MILC results for light pseudoscalars

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- MILC Configurations
- Chiral fitting
- “Low mass” SU(3) chiral fits
- “High mass” SU(3) chiral fits
- SU(2) chiral fits
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- Preliminary results from the SU(2) chiral fits
- Summary

Staggered issues

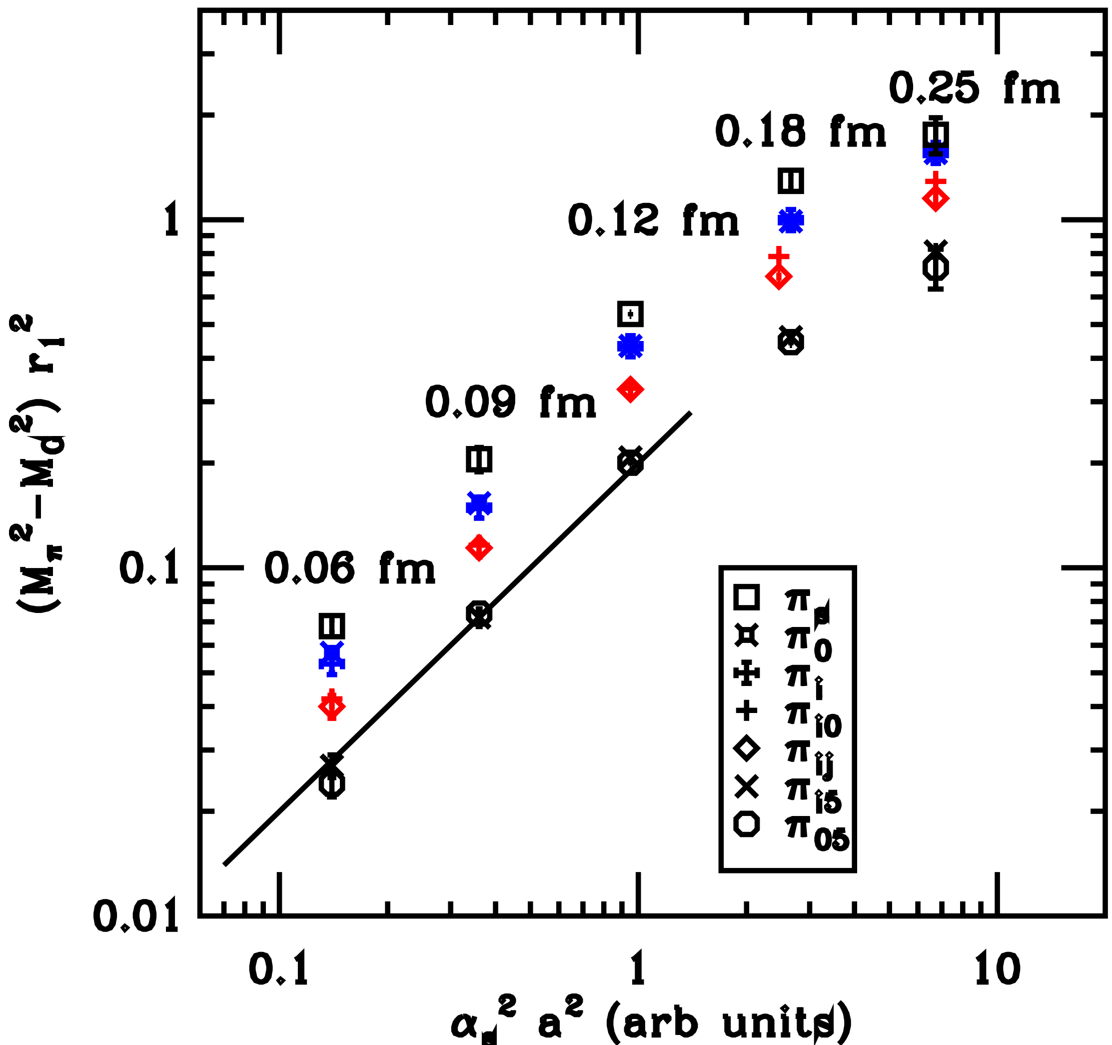
- MILC simulations use improved Kogut-Susskind ("staggered") quarks — a^2-tadpole improved, or "asqtad" quarks
- Staggered quarks are very fast
- Staggered quarks have an exact, non-singlet, axial symmetry on the lattice; have an exact, non-singlet, (pseudo-) Goldstone pion
- BUT: one staggered fermion field (1 "flavor") represents 4 "tastes" — 4-fold remnant of lattice doubling symmetry
- Taste symmetry is broken on the lattice at $\mathcal{O}(\alpha_s^2 a^2)$
  $\Rightarrow$ At finite lattice spacing, extra tastes cannot be trivially accounted for and removed
- MILC simulations use $\sqrt[4]{\mathrm{Det}(\not{D} + m)}$ to get a single taste per flavor in the continuum limit.
  ("Fourth-root procedure" due to Marinari, Parisi & Rebbi.)

Staggered issues

- Normal (unrooted) staggered fermions almost certainly OK in perturbation theory (PT): $\sqrt[4]{\mathrm{Det}}$ is trivially correct to all orders in PT
- But concern that nonperturbatively $\sqrt[4]{\mathrm{Det}}$ produces violations of locality (and hence universality) in the continuum limit
  - In fact, Bernard, Golterman & Shamir showed that fourth-root is non-local at $a \neq 0$
  - If non-locality persisted as $a \to 0$, staggered theory would not reproduce QCD
- Recent arguments & results look very positive, though:
  - Shamir: renormalization group analysis
  - Bernard: chiral perturbation theory analysis
  - See recent reviews by Sharpe and Kronfeld
  - Growing body of numerical checks by Dürr & Hoelbling; Follana, Hart & Davies; MILC

Staggered issues

- Crucial for validity of fourth root procedure that taste violations vanish in the continuum limit.

