.452	0.053	0.027
NLO ($L_i^r = 0$)	235	-1.263	0.407	0.050	0.015
NNLO	538	-1.271	0.394	0.055	0.025
NNLOp (y from T^0)	574	-1.229	0.366	0.052	0.023
NNLOq (incl $(x,y)^4$)	535	-1.257	0.397	0.076	0.004
NNLO ($\mu = 0.6$ GeV)	543	-1.300	0.415	0.055	0.024
NNLO ($\mu = 0.9$ GeV)	548	-1.241	0.374	0.054	0.025
NNLO ($C_i^r = 0$)	465	-1.297	0.404	0.058	0.032
NNLO ($L_i^r = C_i^r = 0$)	251	-1.241	0.424	0.050	0.007
dispersive (KWW)	—	-1.33	0.26	0.10	—
tree dispersive	—	-1.10	0.33	0.001	—
absolute dispersive	—	-1.21	0.33	0.04	—
error	18	0.075	0.102	0.057	0.160

NLO to NNLO: Little change

Error on $|M(s,t,u)|^2$:

$|M^{(6)} M(s,t,u)|$

Experiment: neutral

Exp.	α
KLOE 2007	$-0.027 \pm 0.004^{+0.004}_{-0.006}$
KLOE (prel)	$-0.014 \pm 0.005 \pm 0.004$
Crystal Ball	-0.031 ± 0.004
WASA/CELSIUS	$-0.026 \pm 0.010 \pm 0.010$
Crystal Barrel	$-0.052 \pm 0.017 \pm 0.010$
GAMS2000	-0.022 ± 0.023
SND	$-0.010 \pm 0.021 \pm 0.010$

	$\overline{A}_0^2$	α
LO	1090	0.000
NLO	2810	0.013
NLO ($L_i^r = 0$)	2100	0.016
NNLO	4790	0.013
NNLOq	4790	0.014
NNLO ($C_i^r = 0$)	4140	0.011
NNLO ($L_i^r = C_i^r = 0$)	2220	0.016
dispersive (KWW)	—	$-(0.007$—$0.014)$
tree dispersive	—	-0.0065
absolute dispersive	—	-0.007
Borasoy	—	-0.031
error	160	0.032

Note: NNLO ChPT gets a_0^0 in $\pi\pi$ correct

α is difficult

Expand amplitudes and isospin:

$$M(s,t,u) = A\left(1+\tilde{a}(s-s_0)+\tilde{b}(s-s_0)^2+\tilde{d}(u-t)^2+\cdots\right)$$

$$\overline{M}(s,t,u) = A\left(3+\left(\tilde{b}+3\tilde{d}\right)\left((s-s_0)^2+(t-s_0)^2+(u-s_0)^2\right)+$$

Gives relations ($R_\eta = (2m_\eta Q_\eta)/3$)

$$a = -2R_\eta\,\mathrm{Re}(\tilde{a})\,, \quad b = R_\eta^2\left(|\tilde{a}|^2+2\mathrm{Re}(\tilde{b})\right)\,, \quad d = 6R_\eta^2\,\mathrm{Re}(\tilde{d})\,.$$

$$\alpha = \frac{1}{2}R_\eta^2\,\mathrm{Re}\left(\tilde{b}+3\tilde{d}\right) = \frac{1}{4}\left(d+b-R_\eta^2|\tilde{a}|^2\right) \le \frac{1}{4}\left(d+b-\frac{1}{4}a^2\right)$$

equality if Im$(\tilde{a})=0$

Large cancellation in α, overestimate of b likely the problem

r and decay rates

$$r \equiv \frac{\Gamma(\eta \to \pi^0\pi^0\pi^0)}{\Gamma(\eta \to \pi^+\pi^-\pi^0)}$$

$$\begin{aligned} r_{\rm LO} &= 1.54 \\ r_{\rm NLO} &= 1.46 \\ r_{\rm NNLO} &= 1.47 \\ r_{{\rm NNLO}\,C_i^r=0} &= 1.46 \end{aligned}$$

