Lattice Study of χPT Beyond QCD

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Outline

Lattice Strong Dynamics (LSD) Collaboration

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Introduction and Motivation

- "Beyond QCD": Yang-Mills gauge theory, but varying the number of colors N_c, number of light fermions N_f, and fermion representation.
- Why? Many models for physics beyond the Standard Model involve strongly-interacting gauge theories (see plenary by G. Isidori.) But most existing analysis relies heavily on QCD phenomenology.
- Strongly coupled YM gauge theories can look very different from QCD in some cases - no confinement, no spontaneous χSB! [1] [2]
- Fix $N_c = 3$, fundamental representation, study what happens as we change N_f.

[1] W. Caswell, Phys. Rev. Lett. 33:244,1974

[2] T. Banks and A. Zaks, Nucl. Phys. B 196:189, 1982

Technicolor and Extended Technicolor

- Technicolor: replace the Higgs mechanism with new strong interaction. Electroweak symmetry is spontaneously broken.
- Simplest example: scaled-up QCD ($N_{TC} = 3, N_{TF} = 2.$) χSB gives three Goldstone bosons which are eaten by the W/Z.
- To give particle masses with no Higgs, include additional fields and interactions called extended technicolor (ETC)[3].
- Problem: ETC interactions responsible for mass generation can lead to large flavor-changing neutral currents! E.g. quark masses $m_q \sim \langle \overline{\psi}\psi \rangle / \Lambda^2_{ETC}$. FCNC experimental bounds require large Λ_{ETC}, which in scaled-up QCD forces m_q too small.
- Solution: if $\langle \overline{\psi}\psi \rangle$ is enhanced relative to QCD, then we can get around FCNCs.

[3] T. Appelquist, M. Piai, R. Shrock, Phys. Rev. D 69,105002 (2004)

NLO χPT for general N_f [4] [5]

$$M_m^2/m = 2B\left\{1 + \frac{2mB}{(4\pi F)^2}\left[2\alpha_8 - \alpha_5 + N_f(2\alpha_6 - \alpha_4) + \frac{1}{N_f}\log\frac{2mB}{(4\pi F)^2}\right]\right\}$$

$$F_\pi = F\left\{1 + \frac{2mB}{(4\pi F)^2}\left[\frac{1}{2}(\alpha_5 + N_f\alpha_4) - \frac{N_f}{2}\log\frac{2mB}{(4\pi F)^2}\right]\right\}$$

$$\langle\overline{\psi}\psi\rangle = F^2B\left\{1 + \frac{2mB}{(4\pi F)^2}\left[\frac{1}{2}(2\alpha_8 + \eta_2) + 2N_f\alpha_6 - \frac{{N_f}^2 - 1}{N_f}\log\frac{2mB}{(4\pi F)^2}\right.\right.$$

- α_i, η_i are low- and high-energy constants, rescaled: $\alpha_i = 8(4\pi)^2 L_i$, $\eta_i = 8(4\pi)^2 H_i$.
- N_f appears in both analytic terms and chiral logs.

[4] J. Gasser and H. Leutwyler, Phys. Lett. B 184:1 (1987)

[5] D. R. Nelson, Ph.D. thesis 2002, hep-lat/0212009

NLO χPT for general N_f [4] [5]

$$M_m^2/m = 2B\left\{1 + \frac{2mB}{(4\pi F)^2}\left[\alpha_m + \frac{1}{N_f}\log\frac{2mB}{(4\pi F)^2}\right]\right\}$$

$$F_\pi = F\left\{1 + \frac{2mB}{(4\pi F)^2}\left[\alpha_F - \frac{N_f}{2}\log\frac{2mB}{(4\pi F)^2}\right]\right\}$$

$$\langle\overline{\psi}\psi\rangle = F^2 B\left\{1 + \frac{2mB}{(4\pi F)^2}\left[\alpha_C - \frac{N_f{}^2 - 1}{N_f}\log\frac{2mB}{(4\pi F)^2}\right]\right\}$$

- Can't distinguish the individual LEC's at this point, so combine into a single fit parameter per observable.
- α_C contains η_2, which contains UV-divergent "contact term" $m\Lambda^2 \sim m/a^2$ - expect large contribution.

