Universal Correlations in Pion-less EFT with the Resonating Group Model

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July 6, 2009

Ultimate Goals

- analysis of **universal correlations** between bound & scattering observables in $A > 4$ systems

  $\Rightarrow$ objects which **differ on a microscopic scale** exhibit the **same low-energy behavior**

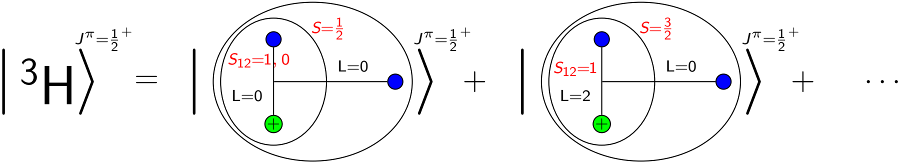
- How far in A (and density) can one **push the pionless EFT**?

  $\Rightarrow$ development of the most **simple** theory, **rooted in QCD**, appropriate for the description of low-energy nuclear properties

(Refined) Resonating Group Model

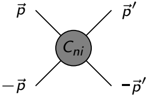
see e.g. H. M. Hofmann, proceedings of Models and Methods in Few-Body Physics, 1986.

- **versatile** method:
  - non-orthogonal expansion of wave function in **Gaussian** basis
    ↷ suited for calculations of **bound** & **scattering** observables
  - applicable to other **degrees offreedom**, *e.g.*, (α, n) & $\left({}^{9}\mathrm{Li}, t, n\right)$
  - Coulomb interaction included
- with $\mathrm{EFT}_{\not\pi}^{\mathrm{NLO}}$ relatively modest **CPU time** requirements



NN potential & LEC determination

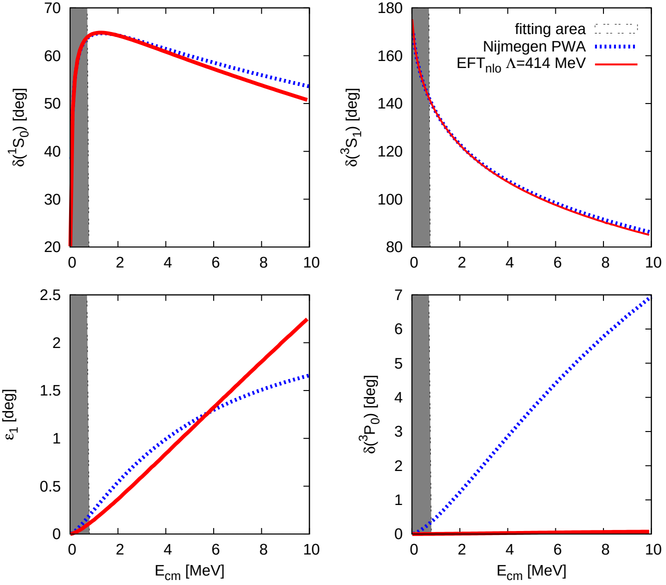
$$
\begin{aligned}
V^{\mathrm{NLO}}_{\not\pi}(\vec{r}) \;=\;& l_0(r)\left(A_1 + A_2\vec{\sigma}_1\cdot\vec{\sigma}_2\right) + \left(A_3 + A_4\vec{\sigma}_1\cdot\vec{\sigma}_2\right)\left\{e^{-\frac{\Lambda^2}{4}\vec{r}^2}, \vec{\nabla}^2\right\} + \\
& l_0(r)\left(A_5 + A_6\vec{\sigma}_1\cdot\vec{\sigma}_2\right)\vec{r}^2 + l_0(r)A_7\vec{L}\cdot\vec{S} + l_0(r)A_8\left[\vec{\sigma}_1\cdot\vec{r}\,\vec{\sigma}_2\cdot\vec{r} - \frac{1}{3}\vec{r}^2\vec{\sigma}_1\cdot\vec{\sigma}_2\right] \\
& -A_9\left\{e^{-\frac{\Lambda^2}{4}\vec{r}^2}, \left[\left[\partial^r\otimes\partial^s\right]^2\otimes\left[\sigma_1^p\otimes\sigma_2^q\right]^2\right]^{00}\right\}
\end{aligned}
$$



