Light quark results from a mixed lattice action

Jack Laiho
(work done in collaboration with
Christopher Aubin and Ruth Van de Water)

Workshop on Chiral Dynamics,
University of Bern
July 9, 2009

Mixed action simulations

Simulations use MILC lattices with 2+1 improved (Asqtad) staggered quarks in the sea sector and domain wall quarks in the valence sector.

Advantages

- A large number of ensembles with different volumes, sea quark masses and lattice spacings exist and are publicly available.
- The existing ensembles have 2+1 flavors of light sea quarks ($m_{strange}/10$ for the lightest quarks)
- The good chiral properties of the valence sector make things much simpler than the staggered case. There are only two additional parameters (over pure domain wall) that appear at one loop in the mixed action ChPT for m_π, f_π, and B_K. They can both be obtained from spectrum calculations.
- Non-perturbarbative renormalization can be carried through in the same way as in purely chiral fermion calculations.

Mixed action calculations

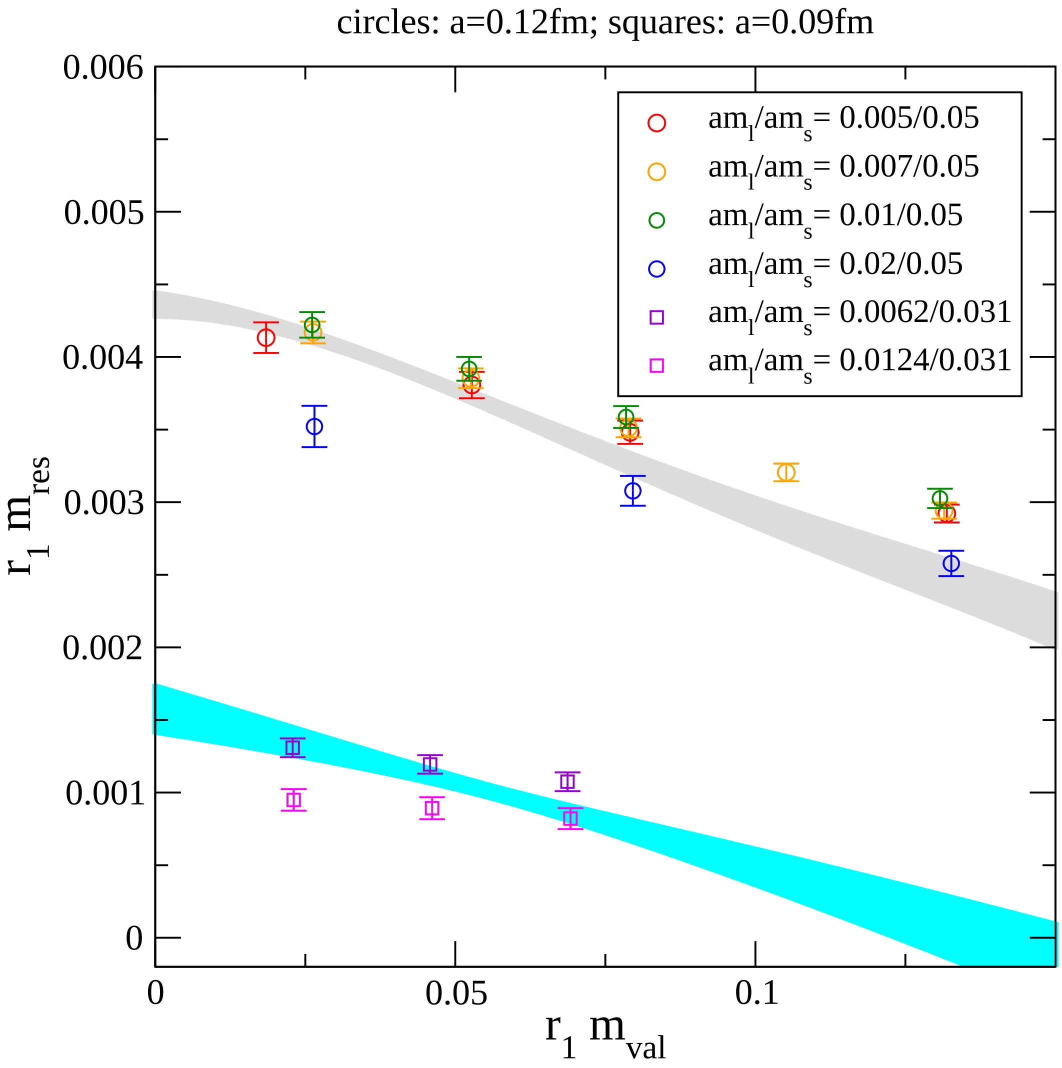
In 1-loop Mixed Action χPT only two parameters beyond those of domain-wall:

$$m_{dw}^2 = 2\mu_{\rm dw}(m_v + m_{\rm res}) \, , \tag{1}$$

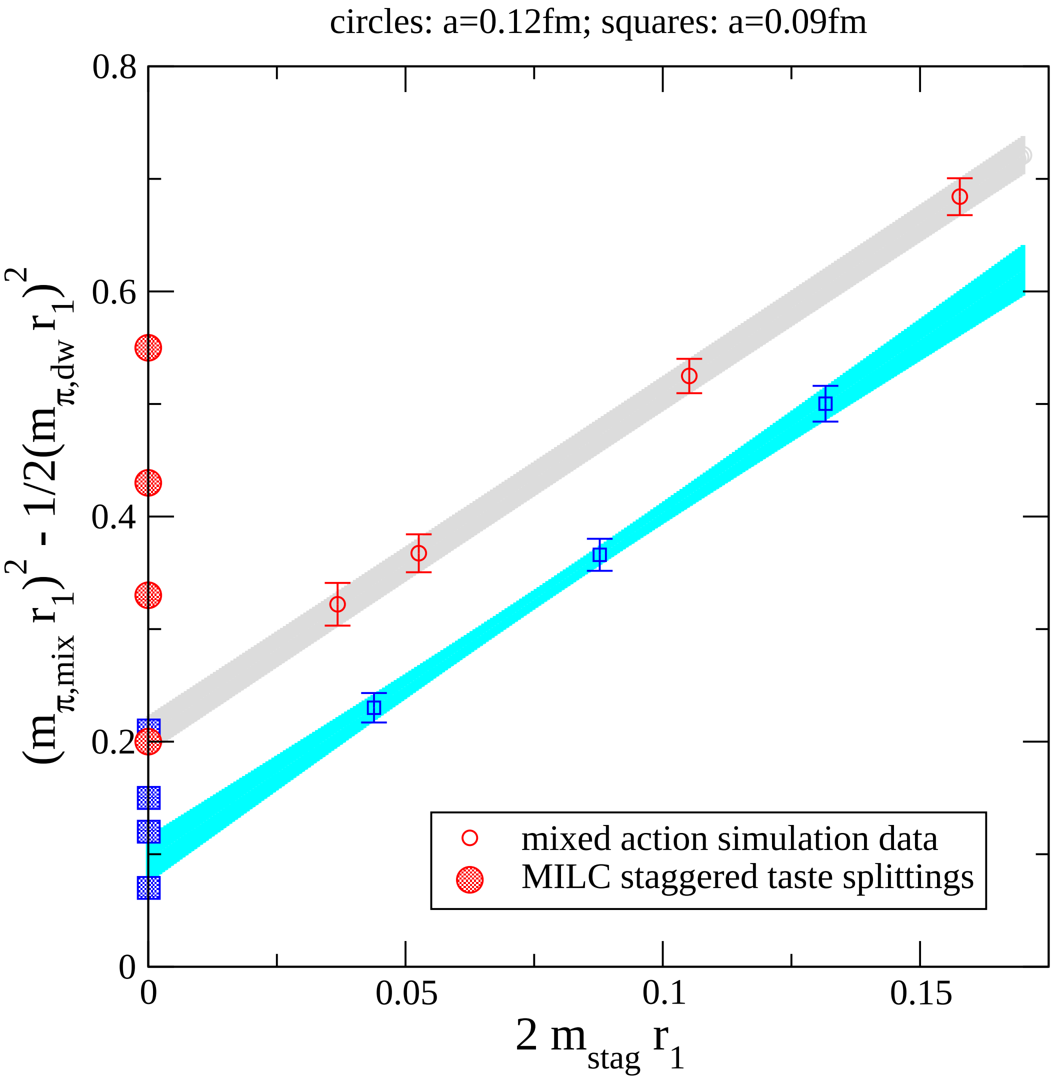
$$m_I^2 = 2\mu_{\rm stag} m_s + a^2 \Delta_I \, , \tag{2}$$

$$m_{mix}^2 = \mu_{\rm dw}(m_v + m_{\rm res}) + \mu_{\rm stag} m_s + a^2 \Delta_{\rm mix}, \tag{3}$$

