Lattice QCD simulations of baryon-baryon interactions (hyperon-nucleon and hyperon-hyperon)

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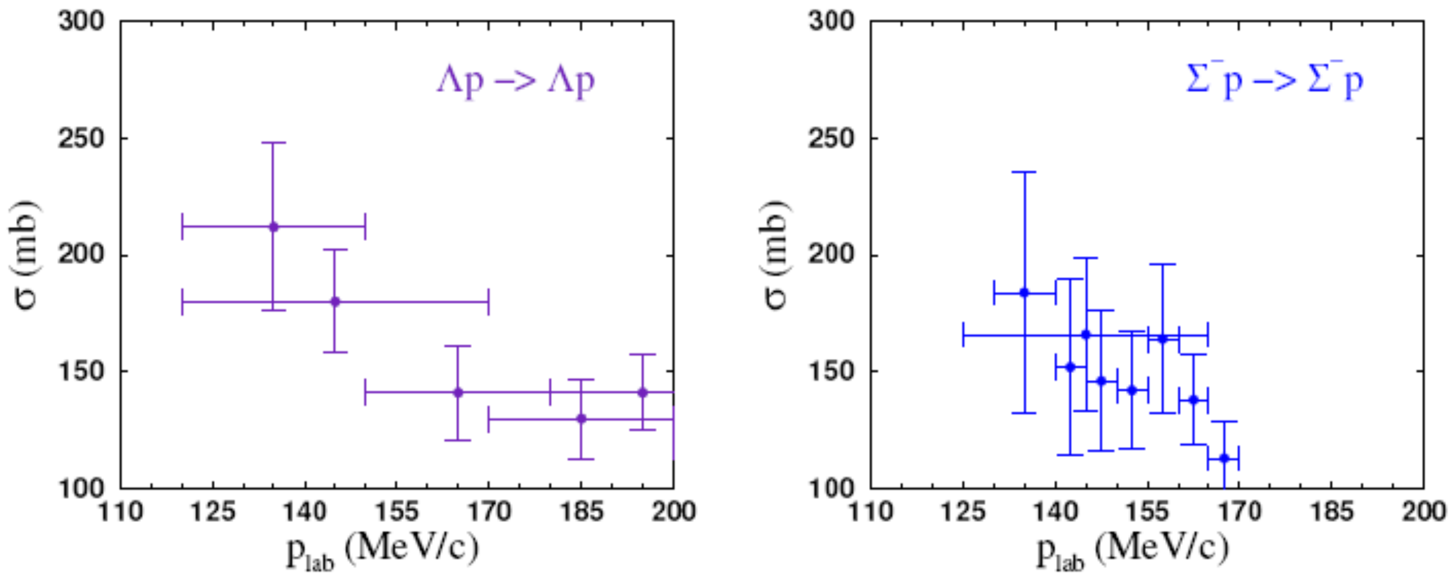
Study of the baryonic interactions in the strange sector with LQCD

provide complementary information to experiment (ΛN, ΣN, ΛΛ, ΣΣ, ΞΞ, …)

In the low-energy regime... around half of pion production threshold...

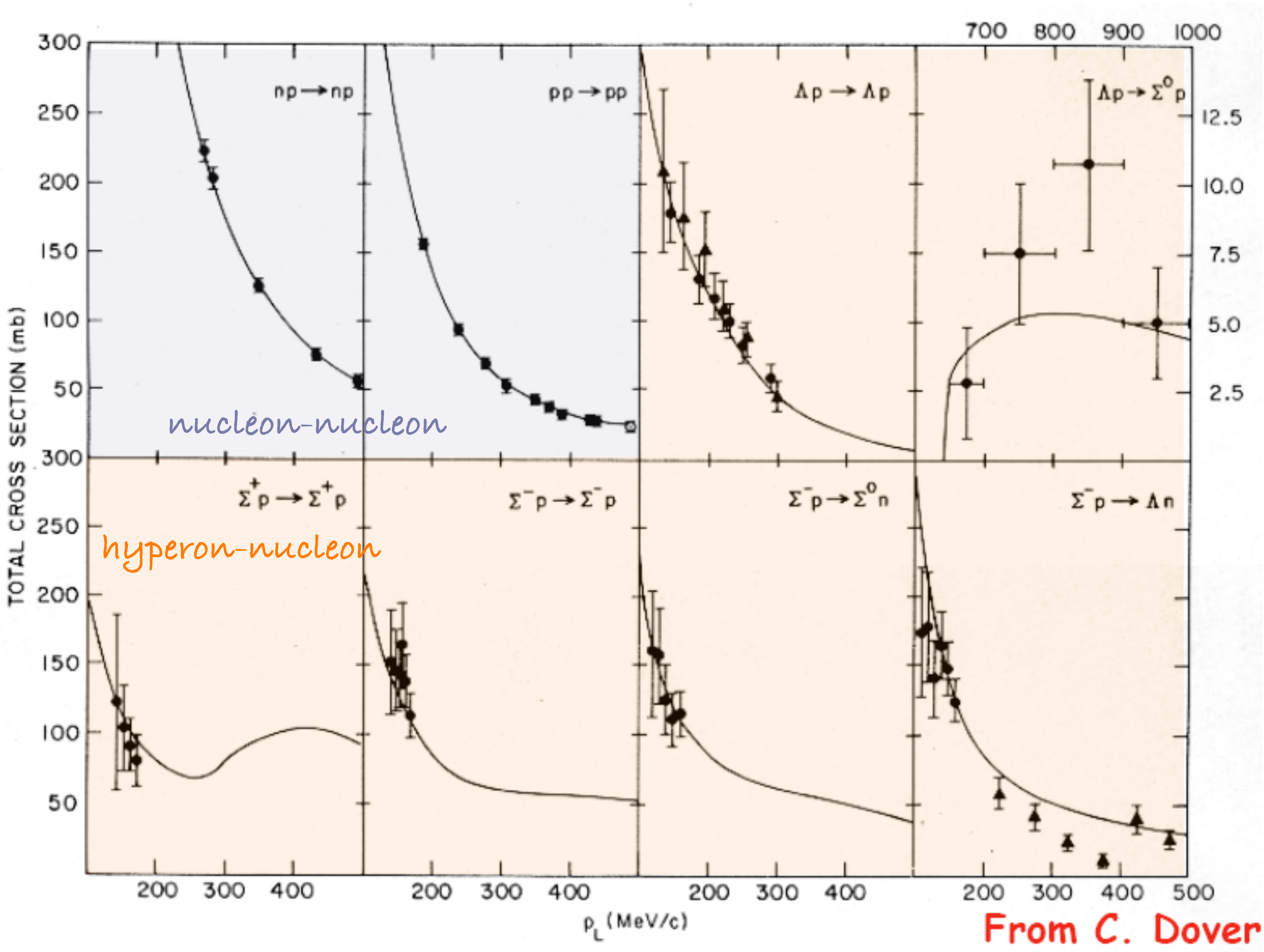


In general, hyperon-nucleon data show:

◇ Large error bars
◇ Absence of truly low-energy cross sections

more info at http://nn-online.sci.kun.nl

scattering events…



The low-energy YN "database" (Rob Timmermans)

~ 35 data points (many pre-1971) with large errors

Λp		# = 12	$6.5\ \text{MeV} < T_{lab} < 50\ \text{MeV}$
$\Sigma^- p \rightarrow \Sigma^- p$		# = 6	
	Λn	# = 6	$9\ \text{MeV} < \text{Tlab} < 12\ \text{MeV}$
	$\Sigma^0 n$	# = 6	
$\Sigma^+ p$		# = 4	$9\ \text{MeV} < \text{Tlab} < 13\ \text{MeV}$ + 3 data from KEK-E289

"Ratio at rest" (inelastic capture ratio) of stopped Σ^- by protons:

$$r_R = \#\,\Sigma^0 / (\#\Sigma^0 + \#\Lambda) = 0.468(10)$$

Some differential cross sections of low quality

Additional information:

YN $\rightarrow$ Light hypernuclei: $^3H_\Lambda$, $^4He_\Lambda$, $^4H_\Lambda$, $^5He_\Lambda$

YY $\rightarrow$ $^6He_{\Lambda\Lambda}$, $^{10}Be_{\Lambda\Lambda}$, $^{13}B_{\Lambda\Lambda}$ ...

B. Sechi-Zorn, B. Kehoe, J. Twitty and R.A. Burnstein, Phys. Rev. 175, 1735 (1968):

$$0.0 > a^{(^1S_0)} > -15 \text{ fm} \qquad 0.0 > r^{(^1S_0)} > 15 \text{ fm}$$

$$-0.6 > a^{(^3S_1)} > -3.2 \text{ fm} \qquad 2.5 > r^{(^3S_1)} > 15 \text{ fm}$$

Alexander, Karshon, Shapira, Yekutieli, Engelmann, Filthuth, Lughofer, Phys. Rev. 173, 1452 (1968):

Best fit: $a^{(^1S_0)} = -1.8$ fm, $r^{(^1S_0)} = 2.8$ fm, $a^{(^3S_1)} = -1.8$ fm, $r^{(^3S_1)} = 3.3$ fm

Very poor statistics

Effective range parameters fit to data highly correlated

It is safe to say:

⇒ There is no hyperdeuteron $\Rightarrow\ a^{(^1S_0)} < 0, \quad a^{(^3S_1)} < 0$

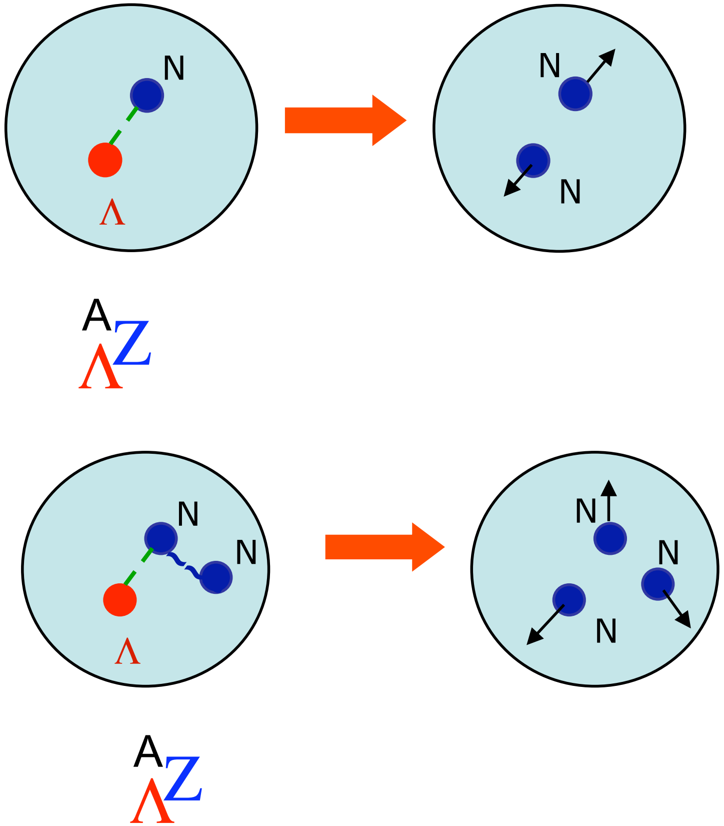
⇒ Consistency of potential models with the hypertriton data (b.e., spin)

