Chiral expansions of the π^0 decay amplitude

Karol Kampf (PSI, Villigen)
Bachir Moussallam (IPN, Orsay)

Outline

1) Chiral structure of amplitude

m_u, m_d expansion: LO, NLO, NNLO (new)

2) Phenomenology

m_s expansion (updated)

Chiral structure of the decay amplitude:

- Definition

$$\mathcal{T}_{\pi^0 \to \gamma(k_1)\gamma(k_2)} = e^2 \epsilon(e_1, k_1, e_2, k_2)\, T$$

1) LO (p^4) [Chiral Lagrangian with anomaly:

Wess-Zumino (1971),Witten (1983)

Simplified $SU(2)$: Kaiser (2001)]

$$T_{LO} = \frac{1}{4\pi^2 F}$$

→ $F = \lim_{m_u = m_d = 0} F_\pi$

→ Exact in chiral limit (LE theorem)

2) NLO (p^6)

→ One-loop diagrams [Donoghue,Holstein,Lin (1985)
Bijnens,Bramon,Cornet (1988)]

→ Tree diagrams [Bijnens,Girlanda,Talavera (2002)
Eberthauser,Fearing,Scherer (2002)]

SU(2) : 13 terms (c_i^W)
SU(3) : 23 terms (C_i^W)

Little is known about these

NLO Result

T_{LO}: $F \longrightarrow F_\pi$

$$T_{NLO} = \frac{16}{3F}\Big\{ m_\pi^2 \left(-4c_3^{W\,r} - 4c_7^{W\,r} + c_{11}^{W\,r}\right) + \frac{4}{3}B(m_d - m_u)\left(5c_3^{W\,r} + c_7^{W\,r} + 2c_8^{W\,r}\right)\Big\}$$

Remarks

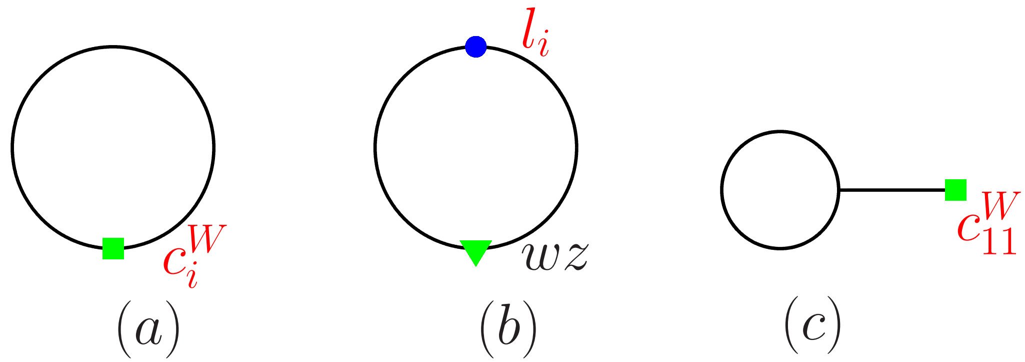
- $m_d - m_u$ contribution (e^2 must be checked)
- No chiral logs: (c_i^W combinations finite)

questions: are there any at NNLO ?

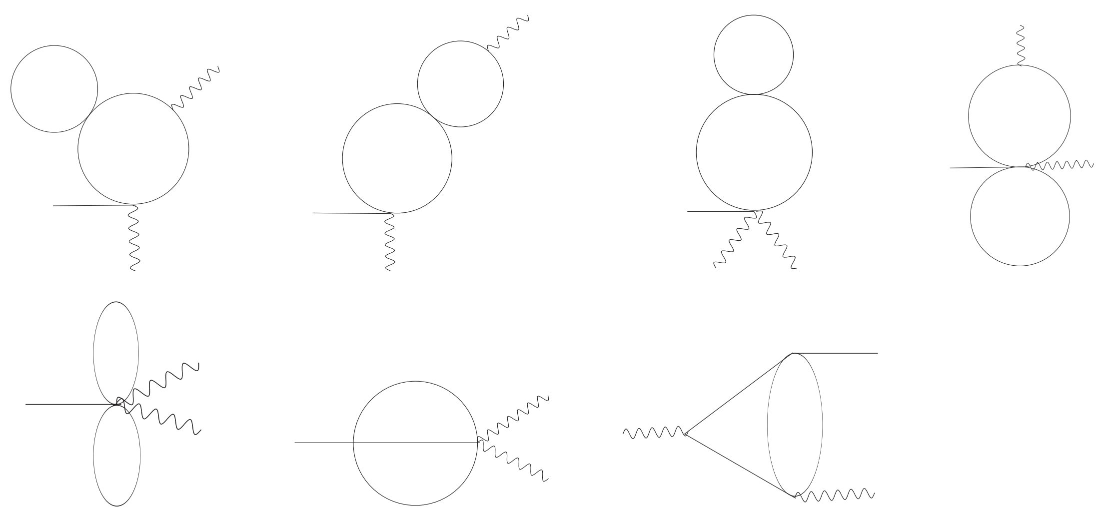
are they large ?