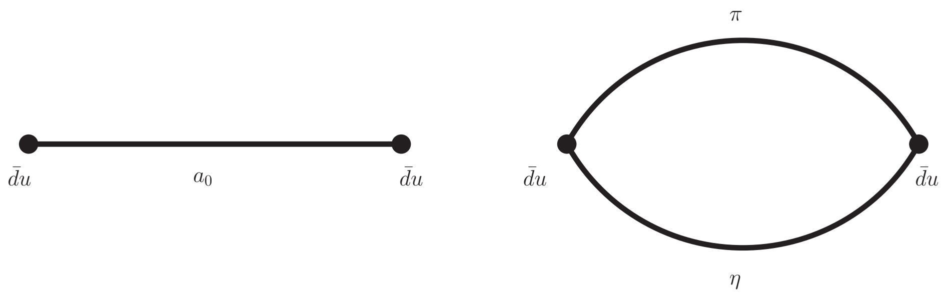
The residual mass



Determining the splitting Δ_{mix}



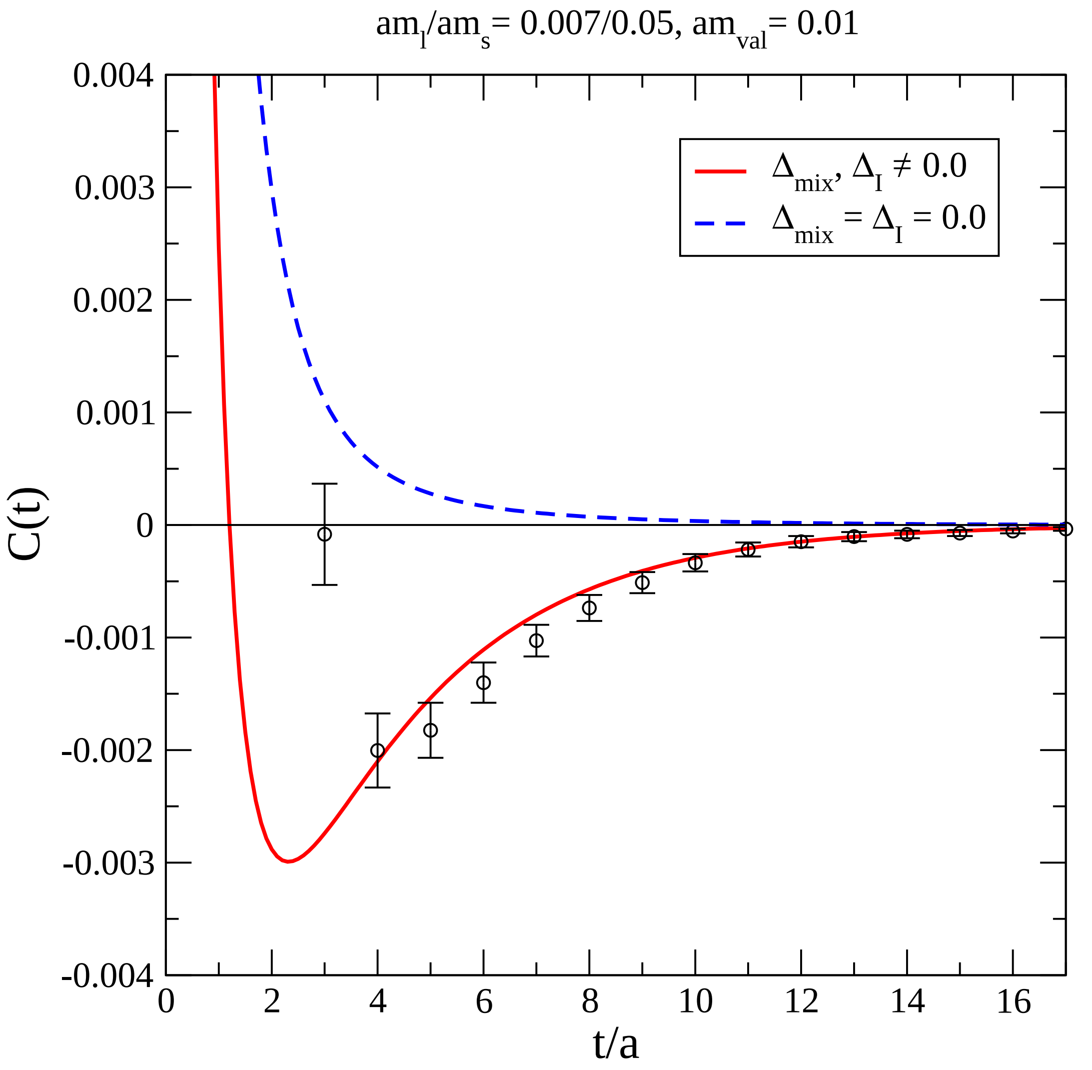
Scalar bubble prediction



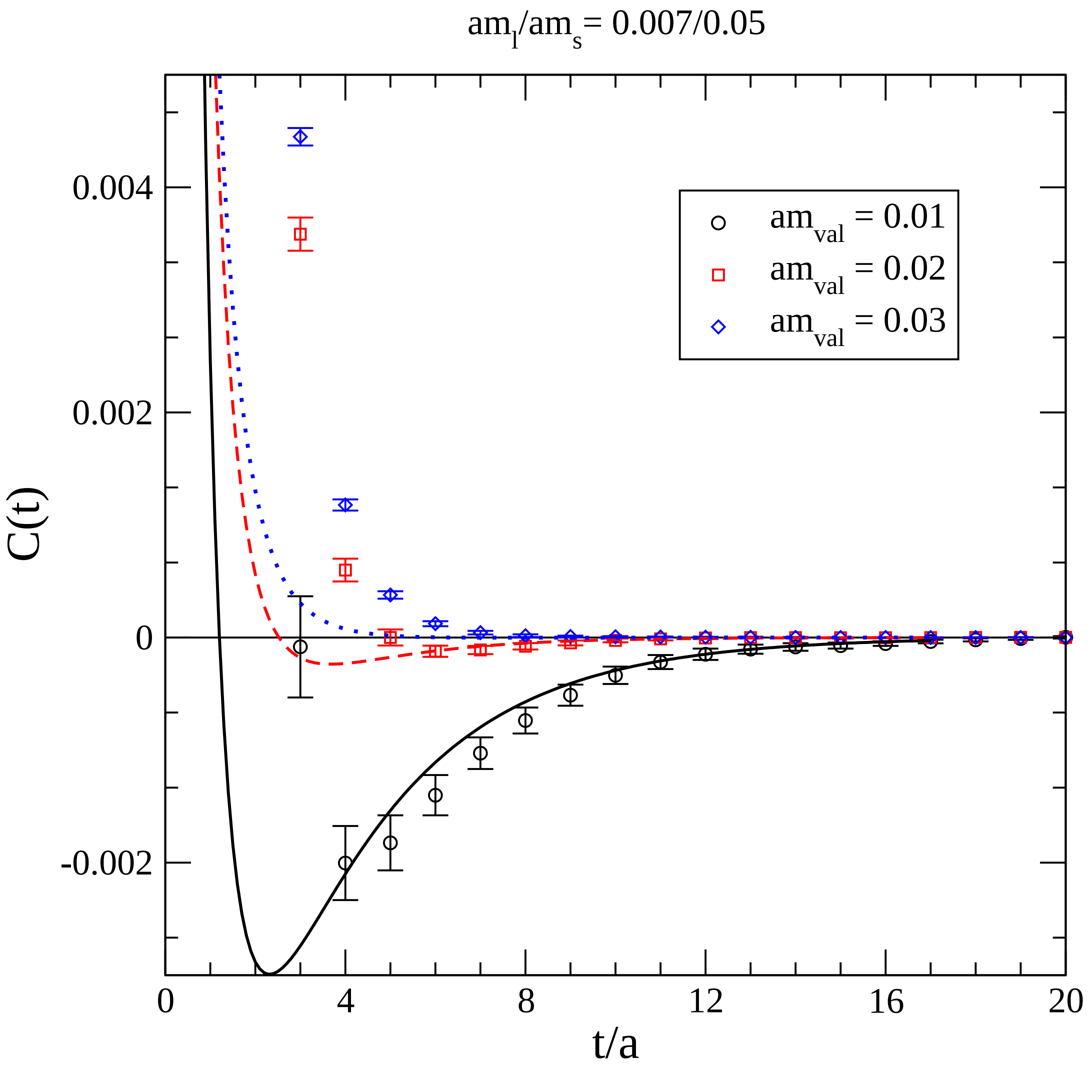
$$\begin{aligned} B(t) \quad = \quad & \frac{\mu^2}{3L^3}\sum_{\mathbf{k}}\Bigg[\frac{2}{9}\,\frac{e^{-(\omega_{vv}+\omega_{\eta_I})t}}{\omega_{vv}\omega_{\eta_I}}\,\frac{(m_{S_I}^2-m_{U_I}^2)^2}{(m_{vv}^2-m_{\eta_I}^2)^2} \\ & -\frac{e^{-2\omega_{vv}t}}{\omega_{vv}^2}\left[\frac{3m_{vv}^2(m_{vv}^2-2m_{\eta_I}^2)+2m_{S_I}^4+m_{U_I}^4}{3(m_{\eta_I}^2-m_{vv}^2)^2}\right] \\ & -\frac{e^{-2\omega_{vv}t}}{2\omega_{vv}^4}\,(\omega_{vv}t+1)\,\frac{(m_{U_I}^2-m_{vv}^2)(m_{S_I}^2-m_{vv}^2)}{m_{\eta_I}^2-m_{vv}^2}+\frac{3}{2}\,\frac{e^{-2\omega_{vu}t}}{\omega_{vu}^2}+\frac{3}{4}\,\frac{e^{-2\omega_{vs}t}}{\omega_{vs}^2}\Bigg] \end{aligned}$$