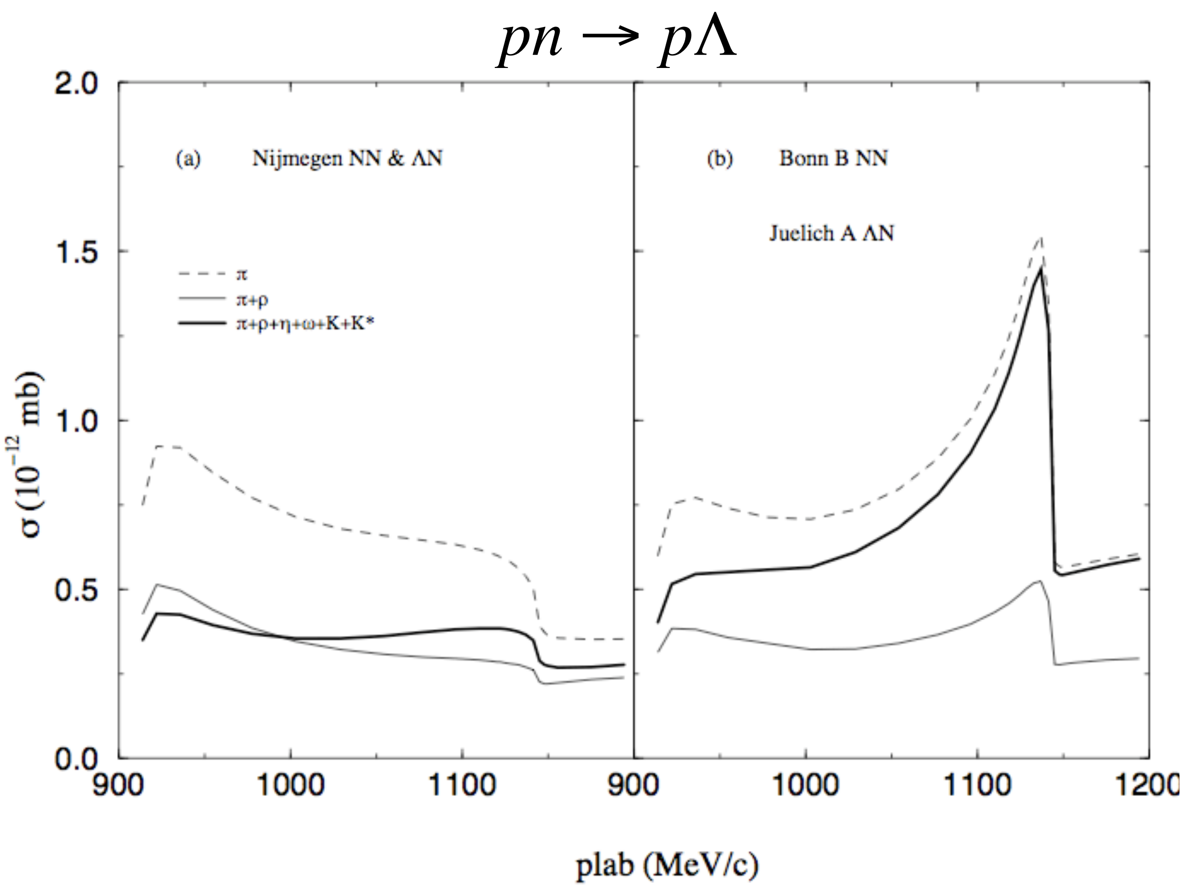
$$\Rightarrow |a^{(^1S_0)}| > |a^{(^3S_1)}|$$

LQCD $\longleftrightarrow$ first principles calculation with smaller uncertainties

Different problems in physics require as input the YN and YY potentials

|ΔS|=1 ΛN →NN ⟷ Hypernuclear structure
Hypernuclear decay

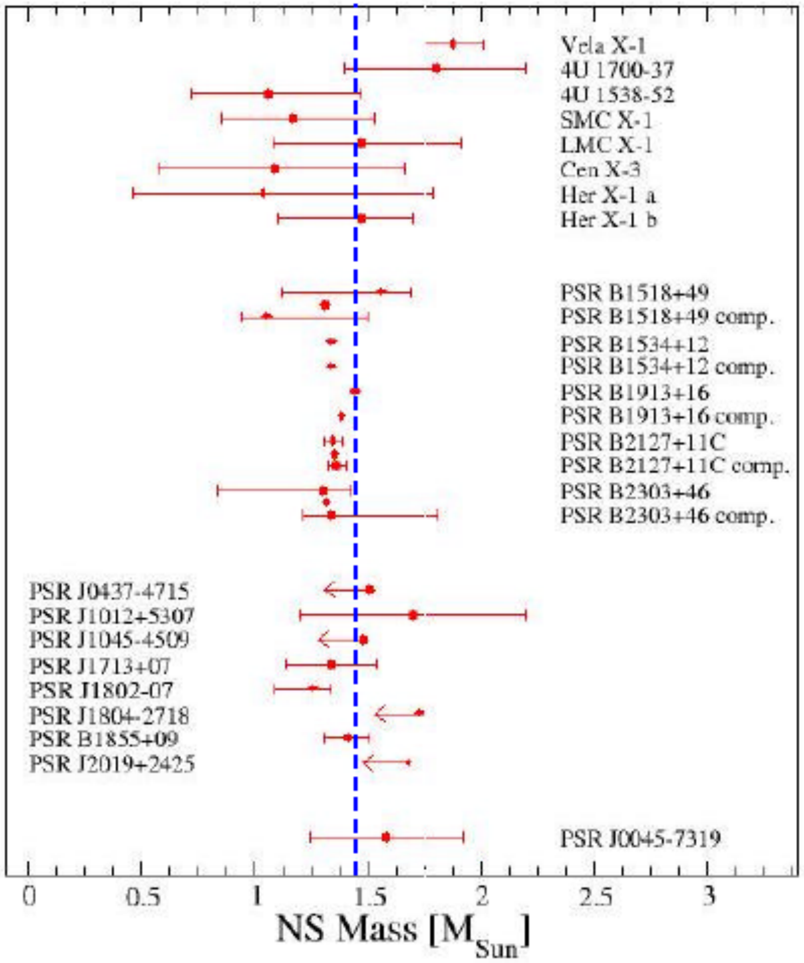


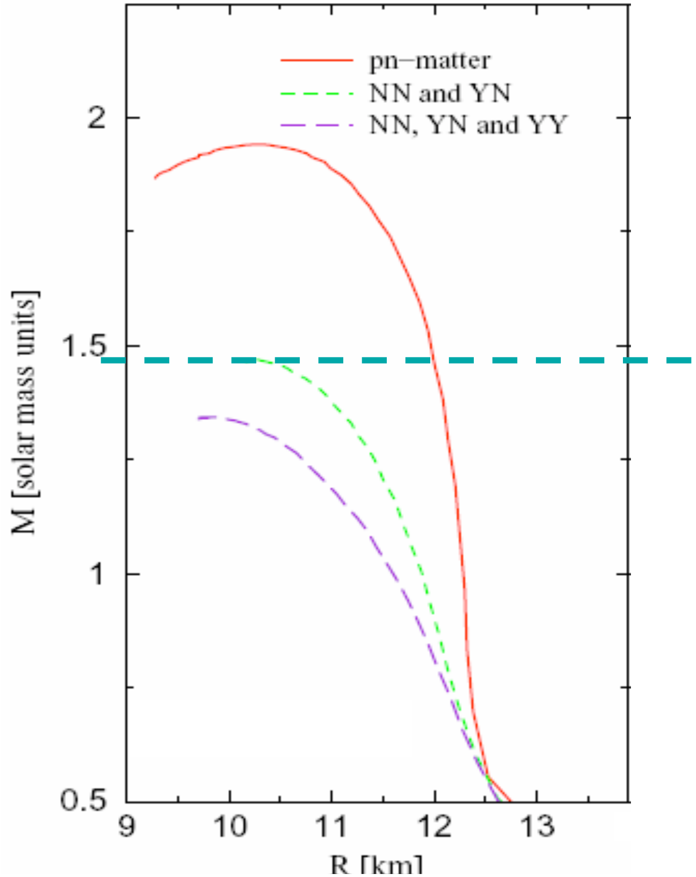


Parreño, Ramos, Kelkar, Bennhold, Phys.Rev.C59:2122-2129,1999

- Neutron stars dynamics (EOS)

Measured NS masses





Influence of hyperons:

lower maximum masses
higher central densities
more compact (smaller radius)

But microscopic EOS for hyperonic matter are “too” soft!

Need for extra pressure at high density: Improved YN, YY two-body interaction

Three-body forces: NNY, NYY, YYY

Extracting masses and energy shifts

$$p_i(t,\vec{x}) = \varepsilon_{abc}\, d_i^a(t,\vec{x})\,(\,d^{bT}(t,\vec{x})\, C\,\gamma_5\, u^c(t,\vec{x})\,)$$

One-baryon correlator:

mass

$$C_A(t) = \sum_{\vec{x}} \left\langle A(t,\vec{x})\, A^\dagger(0,\vec{0}) \right\rangle = \sum_n C_A^n\, e^{-E_A^n t} \longrightarrow C_A\, e^{-M_A t}$$

2-baryon correlator:

$$C_{AB}(t) = \sum_{\vec{x},\vec{y}} \left\langle A(t,\vec{x})\, B(t,\vec{x})\, B^\dagger(0,\vec{0})\, A^\dagger(0,\vec{0}) \right\rangle = \sum_n C_{AB}^n\, e^{-E_{AB}^n t} \longrightarrow C_{AB}\, e^{-E_{AB} t}$$

Energy shift: $\Delta E = E_{AB} - M_A - M_B$

$$G_{AB}(t) = \frac{C_{AB}(t)}{C_A(t)\, C_B(t)} = \sum_n C^n e^{-\Delta E^n t} \longrightarrow C e^{-\Delta E\, t}$$

$$\Delta E \equiv \sqrt{p^2 + M_A^2} + \sqrt{p^2 + M_B^2} - M_A - M_B$$

(below inelastic thresholds)

obtained from the simulation

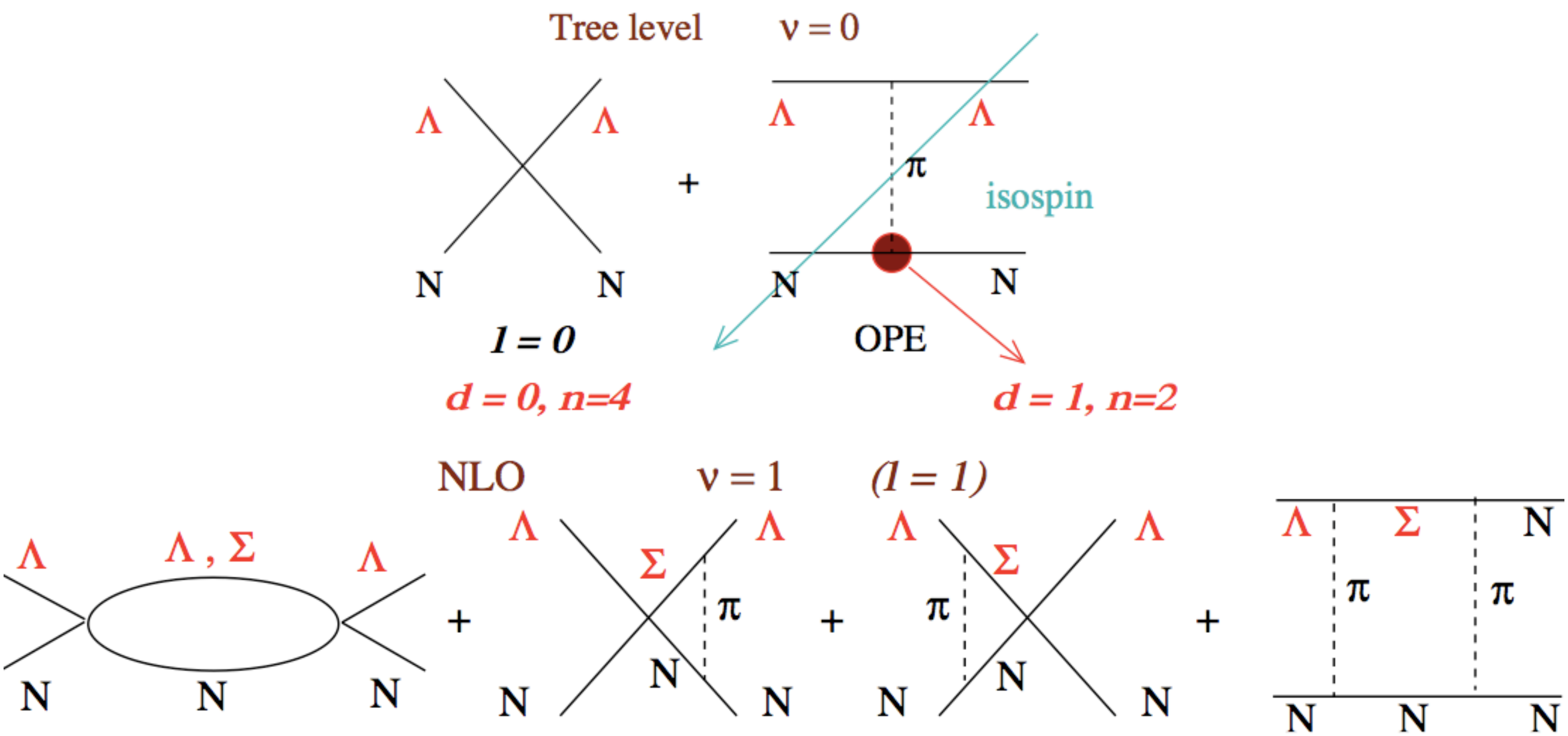
$$S\left(\eta^2 = \frac{p^2 L^2}{4\pi^2}\right) \equiv \sum_{j}^{|\vec{j}|<\Lambda} \frac{1}{|\vec{j}|^2 - \eta^2} - 4\pi\Lambda$$

u.v. regulator

$$-\frac{1}{a} + \frac{1}{2} r_0 p^2 = \quad \Leftarrow \quad p\cot\delta(p) = -\frac{1}{\pi L} S\left(\frac{p^2 L^2}{4\pi^2}\right)$$

