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Δ(1232) FORM FACTORS AND TRANSVERSE DENSITY DISTRIBUTIONS FROM LATTICE QCD

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with

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NPA825, 115 (2009) & PRD79, 014507 (2009)

Chiral Dynamics

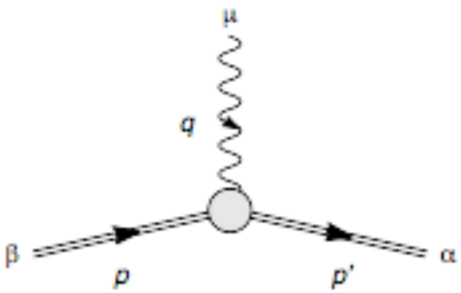
Bern, 5-10 July 2009

Outline

- Introduction
- Density distributions in the infinite momentum frame
- Lattice evaluation of Δ electromagnetic form factors
- Results
- Conclusions

$\gamma^*\Delta\Delta$ form factors

$$\langle \Delta(p',\lambda') \,|\, J^\mu(0) \,|\, \Delta(p,\lambda)\rangle = -\bar{u}_\alpha(p',\lambda') \left\{ \left[F_1^*(Q^2) g^{\alpha\beta} + F_3^*(Q^2) \frac{q^\alpha q^\beta}{(2M_\Delta)^2} \right] \gamma^\mu + \left[F_2^*(Q^2) g^{\alpha\beta} + F_4^*(Q^2) \frac{q^\alpha q^\beta}{(2M_\Delta)^2} \right] \frac{i\sigma^{\mu\nu} q_\nu}{2M_\Delta} \right\} u_\beta(p,\lambda)$$



$$G_{E0} = (F_1^* - \tau F_2^*) + \frac{2}{3}\tau G_{E2},$$

$$G_{E2} = (F_1^* - \tau F_2^*) - \frac{1}{2}(1+\tau)\,(F_3^* - \tau F_4^*)\,.$$

$$G_{M1} = (F_1^* + F_2^*) + \frac{4}{5}\tau G_{M3},$$

$$G_{M3} = (F_1^* + F_2^*) - \frac{1}{2}(1+\tau)\,(F_3^* + F_4^*)\,.$$

At Q^2=0

$$e_\Delta = G_{E0}(0) = F_1^*(0),$$

$$\mu_\Delta = \frac{e}{2M_\Delta} G_{M1}(0) = \frac{e}{2M_\Delta}\left[e_\Delta + F_2^*(0)\right],$$

$$Q_\Delta = \frac{e}{M_\Delta^2} G_{E2}(0) = \frac{e}{M_\Delta^2}\left[e_\Delta - \frac{1}{2}F_3^*(0)\right],$$

$$O_\Delta = \frac{e}{2M_\Delta^3} G_{M3}(0) = \frac{e}{2M_\Delta^3}\left[e_\Delta + F_2^*(0) - \frac{1}{2}\left(F_3^*(0) + F_4^*(0)\right)\right].$$

In terms of the covariant vertex functions a_1, a_2, c_1 and c_2

$F^*_1=a_1+a_2$, $F^*_2=-a_2$, $F^*_3=c_1+c_2$, $F^*_1=-c_2$ ← **these is what we compute on the lattice**

Δ transverse charge densities

Consider a frame where the Δ's have large momentum along (p+p')/2 taken to be the z-axis.
In addition take the baryon light-front + component of q, $q^+=0$ so that the virtual photon has a transverse momentum in the xy-plane

→ $q^2 = \vec{q}_\perp^{\,2} = -Q^2$

→ **virtual photon only couples to forward moving partons**

→ **EM current $j^+(0)$ has the interpretation of a charge density operator:** $\bar{\psi}\gamma^+\psi \propto |\gamma^+\psi|^2$

$$\rho_\lambda^\Delta(b) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\, e^{-i\,\vec{q}_\perp\cdot\vec{b}}\, \frac{1}{2P^+}\langle P^+, \frac{\vec{q}_\perp}{2}, \lambda | J^+ | P^+, \frac{-\vec{q}_\perp}{2}, \lambda\rangle$$
$$= \int_0^\infty \frac{dQ}{2\pi} Q\, J_0(Qb)\, A_{\lambda\lambda}(Q^2).$$

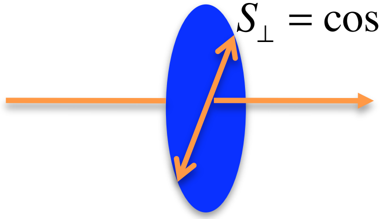
two independent quark densities, λ=3/2 & 1/2

defined in terms of the form factors F*

Δ transverse charge densities

Δ charge densities with transverse spin:

$$\vec{S}_\perp = \cos\varphi\,\hat{e}_x + \sin\varphi\,\hat{e}_y$$



$$\rho^{\Delta}_{T\,s_\perp}(\vec{b}) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\, e^{-i\,\vec{q}_\perp\cdot\vec{b}}\, \frac{1}{2P^+}\langle P^+, \frac{\vec{q}_\perp}{2}, s_\perp \,|\, J^+(0)\,|\, P^+, -\frac{\vec{q}_\perp}{2}, s_\perp\rangle.$$

$$\begin{aligned}\rho^{\Delta}_{T\,\frac{3}{2}}(\vec{b}) = \int_0^{+\infty}\frac{dQ}{2\pi}\,Q \quad \Big[\quad & J_0(Qb)\,\frac{1}{4}\left(A_{\frac{3}{2}\frac{3}{2}} + 3A_{\frac{1}{2}\frac{1}{2}}\right) \\ - \quad & \sin(\phi_b - \phi)\,J_1(Qb)\,\frac{1}{4}\left(2\sqrt{3}A_{\frac{3}{2}\frac{1}{2}} + 3A_{\frac{1}{2}-\frac{1}{2}}\right) \\ - \quad & \cos[2(\phi_b - \phi)]\,J_2(Qb)\,\frac{\sqrt{3}}{2}A_{\frac{3}{2}-\frac{1}{2}} \\ + \quad & \sin[3(\phi_b - \phi)]\,J_3(Qb)\,\frac{1}{4}A_{\frac{3}{2}-\frac{3}{2}}\Big],\end{aligned}$$