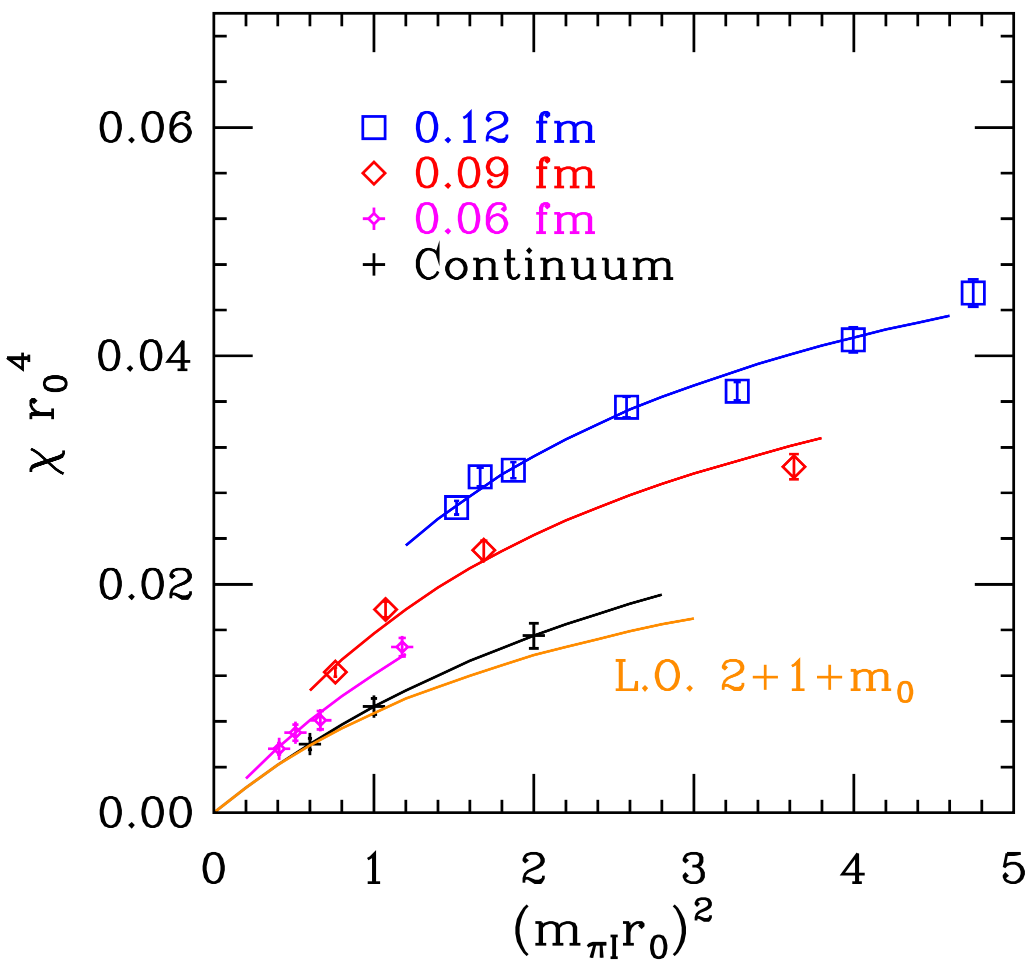


Staggered issues

- Assuming fourth root procedure is valid, taste-violations, and accompanying non-localities at $a \neq 0$ give the dominant lattice spacing errors
- Need to take taste-violations into account for continuum and chiral extrapolations
- Incorporate the staggered discretization errors into chiral perturbation theory: get "staggered χPT" (SχPT), and, taking rooting into account "rooted SχPT" (rSχPT)
- All fits described here use rSχPT forms at NLO

Staggered issues

- Behavior of topological susceptibility is strongly n_f dependent: Good test of fourth root procedure
- Analyze with SχPT — taste singlet pion should be used



MILC Configurations

- Since 1999, MILC Collaboration has been generating asqtad configurations with 2+1 sea-quark flavors, degenerate u and d, and a heavier s
- Lattices are referred to as:
  - $a \approx 0.18$ fm = “extra coarse” — not used here
  - $a \approx 0.15$ fm = “coarser” — not used here
  - $a \approx 0.12$ fm = “coarse” — not used here
  - $a \approx 0.09$ fm = “fine”
  - $a \approx 0.06$ fm = “super-fine”
  - $a \approx 0.045$ fm = “ultra-fine”
- Simulation strange quark masses (m'_s) are in range $0.6 m_s \lesssim m'_s \lesssim 1.2 m_s$, and even $m'_s \approx 0.12 m_s$ ($= \hat{m}'$)
- Lowest $m_\pi \approx 180\,\mathrm{MeV}$ ($\hat{m}' = m_{u,d} \sim 6\,\mathrm{MeV}$) on fine, $m_\pi \approx 240\,\mathrm{MeV}$ on others, except for ultra-fine

MILC Configurations

- Physical volumes range from $\approx (2.4\,\mathrm{fm})^3$ to $\approx (5.4\,\mathrm{fm})^3$ (at lightest masses), all with $m_\pi L > 4$
- The MILC configurations are publicly available at: `http://qcd.nersc.gov/`
- Show table with new, or substantially enlarged, ensembles since "Chiral 06" next:

MILC Configurations

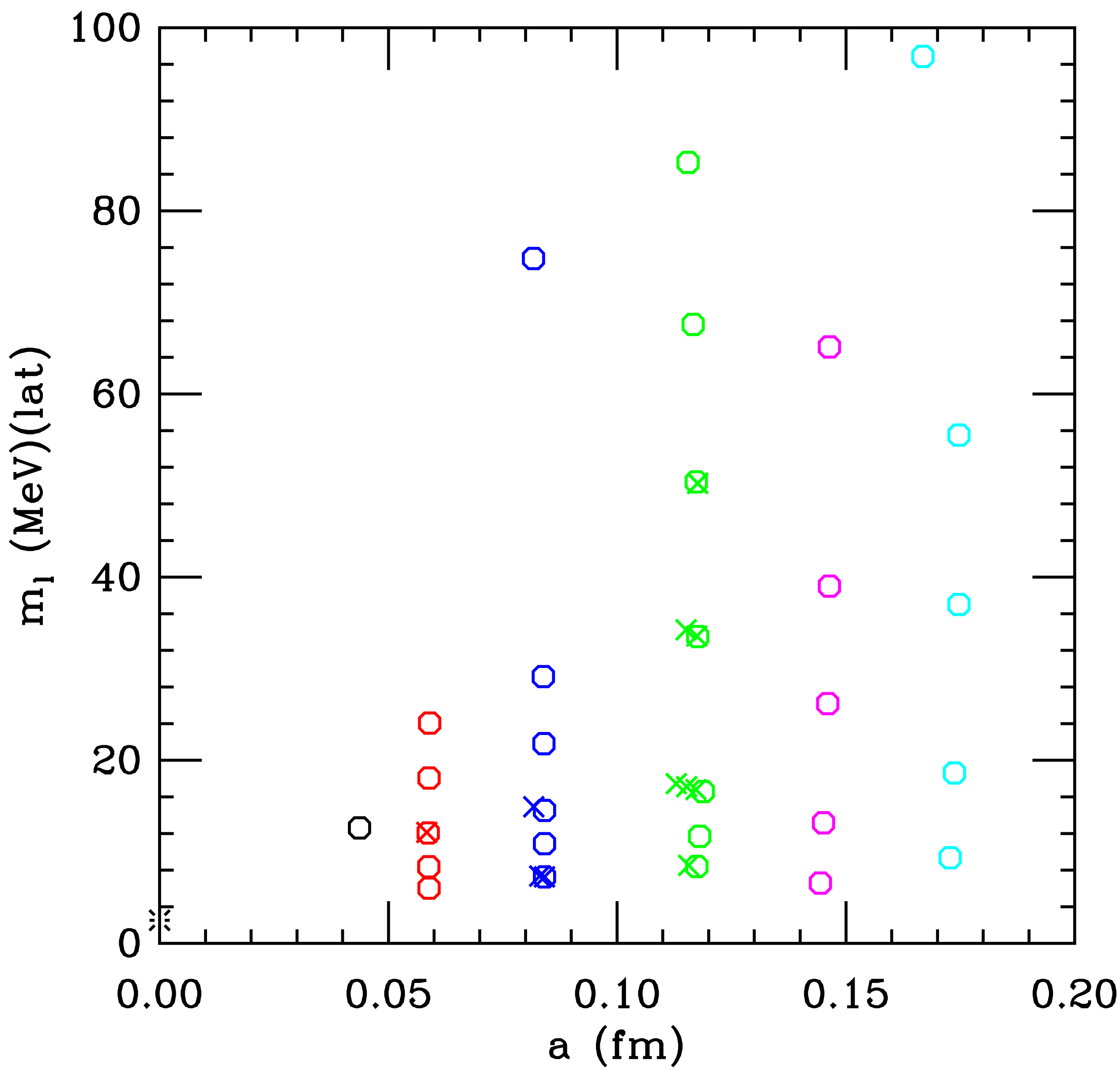
a (fm)	$a\hat{m}'$ / am'_s	$10/g^2$	size	# lats.	$m_\pi L$
≈ 0.12	0.03 / 0.05	6.81	$20^3 \times 64$	362	7.56
≈ 0.12	0.02 / 0.05	6.79	$20^3 \times 64$	485	6.22
≈ 0.12	0.01 / 0.05	6.76	$20^3 \times 64$	894	4.48
≈ 0.12	0.007 / 0.05	6.76	$20^3 \times 64$	836	3.78
≈ 0.12	0.005 / 0.05	6.76	$24^3 \times 64$	527	3.84
≈ 0.12	0.03 / 0.03	6.79	$20^3 \times 64$	360	7.56
≈ 0.12	0.01 / 0.03	6.75	$20^3 \times 64$	349	4.48
≈ 0.12	0.005 / 0.005	6.715	$32^3 \times 64$	701	5.15
≈ 0.09	0.0124 / 0.031	7.11	$28^3 \times 96$	531	5.78
≈ 0.09	0.0093 / 0.031	7.10	$28^3 \times 96$	1124	5.04
≈ 0.09	0.0062 / 0.031	7.09	$28^3 \times 96$	591	4.14
≈ 0.09	0.00465 / 0.031	7.085	$32^3 \times 96$	480	4.11
≈ 0.09	0.0031 / 0.031	7.08	$40^3 \times 96$	945	4.21
≈ 0.09	0.00155 / 0.031	7.075	$64^3 \times 96$	491	4.80

MILC Configurations

Continued:

a (fm)	$a\hat{m}'$ / am'_s	$10/g^2$	size	# lats.	$m_\pi L$
≈ 0.09	0.0062 / 0.0186	7.10	$28^3 \times 96$	985	4.09
≈ 0.09	0.0031 / 0.0186	7.06	$40^3 \times 96$	580	4.22
≈ 0.09	0.0031 / 0.0031	7.045	$40^3 \times 96$	380	4.20
≈ 0.06	0.0072 / 0.018	7.48	$48^3 \times 144$	625	6.33
≈ 0.06	0.0054 / 0.018	7.475	$48^3 \times 144$	465	5.48
≈ 0.06	0.0036 / 0.018	7.47	$48^3 \times 144$	751	4.49
≈ 0.06	0.0025 / 0.018	7.465	$56^3 \times 144$	768	4.39
≈ 0.06	0.0018 / 0.018	7.46	$64^3 \times 144$	826	4.27
≈ 0.06	0.0036 / 0.0108	7.46	$64^3 \times 144$	601	5.96
≈ 0.045	0.0028 / 0.014	7.81	$64^3 \times 192$	801	4.56

MILC Configurations



Chiral fitting

- All fits described here use rSχPT forms at NLO
- The measured taste breakings of the pseudoscalar masses are used in the rSχPT forms
- Continuum NNLO chiral logs are included

  J. Bijnens, N. Danielsson and T.A. Lähde, Phys. Rev. D70 (2004) 111503 [hep-lat/0406017]

  J. Bijnens and T.A. Lähde, Phys. Rev. D71 (2005) 094502 [hep-lat/0501014]

  J. Bijnens, N. Danielsson and T.A. Lähde, Phys. Rev. D73 (2006) 074509 [hep-lat/0602003]
- Thanks to J. Bijnens for providing code for partially quenched NNLO logs
- For the mass in the NNLO chiral logs we use the root mean square (over tastes) pseudoscalar mass
- The NNLO chiral logs include one-loop logs with NLO LECs at one vertex