Prelovsek, PR D73, 014506 (2006), hep-lat/0510080

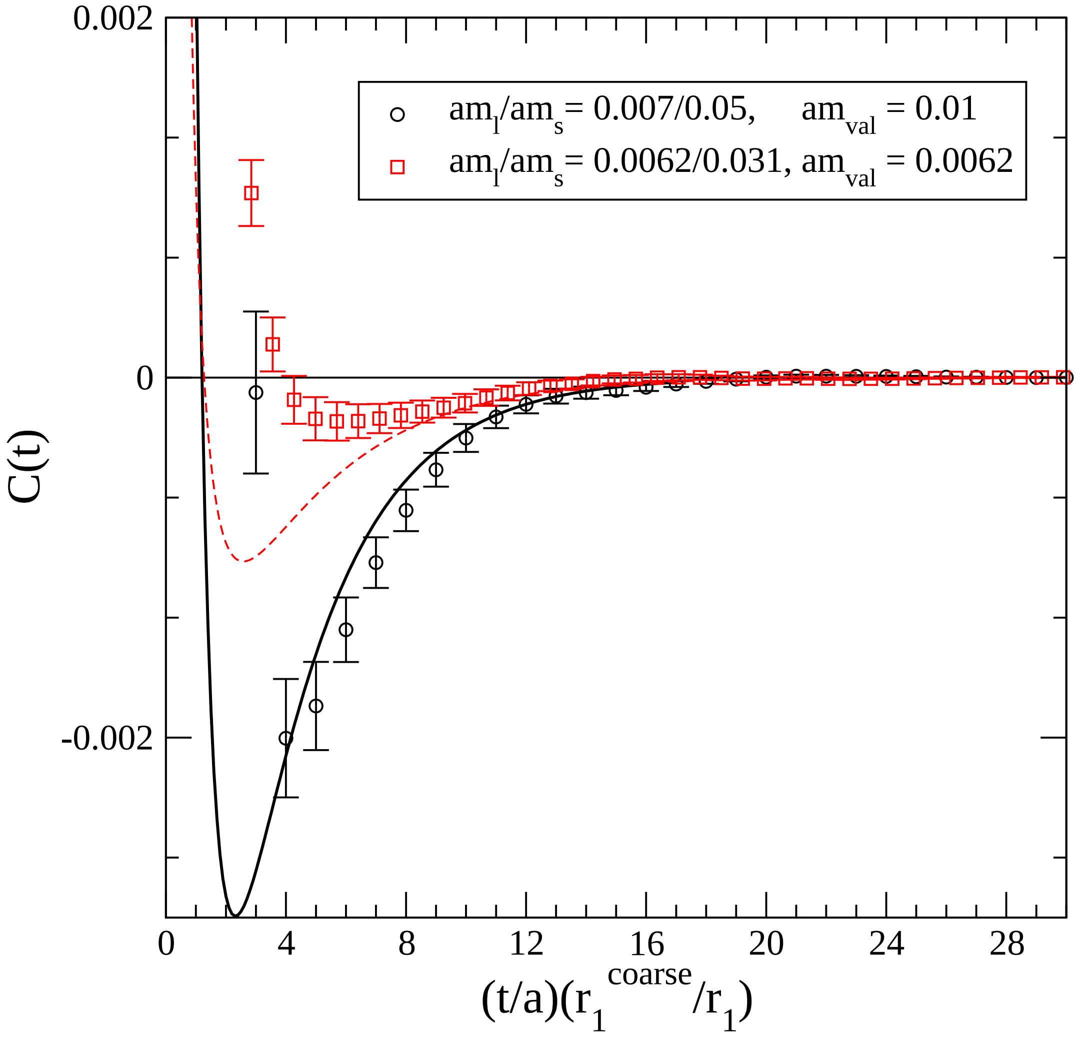
Scalar bubble prediction



Scalar bubble prediction vs. data



Scalar bubble prediction vs. data



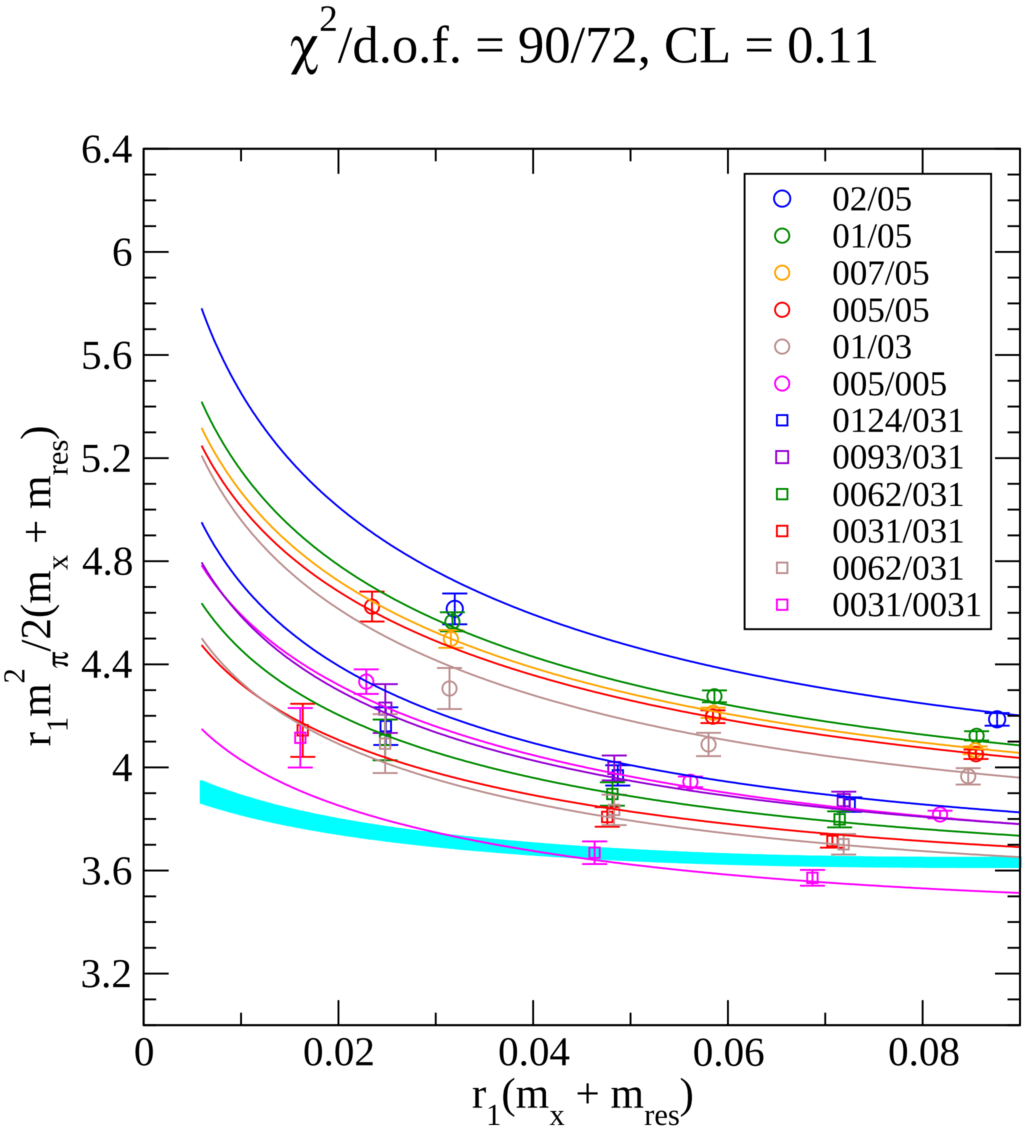
Approach to chiral fits

We have generated data with relatively high statistics so that we can resolve a correlation matrix and obtain reliable confidence levels in fits.