With a corresponding expression for $\rho^{\Delta}_{T\frac{1}{2}}(\vec{b})$

ELECTRIC QUADRUPOLE MOMENT

if $\vec{S}_\perp = \hat{e}_x$ $\quad Q^{\Delta}_{s_\perp} \equiv e \int d^2\vec{b}\, (b_x^2 - b_y^2)\, \rho^{\Delta}_{T\, s_\perp}(\vec{b})$

$$\Rightarrow \quad Q^{\Delta}_{\frac{3}{2}} = -Q^{\Delta}_{\frac{1}{2}} = \frac{1}{2}\left\{2\left[G_{M1}(0) - 3e_\Delta\right] + \left[G_{E2}(0) + 3e_\Delta\right]\right\}\left(\frac{e}{M_\Delta^2}\right)$$

For a spin-3/2 particle without internal structure: $G_{M1}(0)=3e_\Delta$ and $G_{E2}(0)= - 3e_\Delta$

→ $Q_{3/2}$ =0

3d-charge distribution with spin along x-axis

$$Q_{3d} \equiv \int dxdydz\, (3x^2 - r^2)\, \rho_{3d}(x,y,z)$$

if invariant around x-axis

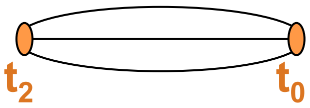
$$Q_{3d} = 2\int dxdydz\, (x^2 - y^2)\, \rho_{3d}(x,y,z)$$

$$\longrightarrow \rho_{2d}(x,y) = \int dz\, \rho_{3d}(x,y,z)$$

with $Q_{2d} \equiv \int dxdy\, (x^2 - y^2)\, \rho_{2d}(x,y)$ $\longrightarrow$ **$Q_{3d}= 2Q_{2d}$**

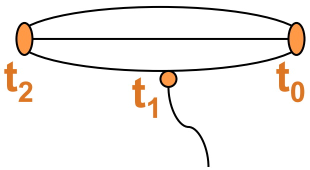
HADRON MASSES AND FORM FACTORS ON THE LATTICE

- **Masses: two-point functions:** $G(\Gamma^\nu, \mathbf{p}, t) = \sum_{\mathbf{x_2}} e^{-i\mathbf{x}_2\cdot\mathbf{p}} \Gamma^\nu \langle J(\mathbf{x}_2, t_2)\bar{J}(\mathbf{x}_0, t_0)\rangle$



- **Form factors, GPDS: three-point functions:**

$$\langle G(t_2, t_1; \mathbf{p}\,', \mathbf{p}; \Gamma^\nu)\rangle = \sum_{\mathbf{x}_2,\, \mathbf{x}_1} e^{-i\mathbf{p}\,'\cdot\mathbf{x}_2} e^{+i(\mathbf{p}\,'-\mathbf{p})\cdot\mathbf{x}_1}\, \Gamma^\nu \langle J(\mathbf{x}_2, t_2)\mathcal{O}(\mathbf{x}_1, t_1)\bar{J}(\mathbf{x}_0, 0)\rangle$$



e.g. $\mathcal{O}$ for EM is $V_\mu(x) = \sum_f q_f \bar{\psi}^f(x)\gamma_\mu\psi(x)^f$

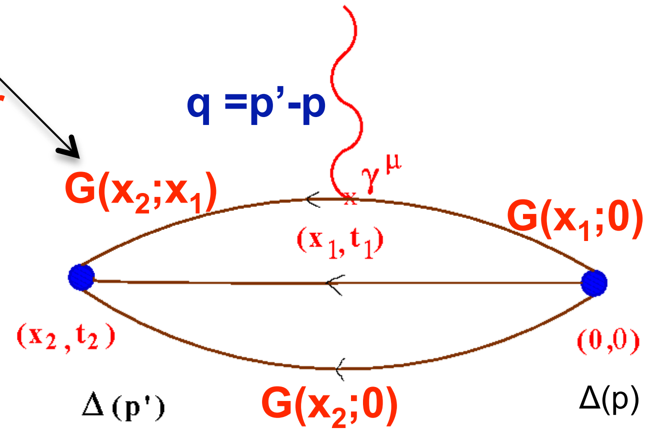
Form factors provide information about the size, magnetization, deformation, density distributions of hadrons.

→ Use lattice to predict Δ form factors – provide input for experiment and phenomenology

Hadron matrix elements on the lattice

all-to-all propagator

$$\sum_{x_1, x_2} e^{-ip'.x_2} e^{iq.x_1}$$

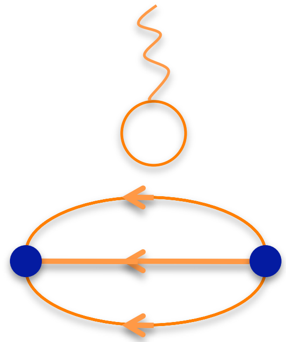


Create from vacuum the correct hadron state: $J^+|\Omega\rangle$

$\langle\Omega|J_\Delta|\Delta\rangle = Z_\Delta u_\tau(p,s)$

where $u_\tau(p,s)$ is a Schwinger-Rarita spinor:: - each vector satisfies the Dirac equation
- and $\gamma_\mu u^\mu(p,s) = 0, \quad p_\mu u^\mu(p,s) = 0$

Disconnected diagram is not included → calculate isovector form factors



For Δ^+: $$J_{\Delta^+}(x) = \frac{1}{\sqrt{3}}\epsilon^{abc}\left\{\left[2\mathbf{u}^{aT}(x)\mathcal{C}\gamma_\tau\mathbf{d}^b(x)\right]\mathbf{u}^c(x) + \left[\mathbf{u}^{aT}(x)\mathcal{C}\gamma_\tau\mathbf{u}^b(x)\right]\mathbf{d}^c(x)\right\}$$

