Baryons (and Mesons) on the Lattice

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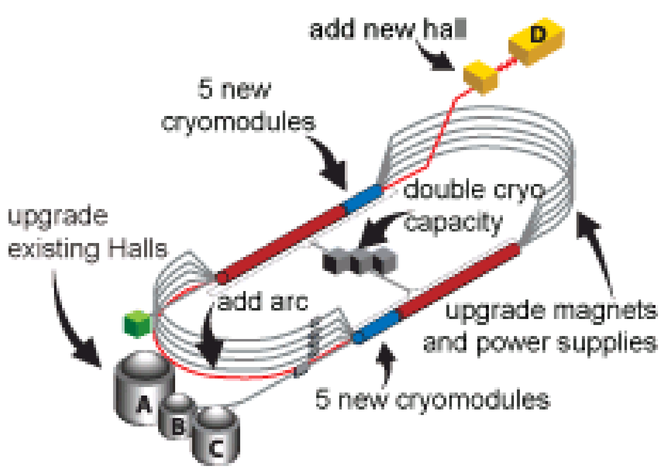
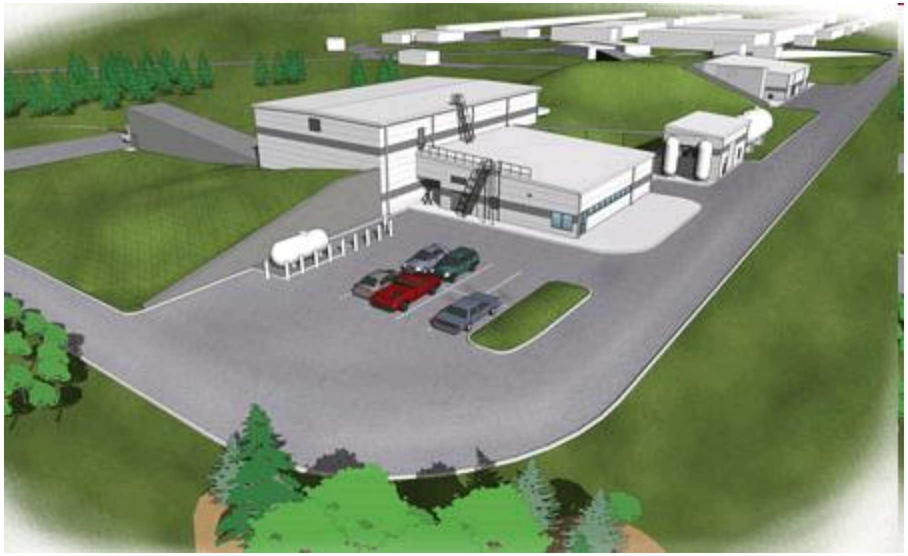
Chiral Dynamics
July 2009

Nuclear Physics & Jefferson Lab

CD-3
JLab Receives DOE Approval to Start Construction of $310 Million Upgrade





- **Lab doubling beam energy to 12GeV**
- **Adding new experimental Hall**

Spectroscopy

Spectroscopy reveals fundamental aspects of hadronic physics

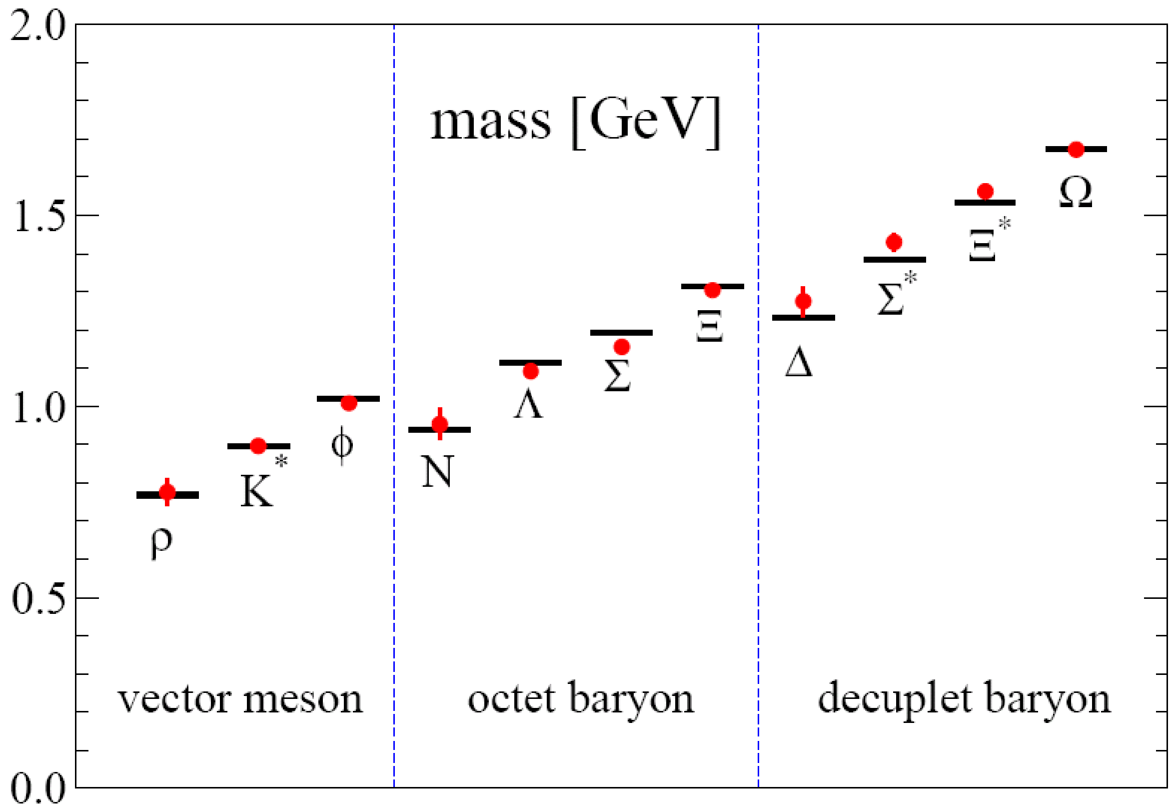
- Essential degrees of freedom?
- Gluonic excitations in mesons - exotic states of matter?

- Status
  - Can extract excited hadron energies & identify spins,
  - Pursuing full QCD calculations with realistic quark masses.

- New spectroscopy programs world-wide
  - E.g., BES III, GSI/Panda
  - Crucial complement to 12 GeV program at JLab.
    - Excited nucleon spectroscopy (JLab)
    - JLab GlueX: search for gluonic excitations.

Some Ground State Masses

Some of the ground state masses

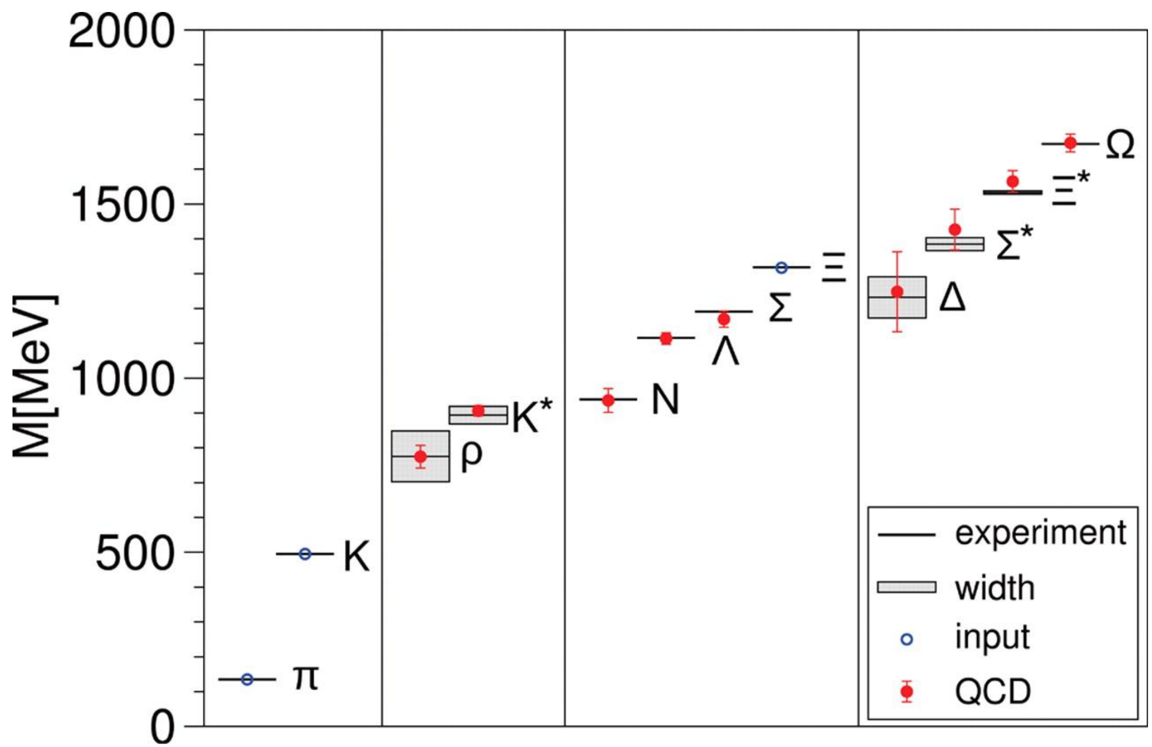


Lowest m_π=156MeV, single a=0.0907fm . PAC-CS collab.

PAC-CS, arXiv:0807.1661, PRD

Some Ground State Masses

Some of the ground state masses



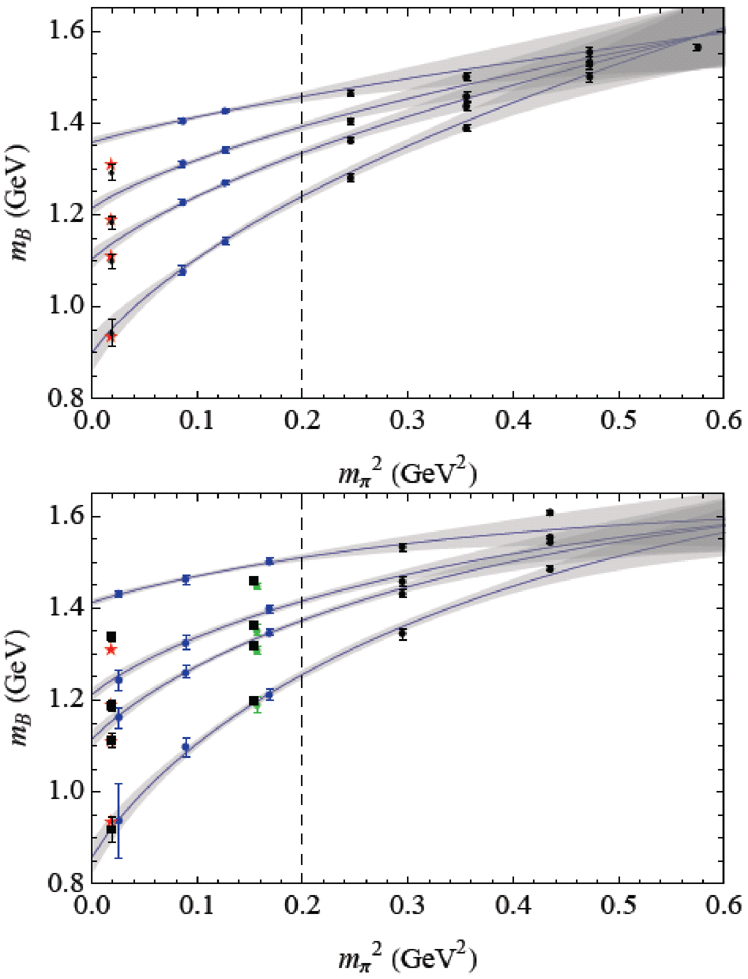
Missing negative parity octet and decuplet – much more to do!