Using SU(3) chiral perturbation theory in order to interpolate about the strange quark mass and extrapolate in the light quark mass. We are using one-loop SU(3) mixed action χPT and higher order analytic terms.

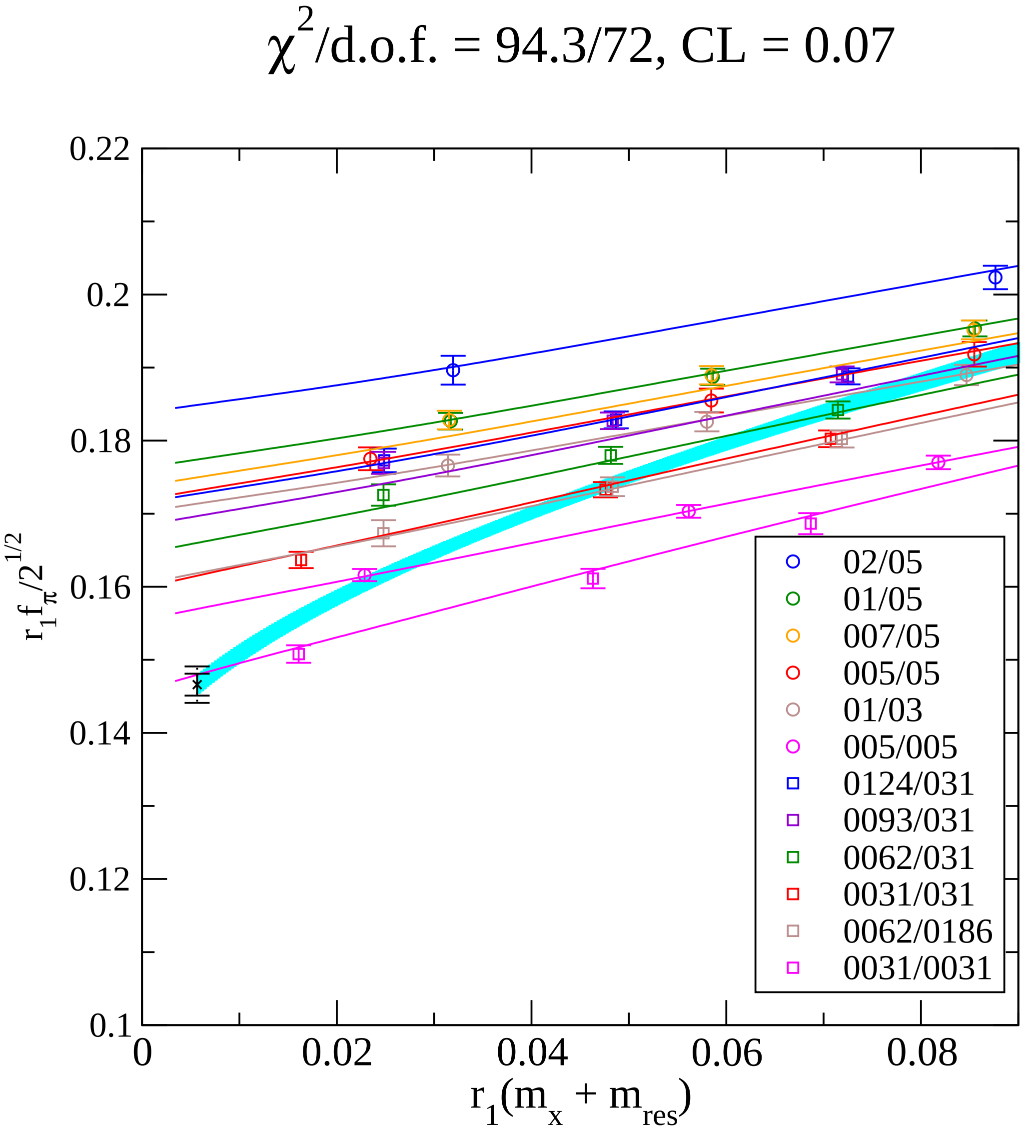
Separate fits to m_π^2/m_q and f_π, where leading order μ is taken from linear fits to m_π^2 data, evaluated in region of data, rather than chiral limit. f_π evaluated at physical pion point.