[4] J. Gasser and H. Leutwyler, Phys. Lett. B 184:1 (1987)
[5] D. R. Nelson, Ph.D. thesis 2002, hep-lat/0212009

The Conformal Window

- In QCD ($N_f = 2$), we have asymptotic freedom ($g \to 0$ in the UV) and confinement ("$g \to \infty$" in the IR). But as we increase N_f, things change:

	Short-distance (UV)	Long-distance (IR)
$0 < N_f < N_f^c$	Free ($g \to 0$)	Confined ($g \to \infty$)
$N_f^c < N_f < 16.5$	Free ($g \to 0$)	Fixed point ($g \to g^\star$)
$N_f > 16.5$	Divergent ($g \to \infty$)	Trivial ($g \to 0$)

[6] T. Appelquist, G. T. Fleming, ETN, Phys. Rev. D 79, 076010 (2009)

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- Theories with an infrared fixed point are said to lie in the conformal window. The gauge coupling never gets strong enough to trigger confinement and chiral symmetry breaking.
- N_f^c is unknown; perturbative study breaks down near the bottom of the window as the fixed-point coupling becomes strong. Previous lattice study indicates $8 \leq N_f^c \leq 12$.[6]

[6] T. Appelquist, G. T. Fleming, ETN, Phys. Rev. D 79, 076010 (2009)

The Conformal Transition and χPT

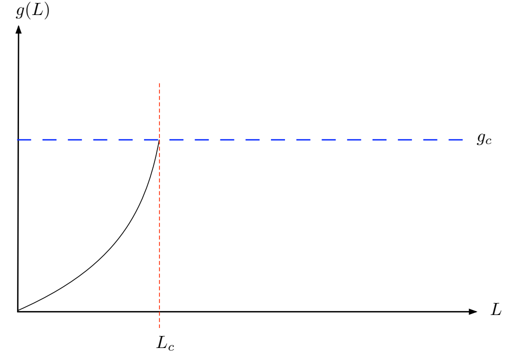
$$M_m^2/m = 2B\left\{1 + \frac{2mB}{(4\pi F)^2}\left[\alpha_m + \frac{1}{N_f}\log\frac{2mB}{(4\pi F)^2}\right]\right\}$$

$$F_\pi = F\left\{1 + \frac{2mB}{(4\pi F)^2}\left[\alpha_F - \frac{N_f}{2}\log\frac{2mB}{(4\pi F)^2}\right]\right\}$$

$$\langle\overline{\psi}\psi\rangle = F^2 B\left\{1 + \frac{2mB}{(4\pi F)^2}\left[\alpha_C - \frac{{N_f}^2 - 1}{N_f}\log\frac{2mB}{(4\pi F)^2}\right]\right\}$$

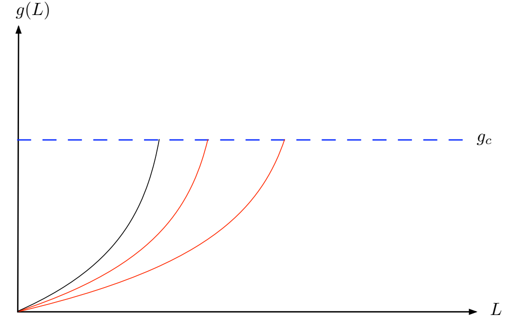
- In the window chiral symmetry is restored, so $F \to 0$ and $\langle\overline{\psi}\psi\rangle \to 0$ (i.e. $F^2 B \to 0$.) But how do F and B evolve as we approach the transition? In particular, does $\langle\overline{\psi}\psi\rangle/F^3 = B/F$ increase (condensate enhancement?) The "walking technicolor" scenario gives some hint of what to expect.

A cartoon of dynamical scales in SU(N) Yang-Mills



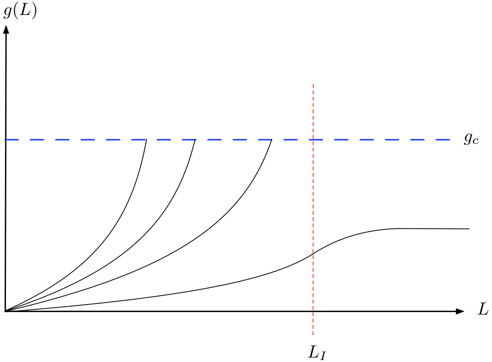
QCD is a one-scale theory, e.g. L_c, the confinement scale, corresponds to the gauge coupling g reaching a critical strength g_c.