3) NNLO (p^8)

→ One-loop diagrams [with one NLO vertex]



→ Two-loop diagrams [with LO vertices]



→ Tree diagrams [vertices from $\mathcal{L}_{(4)}$, $\mathcal{L}_{(6)}$, also $\mathcal{L}_{(8)}^{WZ}$]

- Result:

$$
\begin{aligned}
F\,T_{NNLO} = & -\frac{m_\pi^4}{24\pi^2 F^4}\left(\frac{1}{16\pi^2}\boldsymbol{L}_\pi\right)^2 \\
& +\frac{m_\pi^4}{16\pi^2 F^4}\boldsymbol{L}_\pi\left[\frac{3}{256\pi^4}+\frac{32F^2}{3}\left(2c_2^{W\,r}+4c_3^{W\,r}+2c_6^{W\,r}+4c_7^{W\,r}-c_{11}^{W\,r}\right)\right] \\
& +\frac{32m_\pi^2 B(m_d-m_u)}{48\pi^2F^2}\boldsymbol{L}_\pi\left[-6c_2^{W\,r}-11c_3^{W\,r}+6c_4^{W\,r}-12c_5^{W\,r}-c_7^{W\,r}-2c_8^{W\,r}\right] \\
& +\frac{m_\pi^4}{F^4}\lambda_+ +\frac{m_\pi^2 B(m_d-m_u)}{F^4}\lambda_- +\frac{B^2(m_d-m_u)^2}{F^4}\lambda_{--}\ ,
\end{aligned}
$$

- Remarks:

– Non-local divergences $\dfrac{L_\pi}{d-4}$ cancel (renormalizability)

– Chiral logs are present

Anatomy of of $m^2_{u,d}$ coefficients

$$\lambda_+ = \frac{1}{512\pi^6}\left(-\frac{983}{288} - \frac{4}{3}\zeta(3) + 3\sqrt{3}\,\mathrm{Cl}_2(\tfrac{\pi}{3})\right) \quad \leftarrow \text{2-loops graphs}$$

$$+\frac{1}{\pi^2}\left(-8c_6^r - \frac{1}{4}(l_4^r)^2\right) \quad \leftarrow \text{From } \mathcal{L}_{(6)}, \mathcal{L}_{(4)}, \mathcal{L}_{(6)}^{WZ}$$

$$+\frac{16F^2}{3\pi^2}\left[8l_3^r(c_3^{Wr} + c_7^{Wr}) + l_4^r(-4c_3^{Wr} - 4c_7^{Wr} + c_{11}^{Wr})\right]$$

$$-\frac{2}{3\pi^2}d_+^{Wr}(\mu) \quad \leftarrow \text{From } \mathcal{L}_{(8)}^{WZ}$$

→ Phenomenology: 8 LEC's c_i^{Wr}, need estimates !

Phenomenology

- Perform $SU(3)$ expansion instead of $SU(2)$
$[$only two LEC's involved at NLO $]$

- Starting from $SU(2)$ expression:
use m_s expansion of $SU(2)$ LEC's

$\rightarrow$ Elegant method (matching of generating functionals)

`Gasser,Haefeli,Ivanov,Schmid Phys. Lett. B652 (2007)`

- Matching one-loop functionals in WZ sector: first two-terms in m_s expansion

$$
\begin{aligned}
c_2^{Wr} &= c_0 + C_4^{Wr} - \tfrac{1}{2}C_5^{Wr} + \tfrac{3}{2}C_6^{Wr} + O(m_s) \\
c_3^{Wr} &= -\tfrac{3}{2}c_0 + C_7^{Wr} + 3C_8^{Wr} + O(m_s) \\
c_4^{Wr} &= -\tfrac{1}{2}c_0 + C_9^{Wr} + 3C_{10}^{Wr} + O(m_s) \\
c_5^{Wr} &= C_{11}^{Wr} + \tfrac{1}{8}\tfrac{1}{(32\pi^2)^2}(L_K + 1 + \tfrac{2}{3}L_\eta) + O(m_s) \\
c_6^{Wr} &= -c_0 + C_5^{Wr} - \tfrac{3}{2}C_6^{Wr} - \tfrac{1}{2}C_{14}^{Wr} - \tfrac{1}{2}C_{15}^{Wr} + O(m_s) \\
c_7^{Wr} &= \tfrac{3}{2}c_0 - 3C_8^{Wr} + \tfrac{1}{4}C_{22}^{Wr} + O(m_s) \\
c_8^{Wr} &= \tfrac{3}{4}c_0 + \tfrac{1}{2}C_7^{Wr} + 3C_8^{Wr} - \tfrac{1}{8}C_{22}^{Wr} + O(m_s) \\
c_{11}^{Wr} &= C_{22}^{Wr}
\end{aligned}
$$

with $$c_0 = \frac{1}{32\pi^2}\left[-\frac{1}{16Bm_s} + \frac{2}{F_0^2}\left(3L_7^r + L_8^r - \frac{1}{512\pi^2}(L_K + \frac{2}{3}L_\eta)\right)\right]$$