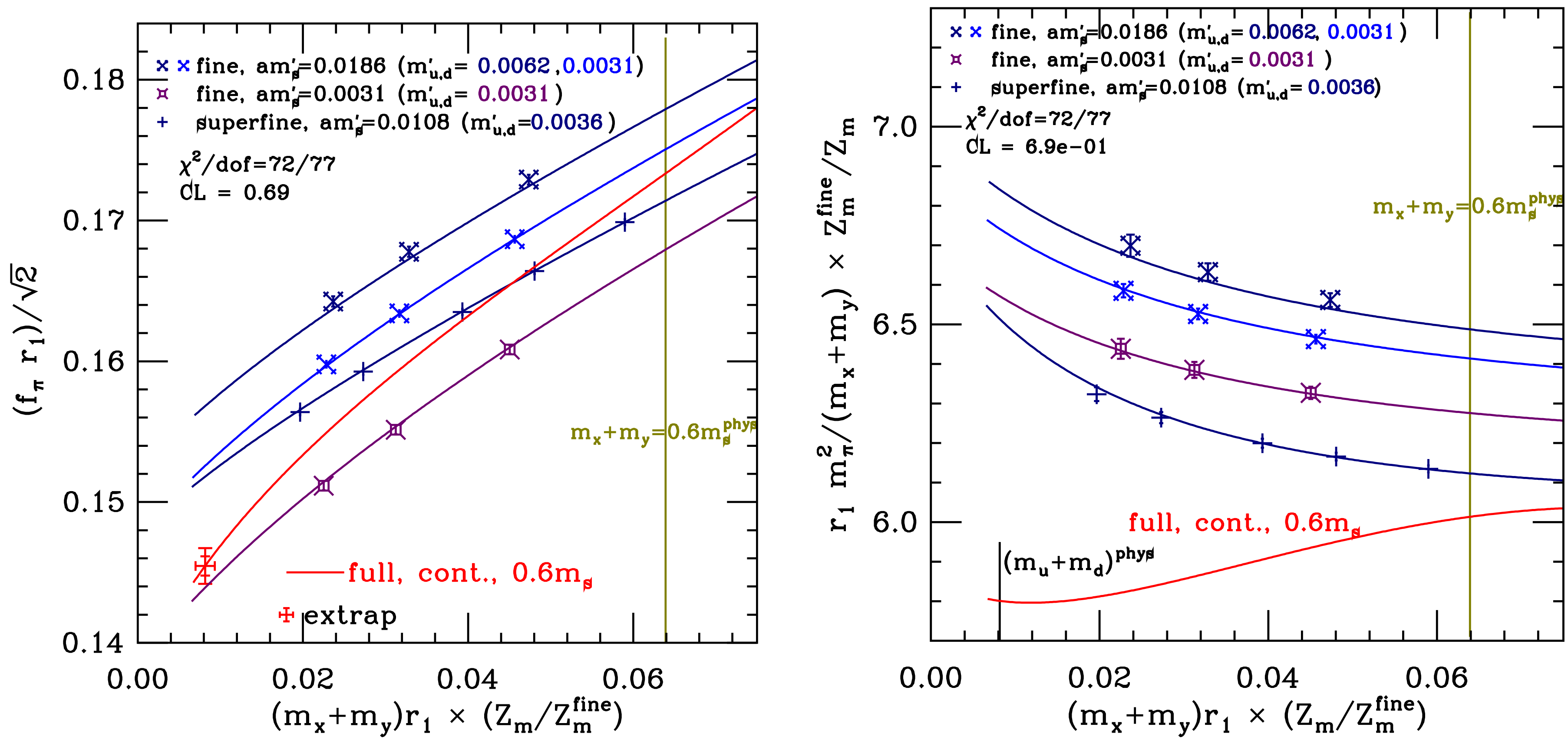
Chiral fitting

- New NLO LECs, that did not contribute to NLO, contribute at NNLO: L_1, L_2, L_3 and L_7 and the partially quenched L_0
- These are not well determined in the fits.
  Priors from Bijnens summary, arXiv:0708.1377, where used for $L_1, \ldots, L_7$.
  For the undetermined L_0, used prior $10^3 L_0 = 0 \pm 2$

“Low mass” SU(3) chiral fits

- Systematic SU(3) chiral fits that use only ensembles with $m_s' \lesssim 0.6 m_s^{phys}$
  This leaves 3 fine and one superfine ensemble
- Valence masses limited by $m_x + m_y \leq 0.6 m_s^{phys}$
- Includes all terms up to NNLO, rSχPT form at NLO, continuum form at NNLO
- For chiral coupling at NNLO, we use a “renormalized” coupling, such as f_π or the decay constant at the lightest valence masses (about 5% bigger than f_π).
  This is equally consistent at this order to using the “bare” (SU(3) chiral limit) coupling f_3, but gives better fits (confidence levels 70% vs 5%).
  Can also let this coupling be a free parameter — fit chooses coupling within 5% to 10% of f_π. Note: $f_\pi / f_3 \approx 1.18$.
- Used to determine LO LECs (B_3 and f_3) and NLO LECs