m_π^2/m_q chiral fit

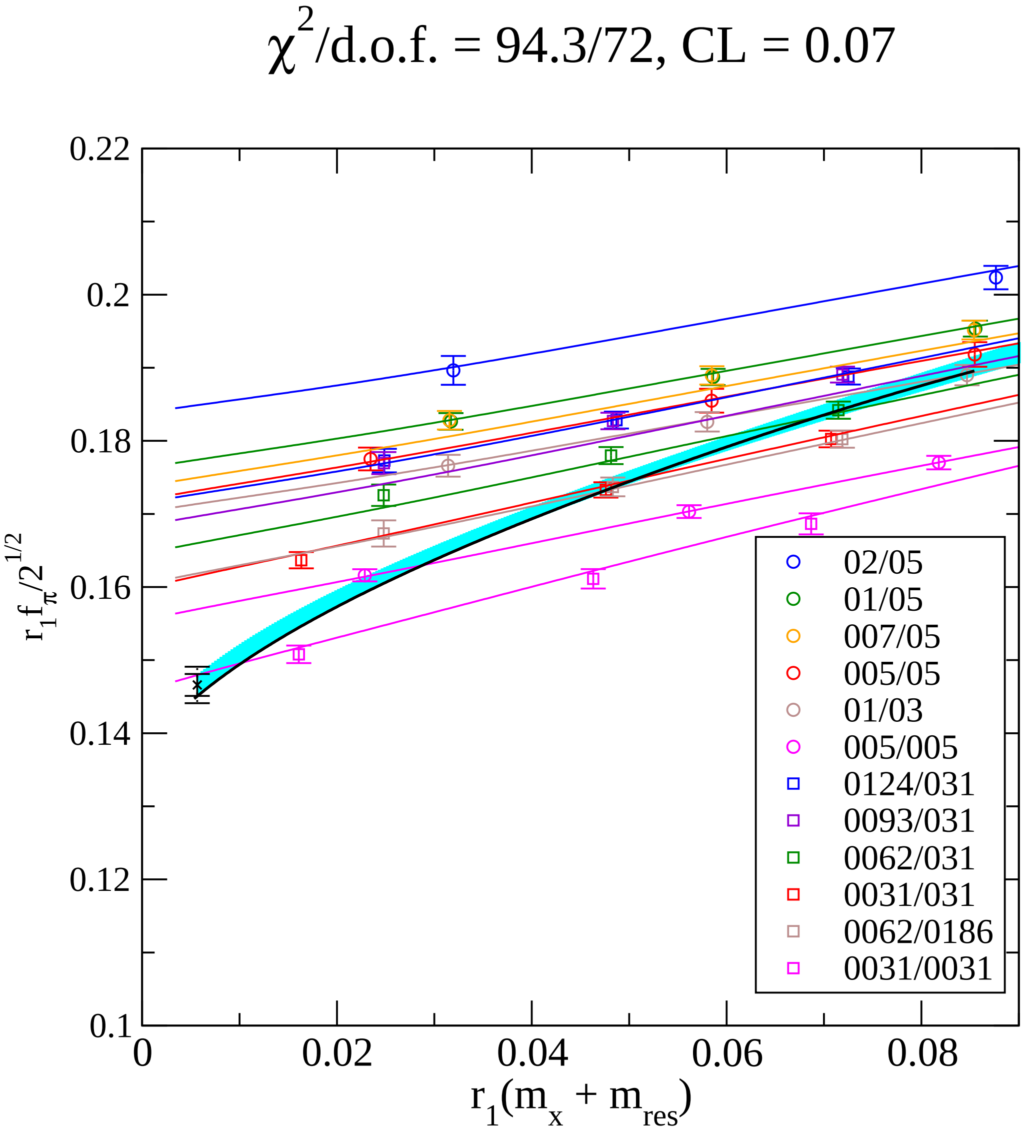


χ^2/d.o.f. = 90/72, CL = 0.11

f_π chiral fit



f_π chiral fit (compared w/ MILC)



χ^2/d.o.f. = 94.3/72, CL = 0.07

Fit results to subset of data

Various “low-mass” fits to NLO χPT + NNLO analytic terms, leaving out different improvements.

type of f_π fit	χ^2/d.o.f.	C.L.
NNLO analytic	0.99	0.54
No NNLO	6.15	9×10^{-41}
No NLO logs	1.22	0.17
No FV	1.34	0.08
No taste-breaking	1.08	0.37

type of m_π^2/m_q fit	χ^2/d.o.f.	C.L.
NNLO analytic	1.12	0.31
No NNLO	6.30	4×10^{-42}
No NLO logs	2.43	4×10^{-7}
No FV	2.34	1×10^{-6}
No taste-breaking	1.50	0.02

Preliminary error budget

Table 1: Uncertainties are shown as percentages.

source	f_K	f_π	f_K/f_π
statistics	0.8	1.0	1.0
input r_1	0.7	0.9	0.3
chiral-continuum extrapolation	0.9	1.1	1.0
finite volume	0.6	0.9	0.9
total error	1.5	2.0	1.7

$$f_\pi = 131.1(13)(22)\ \text{MeV}, \quad f_K = 156.3(13)(20)\ \text{MeV}, \quad f_K/f_\pi = 1.192(12)(16).$$

Preliminary quark masses

Using “partially non-perturbative” method to renormalize quark masses (inspired by Fermilab approach to renormalizing heavy-light currents). The ratio of currents Z_A/Z_S is close to one. The difference from 1 is computed using 1-loop lattice perturbation theory. Z_A is computed non-perturbatively. $Z_m = 1/Z_S$. Quark masses, in $\overline{MS}$ at 2 GeV are:

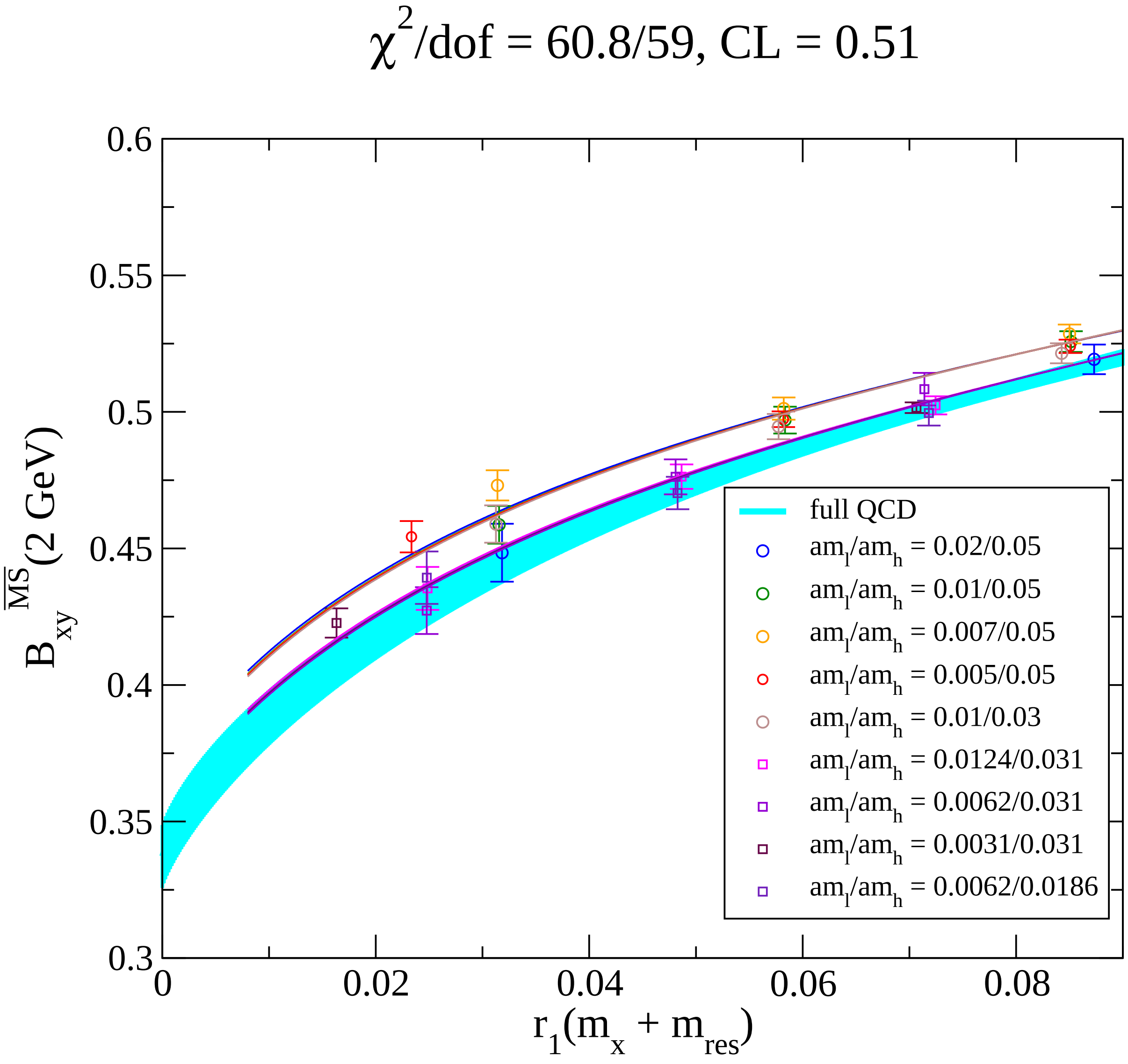
$$\begin{aligned} \widehat{m} &= 3.1(0)(2)(4)(0)\text{MeV}, \\ m_s &= 88(0)(5)(8)(0)\text{MeV}, \\ m_u &= 1.7(0)(2)(2)(1)\text{MeV}, \\ m_d &= 4.4(0)(2)(4)(1)\text{MeV}. \end{aligned} \tag{4}$$