PDG 2006

$$\begin{aligned} r &= 1.49 \pm 0.06 \quad \text{our average}\,. \\ r &= 1.43 \pm 0.04 \quad \text{our fit}\,, \end{aligned}$$

Good agreement

R and Q

	LO	NLO	NNLO	NNLO $(C_i^r = 0)$
R (η)	19.1	31.8	42.2	38.7
R (Dashen)	44	44	37	—
R (Dashen-violation)	36	37	32	—
Q (η)	15.6	20.1	23.2	22.2
Q (Dashen)	24	24	22	—
Q (Dashen-violation)	22	22	20	—

LO from $R = \dfrac{m_{K^0}^2 + m_{K^+}^2 - 2m_{\pi^0}^2}{2\left(m_{K^0}^2 - m_{K^+}^2\right)}$ (QCD part only)

NLO and NNLO from masses: Amorós, JB, Talavera 2001

$$Q^2 = \frac{m_s + \hat{m}}{2\hat{m}} R = 12.7 R \qquad (m_s/\hat{m} = 24.4)$$

≥ 3-flavour: PQChPT

PQChPT: treat closed quark-loops differently from external quarks,

Essentially all manipulations from ChPT go through to PQChPT when changing trace to supertrace and adding fermionic variables

Exceptions: baryons and Cayley-Hamilton relations

So Luckily: can use the n flavour work in ChPT at two loop order to obtain for PQChPT: Lagrangians and infinities

Very important note: ChPT is a limit of PQChPT
$\Longrightarrow$ LECs from ChPT are linear combinations of LECs of PQChPT with the same number of sea quarks.

E.g. $L_1^r = L_0^{r(3pq)}/2 + L_1^{r(3pq)}$

PQChPT

One-loop: Bernard, Golterman, Sharpe, Shoresh, Pallante,...

with electromagnetism: JB,Danielsson, hep-lat/0610127

Two loops:

$m^2_{\pi^+}$ simplest mass case: JB,Danielsson,Lähde, hep-lat/0406017

F_{π^+}: JB,Lähde, hep-lat/0501014

F_{π^+}, $m^2_{\pi^+}$ two sea quarks: JB,Lähde, hep-lat/0506004

$m^2_{\pi^+}$: JB,Danielsson,Lähde, hep-lat/0602003

Neutral masses: JB,Danielsson, hep-lat/0606017

Lattice data: a and L extrapolations needed

Programs available from me (Fortran)

Formulas: `http://www.thep.lu.se/∼bijnens/chpt.html`

Renormalization group

Weinberg 79: nonlocal diverences must cancel $\Longrightarrow$ consistency conditions between graphs with different numbers of loops (but same order in the power counting)

This allows to calculate the leading logarithms to any order from one-loop diagrams Buchler Colangelo 2003

- double logs in $\pi\pi$ Colangelo 95
- all double logs JB, Ecker, Colangelo 1998
- leading logs to five loops for (massless) Scalar two-point function Bissegger Fuhrer 2007
- three loops for generalized GPD Kivel Polyakov 2007
- Recursion relations in the massless $O(N+1)/O(N)$ sigma model for many quantities Kivel, Polyakov, Vladimirov 2008

Renormalization group

Underlying practical problem: the number of needed terms increases fast with order $\Longrightarrow$ need a good way to handle this.

KPV: write the 4-meson vertex using Legendre polynomials
could perform all loopintegrals
$\Longrightarrow$ algebraic recursion relations

It works in the chiral limit since tadpoles vanish:
simplification: the number of external legs in the vertices needed does not go up.

Hard pion ChPT?

- Usual ChPT:
  - everyone soft momentum
  - simple powercounting

Hard pion ChPT?

- Usual ChPT:
  - everyone soft momentum
  - simple powercounting
- (Heavy) Baryon ChPT:
  - Two momentum regions
  - Baryon $p = M_B v + k$
  - Everyone else soft
  - General idea: M_B dependence can always be reabsorbed in LECs, is analytic in the other parts k.
  - Works: baryon lines always go through entire diagram
  - Several different formalisms exist

Hard pion ChPT?