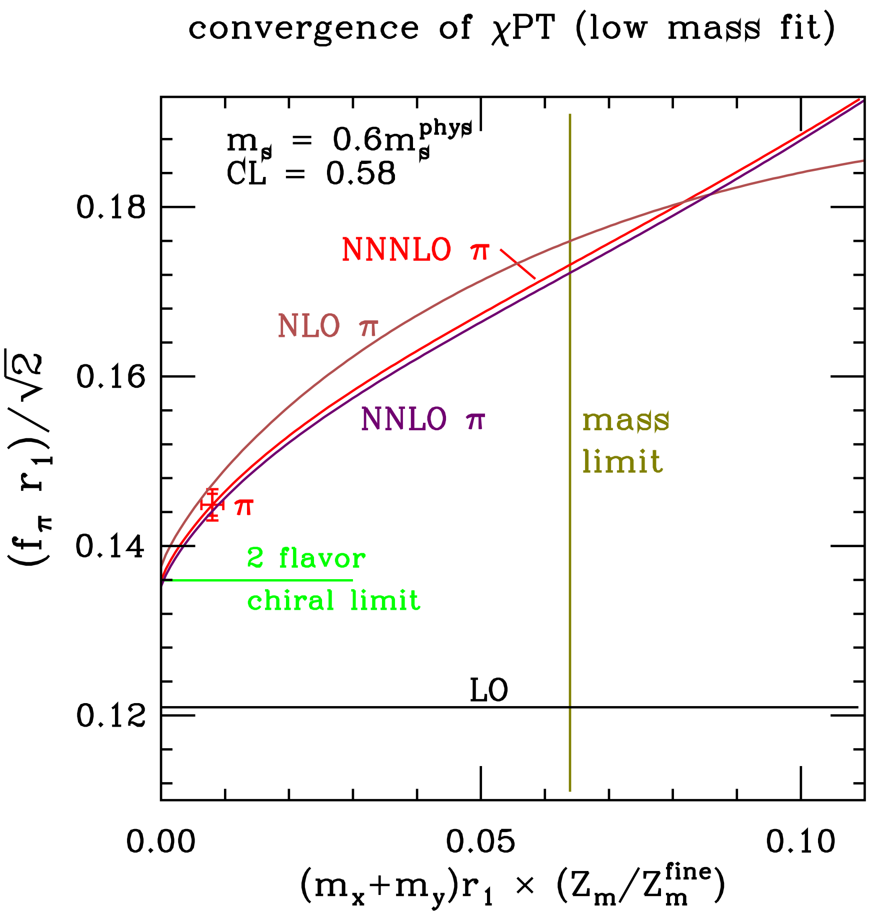
“Low mass” SU(3) chiral fits

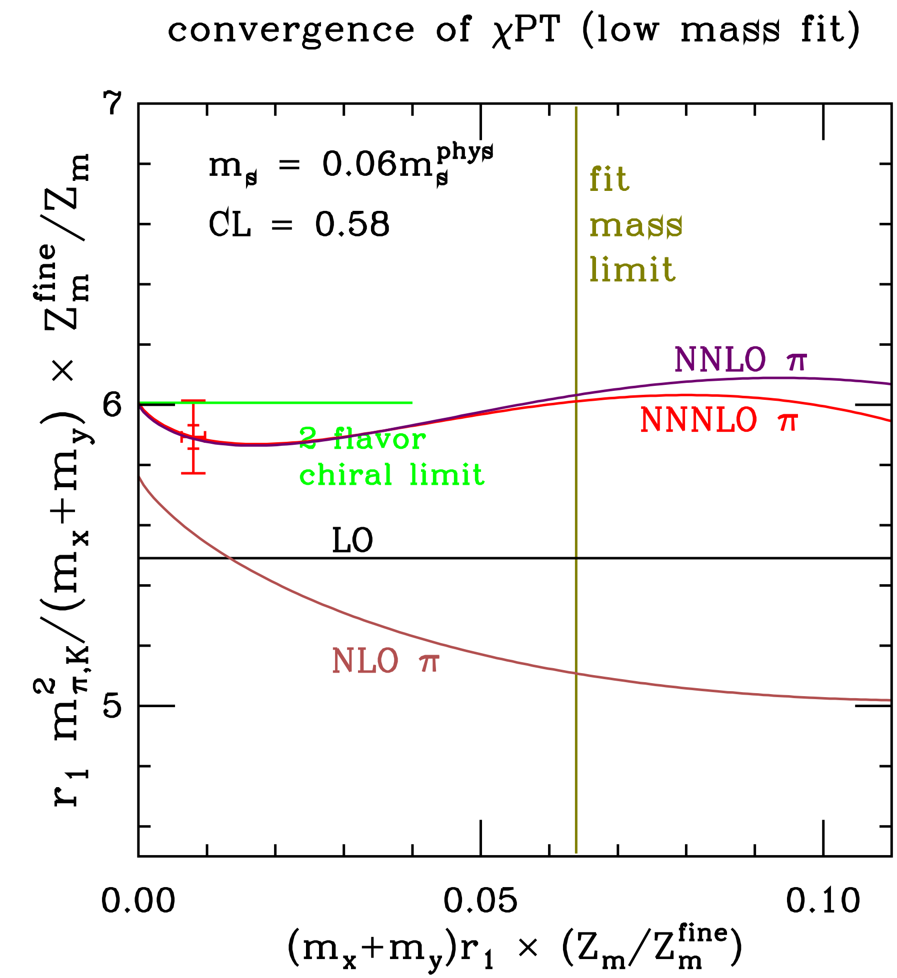


Red curve: continuum extrapolation (a set to zero) and valence and sea quark masses set equal (“full QCD”) with strange mass taken as $0.6m_s^{phys}$

"Low mass" SU(3) chiral fits

Convergence of the low mass SU(3) chiral fits:



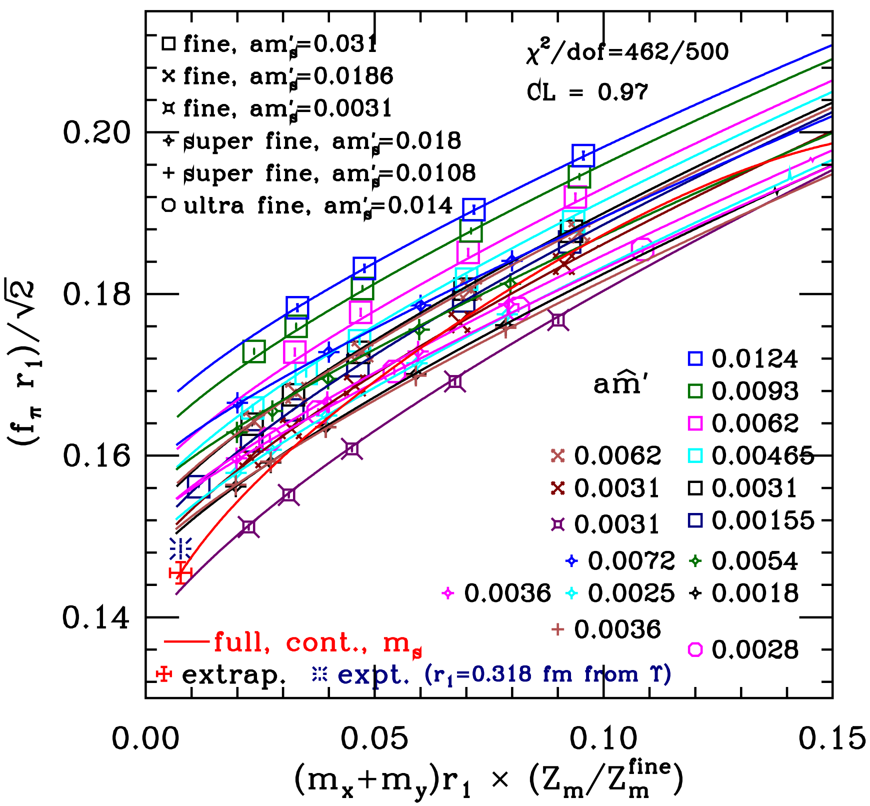


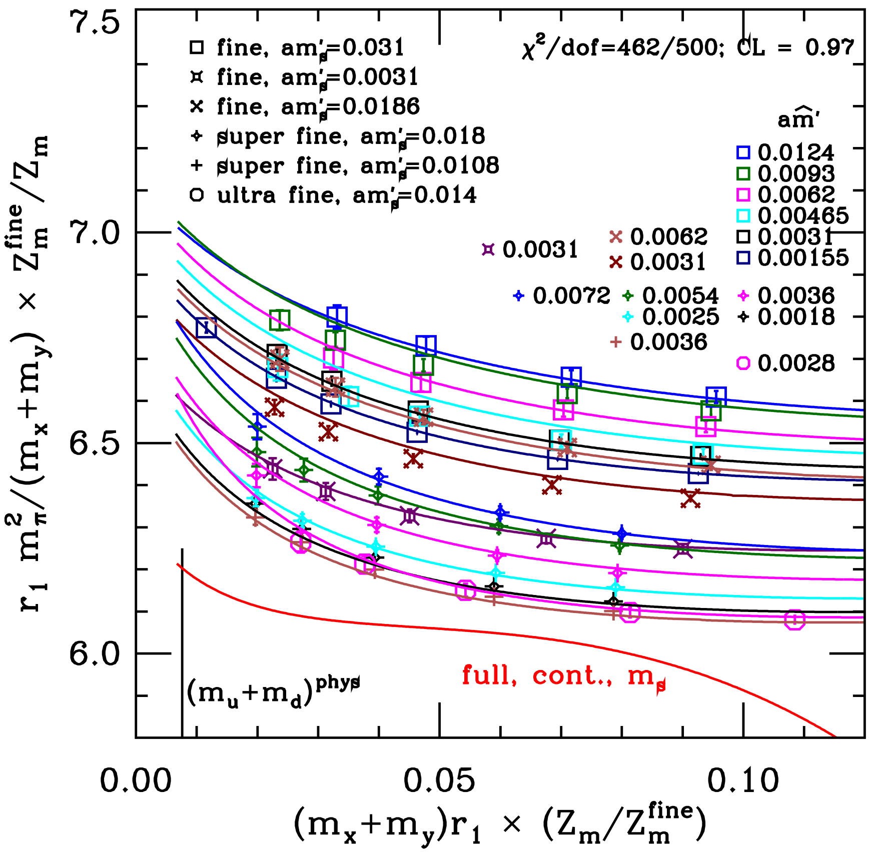
Here, NNNLO analytic terms were added to test convergence.
(Standard fits stop at NNLO.)

“High mass” SU(3) chiral fits

- All fine, superfine and ultrafine ensembles included
- Valence masses limited by $m_x + m_y \leq 1.2 m_s^{phys}$
- LO and NLO LECs fixed from low-mass fits
- NNNLO and NNNNLO analytic terms included, but not the corresponding logs.
  This is needed mostly for interpolating around the strange quark mass.
  Since LO and NLO LECs dominate chiral extrapolation to the physical point, results for decay constants and masses are insensitive to form of these interpolating terms, as long as fit is good.
- Used to give central values of physical decay constant, quark masses and other quantities involving the strange quark mass, like the two-flavor f_2 and B_2.

“High mass” SU(3) chiral fits

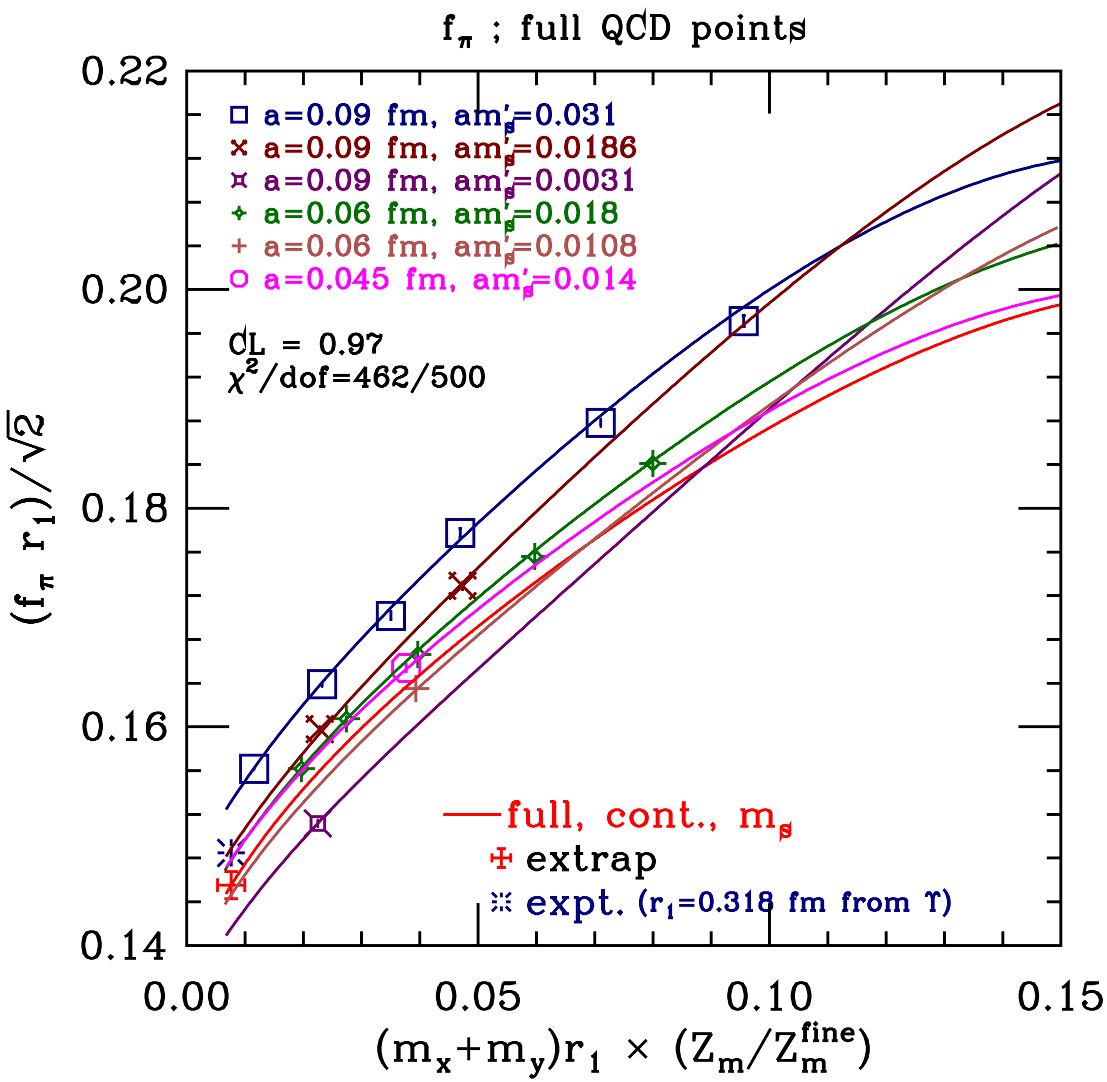




Red curve: continuum extrapolation (a set to zero) and valence and sea quark masses set equal (“full QCD”) with strange mass kept at m_s^{phys}