- Note: $\dfrac{1}{m_s}$ terms: $\pi^0 - \eta$ mixing

- Still too many couplings

→ Introduce counting rule:

$m_u, m_d \sim O(p^2)$	[as usual]
$m_s \quad \sim O(p)$	[since $m_{u,d} << m_s$]

→ Keep terms of order:

$$p,\ p^2,\ p^3 \log(p),\ p^4 \log^2(p)$$

- Then result simplifies

$$
\begin{aligned}
F_\pi T^{SU(3)}_{NLO+} = \; & \frac{1}{16\pi^2}\frac{m_d - m_u}{m_s}\left[1 - \frac{3m_\pi^2}{32\pi^2 F_\pi^2} L_\pi\right] \\
+ \; & \frac{128}{3} B(m_d - m_u)\left[C_7^W + 3C_8^W \left(1 - \frac{3m_\pi^2}{16\pi^2 F_\pi^2} L_\pi\right)\right. \\
& \left. - \frac{3}{64\pi^2 F_\pi^2}\left(3L_7^r + L_8^r - \tfrac{1}{512\pi^2}(L_K + \tfrac{2}{3}L_\eta)\right)\right] \\
- \; & \frac{64}{3} m_\pi^2 C_7^W - \frac{1}{24\pi^2}\left(\frac{m_\pi^2}{16\pi^2 F_\pi^2} L_\pi\right)^2
\end{aligned}
$$

- Remarks

– Same result as $SU(3)$ NLO + log's

– Only two LEC's: C_7^W , C_8^W

- Two inputs

1) Resonance saturation model
(used qualitatively. Note: C_7^W, C_8^W scale independent)

$$C_7^W \simeq \frac{g_{\pi(1300)\gamma\gamma} d_m}{M_{\pi(1300)}^2} \qquad C_8^W \simeq \frac{g_{\eta'\gamma\gamma} \tilde{d}_m}{M_{\eta'}^2}$$

and

$$\left.\begin{array}{ll} \Gamma_{\pi(1300)\to 2\gamma} & < 0.07 \text{ keV (Belle (2006))} \\ \Gamma_{\eta'\to 2\gamma} & = 4.30 \pm 0.15 \text{ keV} \end{array}\right\}$$

$\Rightarrow$ $\boxed{|C_7^W| < 0.1 |C_8^W|}$ (Plausible !)

2) $\eta \to 2\gamma$

Amplitude $SU(3)$ NLO:

$$T_{NLO}^{\eta\to\gamma\gamma} = \frac{1}{\sqrt{3}F_\pi}\left[\frac{F_\pi}{4\pi^2 F_\eta}[1+\sqrt{3}(-\epsilon_1 + e^2(\delta_\eta - \delta_1))] - \frac{64}{3}m_\pi^2 C_7^W + \frac{512}{3}(m_K^2 - m_\pi^2)\left(\frac{1}{6}C_7^W + C_8^W\right)\right] + O(m_s^2)$$

$\Rightarrow$ C_7^W,C_8^W determined

- Input updates

F_π	$= 92.22 \pm 0.07$ MeV	In pure QCD. Updated V_{ud} and improved radiative corrections from χPT(chiral log in Marciano Sirlin (1993) constant)
$\frac{2m_s}{m_u + m_d}$	$= 28.0 \pm 1.5$	From lattice QCD
$\frac{m_d - m_u}{m_s}$	$= (2.29 \pm 0.23)10^{-3}$	From $\eta \to 3\pi$ and χPTp^6 Bijnens, Ghorbani (2007) and p^4
$\Gamma_{\eta \to 2\gamma}$	$= 0.510 \pm 0.026$ keV	Updated in PDG since 2004

Numerical contributions

Orders	Width(eV)	
p^0	7.76	(see F_π)
p	0.09	
p^2	0.29	enhanced : C_8^W (large N_c ?)
e^2	-0.05	Ananthanarayan, BM (2002)
		($\pi^+ - \pi^0$ mass diff. in loops)
$p^3 \log(p)$	0.004	
$p^4 \log^2(p)$	-0.005	No log enhancement !

Finally

$$\Gamma_{\pi\to\gamma\gamma}\text{(eV)} = 8.09 \quad \pm 0.08 \quad \pm 0.10$$

$m_d - m_u$ $\quad m_s^2$

→ Alternative approaches (Chiral + large N_c)

Goity,Bernstein,Holstein (2002)	$\Gamma_{\pi\to\gamma\gamma} = 8.10 \pm 0.08$
Kaiser (2002)	$\Gamma_{\pi\to\gamma\gamma} = 8.07$
Ioffe, Oganessian (2007)	$\Gamma_{\pi\to\gamma\gamma} = 7.93 \pm 0.12$

[η' incorrect ?]

→ **Experiment:** morning talk by A. Bernstein !

Conclusions

- Chiral logs present but numerically small
- Possible lattice QCD results in anomaly sector ?

Cohen,Lin,Dudek,Edwards,arXiv:0810.5550

- More could be investigated in anomaly sector