A cartoon of dynamical scales in SU(N) Yang-Mills



Increasing N_f pushes the confinement scale to longer distances.

A cartoon of dynamical scales in SU(N) Yang-Mills



For large enough N_f, an IR-attractive fixed point appears, and the theory does not confine; chiral symmetry is restored. We can set the scale by e.g. the inflection point at L_I.

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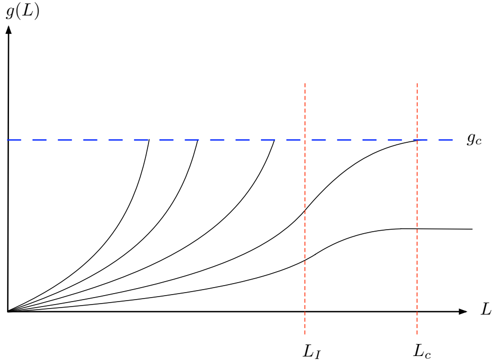


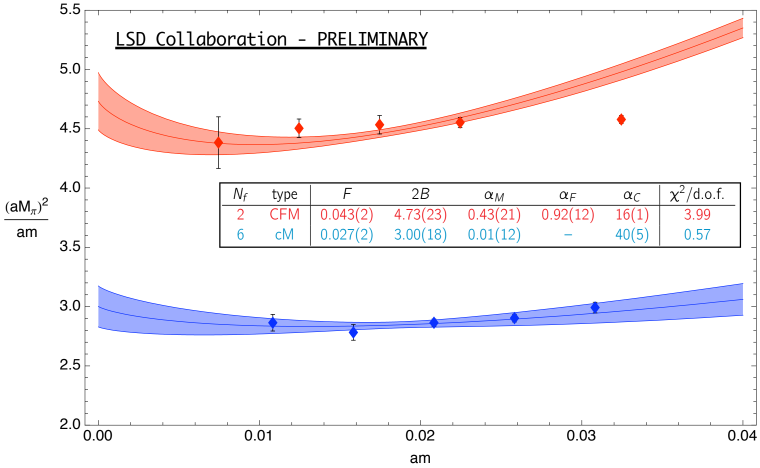
A "walking" theory can have both scales L_I and L_c. Condensates are enhanced by modes between L_I and L_c[a]. Roughly, if $L_c \sim 1/F$ and $L_I \sim 1/B$, then the scale separation enhances $B/F = \langle\overline{\psi}\psi\rangle/F^3$.

[a] Appelquist, Terning, Wijewardhana, Phys. Rev. D **44**, 871 (1991)

χPT and Lattice Simulation

- Our study: dynamical lattice simulation, measurement of spectrum, $\langle\overline{\psi}\psi\rangle$, f_π at various N_f. Use domain wall fermion discretization, which does the least damage to chiral symmetry.
- Results to be shown today: $N_f = 2$ and $N_f = 6$. $N_f = 2$ results shown are exploratory results from a $16^3 \times 32$ volume lattice at a coarse spacing ($am_\rho = 0.535(11)$.) $N_f = 6$ are taken from $32^3 \times 64$ lattices with $am_\rho = 0.192(2)$.
- $N_f = 2$ fits are joint between all three chiral quantities $\langle\overline{\psi}\psi\rangle$ (C), f_π (F), M_π^2/m (M). $N_f = 6$ data admits a joint fit to M and C, but only if C has *no chiral log*. (c) More on this soon.