Beane, Bedaque, Parreño, Savage, Nucl. Phys. A747, 55-74 (2005); nucl-th/0311027

Idea: write down the effective theory for the hyperon-nucleon interaction at low energies (below the pion production threshold)



Beane, Bedaque, Parreño, Savage, Nucl. Phys. A747, 55-74 (2005); nucl-th/0311027

$$a^{(^1S_0)} = -\frac{\mu_{\Lambda N}}{2\pi}\Bigg[\, {}_{\Lambda\Lambda}C_0^{(^1S_0)} - \frac{3}{4\pi}\left({}_{\Sigma\Lambda}C_0^{(^1S_0)}\right)^2 \mu_{\Lambda N}\,\eta + {}_{\Sigma\Lambda}C_0^{(^1S_0)}\,\frac{3g_{\Sigma\Lambda}g_A\mu_{\Lambda N}}{2\pi f^2}\,\frac{\eta^2+\eta\, m_\pi+m_\pi^2}{\eta+m_\pi} - \frac{3g_{\Sigma\Lambda}^2 g_A^2\mu_{\Lambda N}}{4\pi f^4}\,\frac{2\eta^3+4\eta^2 m_\pi+6\eta m_\pi^2+3m_\pi^3}{2(\eta+m_\pi)^2}\Bigg]$$

Extract LECs

$$r^{(^1S_0)} = -\frac{1}{\mu_{\Lambda N}\,\pi}\left[\frac{2\pi}{{}_{\Lambda\Lambda}C_0^{(^1S_0)}}\right]^2\Bigg[\,\frac{3}{8\pi}\left({}_{\Sigma\Lambda}C_0^{(^1S_0)}\right)^2\frac{\mu_{\Lambda N}}{\eta} + {}_{\Sigma\Lambda}C_0^{(^1S_0)}\,\frac{3g_{\Sigma\Lambda}g_A\mu_{\Lambda N}}{2\pi f^2}\,\frac{3\eta^2+9\eta\, m_\pi+8m_\pi^2}{6(\eta+m_\pi)^3} - \frac{3g_{\Sigma\Lambda}^2 g_A^2\mu_{\Lambda N}}{4\pi f^4}\,\frac{6\eta^3+23\eta^2 m_\pi+28\eta m_\pi^2+7m_\pi^3}{12(\eta+m_\pi)^4}\Bigg]$$

Result of the LQCD simulation

Our (NPLQCD) first study of hyperon-nucleon interactions:

Ref: "hyperon-nucleon interactions from Lattice QCD" Nucl. Phys. A794 (2007) 62-72

Lüscher formalism to extract the scattering length

2+1 dynamical flavors

Mixed action: Domain-Wall (valence) on staggered (sea)

Smeared source

MILC 20^3x64 configurations chopped to 20^3x32

Dimensions $L_S^3 \times L_T$ (L_5 = 16)	b (fm)	L (fm)	m_π (MeV)	m_K (MeV)	no. conf x no. src
$20^3 \times 32$ m_l=030 m_s=050	0.125	2.5	591	675	564 x 8
$20^3 \times 32$ m_l=020 m_s=050	0.125	2.5	491	640	486 x 8
$20^3 \times 32$ m_l=010 m_s=050	0.125	2.5	352	595	769 x 8
$20^3 \times 32$ m_l=007 m_s=050	0.125	2.5	291	580	1039 x 8

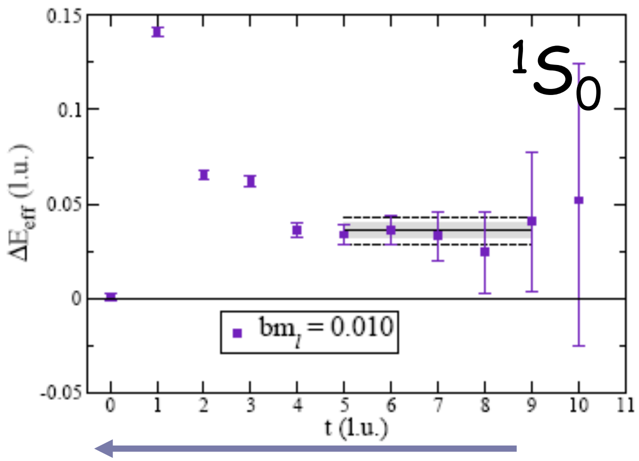
Λn

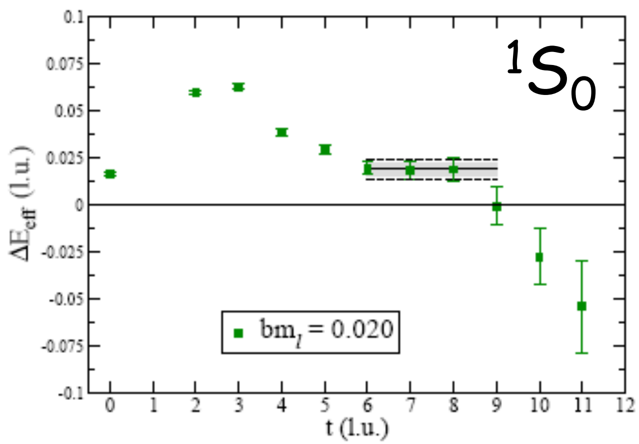
NPLQCD, Nucl. Phys. A794 (2007) 62-72

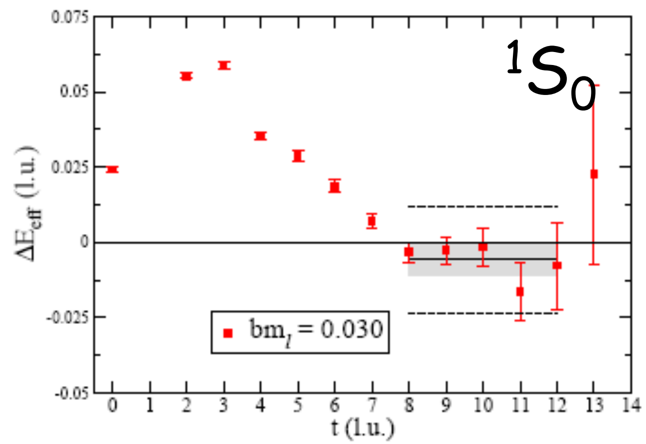
MILC 20^3x32 L = 2.5 fm b ~ 0.125 fm

signal-to-noise ratio ~ $\sqrt{N_{conf}}\; e^{-(M_N + M_\Lambda - 2m_\pi - m_K)t}$