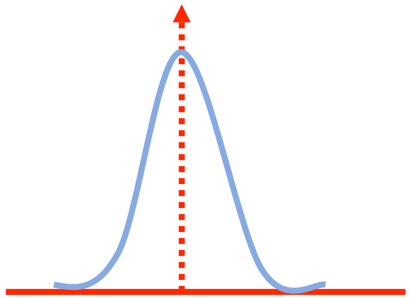
Apply gauge invariant smearing on creation operators to improve ground state dominance

SMEARING

Instead of local source use

$$d^{\mathrm{smear}}(\mathbf{x},t) = \sum_{\mathbf{z}} F(\mathbf{x},\mathbf{z};U(t))\,d(\mathbf{z},t)$$

e.g. Wuppertal smearing



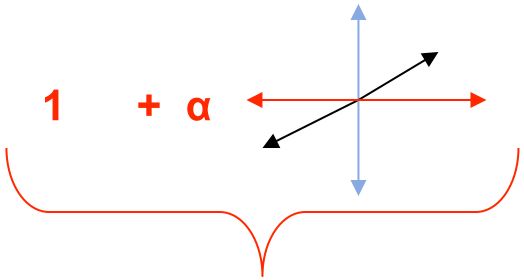
The gauge invariant smearing function can be constructed from the hopping matrix H:

$$F(\mathbf{x},\mathbf{z};U(t)) = (1+\alpha H)^n(\mathbf{x},\mathbf{z};U(t))$$

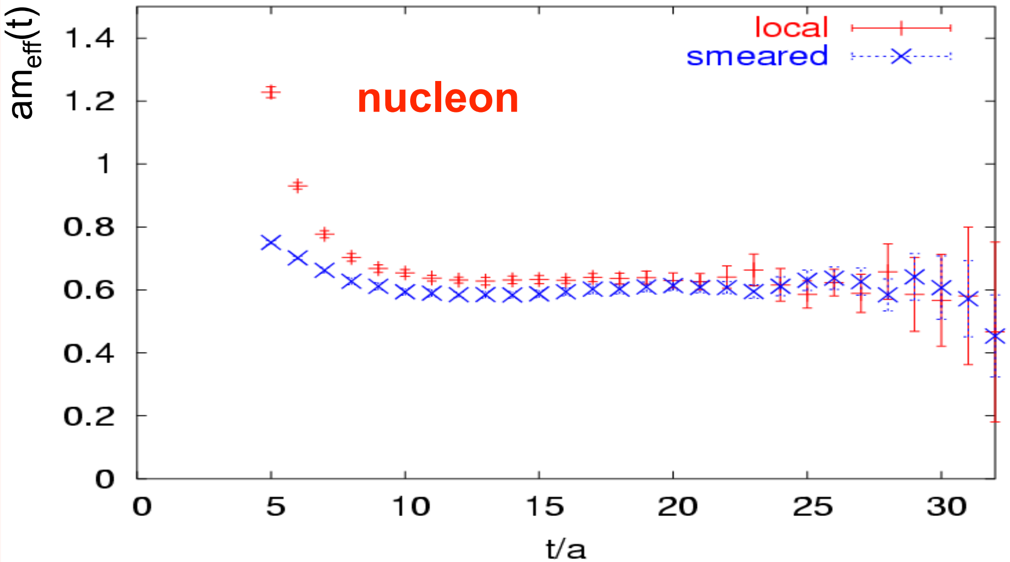
$$H(\mathbf{x},\mathbf{z};U(t)) = \Sigma_{j=1}^{3}\Big(U_j(\mathbf{x})\delta_{\mathbf{x},\mathbf{z}-\hat{j}} + U_j^{\dagger}(\mathbf{x}-\hat{j},t)\delta_{\mathbf{x},\mathbf{z}+\hat{j}}\Big)$$

with optimisation parameters α and n.

n and α smearing parameters optimized for the proton



repeat n-times



α and n fixed by optimizing overlap with nucleon

LATTICE ACTION

- N_F=0 Wilson fermions – a fast way to check the setup on a large, fine lattice
- N_F=2 Wilson fermions
- N_F=2+1 hybrid action for lightest pion mass ~350 MeV

HYBRID ACTION

Improved N_F=2+1 staggered quarks

- Fast to simulate and provided by the MILC collaboration
- Gauge configurations with N_F=2+1 can be downloaded with smallest mass ~300 MeV on (2.5 fm)^3 and (3.5 fm)^3 volumes
- Spectroscopy is however complicated by the four tastes

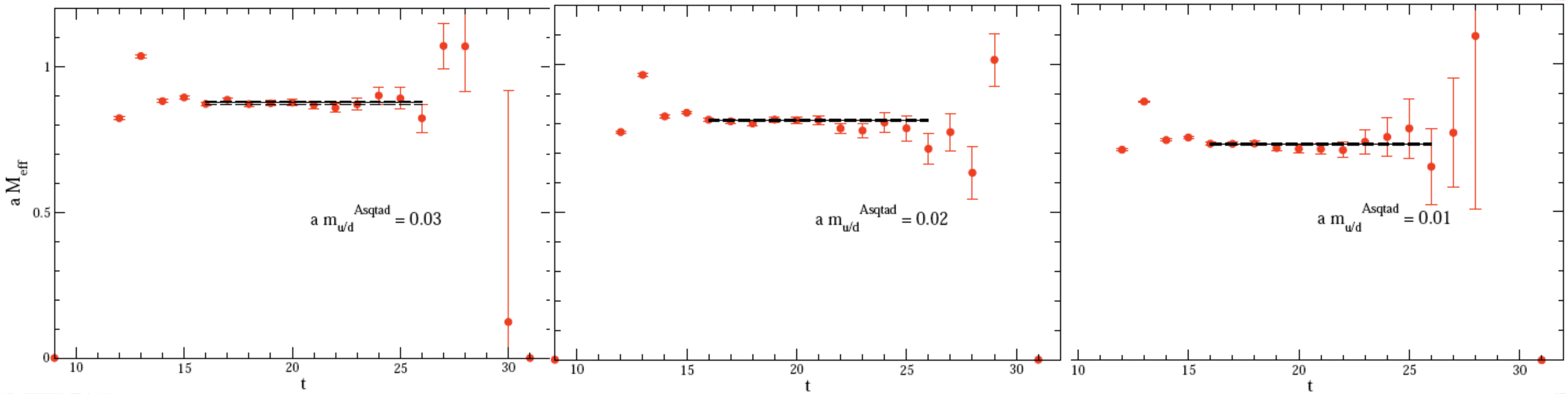
Domain wall valence fermions

- Introduction of a fifth-dimension L_5
- Preserve chiral symmetry even for finite lattice spacing if $L_5 \rightarrow$ infinity
- Tune L_5 so that the explicit symmetry breaking is small → residual mass in the Ward-Takahashi identity is below 10% the quark mass
- Tune the light quark mass to reproduce the Goldstone pion mass obtained with staggered quarks
- Tune the strange quark mass using the N_F=3 simulation and the Goldstone pseudoscalar mass