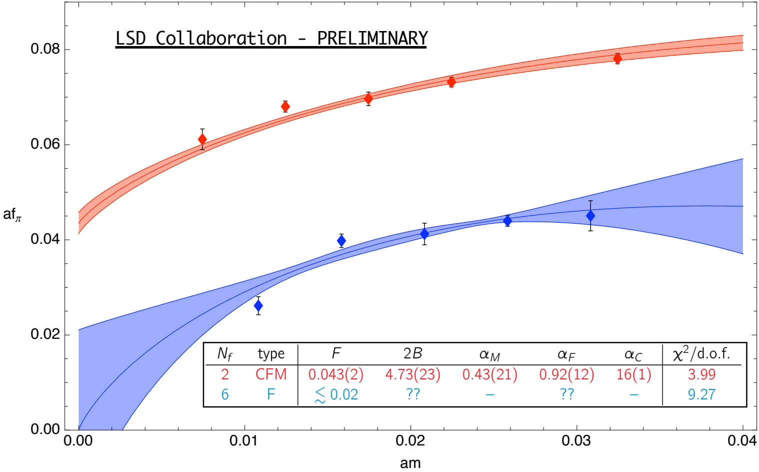
m_π^2 extrapolation



N_f	type	F	$2B$	α_M	α_F	α_C	χ^2/d.o.f.
2	CFM	0.043(2)	4.73(23)	0.43(21)	0.92(12)	16(1)	3.99
6	cM	0.027(2)	3.00(18)	0.01(12)	–	40(5)	0.57

Good chiral fit in both cases (dropping the heaviest point at $N_f = 2$...slope here is strangely constant.) $B/F = \langle\overline{\psi}\psi\rangle/F^3 \sim 54$ or 55 (don't compare - different am_ρ!)

f_π extrapolation

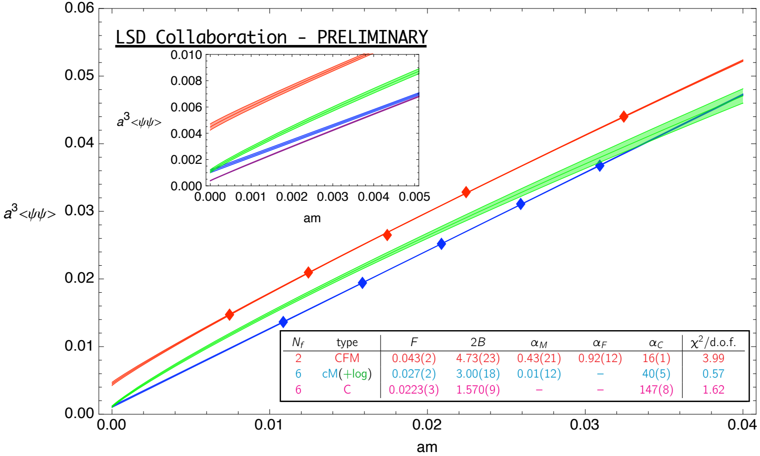


N_f	type	F	$2B$	α_M	α_F	α_C	χ^2/d.o.f.
2	CFM	0.043(2)	4.73(23)	0.43(21)	0.92(12)	16(1)	3.99
6	F	$\lesssim 0.02$	??	–	??	–	9.27

$N_f = 6$ incompatible with other fits, which favor $F > 0.02$. Can force fit with large F intercept, but small B is required to suppress chiral log.

No problems at $N_f = 2$.

$\langle \overline{\psi}\psi \rangle$ extrapolation



N_f	type	F	$2B$	α_M	α_F	α_C	χ^2/d.o.f.
2	CFM	0.043(2)	4.73(23)	0.43(21)	0.92(12)	16(1)	3.99
6	cM(+log)	0.027(2)	3.00(18)	0.01(12)	–	40(5)	0.57
6	C	0.0223(3)	1.570(9)	–	–	147(8)	1.62

$\langle \overline{\psi}\psi \rangle$ highly linear, dominated by (unique!) contact term m/a^2. At $N_f = 6$, things look consistent if we drop the chiral log...but we can't ignore it! Fitting with a log moves the intercept, destroying the joint fit.

Conclusion

- $N_f = 6$ results seem to be inconsistent with NLO χPT so far, although LO values obtained with pure linear extrapolation do look roughly consistent...chiral logs seem smaller than expected. Situation unclear so far.
- Our next steps: matched $N_f = 2$ (same volume, am_ρ as $N_f = 6$), lighter masses at $N_f = 6$ (partial quenching?), move on to $N_f = 8$ and beyond.
- Lots of ground to explore beyond QCD - varying N_c, fermion representation. Lattice studies of chiral properties will need χPT to make sense of results.
- Explicit knowledge of N_f-dependent NNLO chiral log coeffs would be helpful for fitting to lattice data, particularly as we add more mass points.

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