Many of these states decay

BMW Collab, Science 322 (2008)

Sigma terms

SU(3) based chiral extrapolation [g_A fixed, SU(3) couplings/FRR



LHPC

PAC-CS

Thomas&Young, arXiv:0901.3310

Sigma terms

$$\bar{\sigma}_{B_q} = \frac{m_q}{M_B} \frac{\partial M_B}{\partial m_q}$$

B	Mass (GeV)	Expt.	$\bar{\sigma}_{Bl}$	$\bar{\sigma}_{Bs}$
N	0.939(19)(4)(2)	0.939	0.054(7)(2)(2)	0.020(11)(7)(3)
Λ	1.108(11)(10)(1)	1.116	0.0296(31)(5)(10)	0.138(11)(2)(2)
Σ	1.185(9)(2)(1)	1.193	0.0221(20)(7)(7)	0.176(11)(6)(2)
Ξ	1.321(9)(20)(0)	1.318	0.0095(7)(4)(0)	0.236(11)(4)(3)

Thomas/Young
LHPC & PAC-CS results

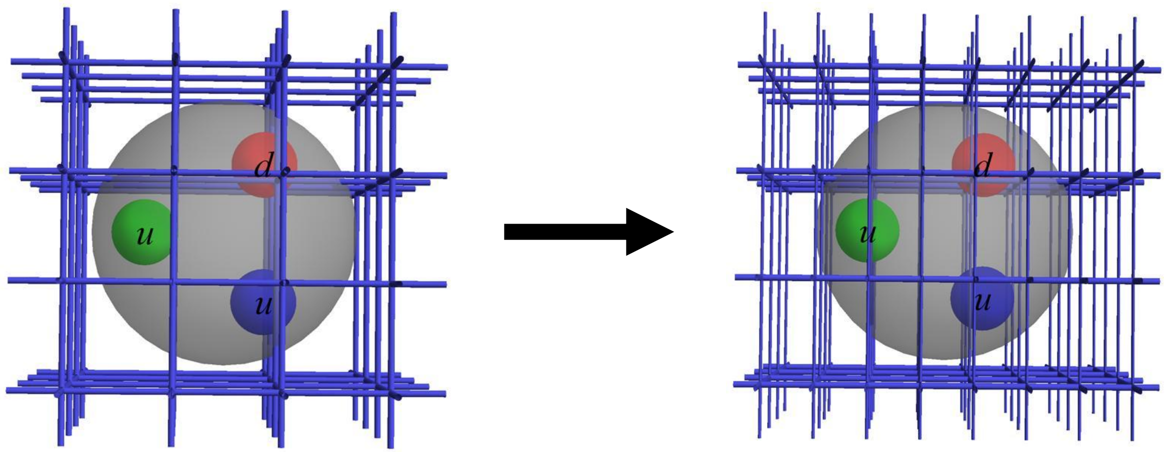
$$\bar{\sigma}_{B_l} = 0.0427(30)$$

QCDSF

Excited states: anisotropy+operators+variational

Make lattice *anisotropic*

- Temporal spacing $a_t < a_s$ (spatial lattice spacing)
- High temporal resolution $\rightarrow$ Resolve noisy states
- Downside: must fine tune anisotropies: $a_t = a_s / \xi$



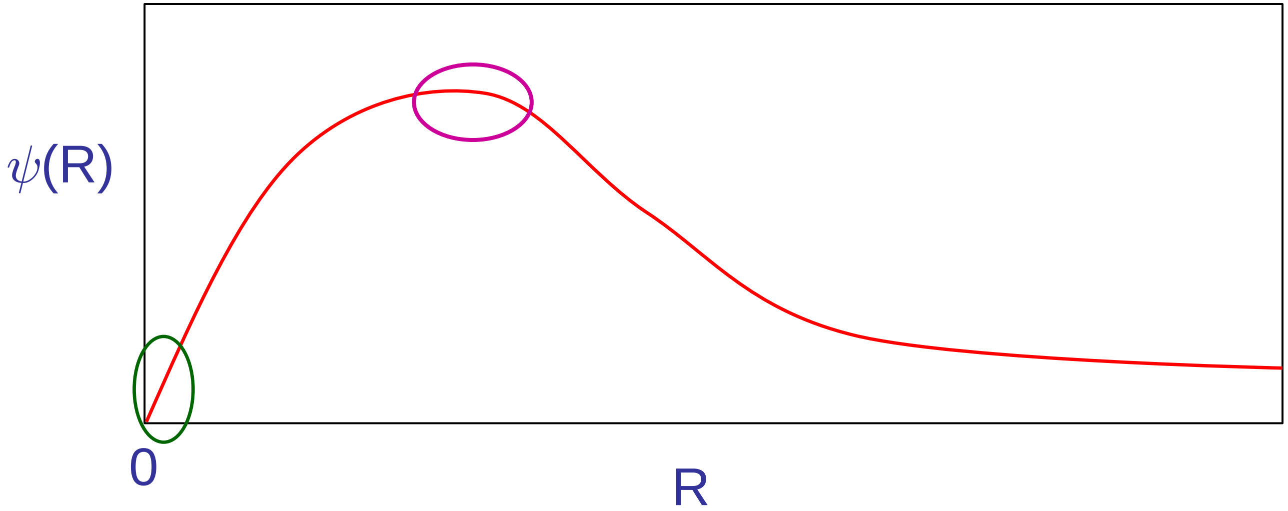
Major project within USQCD – Hadron Spectrum Collab.

Excited states: anisotropy+operators+variational

Extended operators

J^{PC} state: wavefunction

- Short distance: sufficient derivatives – nonzero overlap
- Long distance: different structure



$$\bar{\psi}(x) \quad \text{Gamma's} \times \text{Gauge covariant deriv} \quad \psi(y) \rightarrow \text{Lattice finite diff}$$

Hadron spectrum calculation

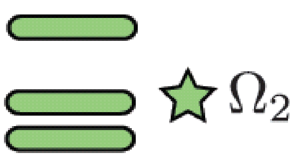
Two-point correlator

$$C(t) = \langle 0 | \Phi'(t)\, \Phi(0) | 0 \rangle$$

$$C(t) = \sum_{\mathfrak{n}} e^{-E_{\mathfrak{n}} t} \langle 0 | \Phi'(0) | \mathfrak{n} \rangle \langle \mathfrak{n} | \Phi(0) | 0 \rangle$$

e.g. pseudoscalars can be 'made' with

$$\bar{\psi}\gamma^5\psi$$
$$\epsilon_{ijk}\, \bar{\psi}\gamma^j\gamma^k(\partial^i - A^i)\psi$$
$$\epsilon_{ijk}\, \bar{\psi}\gamma^i\psi\, F^{jk}$$
$$\vdots$$



Overlap onto tower of pseudoscalar states

Some state $\rightarrow$ optimal linear combination of operators

$$\Omega_{\mathfrak{n}} = v_1^{\mathfrak{n}} \Phi_1 + v_2^{\mathfrak{n}} \Phi_2 + \ldots$$

Finite basis: use variational solution

Variational Method

Matrix of correlators

$$C(t) = \begin{bmatrix} \langle 0|\Phi_1(t)\Phi_1(0)|0\rangle & \langle 0|\Phi_1(t)\Phi_2(0)|0\rangle & \dots \\ \langle 0|\Phi_2(t)\Phi_1(0)|0\rangle & \langle 0|\Phi_2(t)\Phi_2(0)|0\rangle & \dots \\ \vdots & & \ddots \end{bmatrix}$$

e.g. $\bar{\psi}\gamma^5\psi$

$\epsilon_{ijk}\,\bar{\psi}\gamma^j\gamma^k(\partial^i - A^i)\psi$

$\epsilon_{ijk}\,\bar{\psi}\gamma^i\psi\,F^{jk}$

$\vdots$

Variational solution = generalized eigenvalue problem

$$C(t)v^{\mathfrak{n}} = \lambda_{\mathfrak{n}}(t)C(t_0)v^{\mathfrak{n}}$$

Eigenvalue $\rightarrow$ spectrum

$$\lambda_{\mathfrak{n}}(t) \rightarrow e^{-E_{\mathfrak{n}}(t-t_0)}$$

Eigenvectors $\rightarrow$ `optimal' operators

$$\Omega_{\mathfrak{n}} = v_1^{\mathfrak{n}}\Phi_1 + v_2^{\mathfrak{n}}\Phi_2 + \dots$$

Orthogonality needed for near degenerate states

Why all this stuff?

Rewrite:

$$C(t)v^{n} = \lambda_{n}(t)C(t_0)v^{n}$$

Diagonalized the QCD Hamiltonian

$$C(t_0)^{-\frac{1}{2}}C(t)C(t_0)^{-\frac{1}{2}} \to e^{-Ht}$$

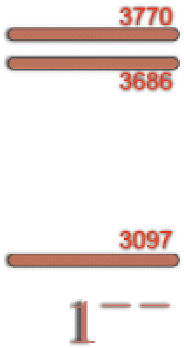
dim(C) # of states

Eigenvectors provide info on spin and glue content of state

Orthogonality needed for near degenerate states

Orthogonality

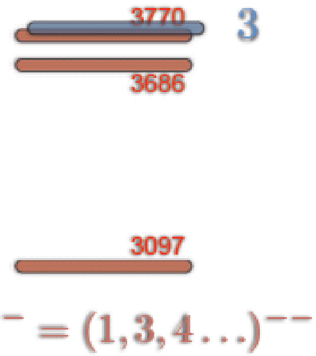
E.g., charmonium vector spectrum

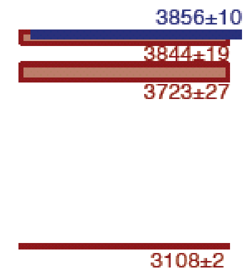


near deg. states are tough to fit

using multi-exponential fits

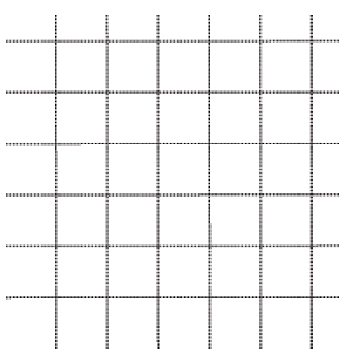
even worse on a cubic lattice





variational solution

$$C(t) = \sum_n e^{-E_n t}\langle 0|\Phi'(0)|n\rangle\langle n|\Phi(0)|0\rangle$$



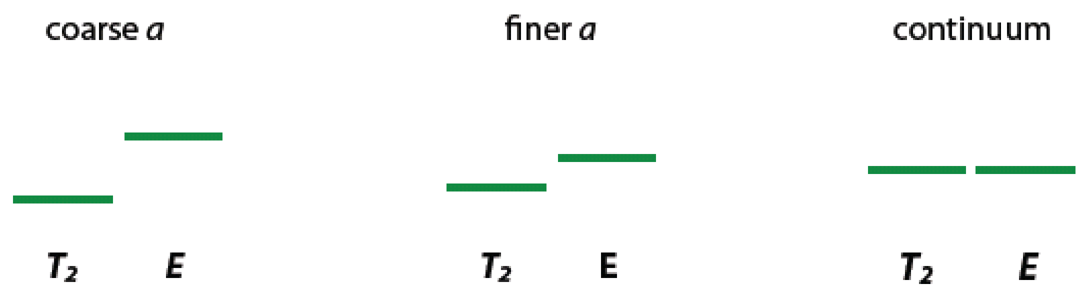
e.g. in two dimensions: $\psi_J(\theta) = e^{iJ\theta}$

so under the allowed π/2 rotations, *J*=0,4,8... indistinguishable

Determining spin on a cubic lattice?

Spin reducible on lattice

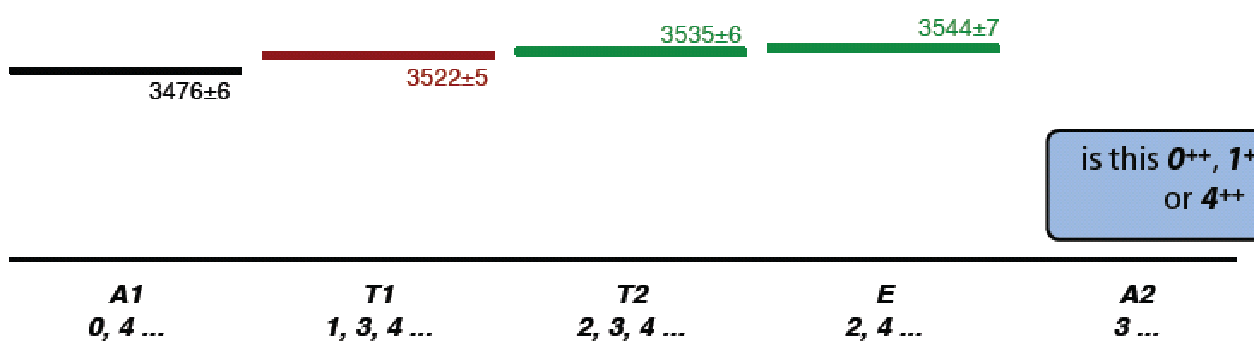
spin 2 $_{(5)}$ → T2 $_{(3)}$ + E $_{(2)}$



Might be dynamical degeneracies

e.g. J^{++}

a_s = 0.1 fm



is this 0^{++}, 1^{++} & 2^{++} or 4^{++} ?