$$\frac{G_{AB}(t)}{G_{AB}(t+1)} = \Delta E$$





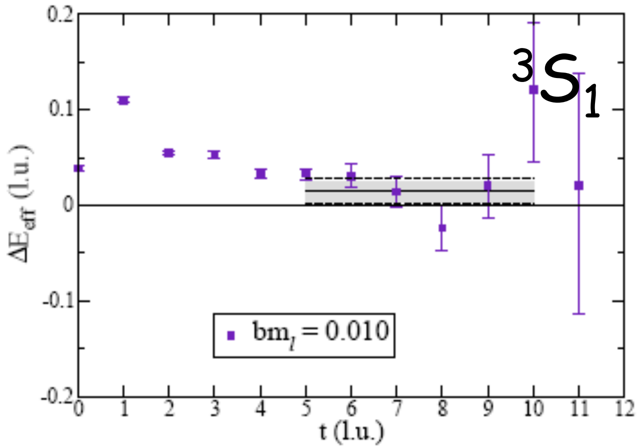


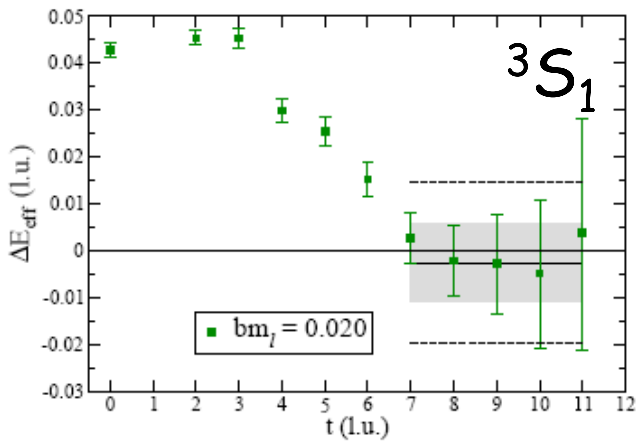
contamination from excited states

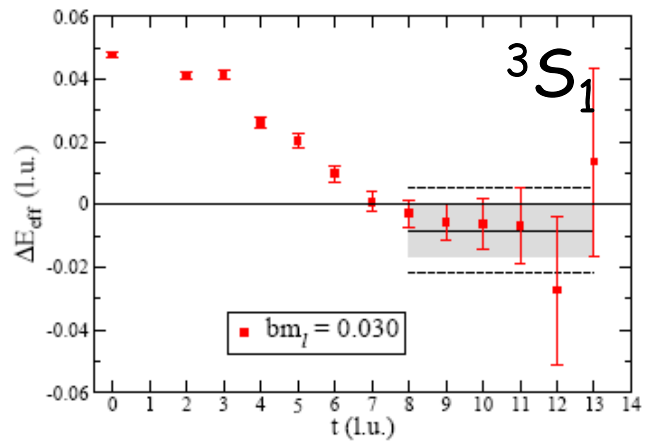
m_π = 350 MeV

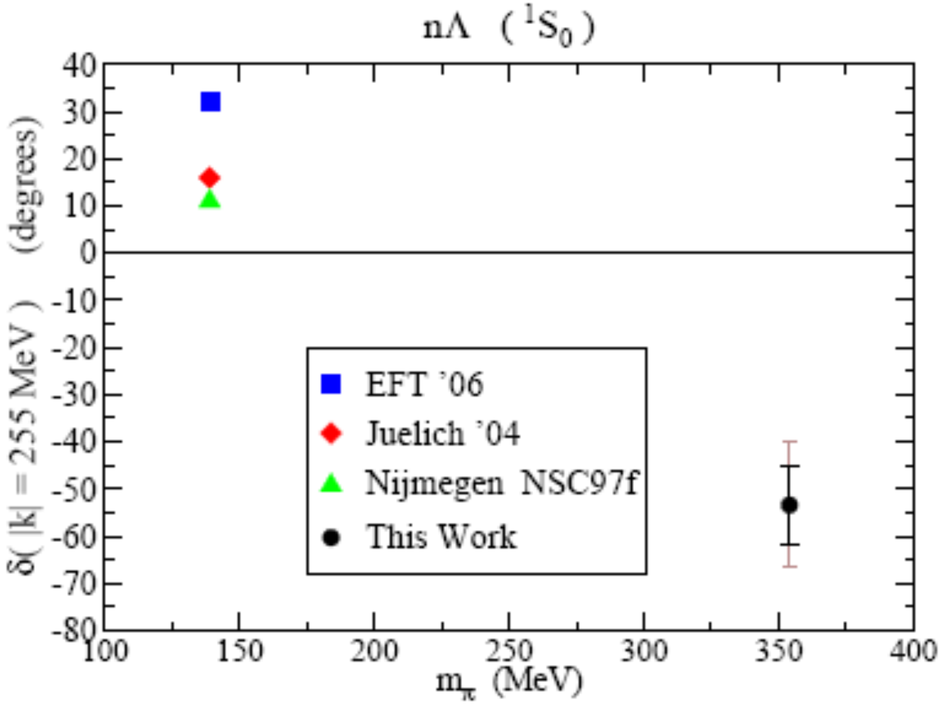
m_π = 490 MeV

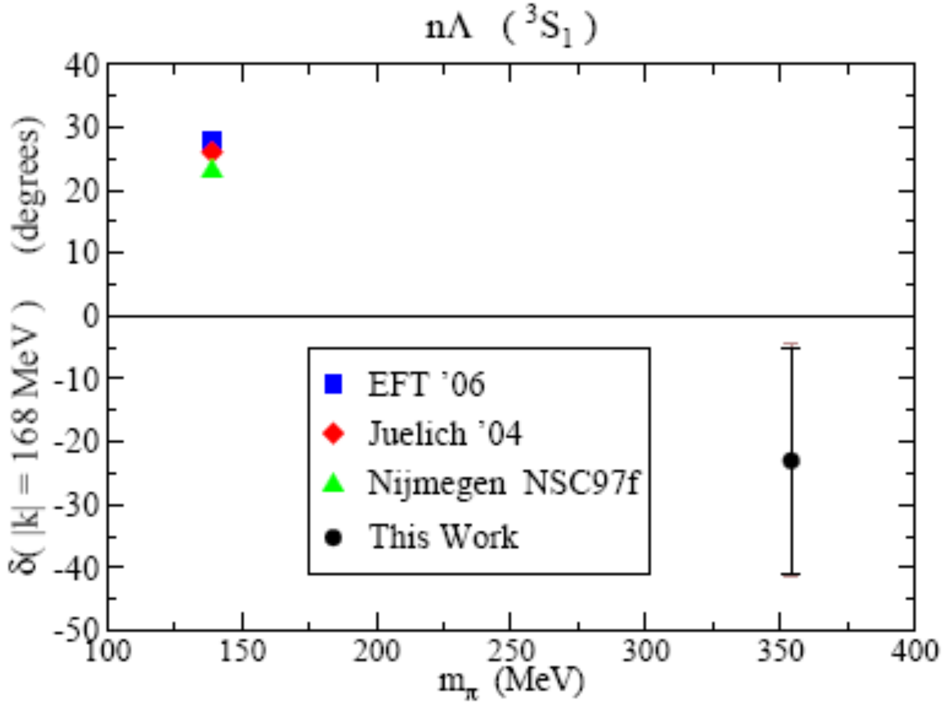
m_π = 590 MeV



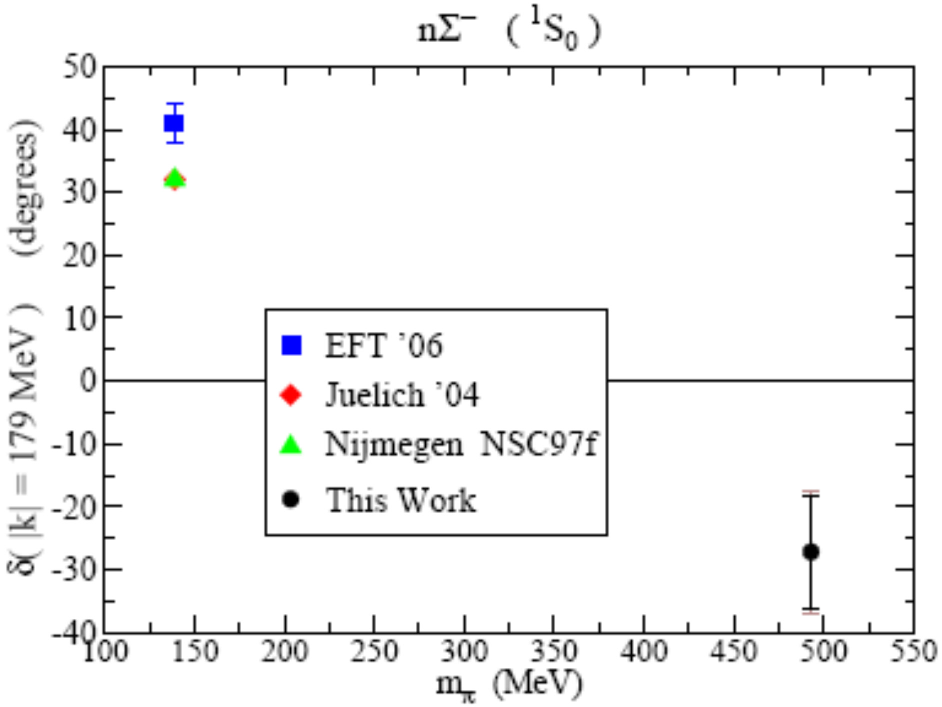


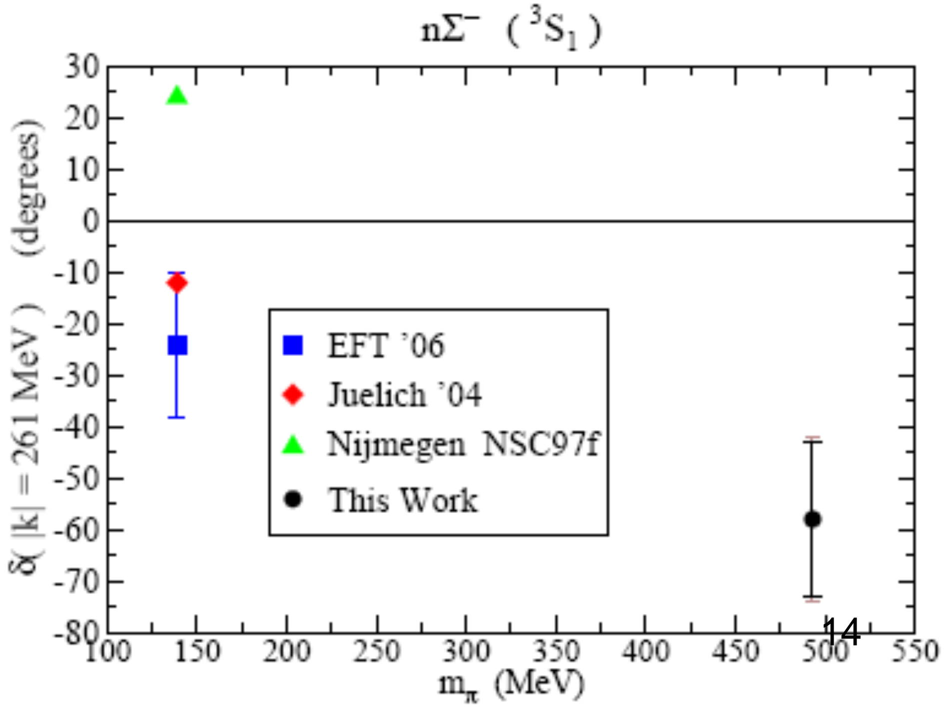






NPLQCD, Nucl. Phys. A 794 (2007) 62-72





Different issues to approach:

use all the temporal extent

go to finer anisotropic lattices (more temporal resolution, reduce the associated systematic error)

go to lighter masses

Dimensions $L_S^3 \times L_T$ ($L_5 = 16$)	b (fm)	L (fm)	m_π (MeV)	m_K (MeV)	no. conf x no. src
$20^3 \times 32$ m_l=0.030 m_s=0.050	0.125	2.5	591	675	564 x 24
$20^3 \times 32$ m_l=0.020 m_s=0.050	0.125	2.5	491	640	486 x 24
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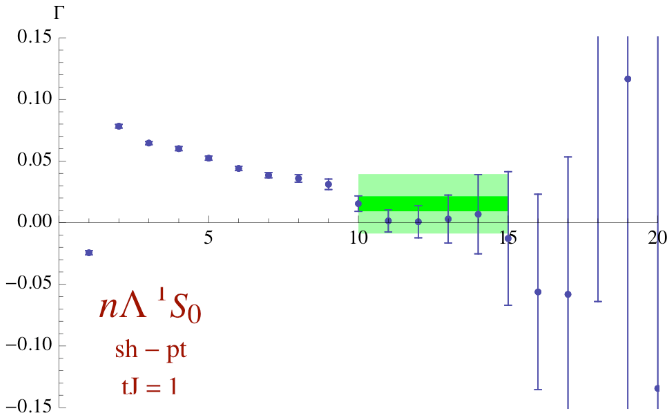
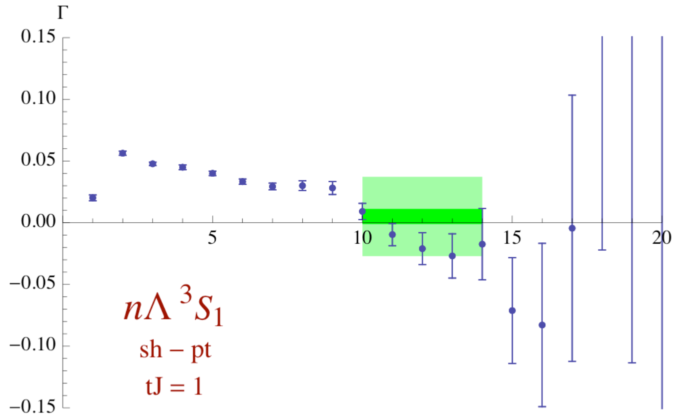
$20^3 \times 32$ m_l=0.007 m_s=0.050	0.125	2.5	291	580	1039 x 24

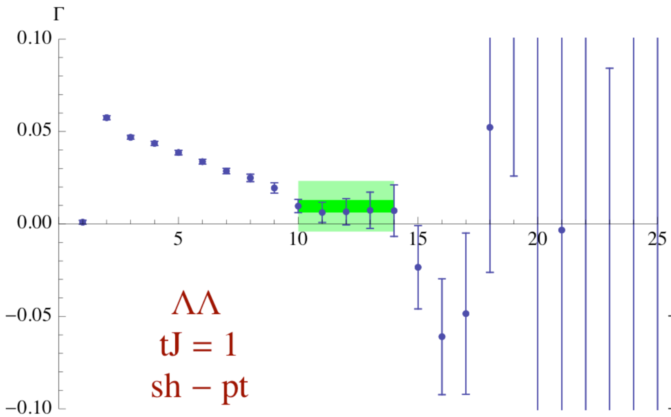
Dimensions $L_S^3 \times L_T$ ($L_5 = 12$)	b (fm)	L (fm)	m_π (MeV)	m_K (MeV)	no. conf x no. src
$28^3 \times 96$ m_l=0.0062 m_s=0.031	0.09	2.5	320	560	1001 x 7
$28^3 \times 96$ m_l=0.0124 m_s=0.031	0.09	2.5	446	578	513 x 3

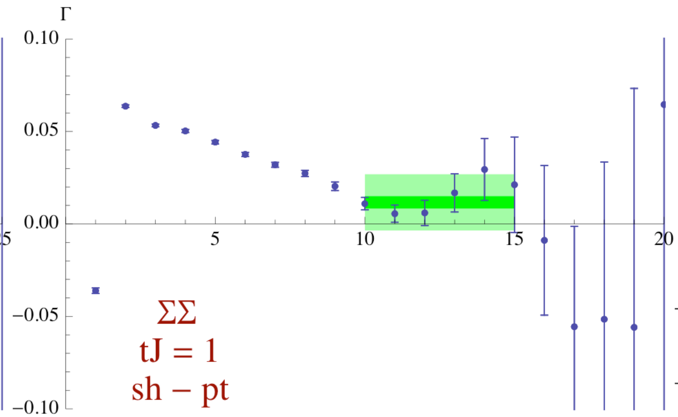
$40^3 \times 96$ m_l=0.0062 m_s=0.031 ($L_5 = 40$)	0.09	2.5	230	539	109 x 1
$40^3 \times 96$ m_l=0.0062 m_s=0.031 ($L_5 = 12$)	0.09	2.5	234	540	109 x 1