Ph. Hägler et al., Phys. Rev. D 78 2008

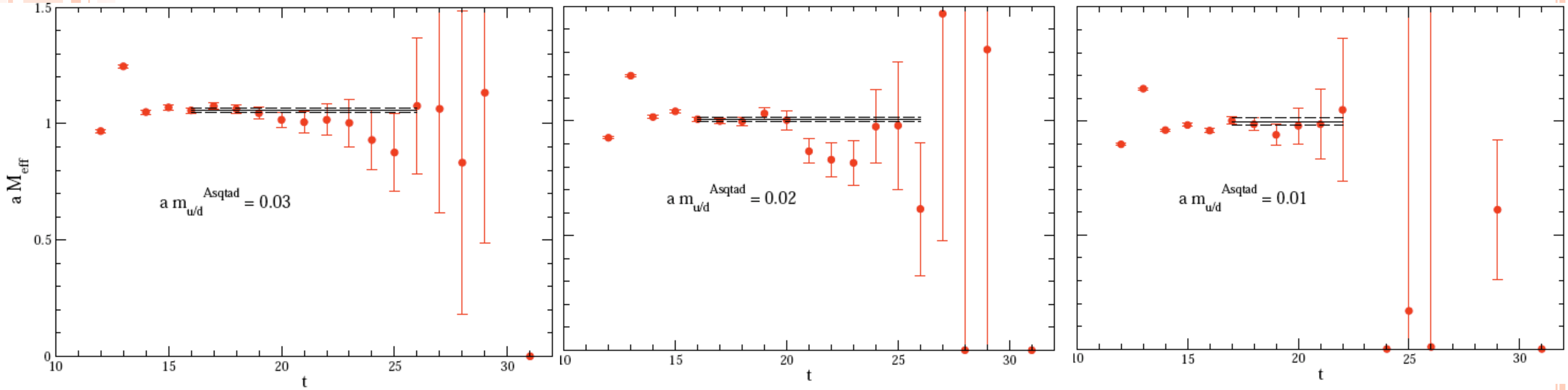
NUCLEON AND DELTA MASS

Nucleon



m_π=596 MeV **m_π=494 MeV** **m_π=355 MeV**

Δ



$m_{eff}(t)=-\log[C(t+1)/C(t)] \rightarrow m$ in the large t-limit when $C(t) \rightarrow Ae^{-mt}$

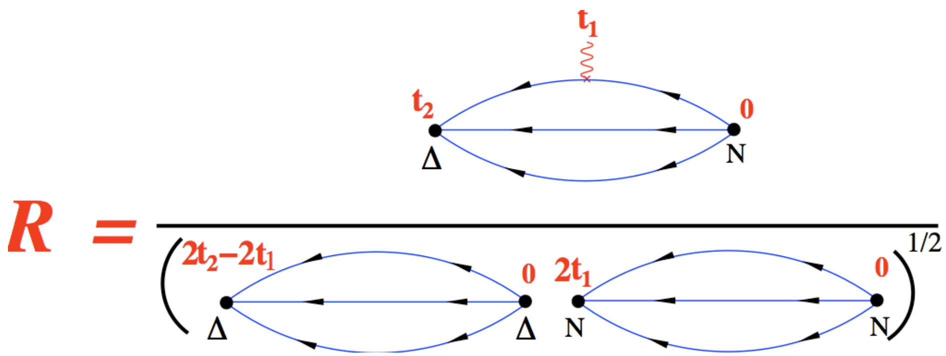
Ratios

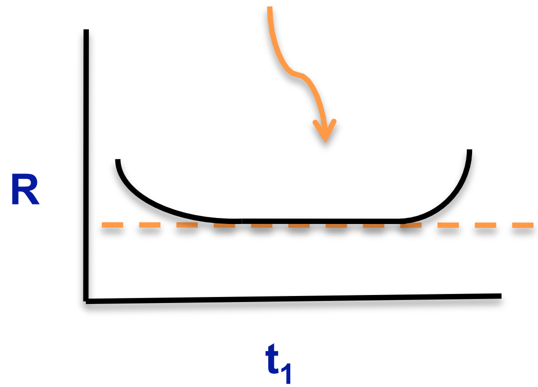
The exponential time dependence and unknown overlaps of the interpolating fields with the physical states cancel by dividing the three-point function with appropriate combinations of two-point functions

e.g.

$$R = \frac{G^{h\mathcal{O}h'}(t_2,t_1;\vec{q})}{\sqrt{G^{h}(2(t_2-t_1);\vec{0})G^{h'}(2t_1;\vec{q})}} \xrightarrow{t_1 \gg 1, t_2 - t_1 \gg 1} < h \,|\, \mathcal{O} \,|\, h' >$$

Smearing enhances ground state dominance i.e plateau for smaller t_1 and t_2-t_1





Optimize R so that two point functions with the shortest possible time separation are involved → less noisy signal