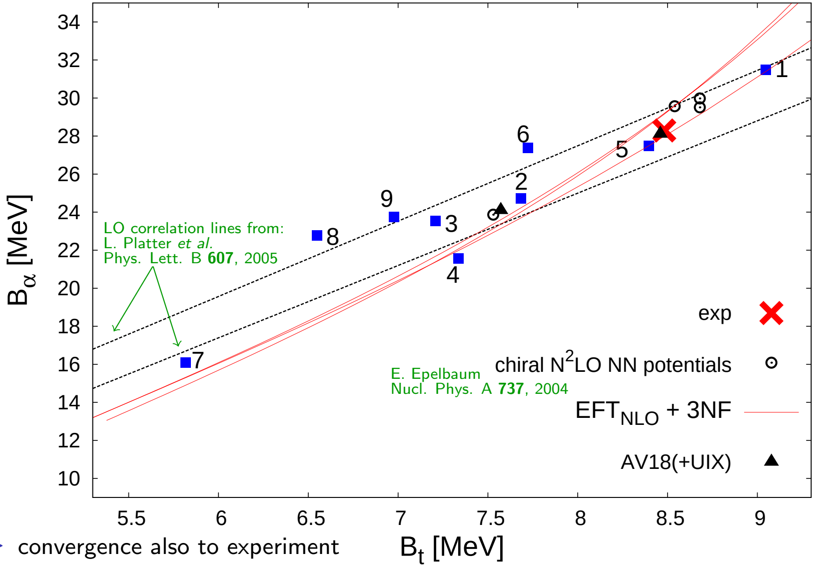
- Gaussian regulator functions $l_0(r,\Lambda) \propto e^{-\frac{\Lambda^2}{4}\vec{r}^2}$
- low-energy-constants depend on cutoff, $A_i = A_i(\Lambda)$
- LEC of leading order 3NF **not** fitted

- low-energy input data: B_d, $\delta\left({}^1S_0, {}^3S_1, {}^{1,3}P_{0,1,2}\right)$, ϵ_1 for $E_{\mathrm{cm}} < 1$ MeV
- two methods to obtain different LEC sets $\leftrightarrow$ different short-range-physics:
  - variation of **cutoff** parameter Λ
  - variation of **low-energy input**