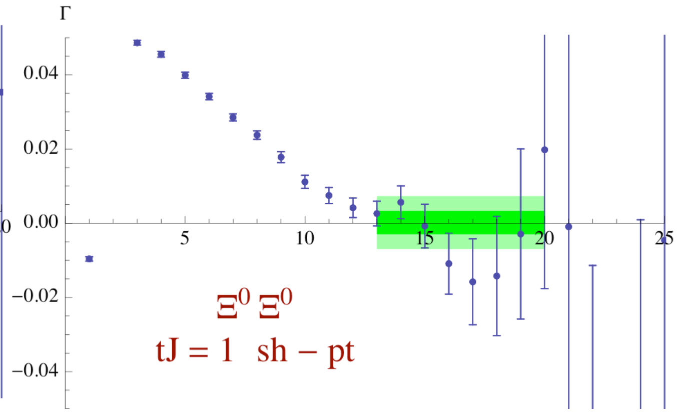
$28^3 \times 96$ m_l=0.0062 m_s=0.031	b=0.09fm	L=2.5 fm	m_π=320 MeV	m_K=560 MEV





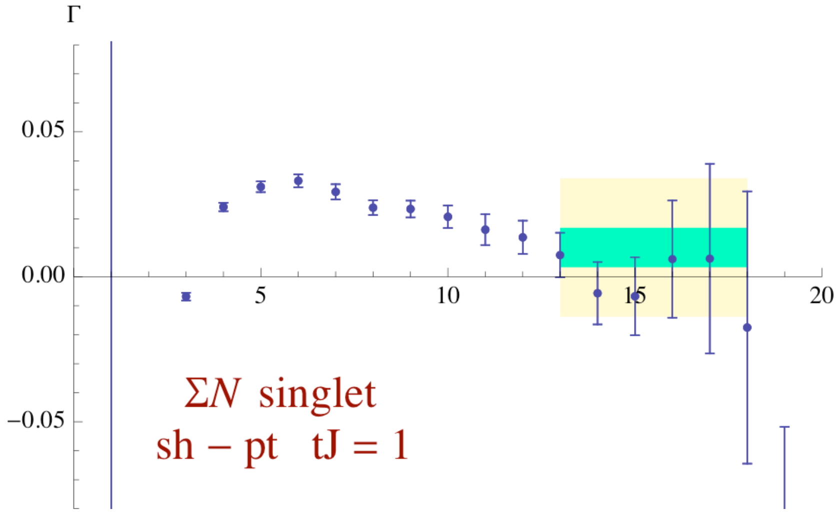


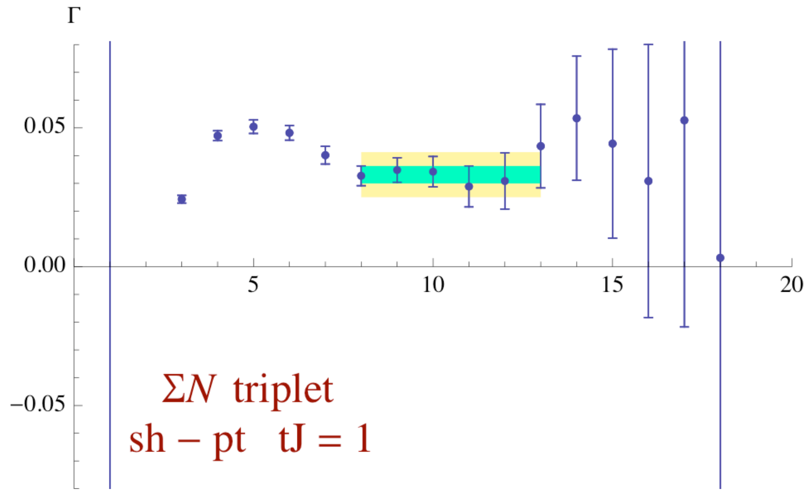


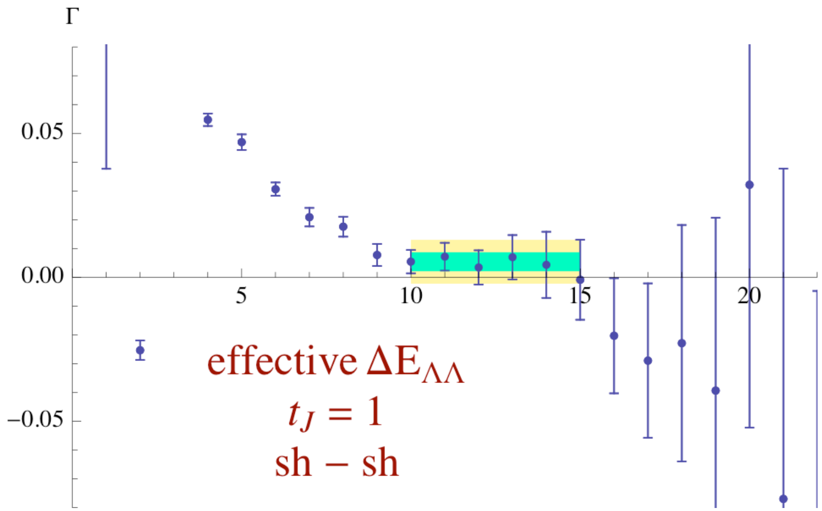


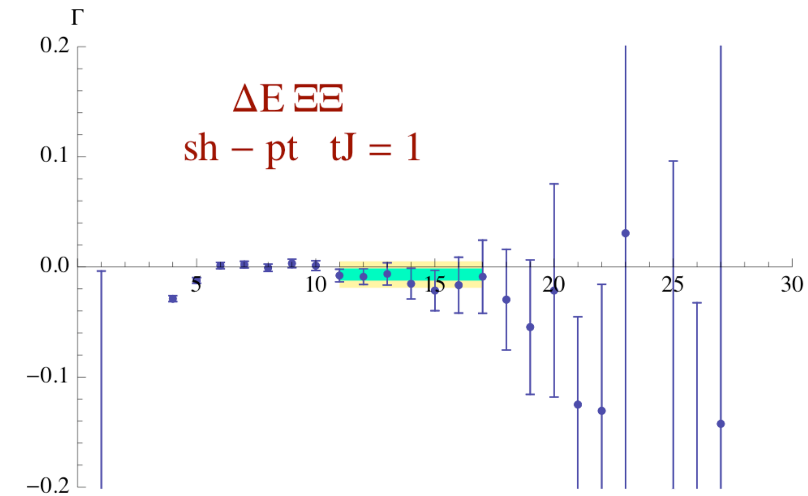
$$\frac{1}{\Delta t}\frac{G_{AB}(t)}{G_{AB}(t+\Delta t)} = \Delta E$$

$28^3 \times 96$ m_l=0.0124 m_s=0.031	b=0.09 fm	L=2.5fm	m_π =446 MeV	m_K =578 MeV



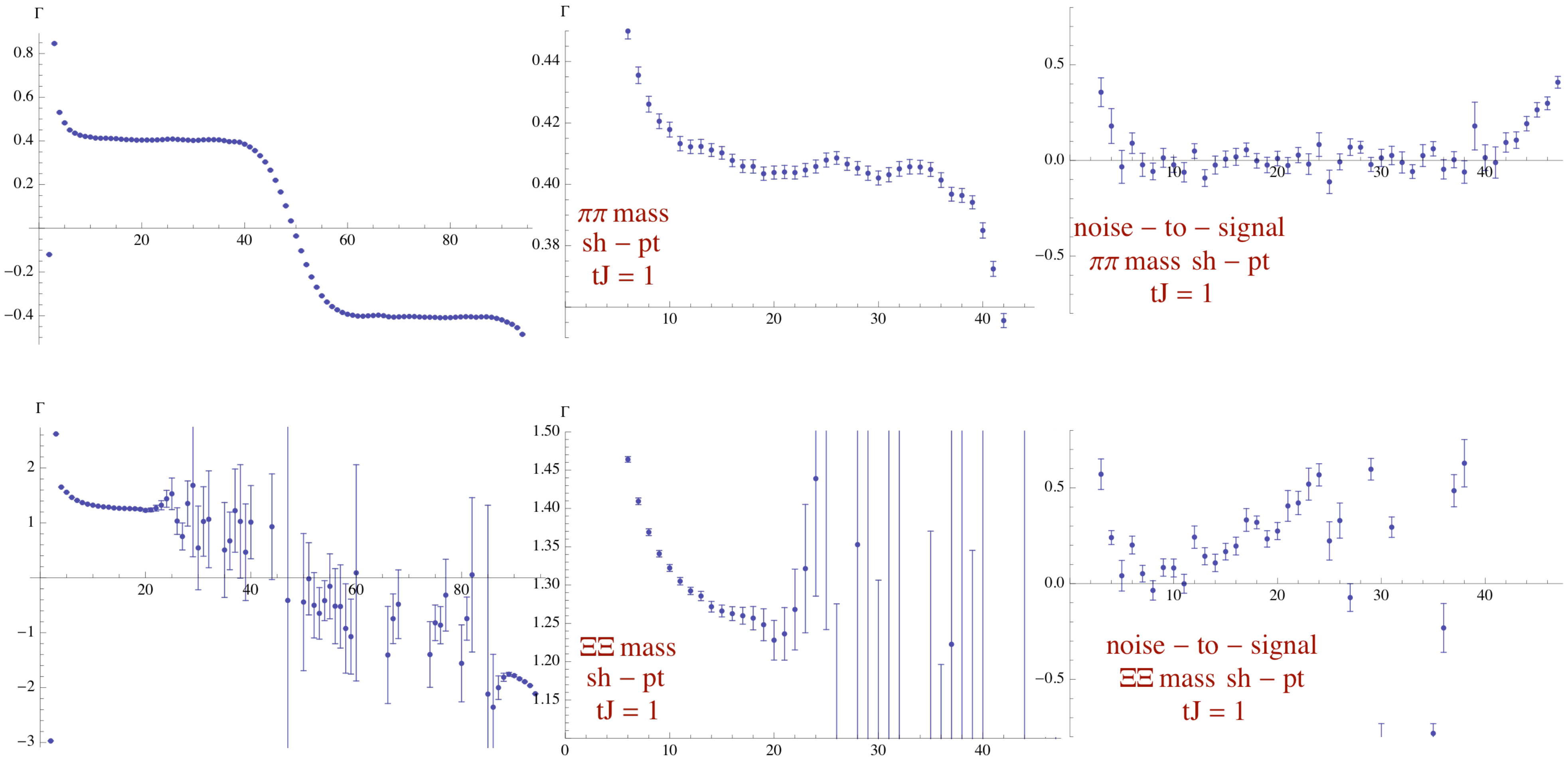






noise-to-signal

$28^3 \times 96$ m_l=0.0124 m_s=0.031	b=0.09 fm	L=2.5fm	m_π =446 MeV	m_K =578 MeV



Hadron Spectrum Collaboration $20^3 \times 128$ lattices

@ m_π =390 MeV and m_K =546 MeV

clover discretization of the fermion action

L = 3.5 fm b ≈ 0.1227 fm

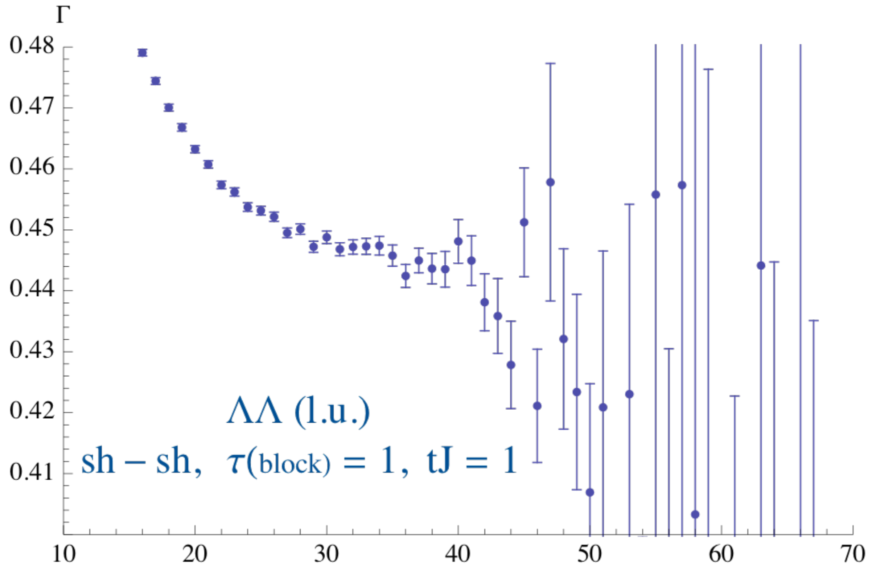
High statistics: 1194 configurations x 245 prop/conf ≈ 292500 measurements