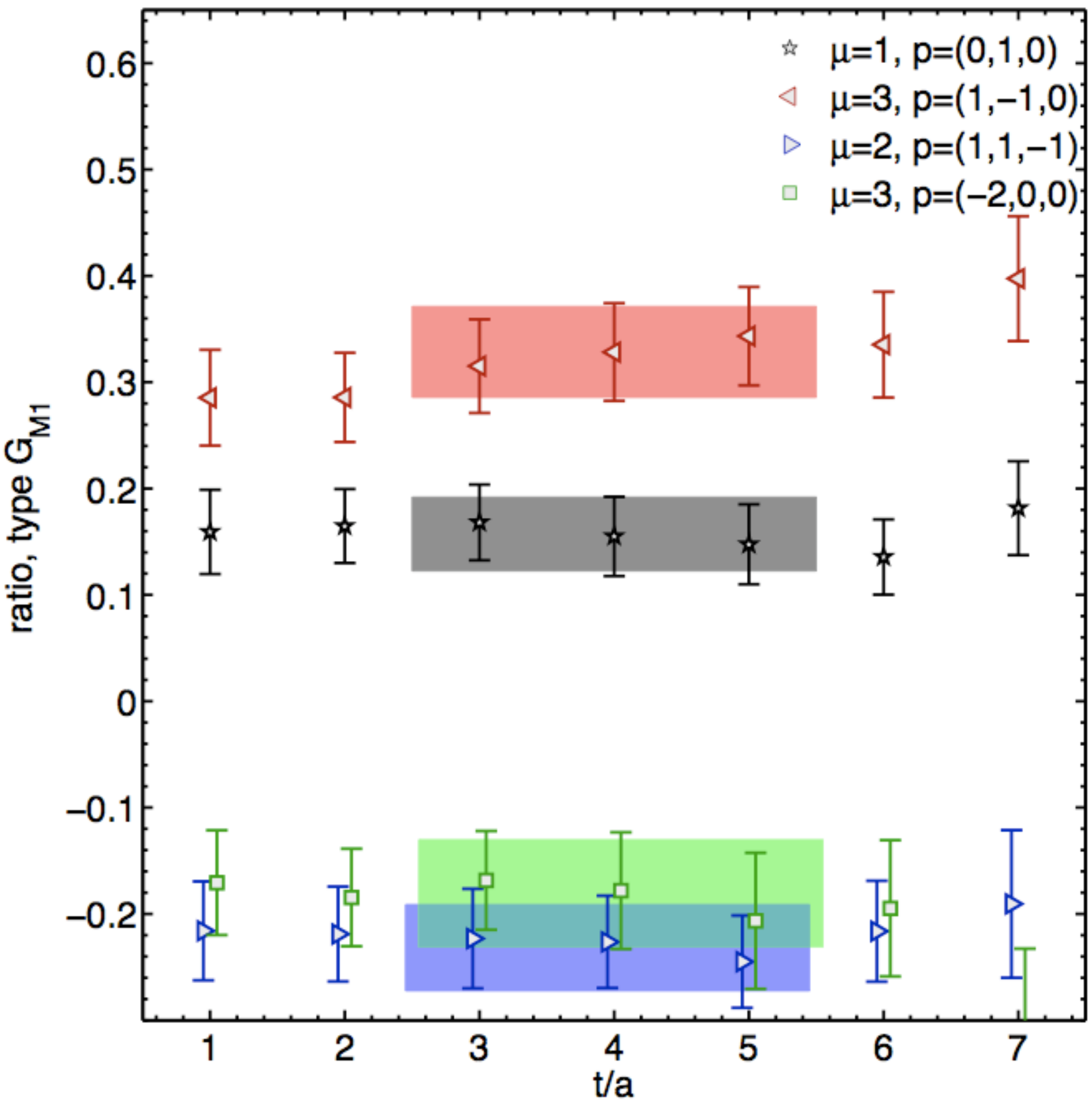
$$R_{\sigma\mu\tau}(\Gamma,\vec{q},t_2,t_1) = \frac{G_{\sigma\mu\tau}(\Gamma,\vec{q},t_1)}{G_{kk}(\Gamma^4,\vec{0},t_2)}\sqrt{\frac{G_{kk}(\Gamma^4,\vec{p}_i,t_2-t_1)G_{kk}(\Gamma^4,\vec{0},t_1)G_{kk}(\Gamma^4,\vec{0},t_2)}{G_{kk}(\Gamma^4,\vec{0},t_2-t_1)G_{kk}(\Gamma^4,\vec{p}_i,t_1)G_{kk}(\Gamma^4,\vec{p}_i,t_2)}}$$

$$\overset{t_2-t_2>>1, t_1>>1}{\Longrightarrow} \boldsymbol{\Pi_{\sigma\mu\tau}(\Gamma,\tilde{q})}$$

$$\Gamma_i = \frac{1}{2}\begin{pmatrix}\sigma_i & 0\\ 0 & 0\end{pmatrix}, \quad \Gamma_4 = \frac{1}{2}\begin{pmatrix} I & 0\\ 0 & 0\end{pmatrix}.$$

Plateau: yields <h|$\mathcal{O}$|h'>

PLATEAUS



Hydrid for pion mass ~350 MeV

EVALUATION OF 3-POINT FUNCTION

Fixed sink

$$D(y_1;y_2)v(y_2)=b(y_1) \quad \rightarrow \quad v(y_1)=D^{-1}(y_1;y_2)b(y_2)$$

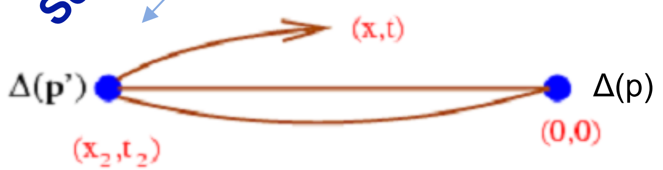
If we take: $b(y_2)=\delta_{y_2,0} \quad \rightarrow \quad v(y_1)=D^{-1}(y_1;0)=G(y_1;0)$ ← **y_1 takes values over all lattice sites**

If we take: $b(y_2;\mathfrak{I}_\alpha)=G(x_2;0)G(x_2;0)\delta_{t,t_2}\,\mathfrak{I}_\alpha$

$$\rightarrow \quad \underbrace{v(y_1;\mathfrak{I}_\alpha)}=\sum_{\vec{x}_2} D^{-1}(y_1;x_2)G(x_2;0)G(x_2;0)\,\mathfrak{I}_\alpha$$

Sequential propagator

Depends on the quantum numbers of the Δ interpolating field i.e. spin index and projection matrices → requires an inversion for each choice



Δ interpolating field is built into the backward sequential propagator with the summation over x_2 and then combine with a photon of any momentum. The final summation over x_1 is done as the last step.