“High mass” SU(3) chiral fits

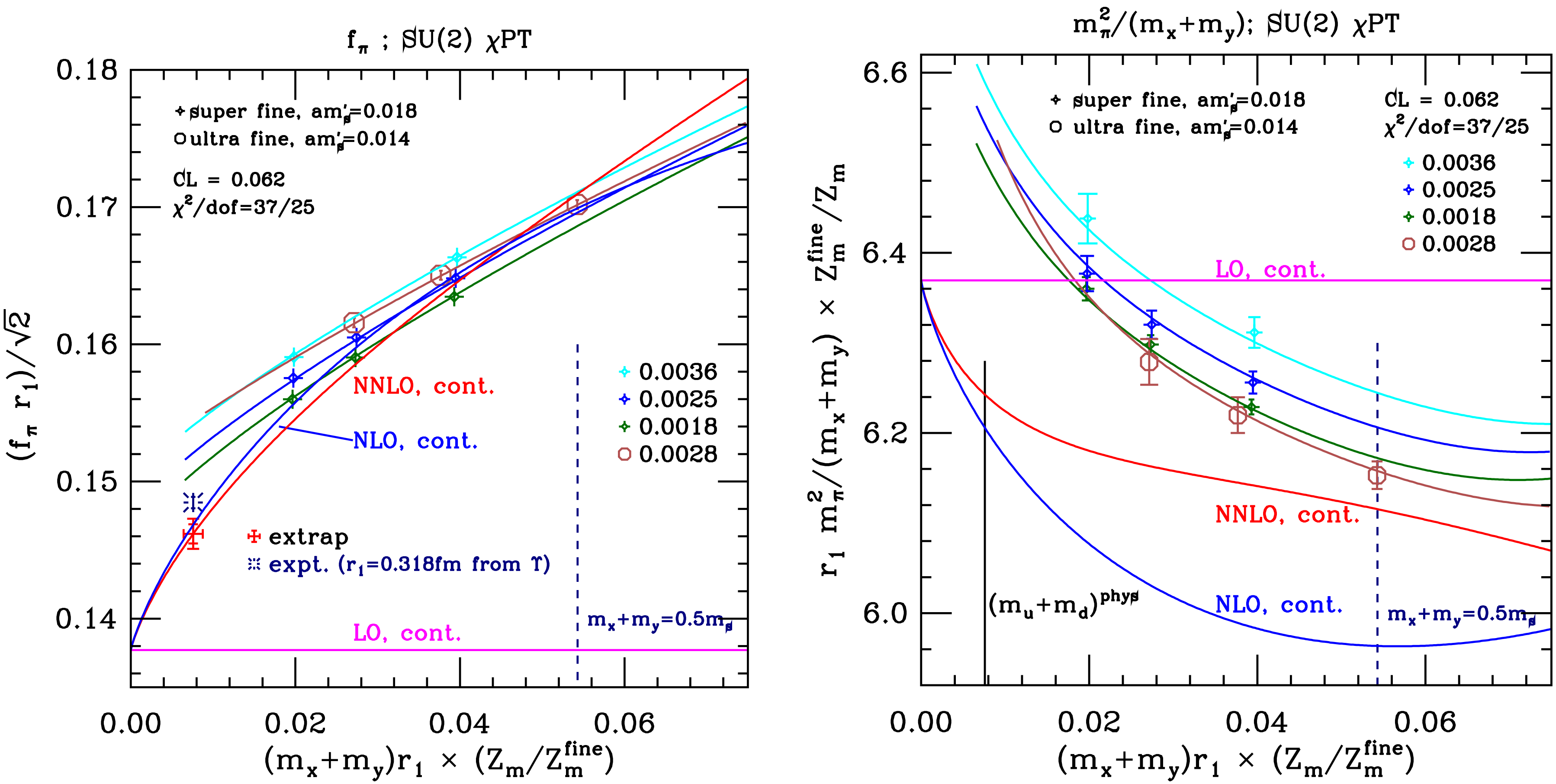
f_π plot showing only the full QCD points (valence mass equal to sea quark mass)



SU(2) chiral fits

- Superfine and ultrafine ensembles included, with $m_s' \approx m_s^{phys}$
- Systematic fits up to NNLO
- Using rSχPT forms at NLO
- With continuum NNLO chiral logs

SU(2) chiral fits



Also shown is the convergence (LO, NLO, NNLO) in the continuum limit.

Preliminary results from the SU(3) chiral fits

With scale $r_1 = 0.318(7)$ fm from Υ-splittings – $r_1^2 F(r_1) = 1$ – we find

$$f_\pi = 128.0 \pm 0.3 \pm 2.9 \text{ MeV} \qquad [128.3 \pm 0.5^{+2.4}_{-3.5} \text{ MeV}]\,,$$

$$f_K = 153.8 \pm 0.3 \pm 3.9 \text{ MeV} \qquad [154.3 \pm 0.4^{+2.1}_{-3.4} \text{ MeV}]\,,$$

$$f_K/f_\pi = 1.201(2)(9) \qquad [1.202(3)(^{+\ 8}_{-14})]\,,$$

where the first error is statistical and the second is systematic, and the results in plum are from our last, 2007, analysis.

PDG 2008: $f_\pi = 130.4 \pm 0.2\,\text{MeV}$

Alternatively, using f_π to set the scale: $\Rightarrow r_1 = 0.3117(6)(^{+12}_{-31})$ fm:

$$f_K = 156.2 \pm 0.3 \pm 1.1 \text{ MeV} \qquad [156.5 \pm 0.4^{+1.0}_{-2.1} \text{ MeV}]\,,$$

$$f_K/f_\pi = 1.198(2)(^{+6}_{-8}) \qquad [1.197(3)(^{+\ 6}_{-13})]\,.$$

Preliminary results from the SU(3) chiral fits

The remainder of our results all use the scale set from f_π.