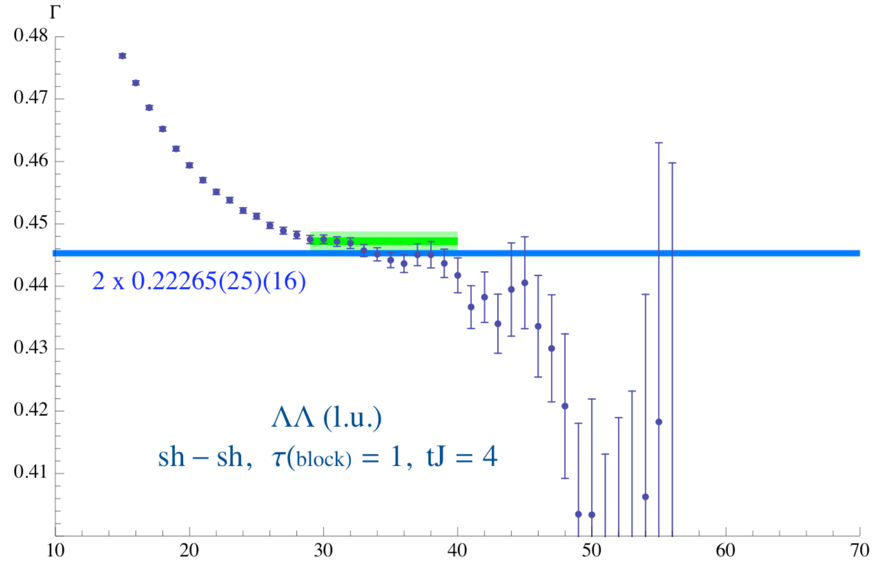
Advantages:

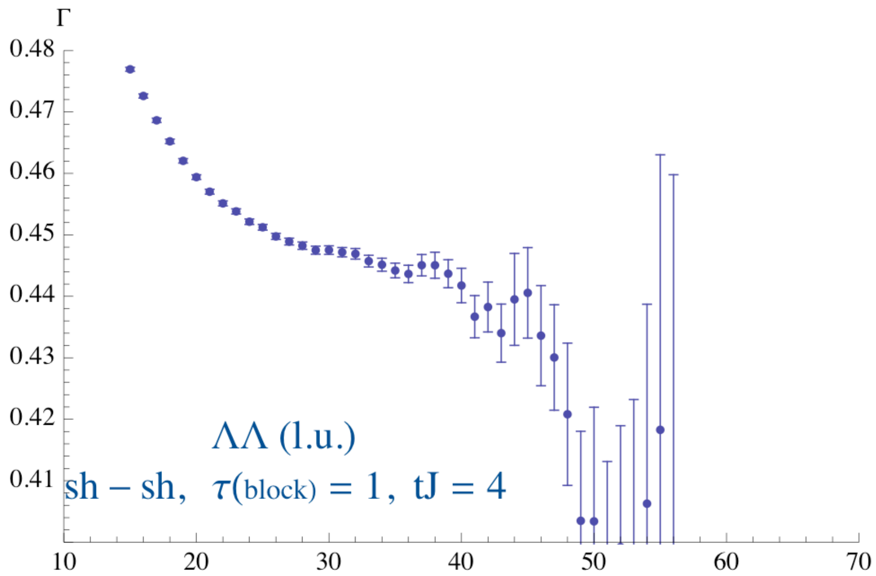
- No mixed action: same fermion action in the gauge-field generation and in the computation of the light and strange quark-propagators
- Faster (4-D compared to 5-D of DW fermions)

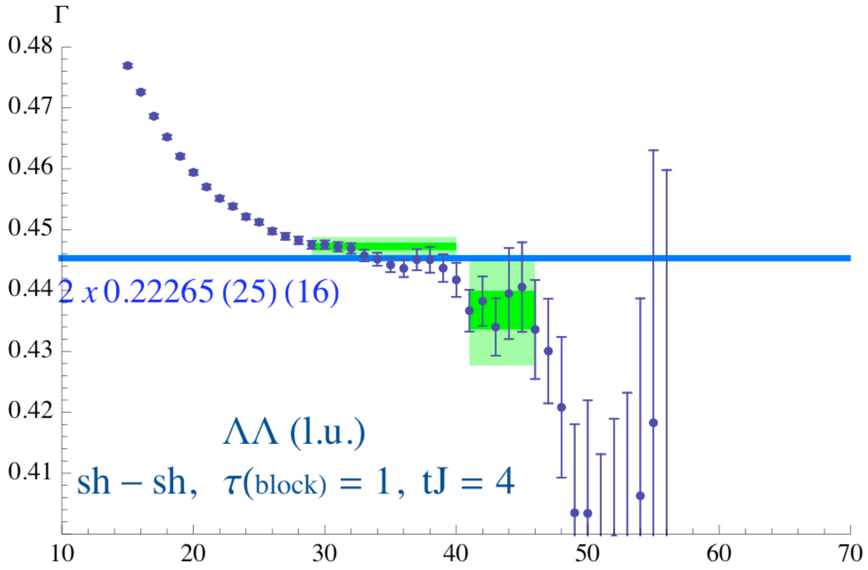
Generalized effective mass plots

$$\frac{G_{AB}(t)}{G_{AB}(t+1)} = \Delta E \quad \Rightarrow \quad \frac{1}{\Delta t}\frac{G_{AB}(t)}{G_{AB}(t+\Delta t)} = \Delta E$$