Spin reduction & (re)identification

Variational solution: v $\rightarrow$ Z

$$C_{ij}(t) = \sum_\alpha Z_i^{(\alpha)} Z_j^{(\alpha)*} e^{-m_\alpha t}$$

Continuum

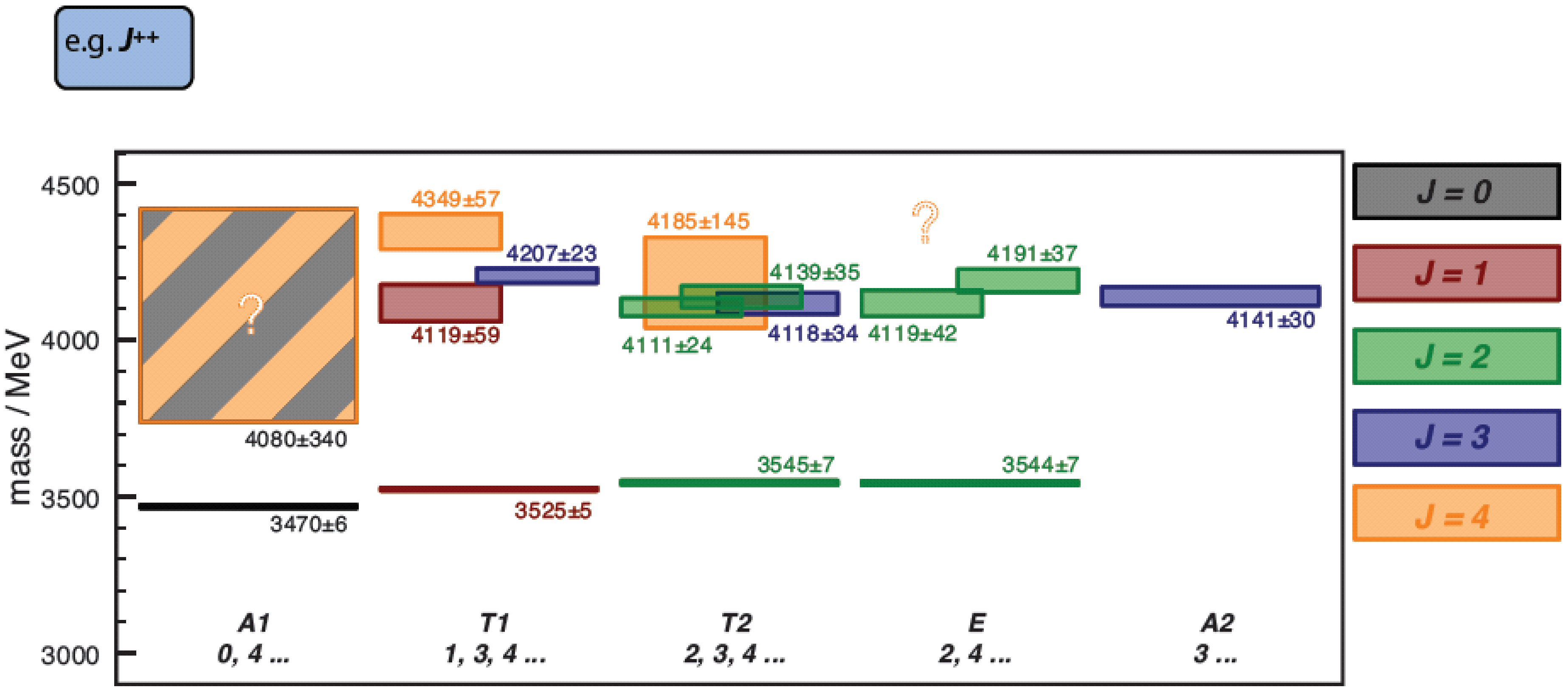
$$\langle 0 \mid \mathcal{O} \mid 2^{++}(\vec{0}, r)\rangle \; = \; Z$$

Lattice

$$\langle 0 \mid \mathcal{O}_{T_2} \mid 2^{++}(\vec{0}, r)\rangle \; = \; Z_{T_2}$$
$$\langle 0 \mid \mathcal{O}_{E} \mid 2^{++}(\vec{0}, r)\rangle \; = \; Z_{E}$$

Method: Check if converse is true

More spectrum

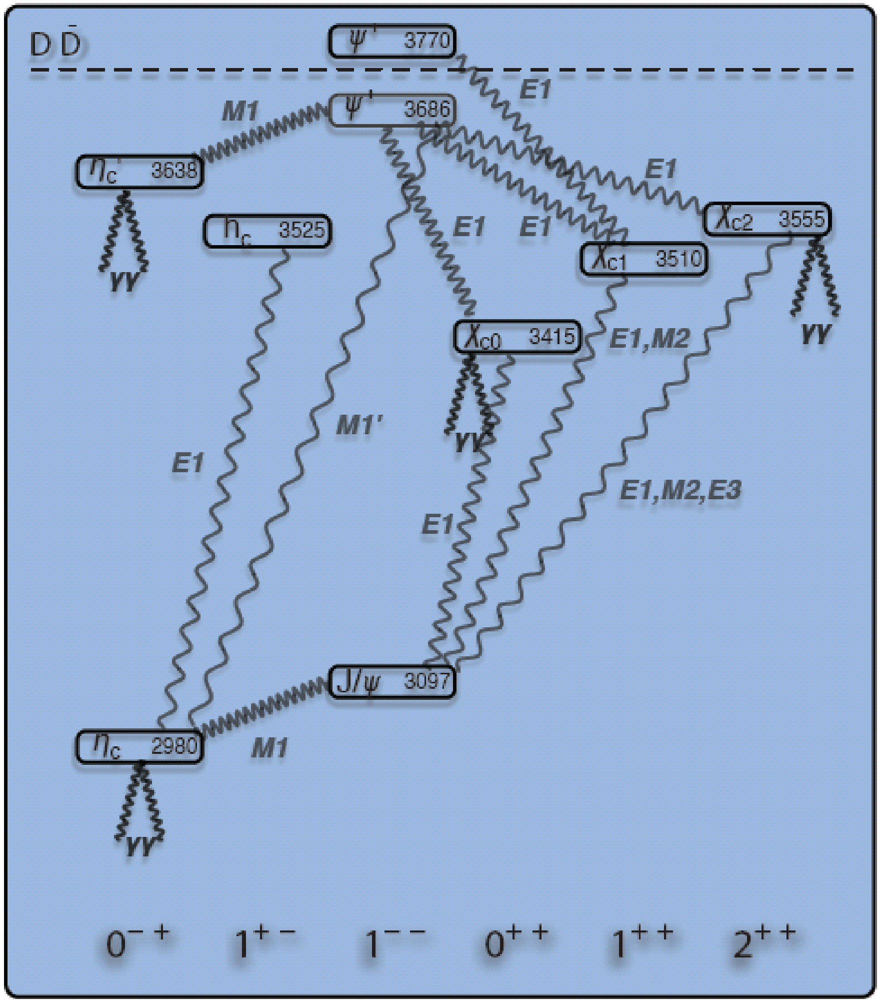


arXiv:0707.4162 & 0902.2241 (PRD)

Radiative decays

Project onto excited states:

compute decays



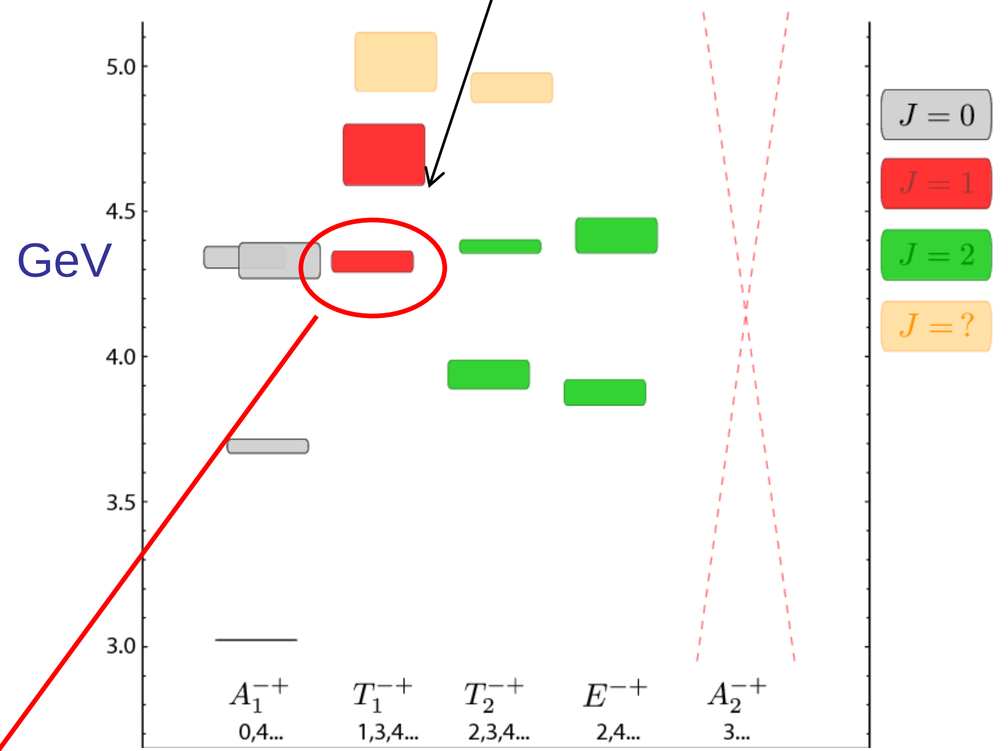
PRL (2007), arXiv:0707.4162 & 0902.2241 (PRD)

Exotic spectrum & decay

Charmonium: J^{-+}

Exotic: $\Gamma(1^{-+} \rightarrow 1^{--}\gamma)$ ~ 100 keV large [c.f. $\Gamma(0^{++} \rightarrow 1^{--}\gamma)$ ~ 100 keV]

Exp unknown: focus of new GSI/Germany

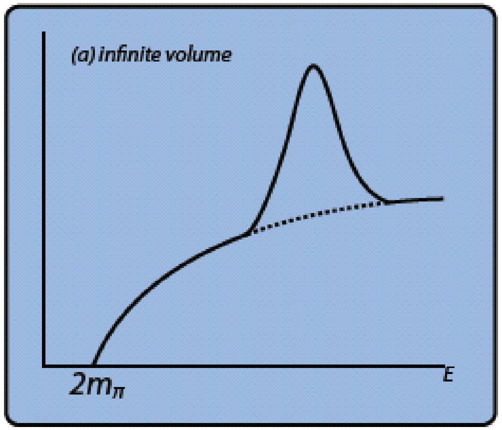


Phys. Rev. D77 034501
& to appear PRD

If true with light quarks: ***produce at JLab Hall D!!***

Resonances in finite volume: cartoon

What does QCD vector spectrum look like?