- Heavy Meson ChPT: B, B^* or D, D^*
  - $p = M_B v + k$
  - Everything else soft
  - Works because b or c number conserved.
  - Decay constant works: takes away all heavy momentum
  - General idea: M_B dependence can always be reabsorbed in LECs, is analytic in the other parts k.

Hard pion ChPT?

- Heavy Meson ChPT: B, B^* or D, D^*
  - $p = M_B v + k$
  - Everything else soft
  - Works because b or c number conserved.
  - Decay constant works: takes away all heavy momentum
  - General idea: M_B dependence can always be reabsorbed in LECs, is analytic in the other parts k.
- (Heavy) (Vector or other) Meson ChPT:
  - (Vector) Meson: $p = M_V v + k$
  - Everyone else soft or $p = M_V + k$
  - General idea: M_V dependence can always be reabsorbed in LECs, is analytic in the other parts k.

Hard pion ChPT?

- (Heavy) (Vector) Meson ChPT:
  - $p = M_V v + k$
  - First: only keep diagrams where vectors always present
  - Applied to masses and decay constants
  - Decay constant works: takes away all heavy momentum
  - *It was argued that this could be done, the nonanalytic parts of diagrams with pions at large momenta are reproduced correctly*
  - Done both in relativistic and heavy meson type of formalism

Hard pion ChPT?

- Heavy Kaon ChPT:
  - $p = M_K v + k$
  - First: only keep diagrams where Kaon always goes through
  - Applied to masses and πK scattering and decay constant Roessl,Allton et al.,. . .
  - Applied to $K_{\ell 3}$ at q^2_{max} Flynn-Sachrajda
  - Works like all the previous *heavy* ChPT

Hard pion ChPT?

- Heavy Kaon ChPT:
  - $p = M_K v + k$
  - First: only keep diagrams where Kaon always goes through
  - Applied to masses and πK scattering and decay constant Roessl,Allton et al.,. . .
  - Applied to $K_{\ell 3}$ at q^2_{max} Flynn-Sachrajda
- Flynn-Sachrajda also argued that $K_{\ell 3}$ could be done for q^2 away from q^2_{max}.
- JB-Celis Argument generalizes to other processes with hard/fast pions and applied to $K \to \pi\pi$
- General idea: heavy/fast dependence can always be reabsorbed in LECs, is analytic in the other parts k.

Hard pion ChPT?

- nonanalyticities in the light masses come from soft lines
- soft pion couplings are constrained by current algebra

$$\lim_{q\to 0}\langle\pi^k(q)\alpha|O|\beta\rangle = -\frac{i}{F_\pi}\langle\alpha|\left[Q_5^k, O\right]|\beta\rangle\,,$$

Hard pion ChPT?

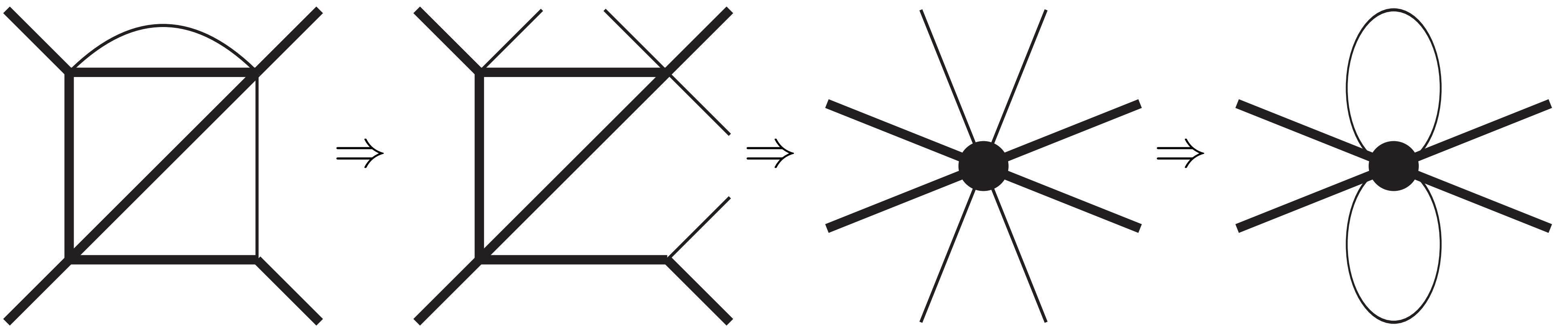
- nonanalyticities in the light masses come from soft lines
- soft pion couplings are constrained by current algebra
  $$\lim_{q\to 0}\langle\pi^k(q)\alpha|O|\beta\rangle = -\frac{i}{F_\pi}\langle\alpha|\left[Q_5^k, O\right]|\beta\rangle\,,$$
- Nothing prevents hard pions to be in the states α or β
- So by heavily using current algebra I should be able to get the light quark mass nonanalytic dependence