Important to select the appropriate combinations for Δ interpolating field

Δ ELECTROMAGNETIC FORM FACTORS

The goal is to extract G_{E0}, G_{M1}, the dominant form factors but also the subdominant G_{E2} connected to an intrinsic Δ quadrupole moment. Determination of G_{M3} is a bonus.

Choose suitable sink combination since for each a sequential inversion is required:

Example: to isolate G_{M1} one can calculate $\Pi_{1\mu 2}(\Gamma^4,\vec{q}) = \mathcal{A}\left(q_1 - q_2\right)\delta_{\mu,3} G_{M1}$

→ But there is only contribution for μ=3 and momenta in x & y directions

Better to choose:

$$\sum_{j,k,l=1}^{3} \epsilon_{jkl}\Pi_{j\mu k}(\Gamma^4,\vec{q}) = \mathcal{A}\, G_{M1}\left[\delta_{1,\mu}(q_3 - q_2) + \delta_{2,\mu}(q_1 - q_3) + \delta_{3,\mu}(q_2 - q_1)\right]$$

this is built into the Δ- sink and requires one inversion

Other optimal combinations:

$$\sum_{k=1}^{3} \Pi_{k\mu k}(\Gamma^4,\vec{q}) \Longrightarrow G_{E0}, G_{E2}$$

$$\sum_{j,k,l=1}^{3} \epsilon_{jkl}\Pi_{jnk}(\Gamma^j,\vec{q}) \Longrightarrow G_{M1}, G_{E2}, G_{M3}$$ **and** $$\sum_{\mathbf{j,k,l=1}}^{\mathbf{3}} \epsilon_{\mathbf{jkl}}\mathbf{\Pi_{j4k}}(\mathbf{\Gamma^j},\tilde{\mathbf{q}}) \Longrightarrow G_{E2}$$

With three inversions we get G_{E0}, G_{M1}, G_{E2} optimally

SIMULTANEOUS OVERCONSTRAINED ANALYSIS

In our analysis all the lattice momentum vectors contributing to a given Q^2 are taken in to account. The overdetermined set of equations to be solved are:

$$S(\vec{q};\mu) = A(\vec{q};\mu).F(Q^2) \qquad F = \begin{pmatrix} \mathcal{G}_{E0} \\ \mathcal{G}_{E2} \\ \mathcal{G}_{M1} \\ \mathcal{G}_{M3} \end{pmatrix}$$

Lattice measurements of the transition matrix elements

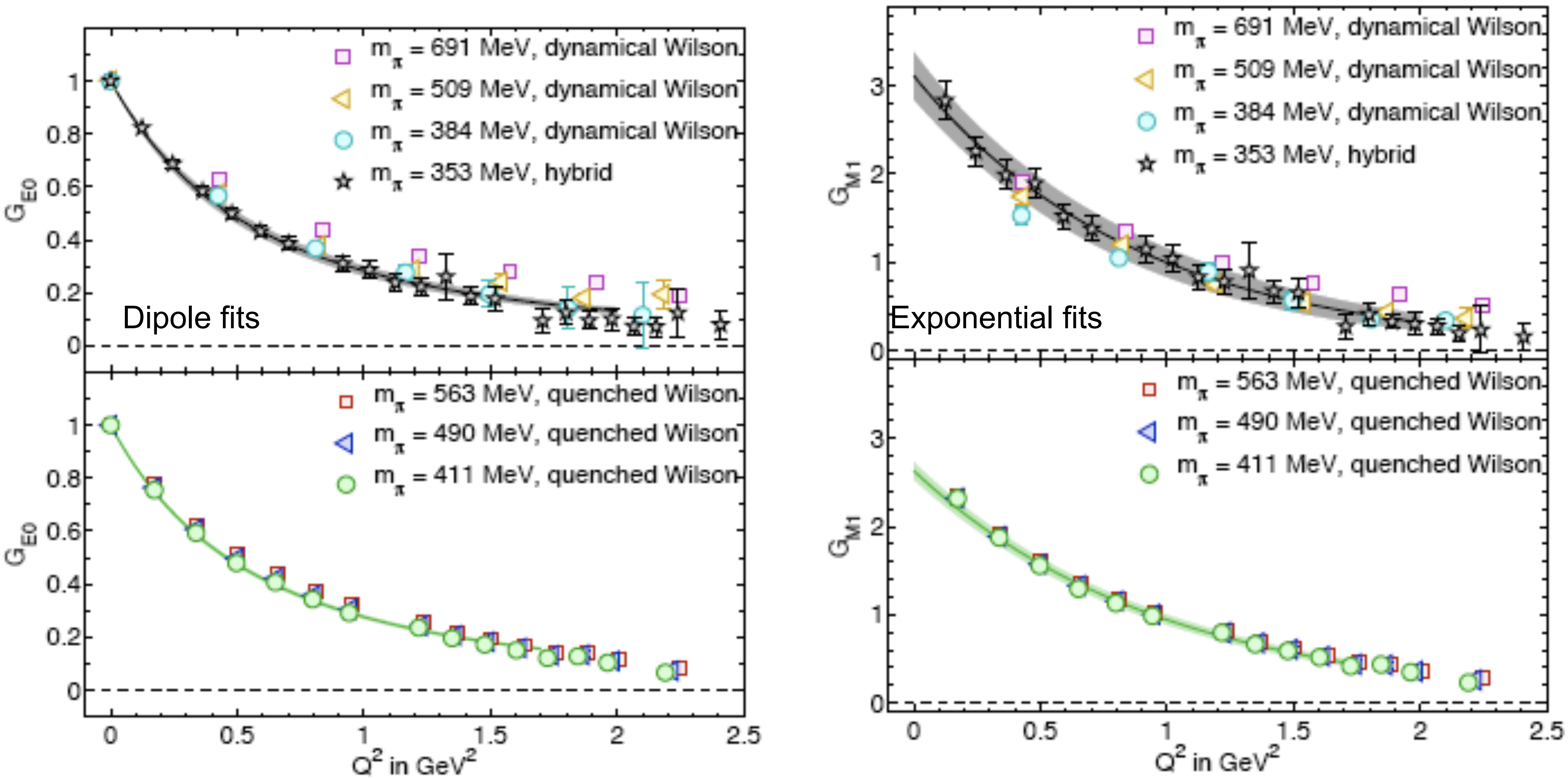
If the number the of current directions μ and the number of momentum vectors contributing to a given Q^2 is N then A is an Nx4 matrix