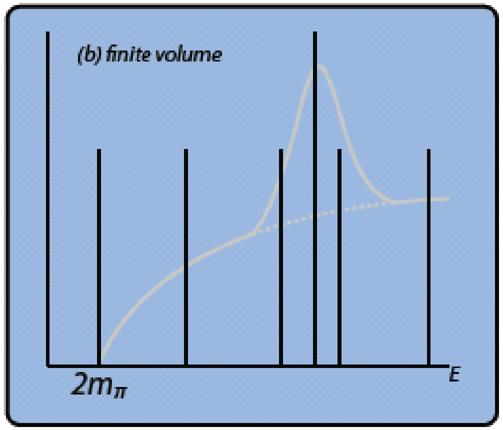


in ***infinite volume***, a continuous spectrum of ππ states

$$E(p) = 2\sqrt{m_\pi^2 + p^2}$$

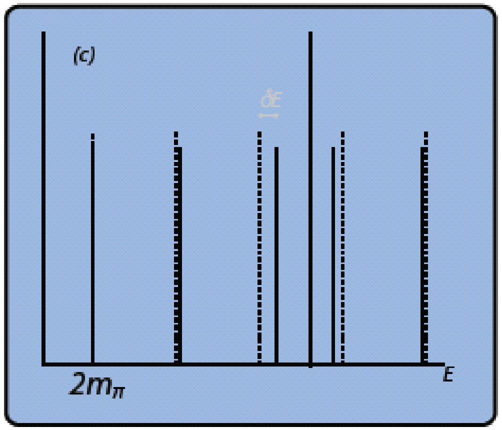
resonance embedded in a continuum of multi-particle states

$$C(\tau) = \int dE\, W(E)\, e^{-E\tau}$$



in ***finite volume***, a discrete spectrum of states

$$C(\tau) = \sum_N W_N e^{-E_N \tau}$$



non-interacting two-particle states have known energies

$$E(p) = 2\sqrt{m_\pi^2 + n\left(\frac{2\pi}{L}\right)^2}$$

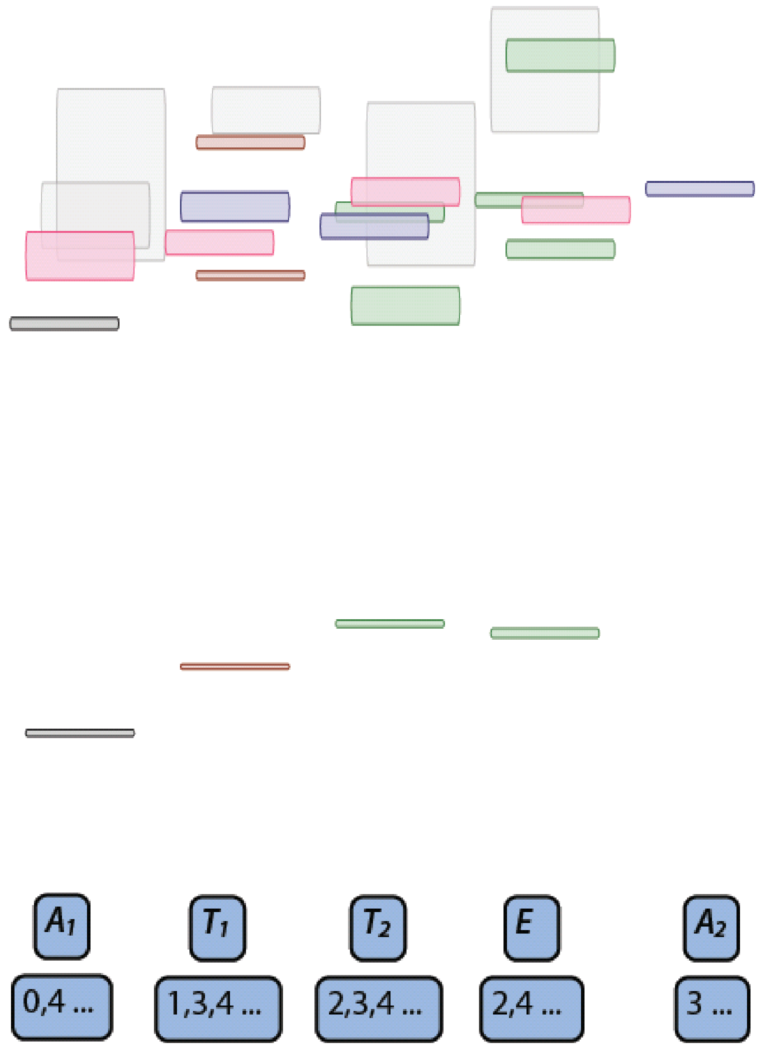
deviation from free energies depends upon the interaction and contains information about the scattering phase shift

δE(L) ↔ δ(E) : Lüscher method

Light & strange quarks

Single particle operators only

J^{++}



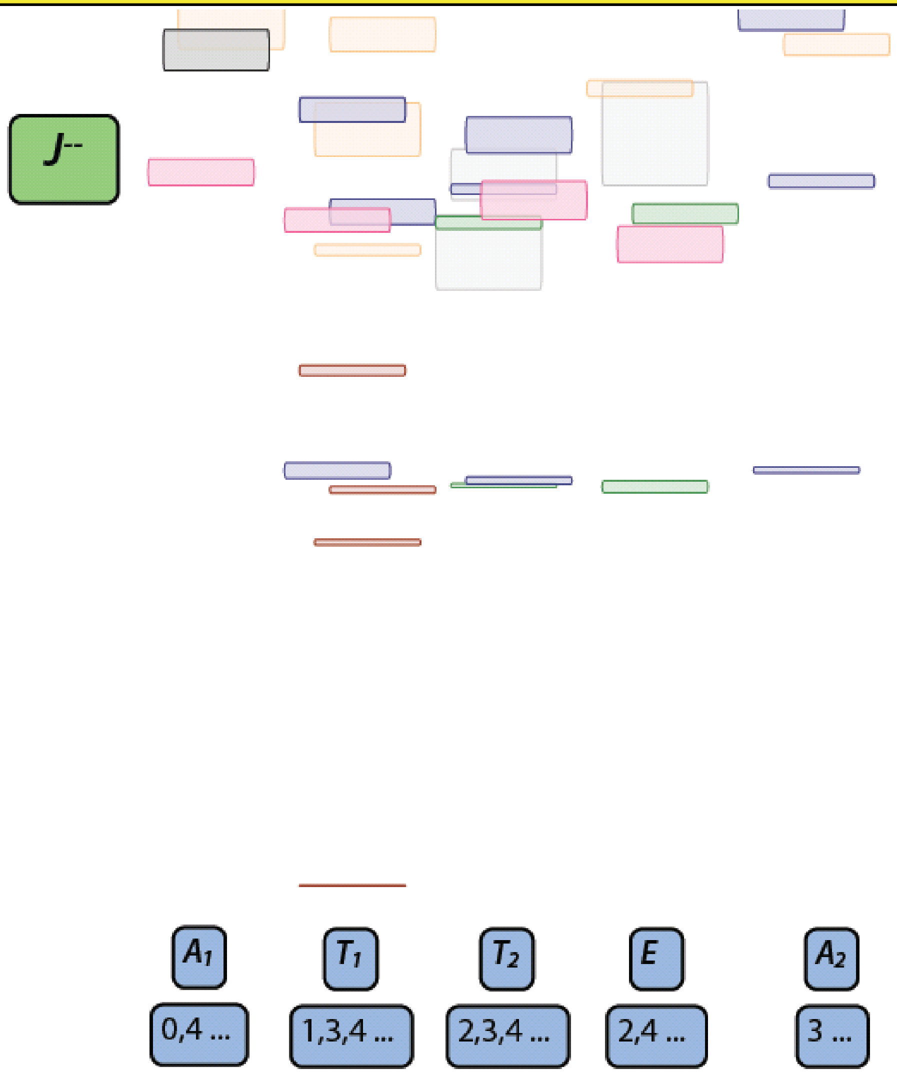
up to three covariant derivatives - operators have continuum overlap up to spin-4

J=0
J=1
J=2
J=3
J=4

arXiv:0905.2160

Light & strange quarks

Single particle operators only



up to three covariant derivatives - operators have continuum overlap up to spin-4

$J=0$

$J=1$

$J=2$

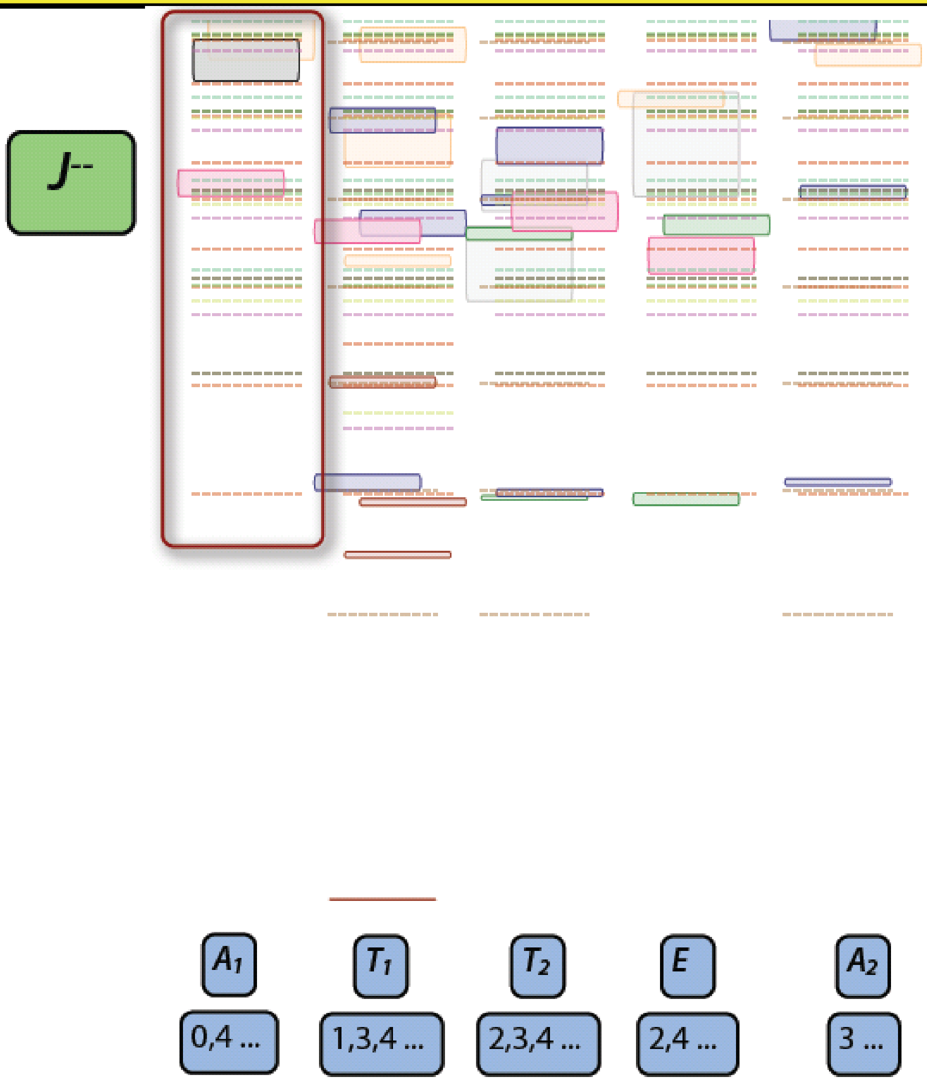
$J=3$

$J=4$

in quark models and their gluonic extensions 0^{--} is rather exotic

Multi-particles?

Would-be (non-interacting) two-meson states:



$$\pi\pi$$
$$\pi\omega$$
$$\rho\rho$$
$$\pi a_1$$
$$\vdots$$

clearly they are not extracted in this way

Why no coupling?

$$C_{ij}(t) = \langle 0 | \Phi_i(t) \Phi_j(0) | 0 \rangle = \langle 0 | \Phi_i(0) e^{-Ht} \Phi_j(0) | 0 \rangle$$

$$= \sum_{N_{\mathrm{QCD}}} \langle 0 | \Phi_i(0) | N_{\mathrm{QCD}} \rangle \langle N_{\mathrm{QCD}} | \Phi_j(0) | 0 \rangle e^{-m_{N_{\mathrm{QCD}}} t}$$

A_1

0,4 ...