Hard pion ChPT?

Field Theory: a process at given external momenta

- Take a diagram with a particular internal momentum configuration
- Identify the soft lines and cut them
- The result part is analytic in the soft stuff
- So should be describably by an effective Lagrangian with coupling constants dependent on the external given momenta
- If symmetries present, Lagrangian should respect them
- Lagrangian should be complete in *neighbourhood*
- Loop diagrams with this effective Lagrangian *should* reproduce the nonanalyticities in the light masses
Crucial part of the argument

Hard pion ChPT?



This procedure works at one loop level, matching at tree level, nonanalytic dependence at one loop:

- Toy models and vector meson ChPT JB, Gosdzinsky, Talavera
- Recent work on relativistic meson ChPT Gegelia, Scherer et al.
- I am not aware of a two-loop check (but thinking)

$K \to 2\pi$ in $SU(2)$ ChPT

Add $K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}$ Roessl

$$\mathcal{L}^{(2)}_{\pi\pi} = \frac{F^2}{4}\left(\langle u_\mu u^\mu\rangle + \langle \chi_+\rangle\right),$$

$$\mathcal{L}^{(1)}_{\pi K} = \nabla_\mu K^\dagger \nabla^\mu K - \overline{M}^2_K K^\dagger K\,,$$

$$\mathcal{L}^{(2)}_{\pi K} = A_1 \langle u_\mu u^\mu\rangle K^\dagger K + A_2 \langle u^\mu u^\nu\rangle \nabla_\mu K^\dagger \nabla_\nu K + A_3 K^\dagger \chi_+ K + \cdots$$

Add a spurion for the weak interaction $\Delta I = 1/2$, $\Delta I = 3/2$

JB,Celis

$$t^{ij}_k \longrightarrow t^{i'j'}_{k'} = t^{ij}_k (g_L)_{k'}{}^k (g_L^\dagger)_i{}^{i'} (g_L^\dagger)_j{}^{j'}$$

$$t^i_{1/2} \longrightarrow t^{i'}_{1/2} = t^i_{1/2} (g_L^\dagger)_i{}^{i'}.$$

$K \to 2\pi$ in $SU(2)$ ChPT

The $\Delta I = 1/2$ terms: $\tau_{1/2} = t_{1/2} u^\dagger$

$$\begin{aligned}\mathcal{L}_{1/2} &= iE_1\,\tau_{1/2}K + E_2\,\tau_{1/2}u^\mu\nabla_\mu K + iE_3\langle u_\mu u^\mu\rangle\tau_{1/2}K \\ &\quad + iE_4\tau_{1/2}\chi_+ K + iE_5\langle\chi_+\rangle\tau_{1/2}K + E_6\tau_{1/2}\chi_- K \\ &\quad + E_7\langle\chi_-\rangle\tau_{1/2}K + iE_8\langle u_\mu u_\nu\rangle\tau_{1/2}\nabla^\mu\nabla^\nu K + \cdots + h.c.\,.\end{aligned}$$