We solve for the form factors by minimizing χ^2

$$\chi^2 = \sum_{k=1}^{N} \left(\frac{\sum_{j=1}^{4} A_{kj} F_j - S_k}{\sigma_k} \right)^2$$

using the singular value decomposition of A.

DOMINANT FORM FACTORS

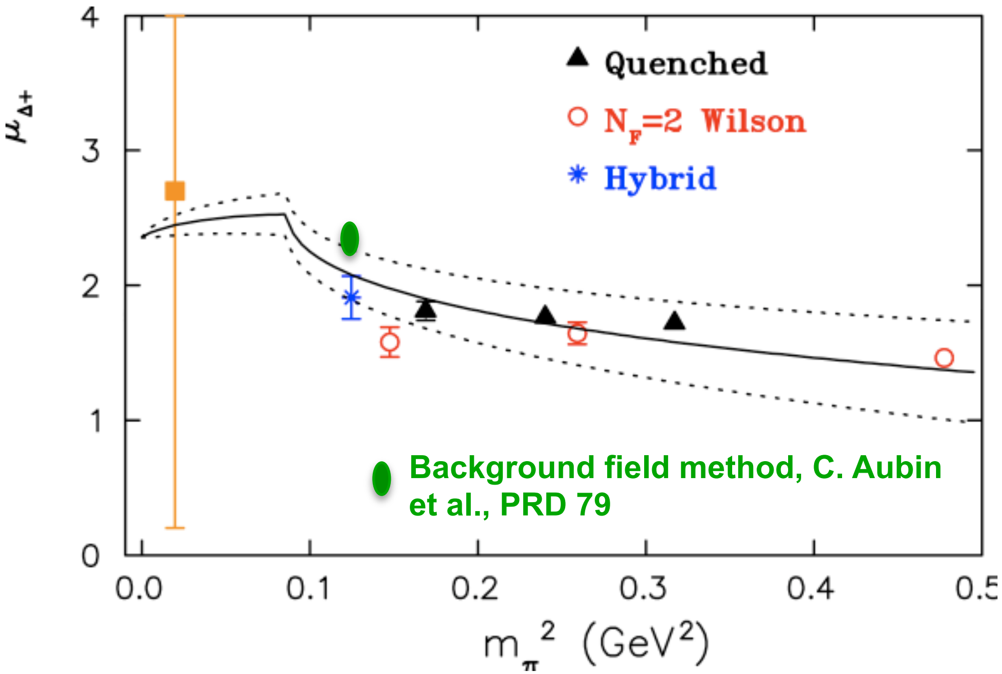


C. A., T. Korzec, G. Koutsou,G. Koutsou, C. Lorec, J. W. Negele, V. Pascalutsa, A. Tsapalis, M. Vanderhaeghen, PRD79:014507(2009)

MAGNETIC MOMENT

NLO relativistic chiral effective field theory in δ-expansion:
$(m_\Delta - m_N)/\Lambda$ counts as one power of δ; (m_π/Λ) counts as two powers of δ

V. Pascalutsa and M. Vanderhaeghen, PRL 94 (2005)



- **Only an overall constant is fitted**
- **The error band is an estimate of the uncertainty in the chiral expansion**

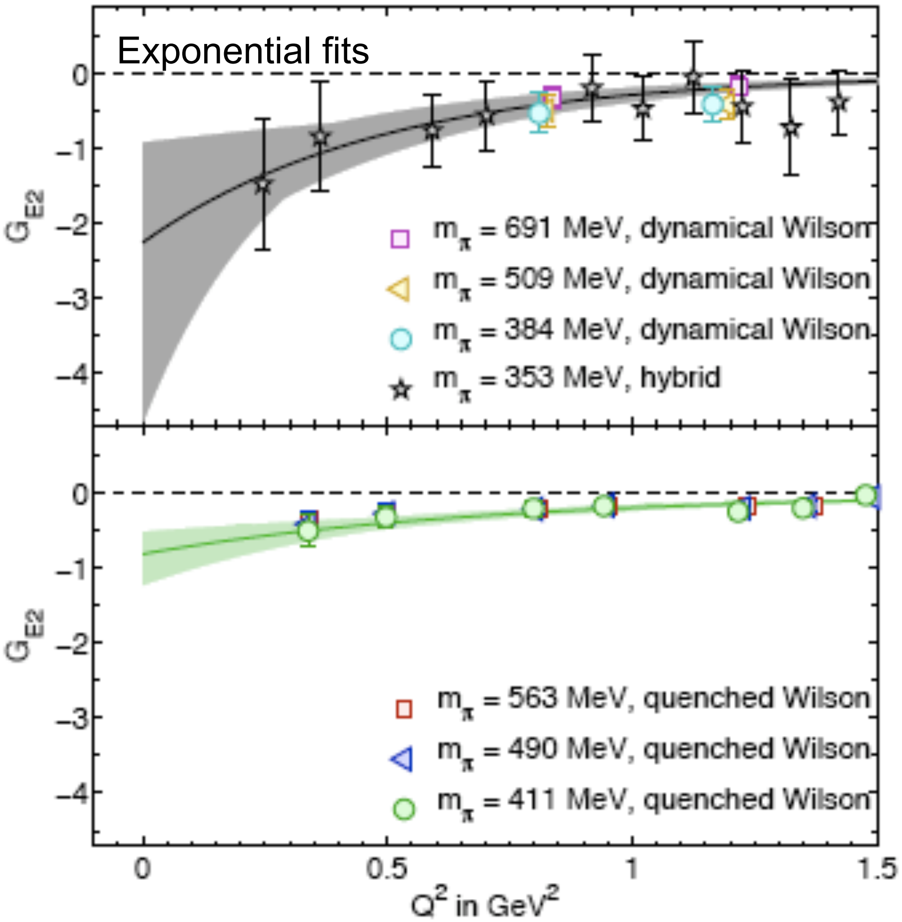
→ talk by Pascalutsa 16:55 WG2

Magnetic moment using the background field method:

Constant magnetic field , N_F=2+1 dynamical Clover fermions

Measure change in mass

Δ ELECTRIC QUADRUPOLE FORM FACTOR

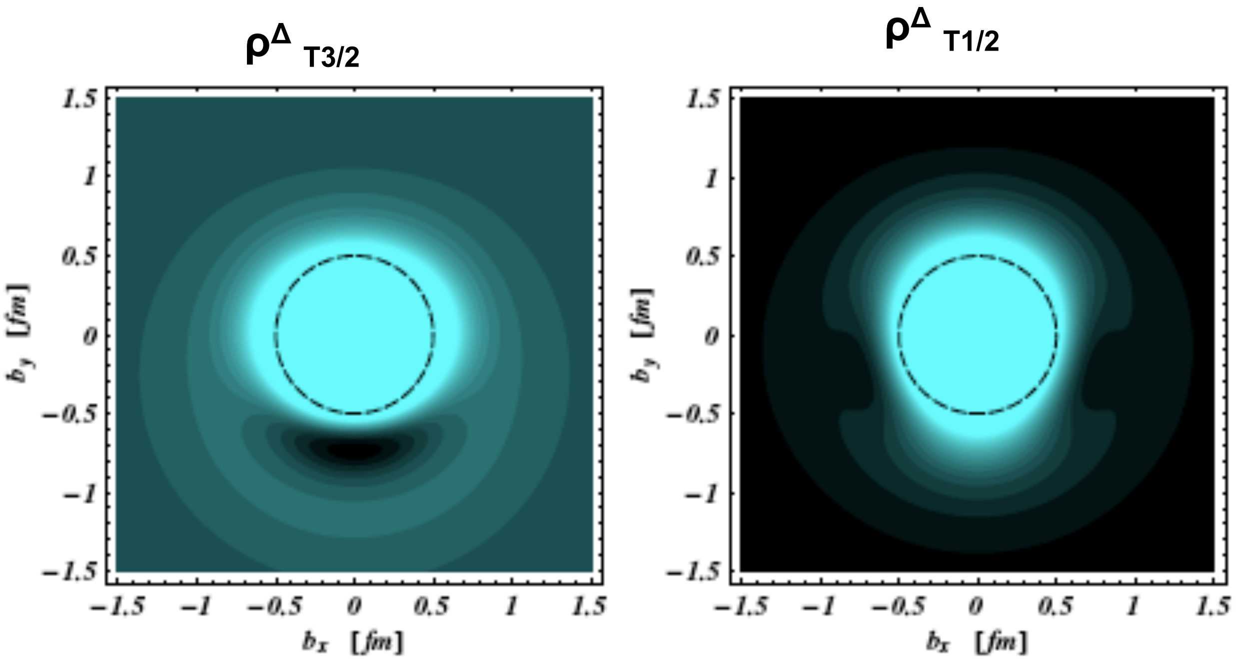


G_{M3} consistent with zero

Also P. Moran et al. [Adelaide], quenched results

QUARK CHARGE DENSITIES

Quark transverse charge densities in Δ^{+} polarized along the x-axis extracted from lattice data



Cyprus-MIT/Mainz

Δ with spin projection 3/2 elongated along spin axis

Δ with spin projection ½ elongated perpendicular to spin axis

C.A., T. Korzec, G. Koutsou, C. Lorce, J.W. Negele, V. Pascalutsa, A. Tsapalis, M. Vanderhaeghe, NPA825, 115 (2009)

CONCLUSIONS

- Improved techniques can yield the subdominant form factors:

  → the Δ electric quadrupole is non-zero

  Can use input from lattice to evaluate the transverse density distribution → well defined in the infinite momentum frame

  → Δ in +3/2 projection prolate

- Calculation of axial form factors requires no new inversions – will yield the Δ axial coupling

THANK YOU FOR YOUR ATTENTION

SPIN-3/2 POINT PARTICLE

$$
\begin{aligned}
\mathcal{L} &= \bar{\psi}_\mu \gamma^{\mu\nu\alpha}(i\partial_\alpha - eA_\alpha)\psi_\nu - m\bar{\psi}_\mu\gamma^{\mu\nu}\psi_\nu + em^{-1}\bar{\psi}_\mu(i\kappa_1 F^{\mu\nu} - \kappa_2\gamma_5\tilde{F}^{\mu\nu})\psi_\nu \\
\gamma^{\mu\nu} &= \frac{1}{2}[\gamma^\mu,\gamma^\nu], \qquad \gamma^{\mu\nu\alpha} = \frac{1}{2}\{\gamma^{\mu\nu},\gamma^\alpha\} \\
F^{\mu\nu} &= \partial^\mu A^\nu - \partial^\nu A^\mu, \\
\tilde{F}^{\mu\nu} &= \epsilon^{\mu\nu\rho\alpha}\partial_\rho A_\alpha
\end{aligned}
$$

It describes a spin-3/2 particle via the Rarita-Schwinger field ψ_ν with mass m coupled to the electromagnetic field A_μ via the minimal coupling and two non-minimal couplings κ_1 and κ_2

Adding gravity in a supersymmetric way to cure pathologies, constrains the non-minimal couplings:

$$\kappa_1=\kappa_2=1$$

and gives

$$\mathbf{G_{E0}(0) = 1,\ \ G_{M1}(0) = 3,\ \ G_{E2}(0) = -3,\ \ G_{M3}(0) = -1}$$