consider a simple Fock state argument

$$\Phi_{\bar{\psi}\psi}(0) = \bar{\psi}_{t=0} \Gamma f(\overleftrightarrow{D}) \psi_{t=0}$$

$$\langle N | \Phi_{\bar{\psi}\psi} | 0 \rangle \sim \langle N | a^\dagger b^\dagger | 0 \rangle$$

suppose ***N*** is a $q\bar{q}$ Fock state

$$\langle N | \Phi_{\bar{\psi}\psi} | 0 \rangle \neq 0$$

suppose ***N*** is a $M\,M = q\bar{q}q\bar{q}$ Fock state

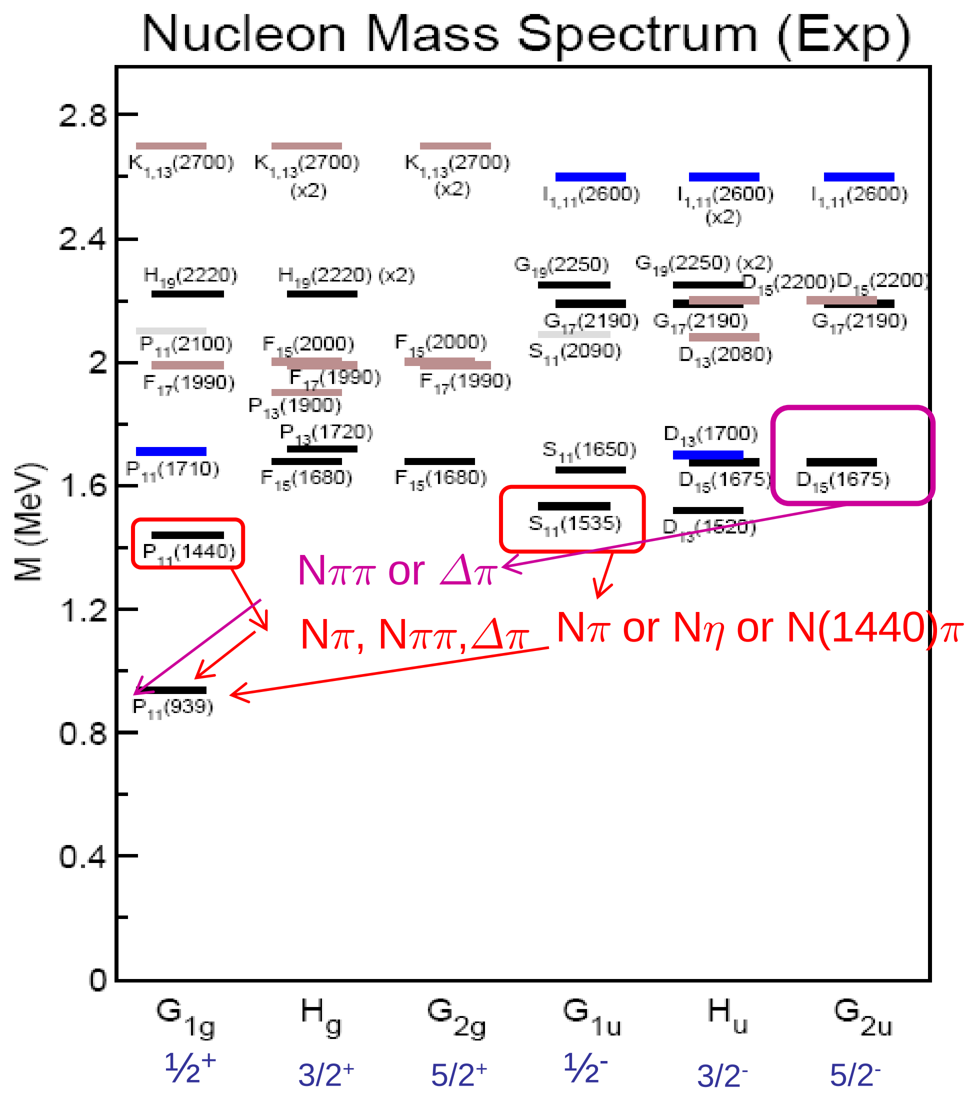
$$\langle N | \Phi_{\bar{\psi}\psi} | 0 \rangle = 0$$

Nucleon spectrum (Experimental)

NP2012 milestone:

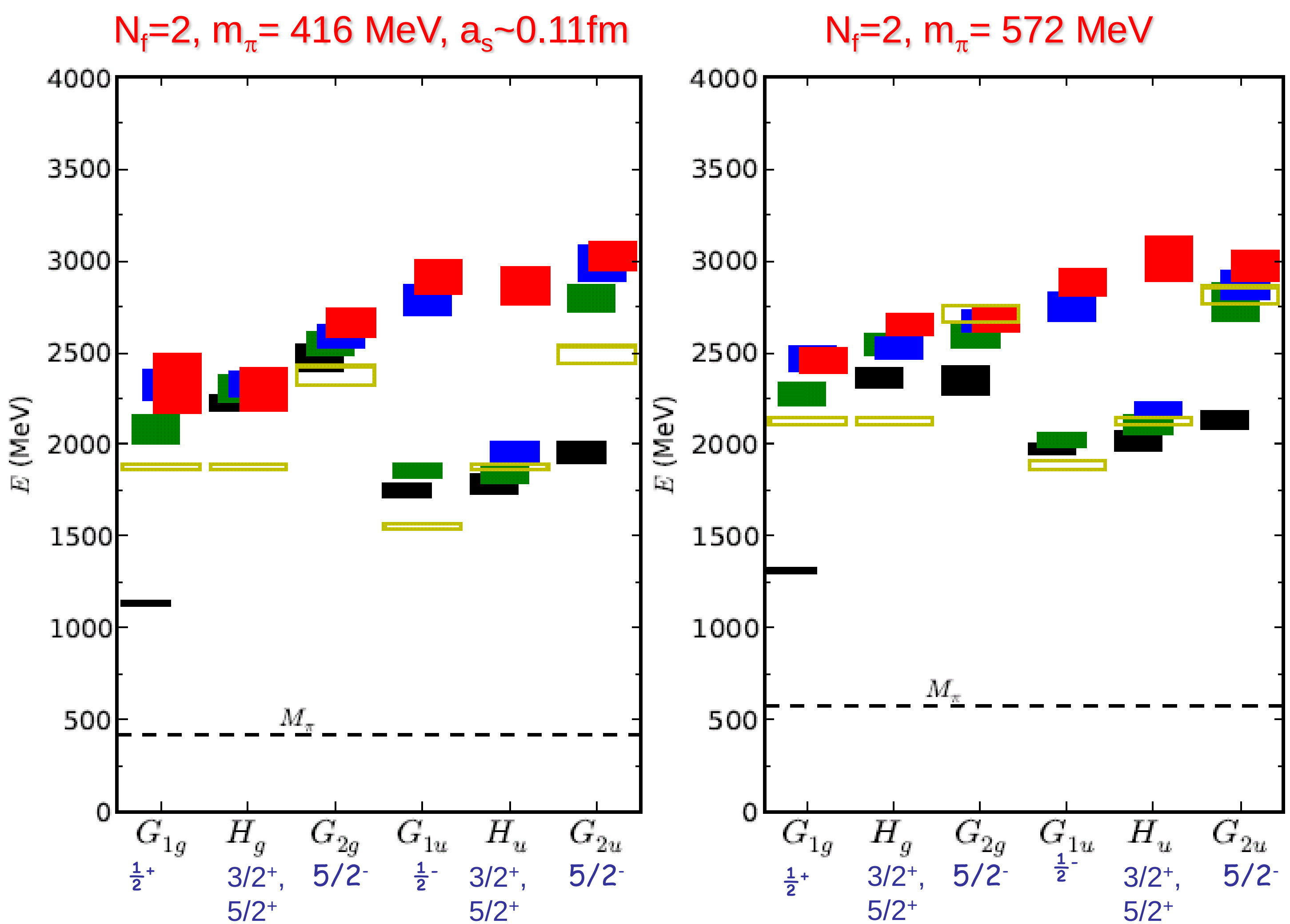
Spectrum & E&M transitions up to $Q^2 = 7\ GeV^2$

- Challenges/opportunities:
  - Compute excited energies
  - Compute decays



N_f=2 Nucleon Spectrum via Group Theory

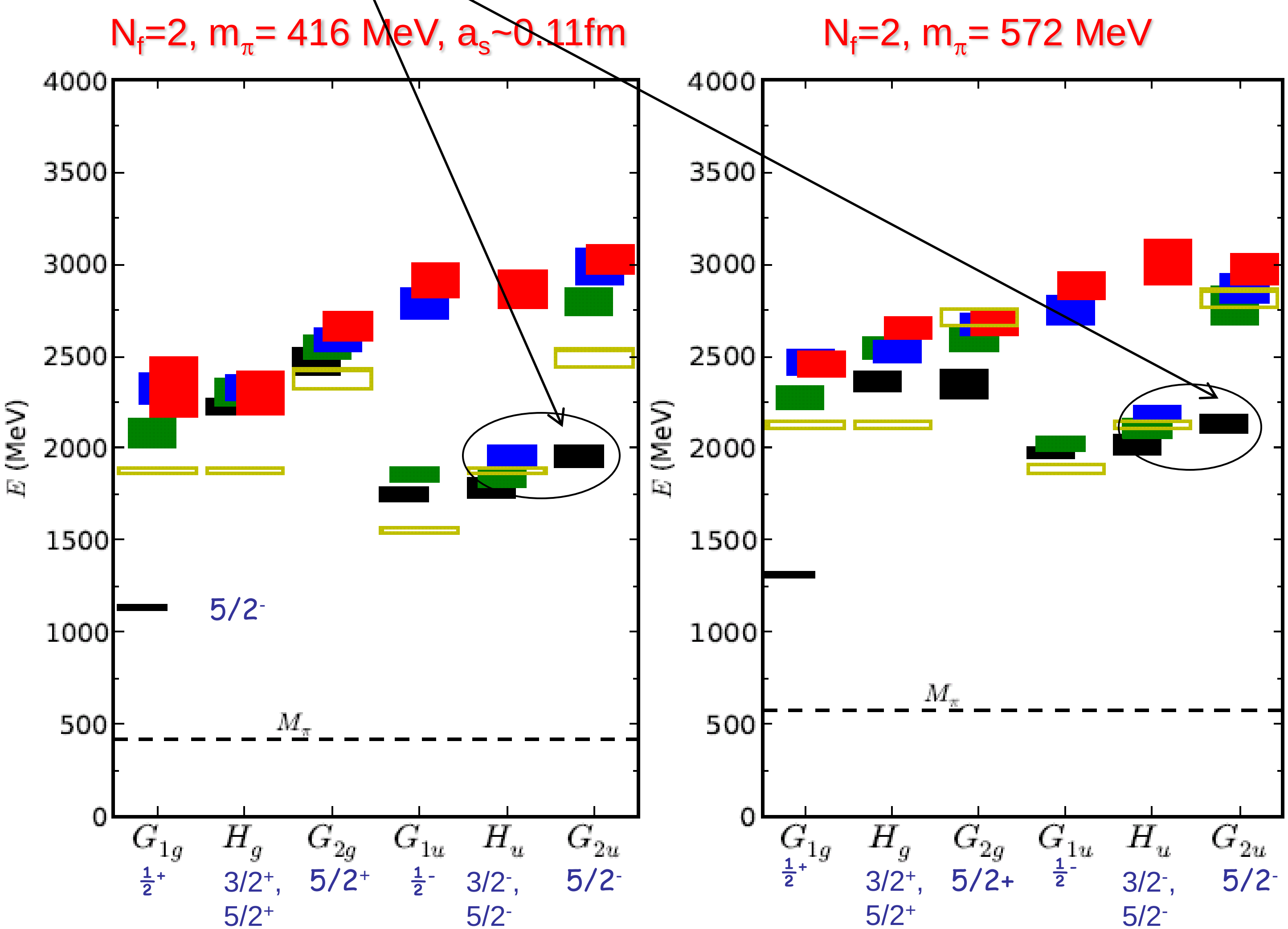
HadSpec 2009



N_f=2 Nucleon Spectrum via Group Theory

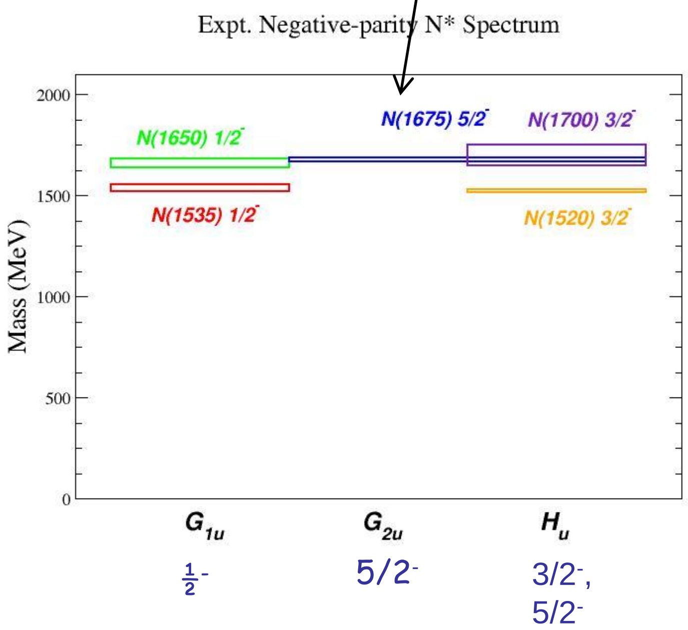
HadSpec 2009

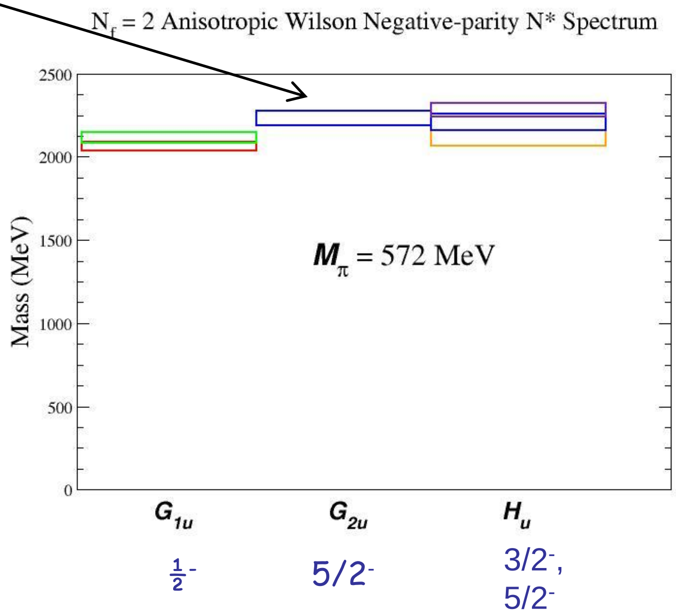
- Possible 5/2⁻ state



N_f=2 Nucleon spectrum

- Possible $5/2^-$ state: pattern similar to exp:

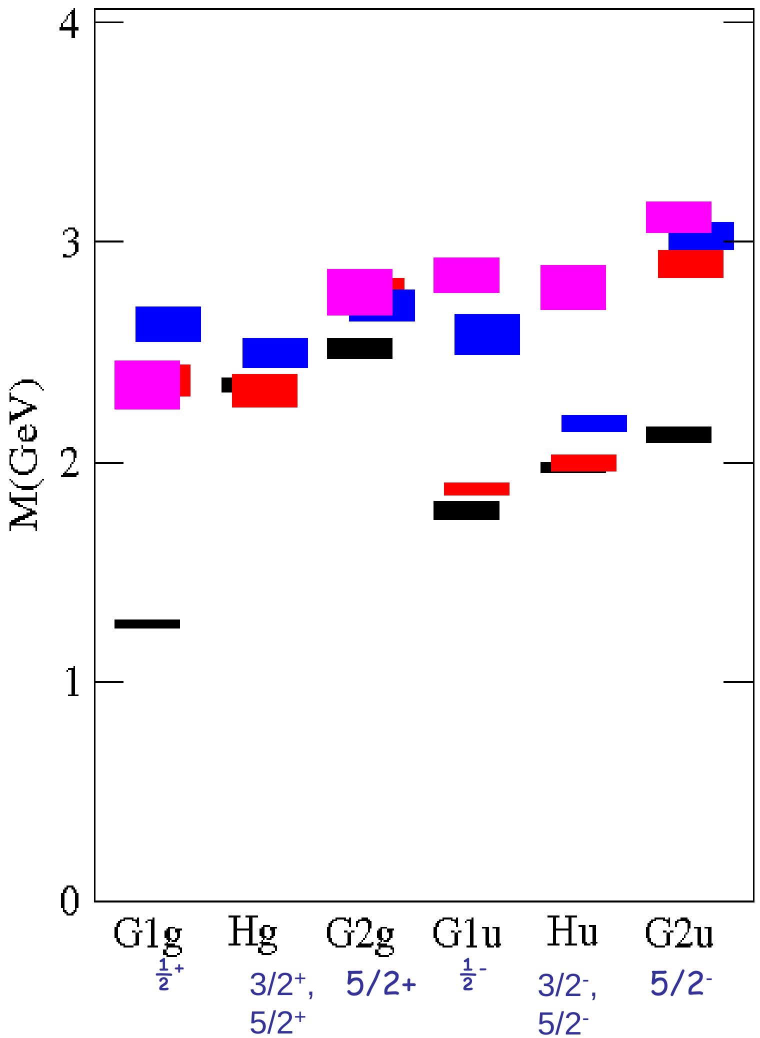




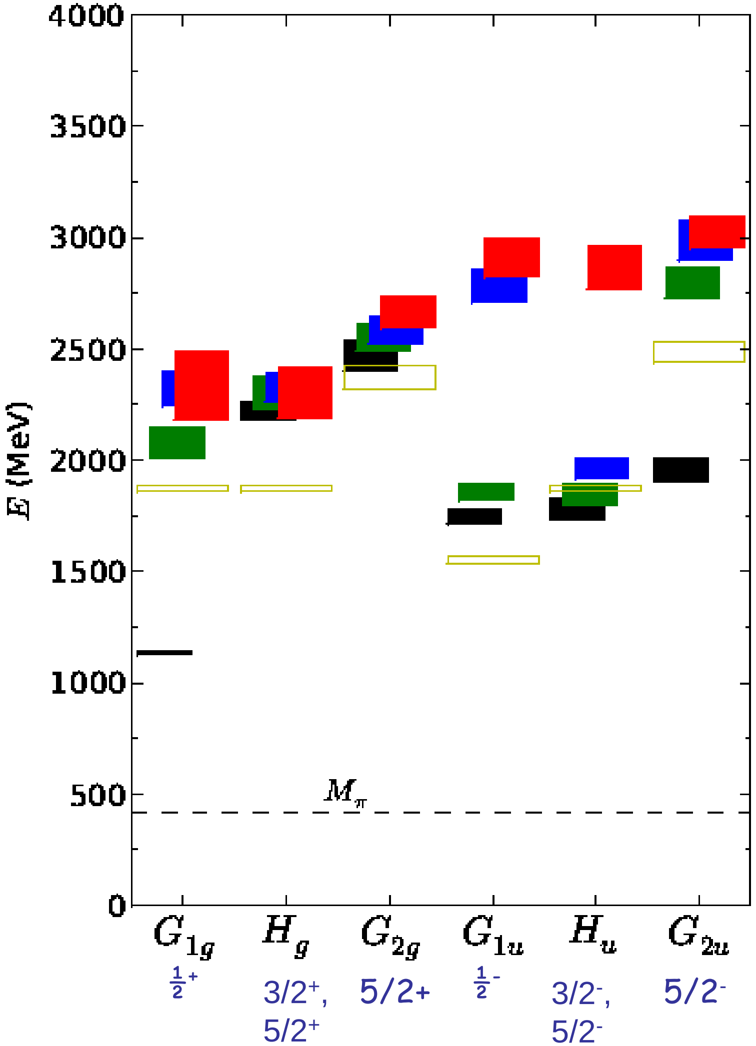
- Future:
  - As expected, most states decaying
  - Multiple volumes for decay analysis

N_f=2+1 Nucleon spectrum

N_f=2+1, m_π= 383 MeV, a_s~0.127fm

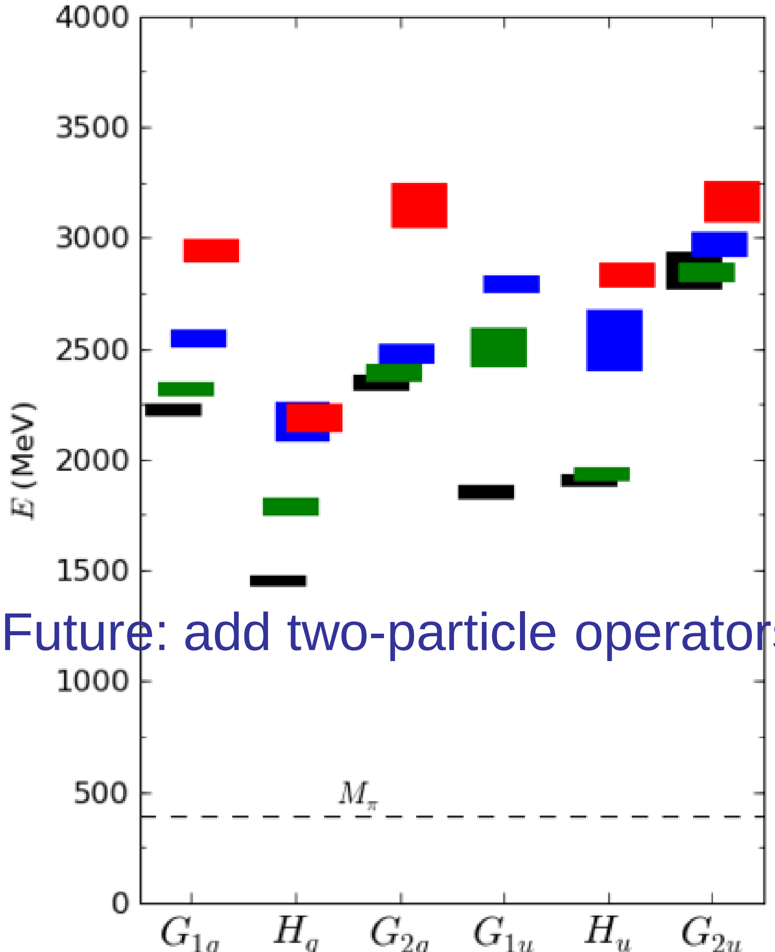


N_f=2, m_π= 416 MeV, a_s~0.11fm



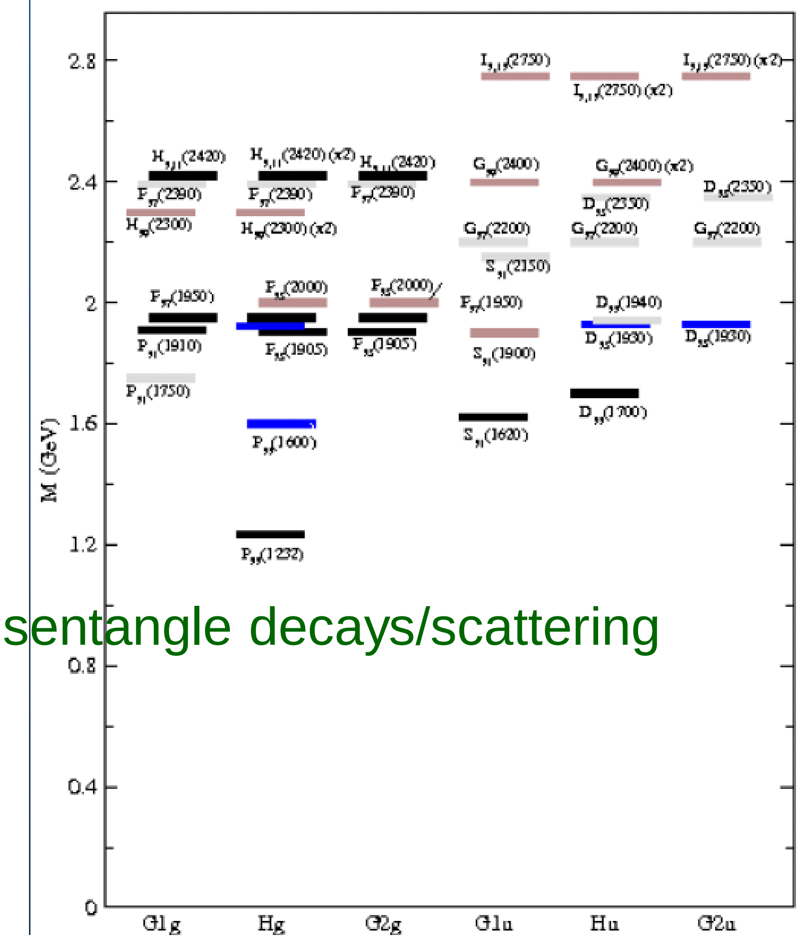
N_f=2+1 Delta spectrum

N_f=2+1, m_π= 383 MeV, a_s~0.127fm



$\frac{1}{2}^+$ | $3/2^+$, $5/2^+$ | 5/2+ | $\frac{1}{2}^-$ | $3/2^-$, $5/2^-$ | $5/2^-$

Delta Mass Spectrum (Exp)



$\frac{1}{2}^+$ | $3/2^+$, $5/2^+$ | 5/2+ | $\frac{1}{2}^-$ | $3/2^-$, $5/2^-$ | $5/2^-$

Future: add two-particle operators: disentangle decays/scattering

Delta (decay)

$\Delta(1232)$ (p-wave). Scattering phase (eff. range expansion)

$$\frac{k^3}{E} \cot \delta_{3/2\,1}(k) = \frac{24\pi}{g^2_{\Delta N\pi}} \left(m^2_\Delta - E^2\right)$$

Here

$$E = \sqrt{k^2 + m_\pi^2} + \sqrt{k^2 + m_N^2}\,, \quad m_\Delta = \sqrt{k_\Delta^2 + m_\pi^2} + \sqrt{k_\Delta^2 + m_N^2}$$

$$\Gamma_\Delta = \frac{g^2_{\Delta N\pi}}{6\pi} \frac{k^3_\Delta}{m^2_\Delta}$$

Free case:

$$k = \frac{2\pi|\vec{n}|}{L}\,, \quad \vec{n} \in \mathbb{N}^3$$

Interacting case:

$$\delta_{11}(k) = \arctan\left\{\frac{\pi^{3/2} q}{\mathcal{Z}_{00}(1, q^2)}\right\} \bmod \pi\,, \quad q = \frac{kL}{2\pi}$$