$$\begin{aligned} \frac{m_s}{\widehat{m}} &= 28.9(3)(14)(0)(0), \\ \frac{m_u}{m_d} &= 0.39(1)(3)(0)(4). \end{aligned} \tag{5}$$

Errors are: statistical, lattice systematic, perturbative, electromagnetic.

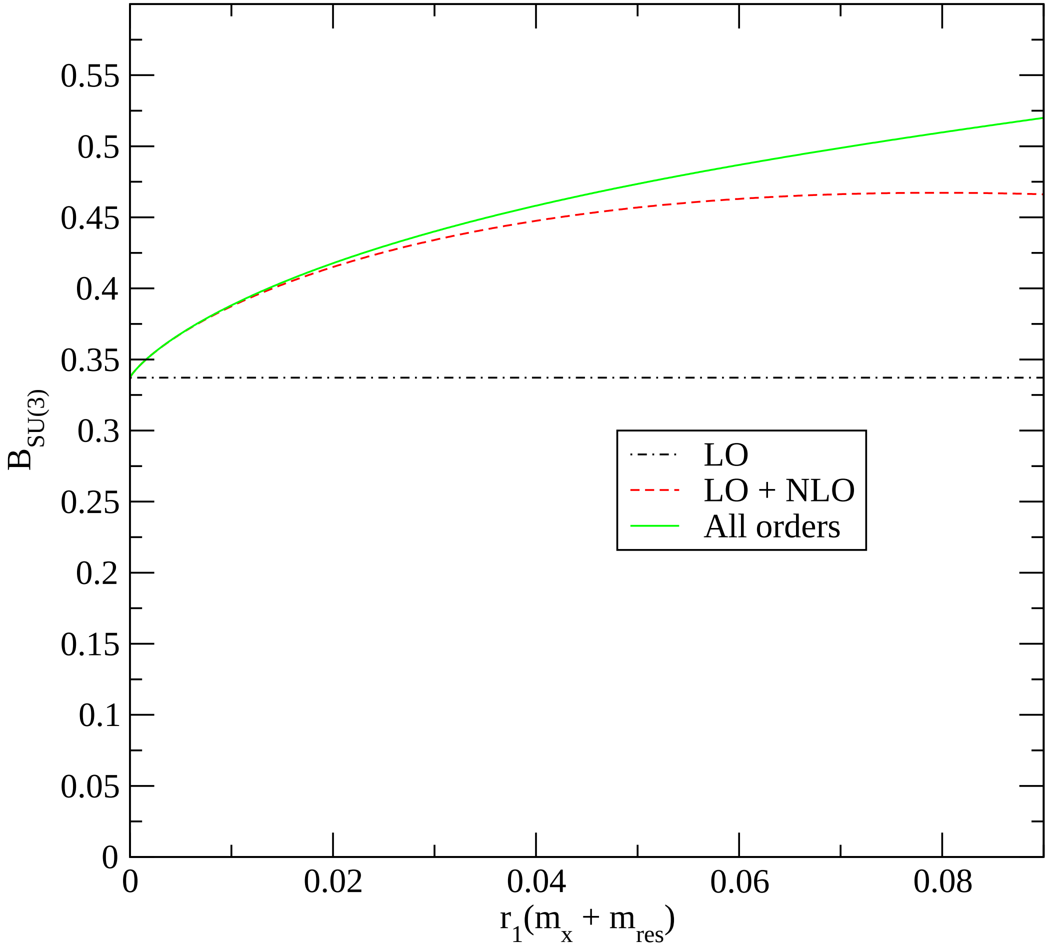
B_K chiral fit

Band is for degenerate valence masses in SU(3) limit



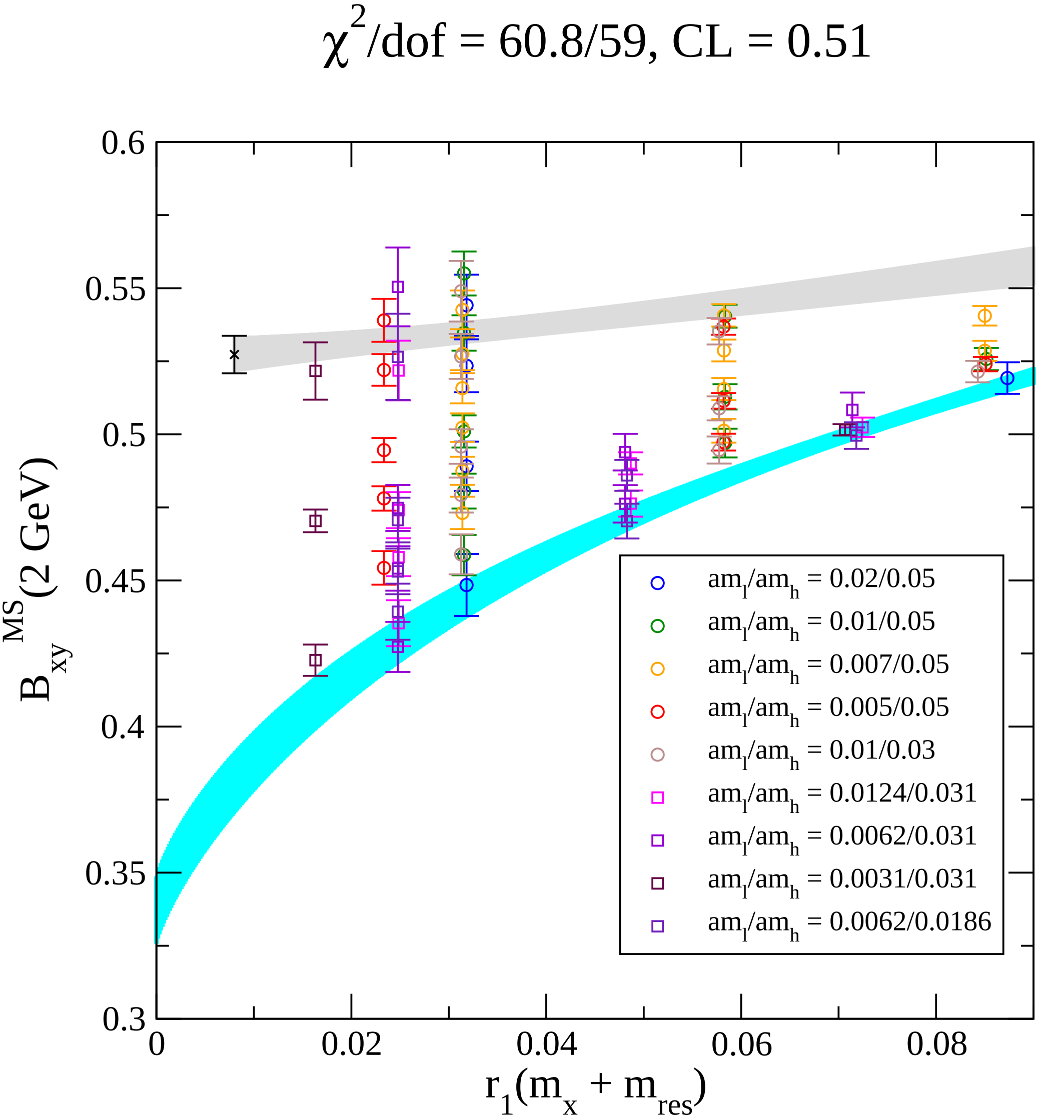
Convergence of ChPT for B_K

Curves are continuum QCD in SU(3) limit

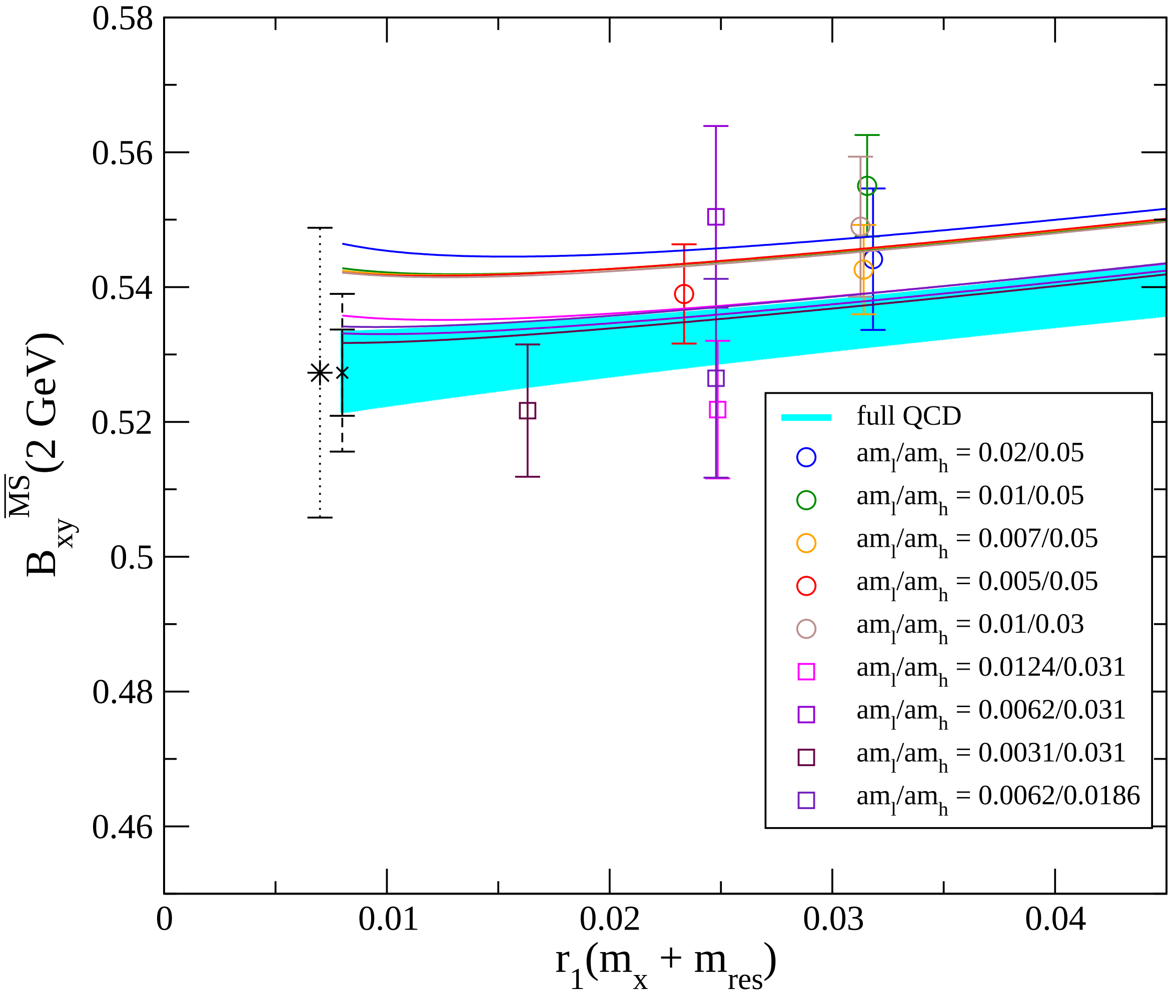


B_K extrapolation to physical point

Grey curve is full QCD at tuned sea and strange masses



B_K extrapolation to physical point



Result for B_K

uncertainty	B_K
statistics	1.2%
chiral & continuum extrapolation	1.9%
scale and quark mass uncertainties	0.8%
finite volume errors	0.6%
renormalization factor	3.3%
total	4.1%

$$\widehat{B}_K = 0.724(8)(28)$$

Compare to $\widehat{B}_K = 0.720(13)(37)$ $[5.6\%$ error] RBC/UKQCD (PRL 100:032001, 2008) and $\widehat{B}_K = 0.83(18)$ $[22\%$ error] HPQCD (PRD 73, 114502, 2006).

Conclusions

Mixed action χPT needed to remove discretization effects and control systematic errors

Many analytic terms necessary to describe large data set with good confidence levels.

Good consistency with MILC calculations on decay constants and quark masses.

Agreement with RBC/UKQCD on B_K.