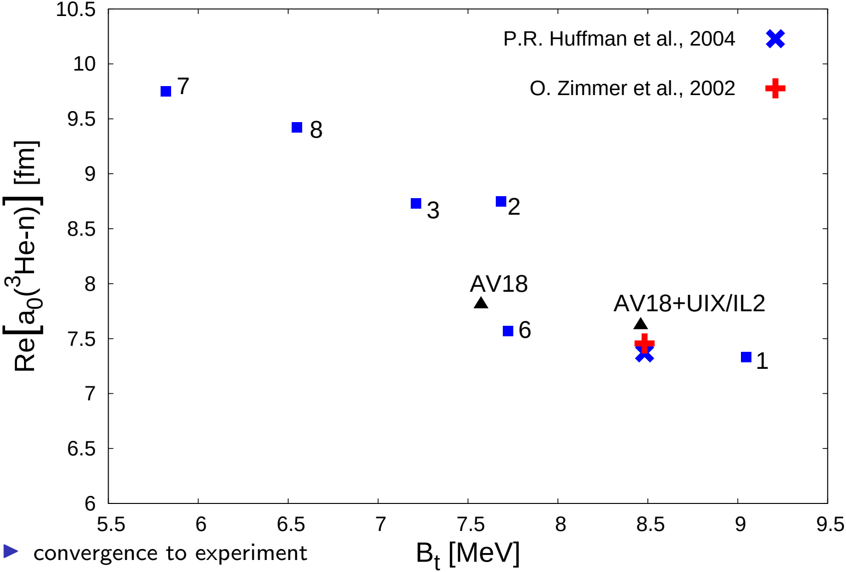
NN phase shifts at NLO



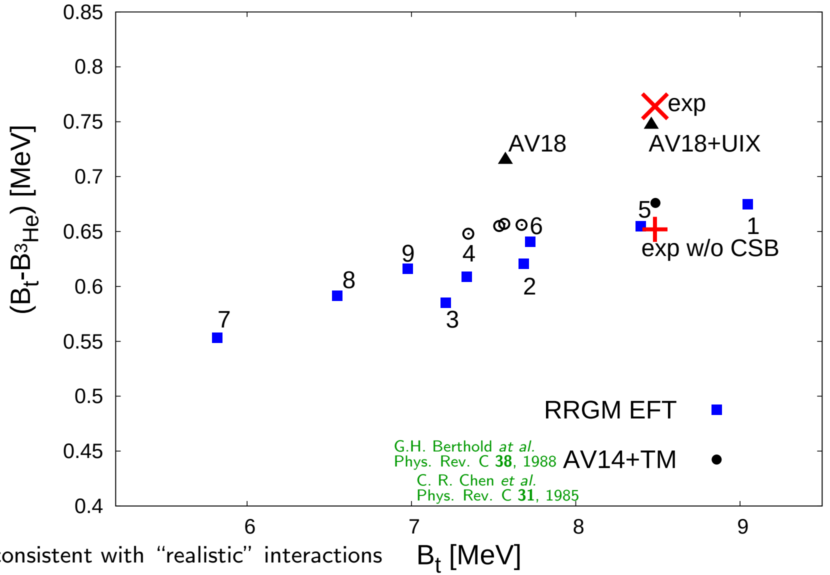
- deviation from $\delta^{\mathrm{Nij}}({}^{1,3}S_{0,1})$ is $\lesssim 10\%$
- deviation from $\epsilon_1^{\mathrm{Nij}}$ is $\lesssim 30\%$
- P-wave phase shifts fitted to various values $< \delta^{\mathrm{Nij}}(P)$



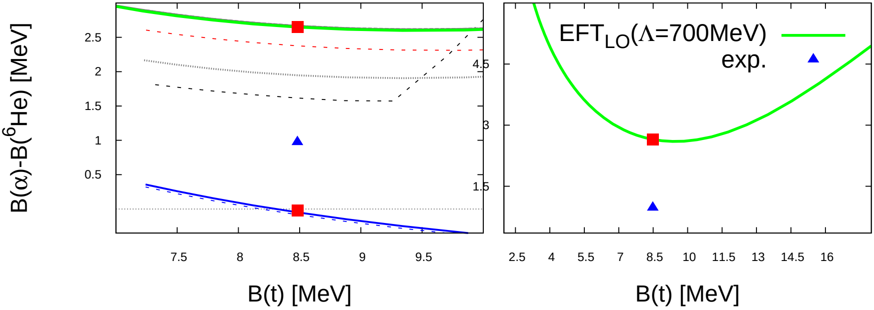
- convergence also to experiment
- NLO band width $\Rightarrow \quad Q \approx \frac{1}{3}$
- **no** four-body force needed at NLO
- one three-body parameter needed for a result independent of short distance physics
- EFT$_{\not\pi}$ still works at NLO



- convergence to experiment
- NLO band width ⇒ $Q \approx \frac{1}{3}$
- **no** four-body force needed at NLO
- calculation **not accurate enough** to discriminate between conflicting data
- EFT$_{\not\pi}$ still[2] works at NLO



- consistent with "realistic" interactions
- discrepancy to experiment due to higher order CSB interactions
- missing contribution consistent with χPT calculations Friar, Payne, van Kolck Phys. Rev. C **71**, 2005



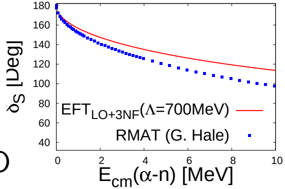
	EFT$_{\not\pi}$ (700 MeV)	exp.
$\langle \vec{r}^2 \rangle_{\text{ch}}^{1/2} \left(^6\text{He}\right)$	3.261 fm	2.054 fm
$\langle \vec{r}^2 \rangle_{\text{m}}^{1/2} \left(^6\text{He}\right)$	5.678 fm	2.30 fm

- halo structure at “real” triton
- cutoff & 3NF strength variation exhibit same qualitative effect
- neutron halo structure still intact at less/more bound “tritons”?