- Using our f_K/f_π, the experimental $B(K \to \ell\nu)/B(\pi \to \ell\nu)$ and the well known Cabbibo angle $V_{ud} = 0.97458(27)$:
  $\Rightarrow V_{us} = 0.2247(^{+16}_{-13}) \qquad [0.2246(^{+25}_{-13})]$
  (includes sys. error of 0.0005 from non-lattice theory)
  PDG 2008 value: $V_{us} = 0.2255(19)$
- Also get (in units of 10^{-3}, at chiral scale m_η):

$$
\begin{aligned}
2L_6 - L_4 &= 0.19(12)(1) \qquad [0.4(1)(^{+2}_{-3})]\,, \\
2L_8 - L_5 &= -0.47(8)(14) \qquad [-0.1(1)(1)]\,, \\
L_4 &= 0.30(13)(4) \qquad [0.4(3)(^{+3}_{-1})]\,, \\
L_5 &= 1.64(12)(17) \qquad [2.2(2)(^{+2}_{-1})]\,, \\
L_6 &= 0.24(10)(3) \qquad [0.4(2)(^{+2}_{-1})]\,, \\
L_8 &= 0.59(5)(2) \qquad [1.0(1)(1)]\,.
\end{aligned}
$$

Preliminary results from the SU(3) chiral fits

- From these we get, using one-loop conversion formulae (in units of 10^{-3}, at chiral scale m_η):

$$l_3 = -0.6(10)(6) ,$$
$$l_4 = 7.7(10)(7) ,$$

- and the scale invariant (and without other factors)

$$\bar{l}_3 = 3.15(64)(42) ,$$
$$\bar{l}_4 = 4.01(16)(13) .$$

- Also look at various chiral limit quantities:
  - The two-flavor chiral limit decay constant f_2:
    $m_u, m_d \to 0$; m_s fixed at physical value.

$$f_2 = 122.8 \pm 0.3 \pm 0.5 \text{ MeV} ,$$
$$f_\pi / f_2 = 1.062(1)(3) .$$

Preliminary results from the SU(3) chiral fits

- - The three-flavor chiral limit decay constant f_3: $m_u, m_d, m_s \to 0$.

$$
\begin{aligned}
f_3 &= 111.0 \pm 2.0 \pm 4.1 \text{ MeV} , \\
f_\pi / f_3 &= 1.174(3)(43) , \\
f_2 / f_3 &= 1.107(3)(39) .
\end{aligned}
$$

  - The two- and three-flavor chiral limit of $m_\pi^2/(m_u + m_d)$, B_2 and B_3. Get (in $\overline{\text{MS}}$ at 2 GeV):

$$
\begin{aligned}
B_2 &= 2.87(1)(4)(14) \text{ GeV} , \\
B_3 &= 2.38(8)(10)(12) \text{ GeV} , \\
B_2 / B_3 &= 1.204(3)(8)(0) .
\end{aligned}
$$

The last error is from perturbation theory, using the two-loop result for Z_m from Q. Mason *et al.*, Phys. Rev. D73 (2006) 114501 [hep-lat/0511160].

Preliminary results from the SU(3) chiral fits

- B's and f's are related to the condensate of a light flavor:

$$\langle \bar{u}u \rangle_2 = -f_2^2 B_2/2 \,,$$
$$\langle \bar{u}u \rangle_3 = -f_3^2 B_3/2 \,,$$

  in the two- and three-flavor chiral limit, respectively.

- Get (in $\overline{\text{MS}}$ at $2\ \text{GeV}$):

$$\langle \bar{u}u \rangle_2 = -[\ 279(1)(2)(4)\ \text{MeV}\ ]^3 \,,$$
$$\langle \bar{u}u \rangle_3 = -[\ 245(5)(4)(4)\ \text{MeV}\ ]^3 \,,$$
$$\frac{\langle \bar{u}u \rangle_2}{\langle \bar{u}u \rangle_3} = 1.47(1)(10)(0) \,.$$

- For the SU(3) NLO correction to the physical pion mass we find:

$$\delta_\pi^{(2)} = 0.06(5)(1) \,.$$

Preliminary results from the SU(3) chiral fits

- Using the two-loop result for Z_m from Q. Mason *et al.*, Phys. Rev. D73 (2006) 114501 [hep-lat/0511160], we also find:

$$
\begin{aligned}
m_s^{\overline{\text{MS}}} &= 89.0(0.2)(1.6)(4.5)(0.1)\ \text{MeV} && [88(0)(3)(4)(0)\ \text{MeV}]\,, \\
\hat{m}^{\overline{\text{MS}}} &= 3.25(1)(7)(16)(0)\ \text{MeV} && [3.2(0)(1)(2)(0)\ \text{MeV}]\,, \\
m_s/\hat{m} &= 27.41(5)(22)(0)(4) && [27.2(1)(3)(0)(0)]\,, \\
m_u^{\overline{\text{MS}}} &= 1.96(0)(6)(10)(12)\ \text{MeV} && [1.9(0)(1)(1)(1)\ \text{MeV}]\,, \\
m_d^{\overline{\text{MS}}} &= 4.53(1)(8)(23)(12)\ \text{MeV} && [4.6(0)(2)(2)(1)\ \text{MeV}]\,, \\
m_u/m_d &= 0.432(1)(9)(0)(39) && [0.42(0)(1)(0)(4)]\,,
\end{aligned}
$$

where the errors are from statistics, simulation systematics, perturbation theory ($2\alpha^3$), and electromagnetic effects, respectively. The renormalization scale of the masses is $2\,\text{GeV}$.

Preliminary results from the SU(2) chiral fits

Using the scale from Υ-splittings we find

$$f_\pi = 128.7 \pm 0.9^{+3.2}_{-2.7}\ \mathrm{MeV} \qquad [128.0 \pm 0.3 \pm 2.9\ \mathrm{MeV}\ ]\ ,$$

where the result from the SU(3) chiral fits is given in plum.
We see good agreement.

Using the more accurate scale from f_π we further obtain

$$\bar{l}_3 = 3.0(6)(^{+9}_{-6}) \qquad [3.15(64)(42)]\ ,$$

$$\bar{l}_4 = 3.9(2)(3) \qquad [4.01(16)(13)]\ ,$$

$$B_2 = 2.87(2)(^{+1}_{-8})(14)\ \mathrm{GeV} \qquad [2.87(1)(4)(14)\ \mathrm{GeV}\ ],$$

$$f_2 = 123.7 \pm 0.8^{+1.3}_{-1.4}\ \mathrm{MeV} \qquad [122.8 \pm 0.3 \pm 0.5\ \mathrm{MeV}]\ ,$$

$$\hat{m} = 3.23(3)(^{+5}_{-3})(16)(0)\ \mathrm{MeV} \qquad [3.25(1)(7)(16)(0)\ \mathrm{MeV}]\ ,$$

with good agreement between SU(2) and SU(3) chiral fits.

Summary

- MILC pushed its analysis of light pseudoscalar mesons to smaller lattices spacing and smaller quark masses
- In particular used smaller strange sea quark masses, including a degenerate three-flavor ensemble with $m_s' \approx 0.12 m_s^{phys}$
- Include (continuum) NNLO chiral logs in the chiral fits
- "Low mass" fits show good convergence; used to determine LO and NLO LECs
- Results consistent with, but more accurate than, our previous ones
- New SU(2) chiral fits show good agreement with SU(3) chiral fits
- All results are still preliminary