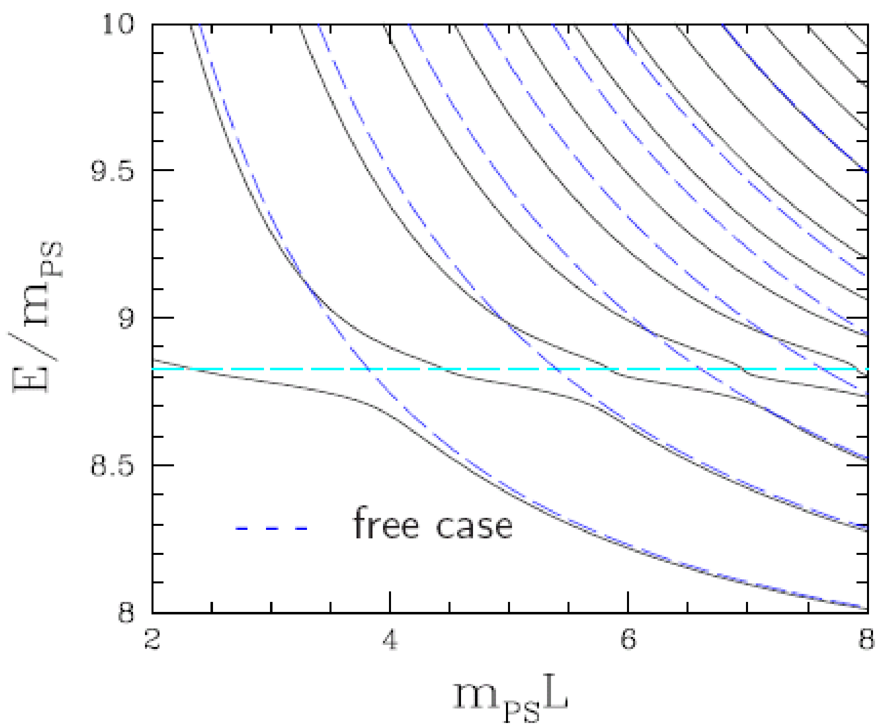
Lüscher; Weise; Bernard, Meißner, Rusetsky (2007)

Delta

Energy levels

$$\frac{k^3}{E}\cot\,\delta_{3/21}(k) = \frac{24\pi}{g^2_{\Delta N\pi}}\left(m^2_\Delta - E^2\right)$$

Physical m_π, m_Δ and Γ_Δ

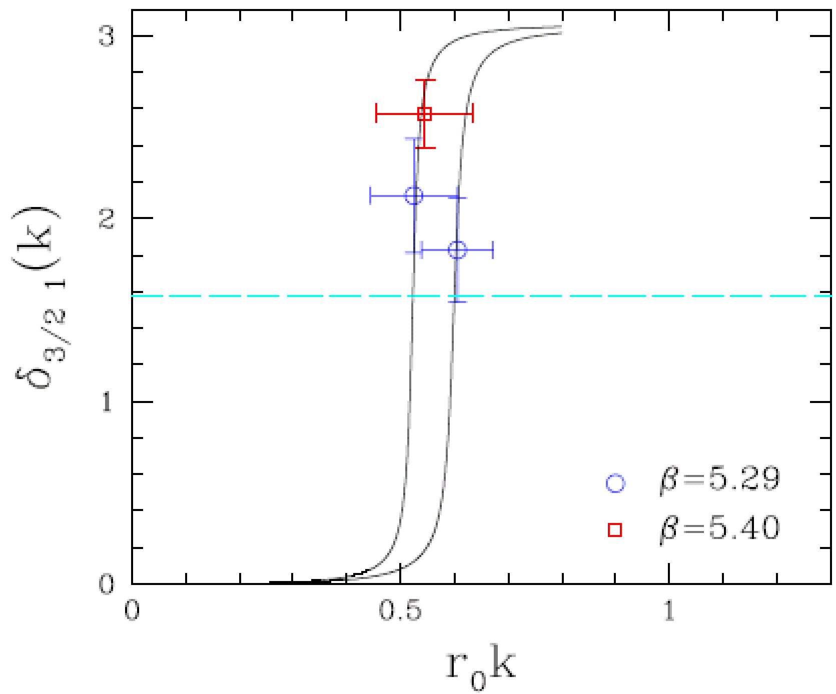


QCDSF

Delta

Phase shift

$$\frac{k^3}{E}\cot\,\delta_{3/21}(k) = \frac{24\pi}{g^2_{\Delta N\pi}}\left(m^2_\Delta - E^2\right)$$

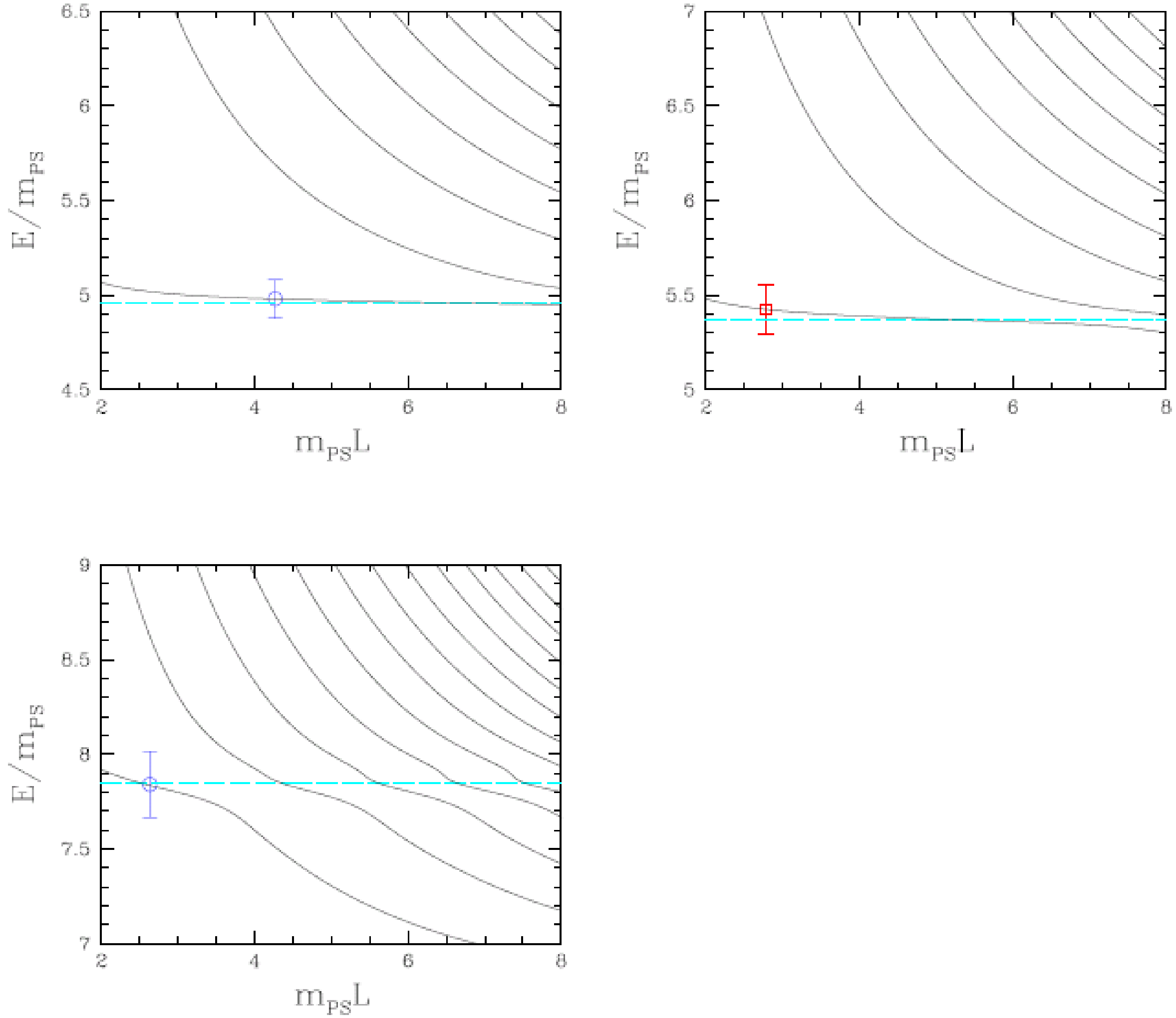


$m_\pi =$ 250 150 MeV

QCDSF

Delta

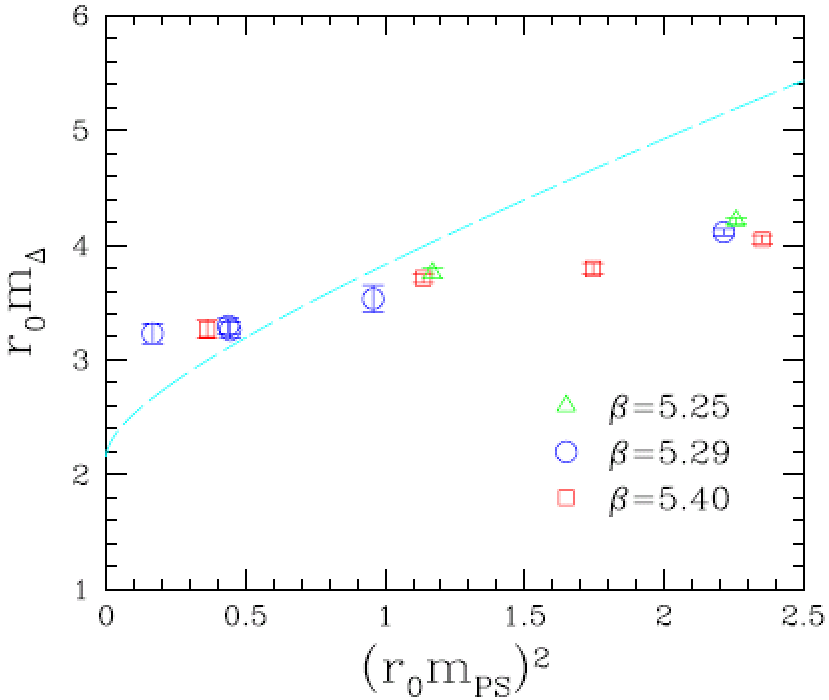
Energy levels (+lattice results @ E, m_π & L)



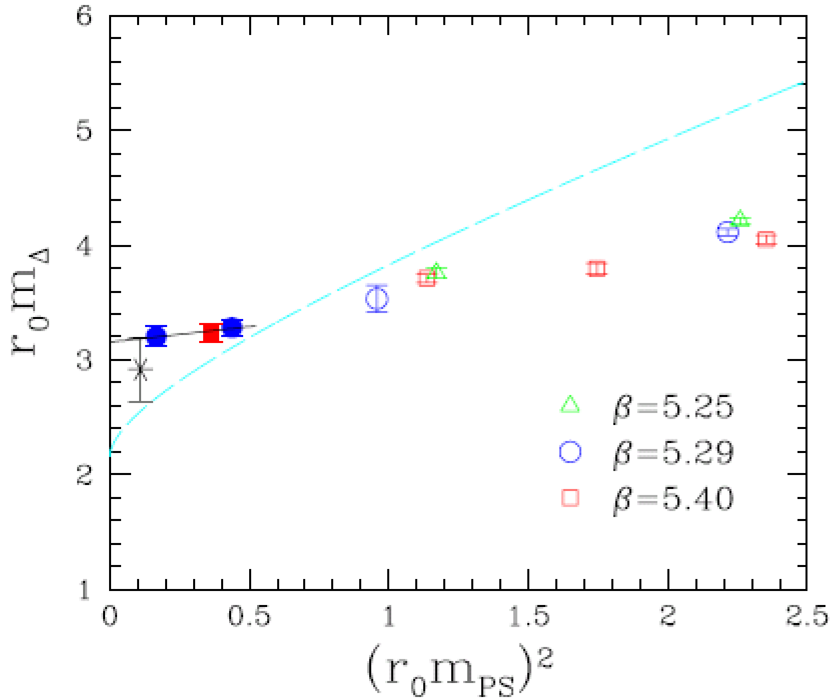
QCDSF

Delta

Lowest energy levels



True Δ mass



Chiral fit: $m_\Delta = m_\Delta^0 - 4c_1 m_\pi^2 + c_2 m_\pi^3$

Bernard 2007, QCDSF 2009

Extensions

Go beyond isolated states, e.g.:

- $[\frac{1}{2}^+]$ $P_{11}(1440) \to N\pi$ or $\Delta\pi$
- $[\frac{1}{2}^-]$ $S_{11}(1535) \to N\pi$ or $N\eta$

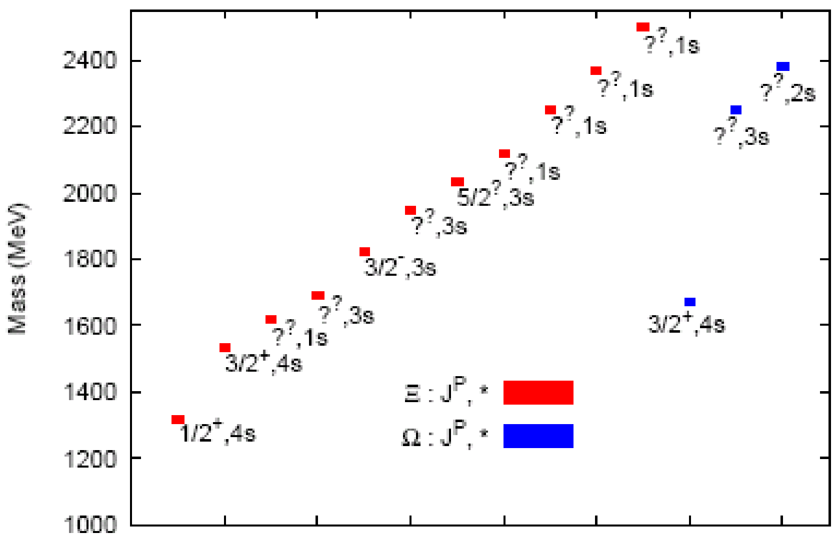
$\to$ multi-channel finite-V analysis

Lage, Meißner, Rusetsky (2009)

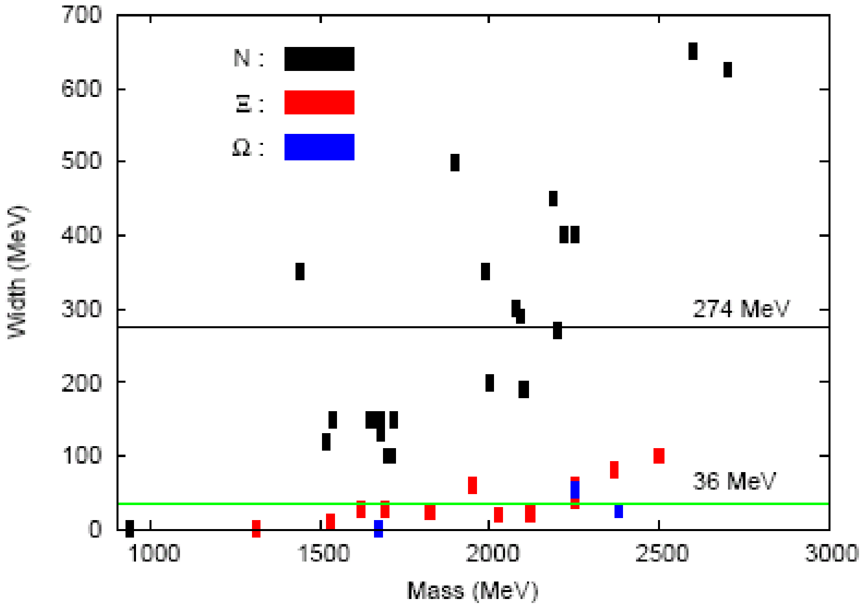
Strange Quark Baryons

Strange quark baryon spectrum poorly known

Ξ & Ω: unknown spin & parities



Widths are small



Future:

- Narrow widths: easy(er) to extract (?)

Current and future work

- Some efforts underway (HadSpec)
  - Strange quark spectrum (hybrids) and radiative transitions
  - Excited light baryon spectrum (N, Δ, Ξ, Σ, Λ)
    - Radiative transitions for $P_{11}(1440)$, $S_{11}(1535)$, $D_{13}(1520)$
    - Q^2 <~ 5 GeV^2
  - Need to disentangle decay states:
    - Two-meson states, I=1 & 0
    - Meson-baryon

Summary

Lattice can handle excited states

Anisotropy+variational method allows for high lying states

Lattice can handle decays (simple ones so far)

Example, ρ & Δ (QCDSF)

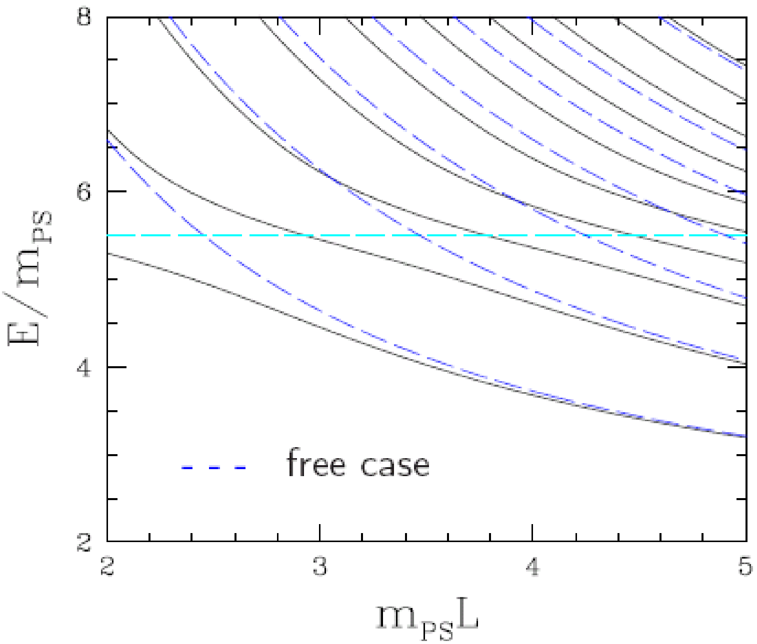
- Early stages
- Start at heavy masses: have some "elastic scattering"
- Will need multi-particle operators

Message:

Needed is multi-channel finite-volume analysis for inelastic scattering

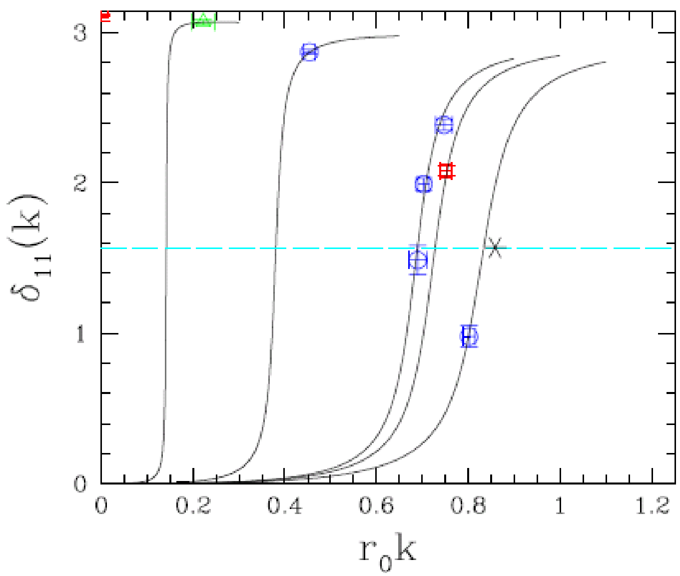
Rho decay

$$\frac{k^3}{W}\cot(\delta_{11}(k)) = \frac{24\pi}{g^2_{\rho\pi\pi}}(k^2_\rho - k^2), \quad W = 2\sqrt{k^2+m^2_\pi}, \quad k_\rho = \frac{1}{2}\sqrt{m^2_\rho - 4m^2_\pi}$$



Useful region

QCDSF

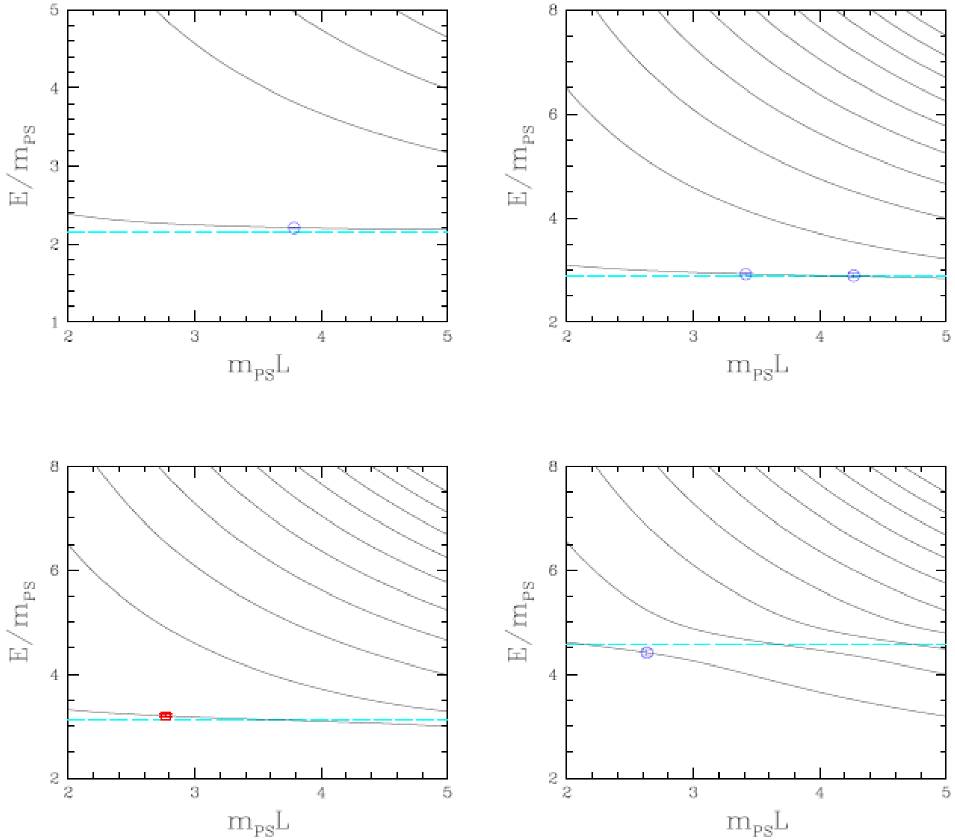


$m_\pi =$ 430 390 250 150 MeV
240

↑
Fit

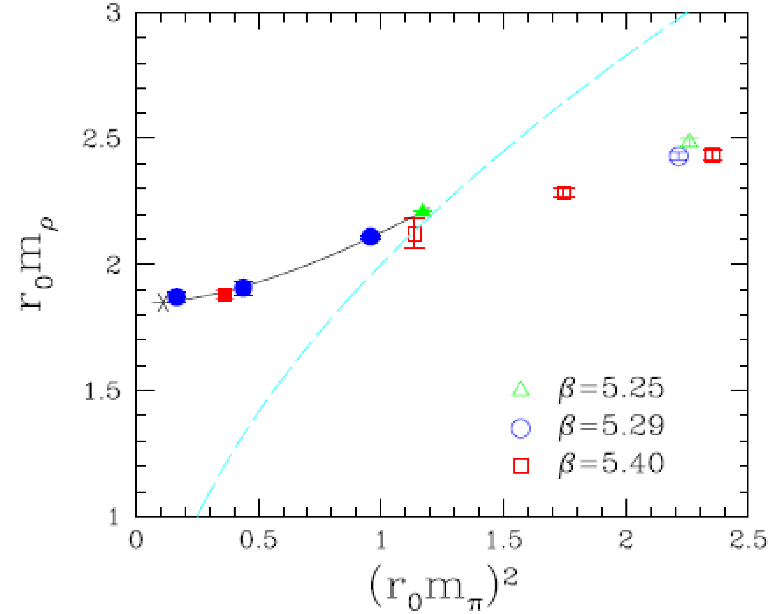
$g_{\rho\pi\pi} = 5.5 \pm 0.5$

Rho decay



5

True ρ mass



Chiral fit: $m_\rho = m_\rho^0 + c_1 m_\pi^2 + c_2 m_\pi^3 + c_3 m_\pi^4 \ln(m_\pi^2)$

Kink ?

Bruns & Meißner

QCDSF

Scaling of costs

- Isotropic: $m_\pi L = 4.2$