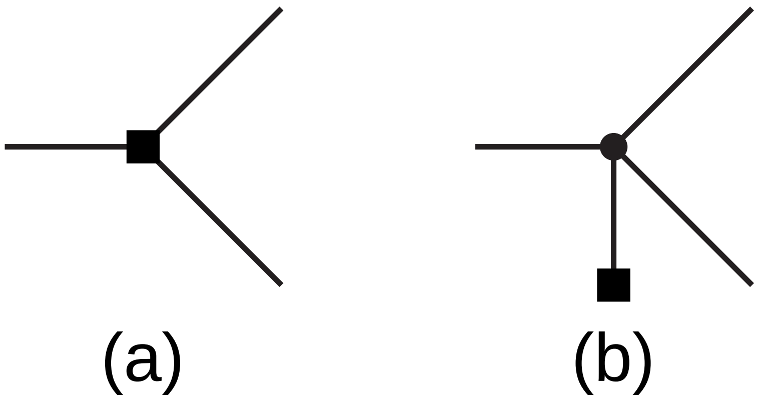
Note: higher order terms kept in both $\mathcal{L}_{1/2}$ and $\mathcal{L}^{(2)}_{\pi K}$ to check the arguments

Using partial integration,. . . :

$$\langle\pi(p_1)\pi(p_2)|O|K(p_K)\rangle =$$
$$f(\overline{M}^2_K)\langle\pi(p_1)\pi(p_2)|\tau_{1/2}K|K(p_K)\rangle + \lambda M^2 + \mathcal{O}(M^4)$$

O any operator in $\mathcal{L}_{1/2}$ or with more derivatives. Similar for $\mathcal{L}_{3/2}$

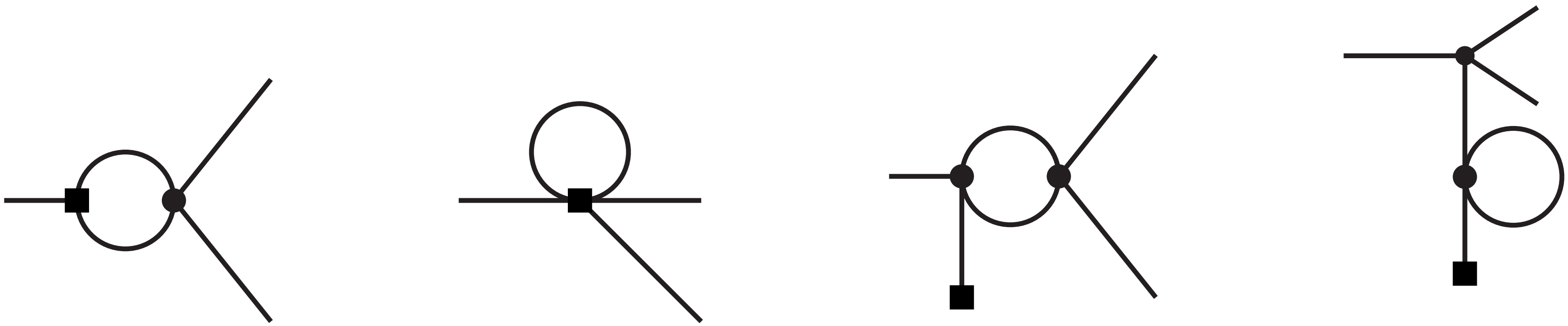
Tree level



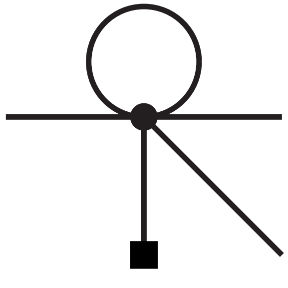
$$A_0^{LO} = \frac{\sqrt{3}i}{2F^2}\left[-\frac{1}{2}E_1 + (E_2 - 4E_3)\,\overline{M}_K^2 + 2E_8\overline{M}_K^4 + A_1E_1\right]$$

$$A_2^{LO} = \sqrt{\frac{3}{2}}\frac{i}{F^2}\left[(-2D_1 + D_2)\,\overline{M}_K^2\right]$$

One loop



(a) (b) (c) (d)



(e) (f)