Conclusions

- evidence for the **applicability** of EFT($\not\pi$) at NLO in the ^{4}He-channel
  - **convergence** from LO to NLO with $Q \approx \frac{1}{3}$
  - new correlation for $a_0\left({}^3\text{He} - n\right)$
  - consistency with high precision models & experiment
  - **four-nucleon** contact interaction **not** necessary at NLO
- model independent calculation of CIB/CSB effect of the Coulomb interaction in ^{3}H and ^{3}He
- $A = 6$ systems accessible with RRGM
- halo structure of ^{6}He not a universal property of the NN-force?
- RRGM and EFT($\not\pi$) *resonate* well

The "next order"



- universality in **halo nuclei** at (*clean*) NLO
- (α-N) scattering system as bridge to **cluster EFT**

RRGM for scattering states:

$$\delta\left(\langle\psi_\lambda|\hat{\mathrm{H}}-E|\psi_\lambda\rangle-\tfrac{1}{2}a_{\lambda\lambda}\right)=0$$

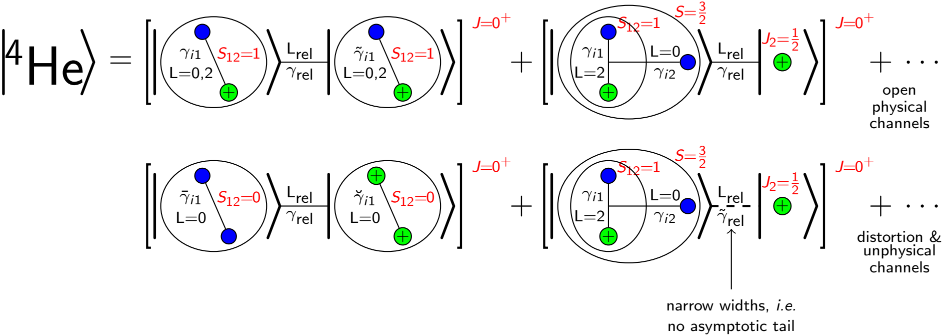
fragment wave functions

$$\psi^{J^\pi}_{\mathrm{SS},\lambda}=\mathcal{A}\sum_j^{n_k}\left[\frac{1}{R_j}Y_{L_j}(\hat{\vec{R}}_j)\otimes\left[\psi_j^{J_1^{\pi 1}}\otimes\psi_j^{J_2^{\pi 2}}\right]^{S_{c_j}}\right]^J\left(\delta_{\lambda j}F_{L_j}(R_j)+a_{\lambda j}\tilde{G}_{L_j}(R_j)+\sum_m b_{\lambda jm}R_j^{L_j+1}e^{-\omega_{jm}\vec{R}_j^2}\right)$$

open & distortion channels

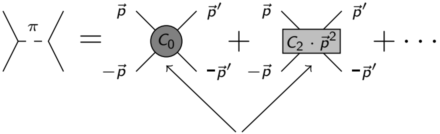
Coulomb functions

approximates state in interaction region



The Effective Field Theory:

- theory breaks down as momenta approach pion mass m_π
- (unnaturally) small deuteron binding energy relative to the triplet $n-p$ effective range
- non-relativistic isospin doublet of Pauli spinors: $N = \begin{pmatrix} p \\ n \end{pmatrix}$ LO: $B_d = \frac{1}{a_t^2 M} \ll \frac{m_\pi^2}{M}$
- Lorentz symmetry at small momentum



truncation based on two estimates:

coupling strength & relative contribution of graphs

naïve dimensional analysis & naturalness

- encode short distance physics
- match to underlying theory or low-energy data

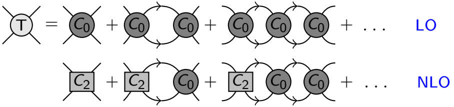
$$\mathcal{L}_{\not\pi} = N^\dagger \left(iD_0 + \frac{\mathbf{D}^2}{2M} \right) N + C_0 (N^\dagger N)(N^\dagger N) + C_2 (N^\dagger N)(N^\dagger \partial_i^2 N) + \ldots$$

$\frac{1}{a} = \mathcal{O}(p_{\text{typ}}) \Rightarrow$ certain operators require resummation

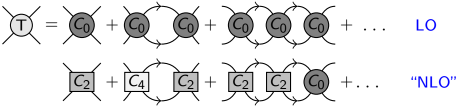
others can still be treated perturbatively

The two philosophies:

- power counting on the Lagrangean level **and** on the graph level:



- iterate **effective potential** by solving Schrödinger/Lippmann-Schwinger equation:



In **both** approaches higher order terms are expected to contribute $\mathcal{O}\left[\left(\frac{p_{\text{typ}}}{m_\pi}\right)^{N>n}\right]$ at order n