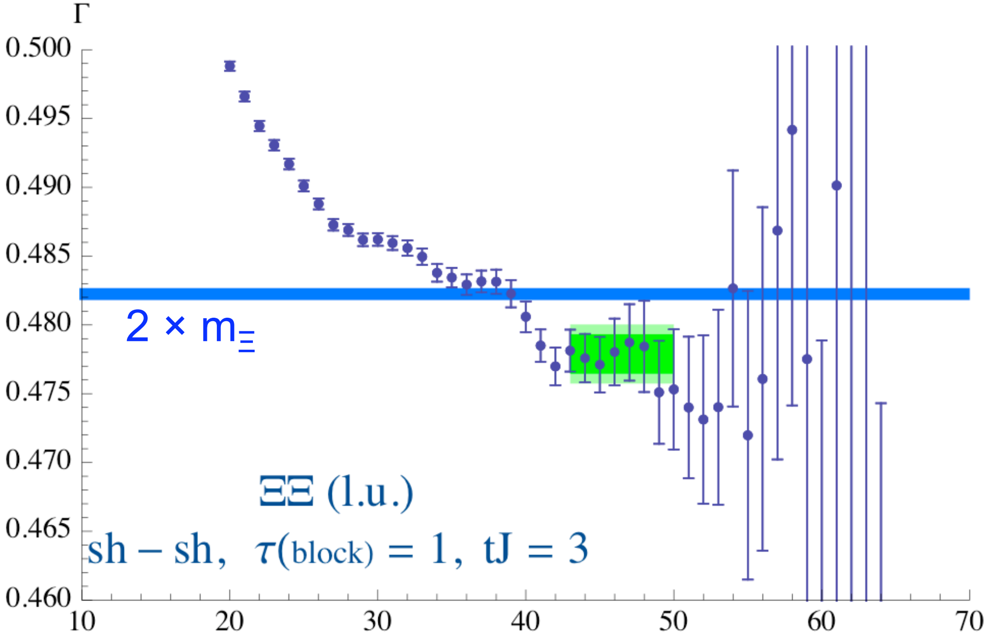


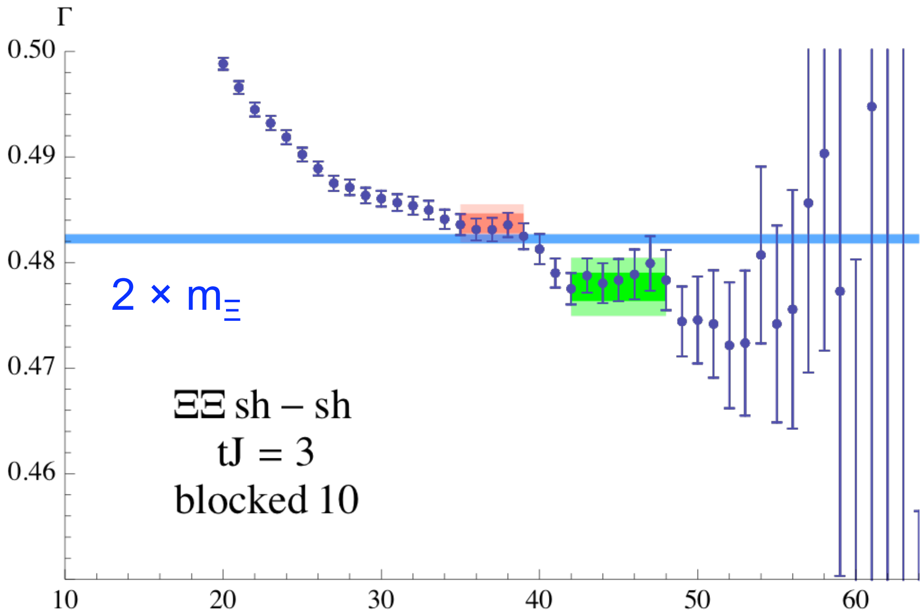


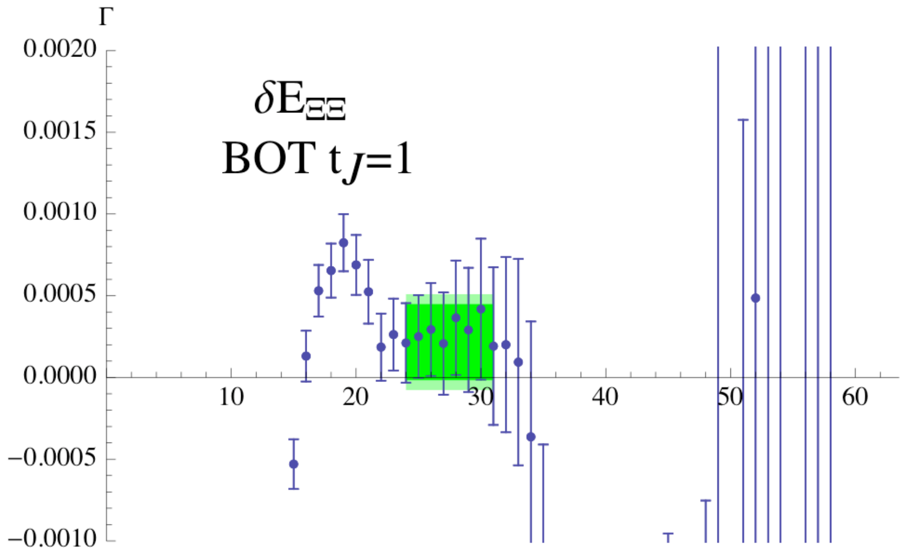


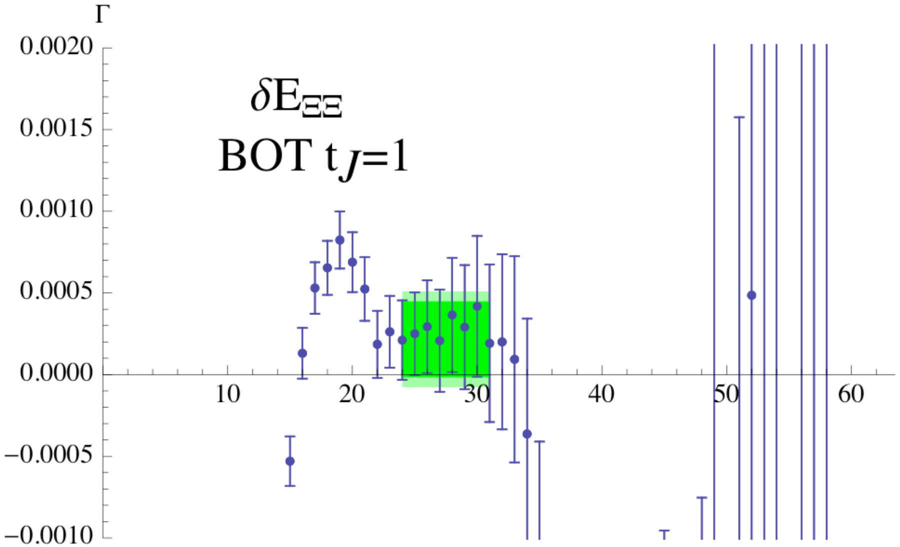


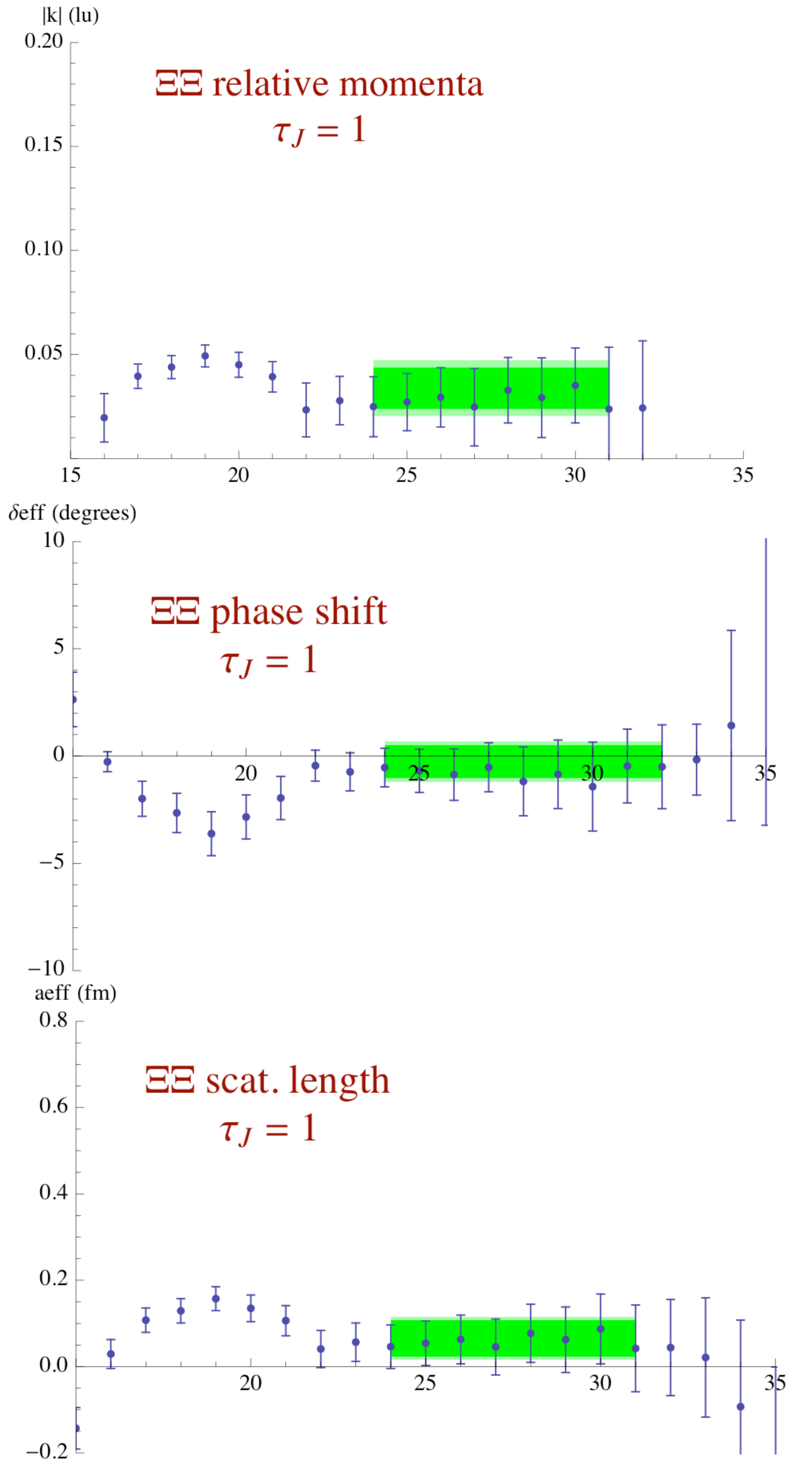
Making an optimal combination of the smeared-smeared and smeared-point correlators







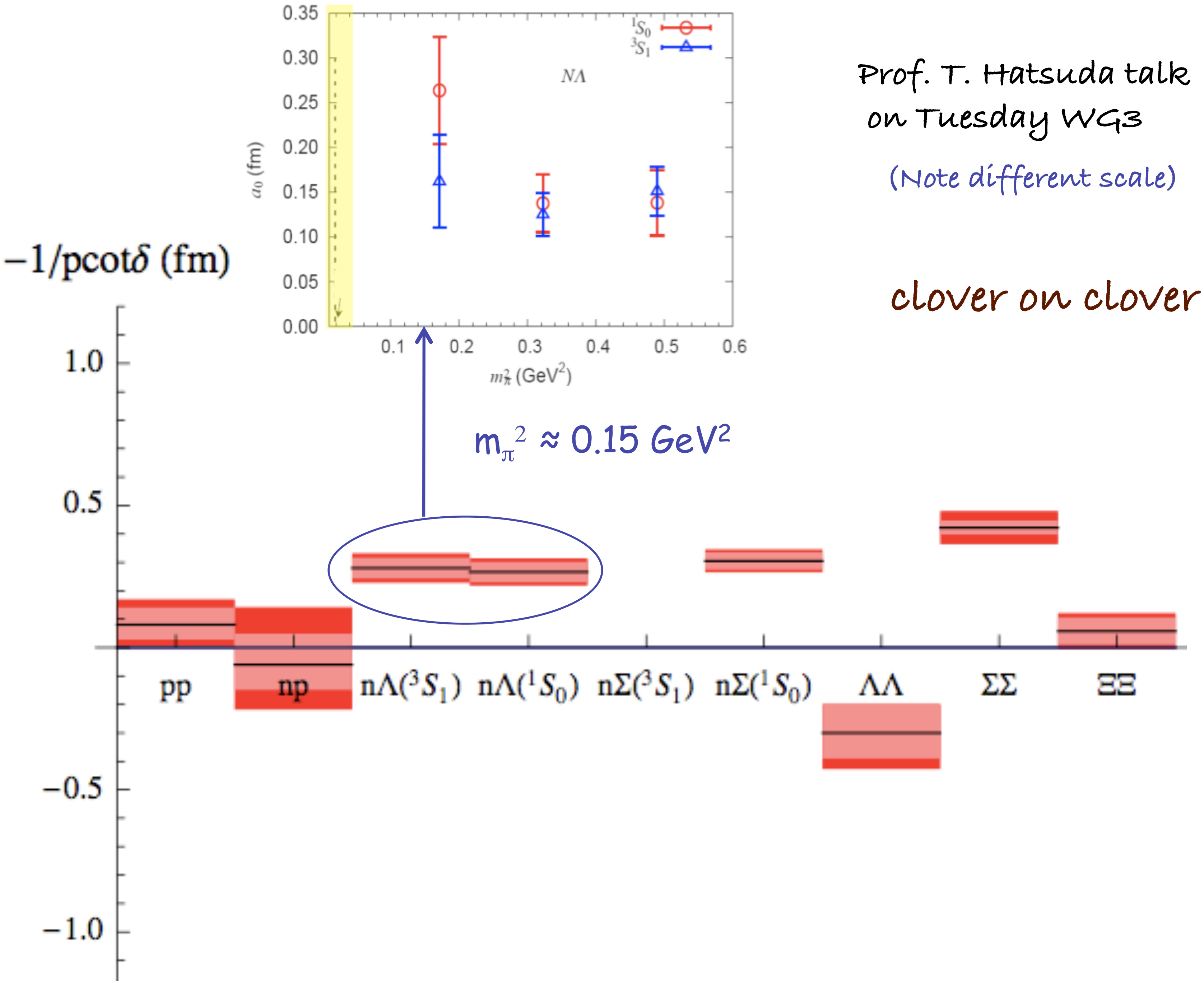




$$p \cot \delta(p) = -\frac{1}{\pi L} S\left(\frac{p^2 L^2}{4\pi^2}\right)$$

$$= -\frac{1}{a} + \frac{1}{2} r_0 p^2$$

Chiral Dynamics, 6-10 July 2009, Bern



Prof. T. Hatsuda talk on Tuesday WG3

(Note different scale)

clover on clover

Preliminary

Conclusions

Mixed action on coarse lattices: the signal to noise ratio scales very poorly with the number of configurations. Extraction of a precise result in the baryonic sector would probably require an exponentially-large number of configurations.

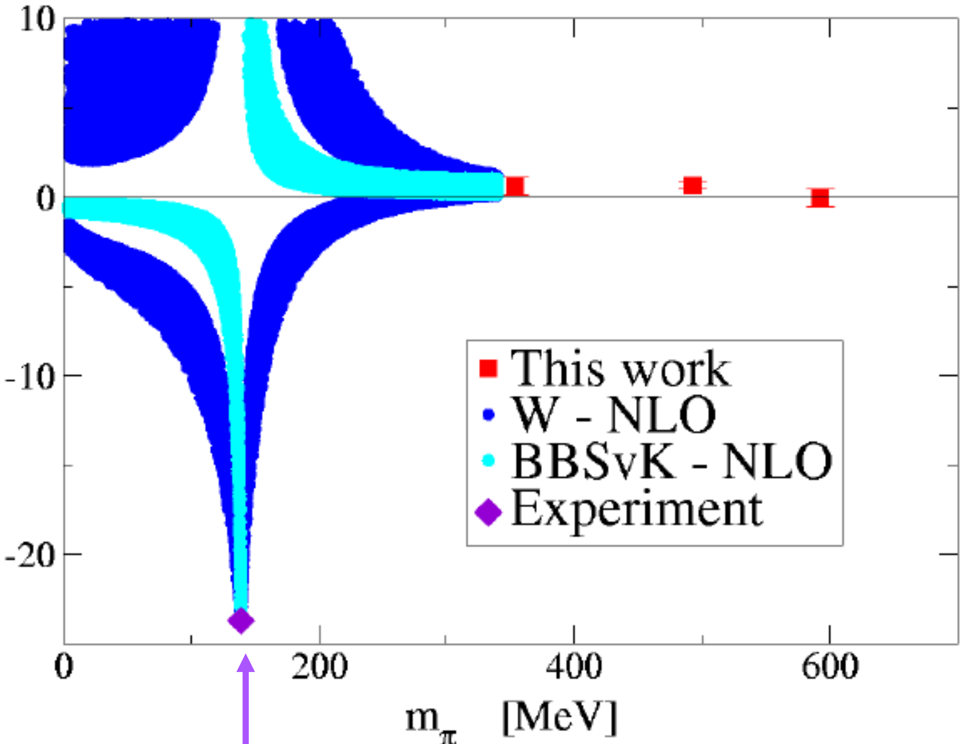
We are increasing our statistics in the anisotropic finer lattices.