One loop

Diagram	A_0	A_2
Z	$-\frac{2F^2}{3}A_0^{LO}$	$-\frac{2F^2}{3}A_2^{LO}$
(a)	$\sqrt{3}i\left(-\frac{1}{3}E_1+\frac{2}{3}E_2\overline{M}_K^2\right)$	$\sqrt{\frac{3}{2}}i\left(-\frac{2}{3}D_2\overline{M}_K^2\right)$
(b)	$\sqrt{3}i\left(-\frac{5}{96}E_1-\left(\frac{7}{48}E_2+\frac{25}{12}E_3\right)\overline{M}_K^2+\frac{25}{24}E_8\overline{M}_K^4\right)$	$\sqrt{\frac{3}{2}}i\left(-\frac{61}{12}D_1+\frac{77}{24}D_2\right)\overline{M}_K^2$
(e)	$\sqrt{3}i\frac{3}{16}A_1E_1$	
(f)	$\sqrt{3}i\left(\frac{1}{8}E_1+\frac{1}{3}A_1E_1\right)$	

The coefficients of $\overline{A}(M^2)/F^4$ in the contributions to A_0 and A_2. Z denotes the part from wave-function renormalization.

- $\overline{A}(M^2)=-\frac{M^2}{16\pi^2}\log\frac{M^2}{\mu^2}$
- $K\pi$ intermediate state does not contribute, but did for Flynn-Sachrajda

One-loop

$$A_0^{NLO} = A_0^{LO}\left(1 + \frac{3}{8F^2}\overline{A}(M^2)\right) + \lambda_0 M^2 + \mathcal{O}(M^4)\,,$$

$$A_2^{NLO} = A_2^{LO}\left(1 + \frac{15}{8F^2}\overline{A}(M^2)\right) + \lambda_2 M^2 + \mathcal{O}(M^4)\,.$$

One-loop

$$A_0^{NLO} = A_0^{LO}\left(1+\frac{3}{8F^2}\overline{A}(M^2)\right)+\lambda_0 M^2+\mathcal{O}(M^4)\,,$$
$$A_2^{NLO} = A_2^{LO}\left(1+\frac{15}{8F^2}\overline{A}(M^2)\right)+\lambda_2 M^2+\mathcal{O}(M^4)\,.$$

Match with three flavour $SU(3)$ calculation Kambor, Missimer, Wyler; JB, Pallante, Prades

$$A_0^{(3)LO} = -\frac{i\sqrt{6}CF_0^4}{\overline{F}_K F^2}\left(G_8+\frac{1}{9}G_{27}\right)\overline{M}_K^2\,, \qquad A_2^{(3)LO} = -\frac{i10\sqrt{3}CF_0^4}{9\overline{F}_K F^2}G_{27}\overline{M}_K^2\,,$$

When using $F_\pi = F\left(1+\frac{1}{F^2}\overline{A}(M^2)+\frac{M^2}{F^2}l_4^r\right)\,,\ F_K = \overline{F}_K\left(1+\frac{3}{8F^2}\overline{A}(M^2)+\cdots\right)\,,$

logarithms at one-loop agree with above

Semileptonic Decays

- $K \to \ell\nu$: known to order p^6 including isospin breaking and electromagnetic corrections. Also important for lepton-universality tests with $\pi_{e2}/\pi_{\mu2}$ and $K_{e2}/K_{\mu2}$

  talk by Rosell
- $K \to \pi\ell\nu$: known to order p^6, isospin breaking included, electromagnetic corrections also studied in detail

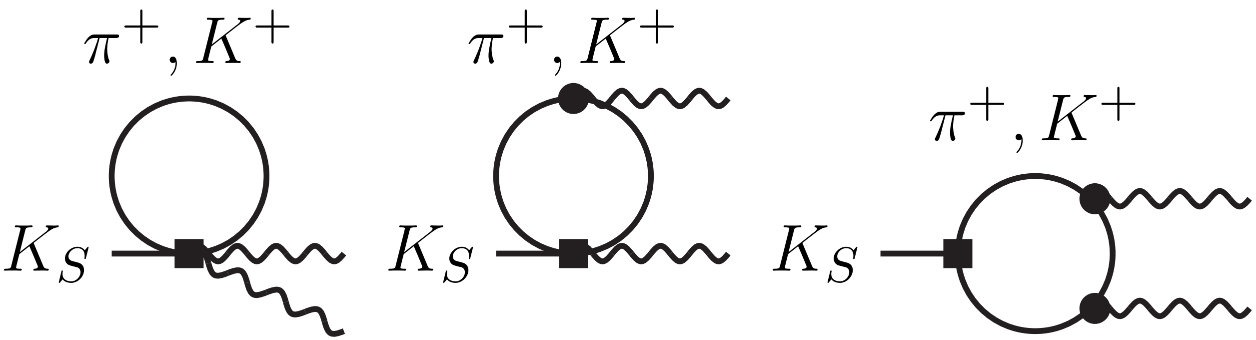
  talk by Neufeld
- $K \to \pi\pi\ell\nu$: F, G and H known to p^6, R only to p^4, isospin breaking studied at one-loop and in nonrelativistic EFT talk by Rusetsky
- $K \to \pi\pi\pi\ell\nu$: known to p^2

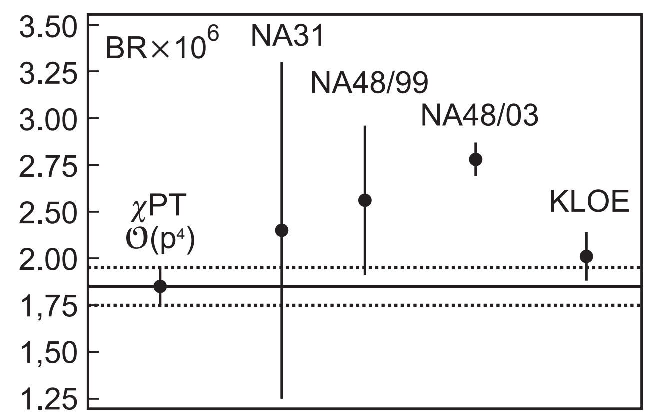
Nonleptonic weak interaction

- Mainly done to one-loop with estimates of higher order corrections
- Big success: prediction of $K_S \to \gamma\gamma$ D'Ambrosio, Espriu, Goity
- Extended to $K \to \pi\ell^+\ell^-$ and $K \to \pi\gamma\gamma$ Ecker, Pich, de Rafael
- Put generally together: Kambor, Missimer, Wyler
- K^0-$\overline{K}^0$, $K \to 2\pi$, $K \to 3\pi$: all done, also including isospin breaking and electromagnetic corrections Kambor, Missimer, Wyler, JB, Pallante, Prades, Dhonte, Borg, Cirigliano, Pich, Ecker
- Already very many parameters at NLO Kambor, Missimer, Wyler, Ecker, Esposito-Farese
- Cusps in $K \to 3\pi$ used for $\pi\pi$ scattering determination Cabbibo, Isidori,... talk by Giudici
- Recent review: D'Ambrosio in EFT09

$K_S \to \gamma\gamma$

Well predicted by CHPT at order p^4 from Goity, D'Ambrosio, Espriu





Prediction was: BR = $2.1 \cdot 10^{-6}$

NA48: $2.78(6)(4) \cdot 10^{-6}$ (PLB 551 2003)
KLOE: $2.26(12)(6) \cdot 10^{-6}$ (JHEP 05 (2008) 051)

No full p^6 calculation exists, FSI effects estimated

Conclusions

- Modern ChPT is doing fine:
- Two flavour ChPT is in good shape: precision science in many ways
- Three flavour ChPT: corrections are larger
  there seem to be some problems, but many parameters (scalar sector) rather uncertain, errors very quantity dependent
- Partially quenched: useful for the lattice
- New application areas continue to be found: examples here RGE and "hard pion ChPT"
- Did not cover isospin breaking: talk by Rusetsky and Neufeld
- Only a very short bit about the weak interaction