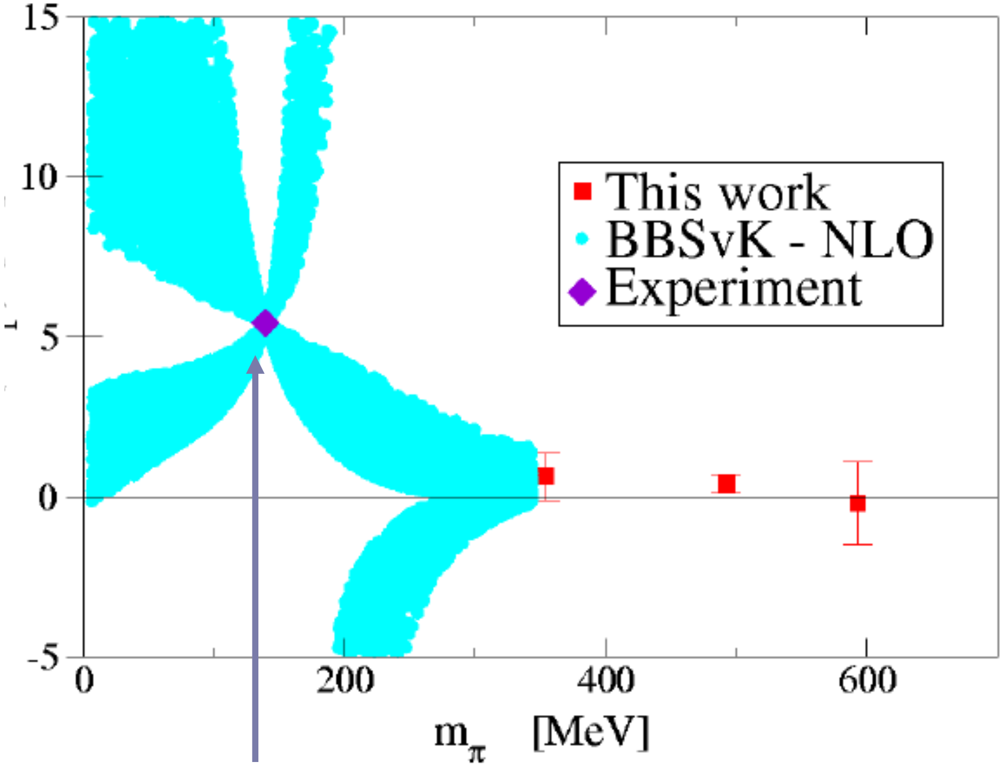
We have currently exploring a number of analysis techniques in order to better isolate the ground state for the two-baryon systems.

We are in the process of analyzing our clover on clover data, where we have a large number of measurements.

Back-up slides

NN Beane, Bedaque, Orginos, Savage, PRL97 012001 (2006)





$a\,(^1S_0)$ [fm]

5.4 fm

$a\,(^3S_1)$ [fm]

- 23.8 fm

@ m_π = 350, 590, 590 MeV

L=2.5 fm

Extracting masses and energy shifts

Generalized effective plots

$$\frac{1}{\Delta t}\log\frac{C_A(t)}{C_A(t+\Delta t)} = m_A \qquad \frac{1}{\Delta t}\log\frac{G_{AB}(t)}{G_{AB}(t+\Delta t)} = \Delta E_{AB}$$

$$G_{AB}(t) = \frac{C_{AB}(t)}{C_A(t)\,C_B(t)} = \sum_n C^n e^{-\Delta E^n t} \longrightarrow \boxed{Ce^{-\Delta E\, t}}$$

$$\log\frac{G_{AB}(t)}{G_{AB}(t+1)} = \Delta E$$

effective plots

alternatives...

$p\,p \rightarrow K^+ \Lambda\, p$
(COSY, Jülich)

Balewski et al. EPJA 2 (1998)
Hinterberger, Sibirtsev, EPJA 21 (2004)
Gasparyan, Haidenbauer, Hanhart, Speth, PRC69 (2004)
Gasparyan, Haidenbauer, Hanhart, PRC72 (2005)

$\gamma\, d \rightarrow K^+ \Lambda\, n$

Gasparyan, Haidenbauer, Hanhart, K. Miyagawa

(CEBAF, ELSA, JLAB, MAMI-C)

Reconstruct the elastic two-body amplitude via the invariant mass dependence of the production amplitude in the region where the YN momentum is small.

$$K^- d \rightarrow n\Lambda\gamma$$

Gall et al., PRC42 (1990)

Gibson et al. BNL report No. 18335(1973)
Gibbs, Coon, Han, Gibson ,PRC61 (2000)

$$a(^1S_0) = -0.15 \rightarrow -5.0$$

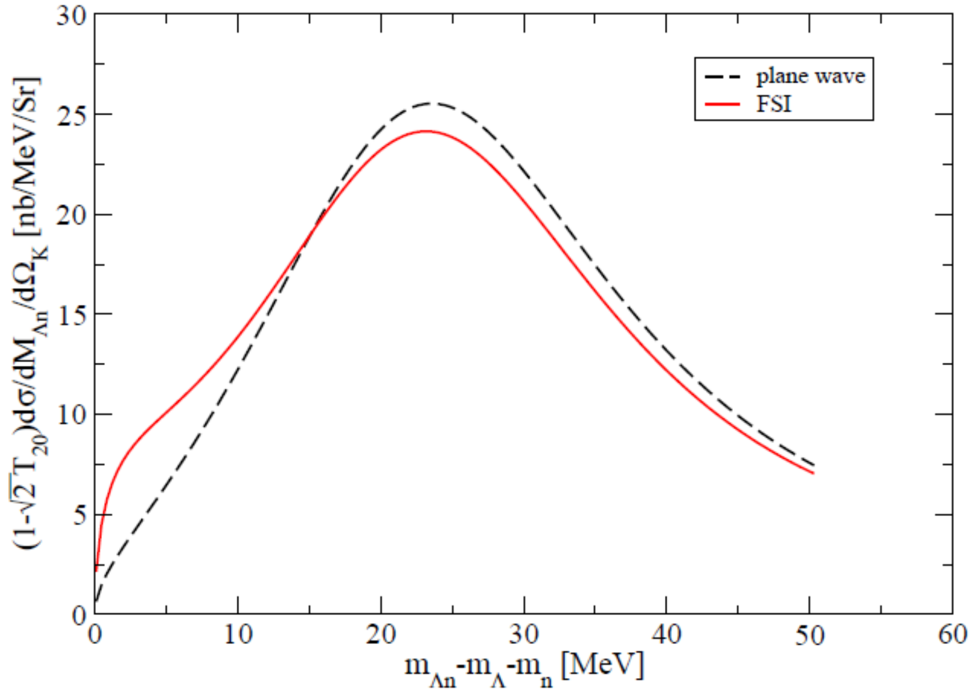
$$a(^3S_1) = -1.3 \rightarrow -2.65$$

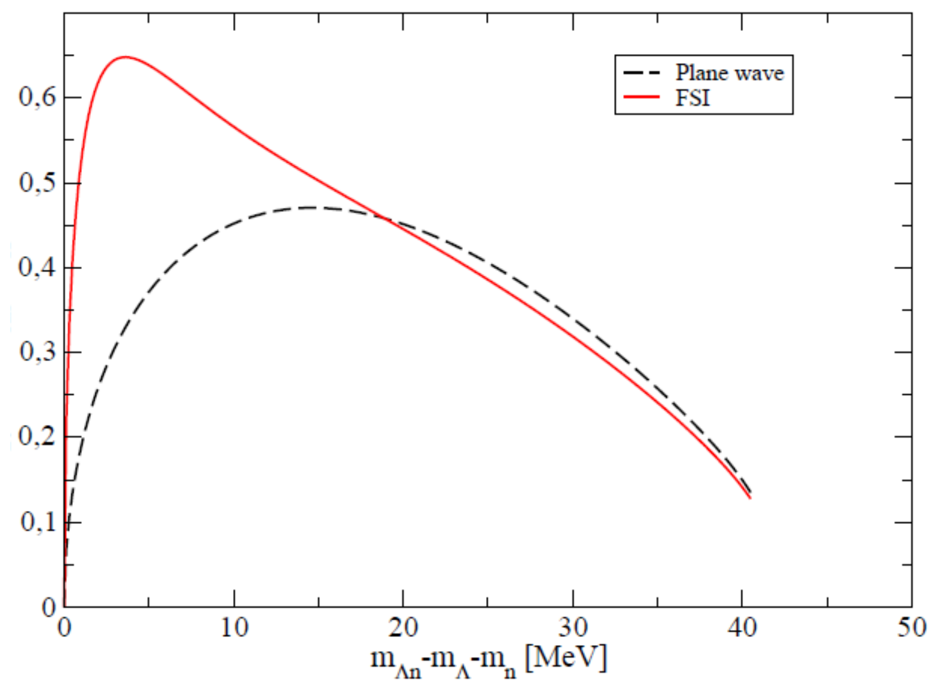
kaon production on the proton

$\gamma d \rightarrow K^+ \Lambda n$

COSY, Jülich (Germany)

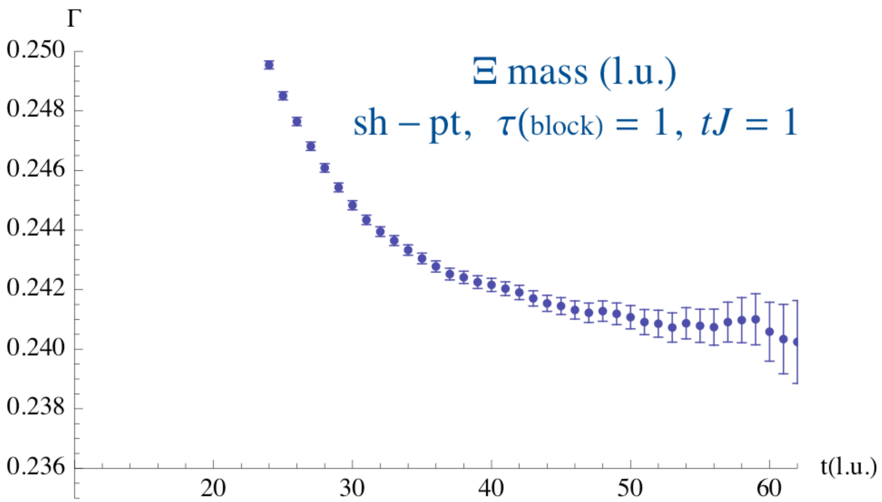
$$1-\sqrt{2}T_{02}^{0}$$

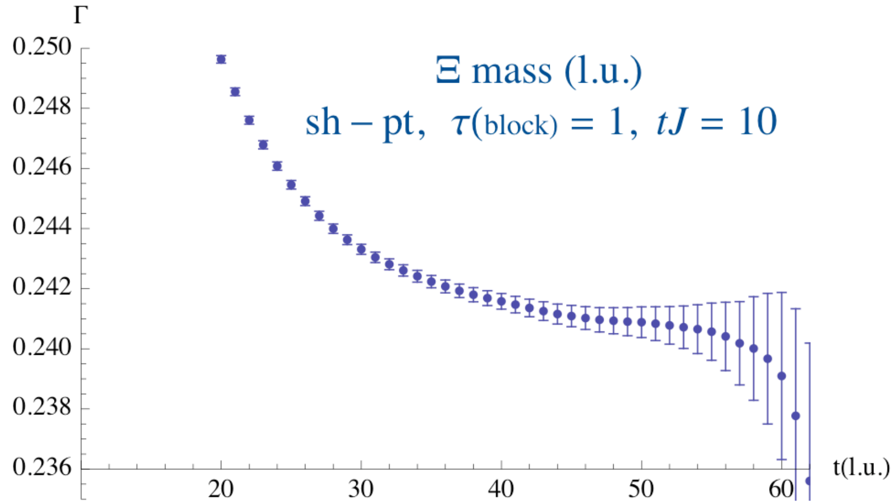




at p_{lab}=1300 MeV and $\theta_K=0^o$

at p_{lab}=850 MeV and $\theta_K=0^o$





$$\frac{G_{AB}(t)}{G_{AB}(t+1)} = \Delta E \quad \Rightarrow \quad \frac{1}{\Delta t}\frac{G_{AB}(t)}{G_{AB}(t+\Delta t)} = \Delta E$$